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Field to thermo-field to thermionic electron emission: A practical guide to evaluation and electron emission from arc cathodes

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This work is concerned with devising a method of evaluation of electron emission in the framework of the Murphy-Good theory, which would be as simple and computationally efficient as possible while being accurate in the full range of conditions of validity of the theory. The method relies on Padé approximants. A comparative study of electron emission from cathodes of arcs in ambient gas and vacuum arcs is performed with the use of this method. Electron emission from cathodes of arcs in ambient gas is of thermionic nature even for extremely high gas pressures characteristic of projection and automotive arc lamps and is adequately described by the Richardson-Schottky formula. The electron emission from vaporizing (hot) cathodes of vacuum arcs is of thermo-field nature and is adequately described by the Hantzsche fit formula. Since no analytical formulas are uniformly valid for field to thermo-field to thermionic emission, a numerical evaluation of the Murphy-Good formalism is inevitable in cases where a unified description of the full range of conditions is needed, as is the general case of plasma-cathode interaction in vacuum arcs, and the technique proposed in this work may be the method of choice to this end. © 2013 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4818325>]

I. INTRODUCTION

Modern multidimensional numerical models of plasma-cathode interaction in vacuum arcs require field to thermo-field to thermionic electron emission current density be evaluated at each iteration at each time step at each point of the arc attachment. Therefore, a fast and accurate evaluation method is of crucial importance. A theoretical description of field to thermo-field to thermionic electron emission from metals in the quasi-classical approximation has been developed by Murphy and Good¹ long ago. However, a question of fast and accurate evaluation in the framework of the formalism¹ is not quite trivial. Different aspects of such evaluation have been considered in many works; see, e.g., Refs. 2–13 and discussion in Ref. 14. One thing that has become clear is that an accurate evaluation method uniformly valid in the full range of conditions of validity of the theory¹ from field to thermo-field to thermionic emission cannot be analytical.

This work is concerned with devising an evaluation method, which would be as simple and computationally efficient as possible while being accurate in the full range of conditions of validity of the theory.¹ The method relies on Padé approximants. Unsurprisingly, the method is not fully analytical and still involves a numerical evaluation of one integral, which is performed by means of Romberg integration. Another objective of this work is a comparative study of electron emission from vaporizing (hot) cathodes of vacuum arcs and from cathodes of arcs in ambient gas, including in cases where the ambient gas pressure is extremely high, as in projection and automotive arc lamps, and physical conditions appear to be not very different from those in cathode attachments of vacuum arcs.

II. EVALUATING ELECTRON EMISSION CURRENT IN THE FRAMEWORK OF THE MURPHY-GOOD FORMALISM

A. The formulas

The Murphy and Good theory¹ and its limitations are well known, so only a summary of relevant formulas is given here. The density of electron emission current is given by the expression

$$j_{em}(T_w, E_w, \phi) = e \int_{-\infty}^{\infty} N(T_w, W, \phi) D(E_w, W) dW, \quad (1)$$

where T_w is the temperature of the surface of the emitter, E_w is the electric field at the surface of the emitter, ϕ is the work function, W has the meaning of the part of the electron energy for the motion normal to the surface measured from zero for a free electron outside the metal, N is the Fermi-Dirac distribution for the free electrons in the metal,

$$N(T_w, W, \phi) = \frac{4\pi m_e k T_w}{h^3} \ln \left[1 + \exp \left(-\frac{W + \phi}{k T_w} \right) \right], \quad (2)$$

and D is the tunnelling probability,

$$D(E_w, W) = \begin{cases} 1 & \text{for } W > W_l, \\ \left[1 + \exp \frac{av(y)}{y^{3/2}} \right]^{-1} & \text{for } W < W_l. \end{cases} \quad (3)$$

Here,

$$W_l = -\sqrt{\frac{e^3 E_w}{8\pi\epsilon_0}}, \quad a = \frac{4\sqrt{2}}{3(4\pi\epsilon_0)^{3/4}} \left(\frac{m_e^2 e^5}{\hbar^4 E_w} \right)^{1/4}, \quad y = \frac{\sqrt{2}W_l}{W} \quad (4)$$

and

$$v(y) = \begin{cases} \sqrt{1+y} \left[E\left(\frac{1-y}{1+y}\right) - yK\left(\frac{1-y}{1+y}\right) \right] & \text{for } y < 1 \\ \sqrt{\frac{y}{2}} \left[2E\left(\frac{y-1}{2y}\right) - (y+1)K\left(\frac{y-1}{2y}\right) \right] & \text{for } y > 1 \end{cases}, \quad (5)$$

where $K = K(m)$ and $E = E(m)$ are complete elliptic integrals of the first and second kinds.¹⁵

Substituting Eqs. (2) and (3) into Eq. (1), one can rewrite the latter equation as

$$\frac{j_{em}(T_w, E_w, \phi)}{A_{em} T_w^2} = I_1 + gI_2, \quad (6)$$

where

$$I_1 = \int_c^\infty \ln(1 + e^{-z}) dz, \quad c = \frac{\phi + W_l}{kT_w}, \quad (7)$$

$$I_2 = \int_{1/\sqrt{2}}^\infty \frac{\ln[1 + \exp(gz - b)]}{1 + \exp[az^{3/2}v(z^{-1})]} dz, \quad g = \frac{-\sqrt{2}W_l}{kT_w},$$

$$b = \frac{\phi}{kT_w}, \quad (8)$$

and $A_{em} = 4\pi m_e k^2 e/h^3 \approx 1.20 \times 10^6 \text{ A m}^{-2} \text{ K}^{-2}$ is the Richardson constant.

A fast and accurate method of evaluation of the integrals I_1 and I_2 based on the use of Padé approximants is given in Secs. II B and II C.

B. Integral I_1

The function $I_1 = I_1(c)$ may be expressed in terms of dilogarithm (Spence's integral for $n=2$) and the latter for $c \geq 0$ may be evaluated with the use of the Chebyshev series [Ref. 16, p. 67]. However, a faster and sufficiently accurate way to evaluate the function $I_1(c)$ for $c \geq 0$ is to use a Padé rational approximation (Padé approximant) over the variable $x = e^c$. The two-term expansions of I_1 for $x \rightarrow 1$ and $x \rightarrow \infty$ read, respectively,

$$I_1 = \frac{\pi^2}{12} - (\ln 2)(x-1) + \dots, \quad I_1 = \frac{1}{x} - \frac{1}{4x^2} + \dots \quad (9)$$

(Note that the latter formula is readily obtained by expanding the logarithm in Eq. (7) in powers of e^{-z} .) The simplest rational approximation which agrees with these expansions may be written as

$$I_1 = \frac{c_1 + c_2(x-1)}{1 + c_3(x-1) + c_2(x^2-1)}, \quad (10)$$

with

$$c_1 = \frac{\pi^2}{12}, \quad c_2 = -\frac{1}{3} \frac{\pi^4 - 144 \ln 2}{-48 + 5\pi^2},$$

$$c_3 = \frac{2}{3} \frac{-6\pi^2 + \pi^4 - 54 \ln 2}{-48 + 5\pi^2}. \quad (11)$$

The relative error of this approximant does not exceed 4.6×10^{-5} for all $c > 0$.

For $c < 0$, the functional relation

$$I_1(c) = \frac{\pi^2}{6} + \frac{c^2}{2} - I_1(-c) \quad (12)$$

can be used. (Note that this relation follows from Ref. 16 (p. 67) or from Ref. 15 [Eqs. (27.3.3) and (27.3.5)].)

Note that before programming Eq. (10), it is advisable to multiply the nominator and denominator by e^{-c} , in order to avoid overflow which may occur in evaluation of the last term of the denominator while calculating $I_1(-c)$ on the rhs of Eq. (12) for very high E_w and low T_w , where $-c$ is very large.

C. Integral I_2

Integral (8) cannot be expressed in terms of conventional special functions. On the other hand, I_2 is governed by three dimensionless parameters (a, b, g), so it is hardly possible to devise an accurate uniformly valid approximate formula. Therefore, the integral has to be evaluated numerically.

Let us consider first evaluation of the function $v(y)$. A straightforward numerical evaluation of this function requires an evaluation of complete elliptic integrals $K(m)$ and $E(m)$. The latter can be performed, e.g., by means of the numerical method described in Ref. 17 [Sec. 6.11] or polynomial approximations [Ref. 15, Eqs. (17.3.33) and (17.3.34)]. However, simple analytical formulas for $v(y)$ are desirable in order for numerical evaluation of integral (8) to be fast. Note that, as pointed out in Ref. 14, $v(y)$ greatly affects the calculated current density and therefore derivation of such formulas requires careful treatment. There are many works in which simple fit formulas of different degrees of accuracy for the function $v(y)$ are suggested; e.g., Refs. 2 and 3, and 8–13. In this work, simple and accurate formulas are derived by means of Padé approximants with the use of results¹³ elucidating the nature of the dependence $v(y)$ for small y .

Relevant for evaluation of the integral I_2 are values of the function $v(y)$ on the interval $0 \leq y \leq \sqrt{2}$. Since $v(y)$ vanishes at $y=1$, it is natural to try to derive separate formulas for the intervals $0 \leq y \leq 1$ and $1 \leq y \leq \sqrt{2}$ requiring that both formulas ensure for $y=1$ not only exact (zero) value of the function $v(y)$ but also the exact value of the derivative. Let us consider first the interval $0 \leq y \leq 1$. [Note that Eqs. (17) and (18) of Ref. 10 could be used on this interval (one should be aware that there is an error in the last line of the former equation). However, the use of Padé approximants jointly with the results¹³ allows one to derive a formula which is somewhat simpler while having a comparable accuracy.] The expansion of function $v(y)$ for $y \rightarrow 0$ reads:¹³

$$v(y) = \left[1 - \left(\frac{9}{8} \ln 2 + \frac{3}{16} \right) w + \dots \right] + \ln w \left[\frac{3}{16} w + \dots \right], \quad (13)$$

where $w = y^2$ and the series in the square brackets involve integer powers of w . The series expansion for $y \rightarrow 1$ can be found with the use of series expansions of functions $K(m)$ and $E(m)$ in powers of m (e.g., Ref. 15, Eqs. (17.3.11) and (17.3.12)) and reads

$$v(y) = \frac{3\pi\sqrt{2}}{8}(1-y) + \dots \quad (14)$$

In view of the structure of expansion (13), it is natural to represent $v(y)$ in the interval $0 \leq y \leq 1$ as $v(y) = v^{(1)}(w) + v^{(2)}(w)\ln w$ and to find Padé approximants of the functions $v^{(1)}(w)$ and $v^{(2)}(w)$. Devising the simplest approximants which agree with expansions (13) and (14), one obtains

$$v(y) = \frac{1-w}{1+c_4w} + \frac{3w \ln w}{16(1+c_5w)}, \quad (15)$$

with

$$c_4 = \frac{9}{8} \ln 2 - \frac{13}{16}, \quad c_5 = \frac{13 - 3c_4 - 3\pi\sqrt{2}(1+c_4)}{3\pi\sqrt{2}(1+c_4) - 16}. \quad (16)$$

The relative error of approximate formula (15) does not exceed 3.7×10^{-4} over the whole range $0 \leq y \leq 1$.

Let us now consider the interval $1 \leq y \leq \sqrt{2}$. A series expansion of $v(y)$ for $y \rightarrow \sqrt{2}$ reads

$$v(y) = \frac{1}{2^{1/4}} [2E(m_0) - (\sqrt{2}+1)K(m_0)] - \frac{3}{2^{5/4}} K(m_0)(y - \sqrt{2}) - \frac{3}{2^{13/4}} [2E(m_0) - K(m_0)](y - \sqrt{2})^2 + \dots, \quad (17)$$

where $m_0 = \frac{2-\sqrt{2}}{4}$. The simplest approximant that agrees with expansions (14) and (17) may be written as

$$v(y) = -\frac{3\pi}{2^{5/2}} \frac{(y-1) + c_6(y-1)^2}{1 + c_7(y-1) + c_8(y-1)^2}, \quad (18)$$

where c_6 , c_7 , and c_8 are numerical coefficients which are expressed in terms of $K(m_0)$ and $E(m_0)$ (these expressions are skipped for brevity) and have numerical values $c_6 = 0.514\,706\,54$, $c_7 = 0.202\,328\,90$, and $c_8 = -0.013\,410\,07$. The relative error of this approximant does not exceed 4.8×10^{-6} over the whole interval $1 \leq y \leq \sqrt{2}$.

Let us proceed to numerical evaluation of integral (8). Under conditions of practical interest, one or more parameters governing the integrand are large and the integrand represents a multi-scale function. Therefore, an efficient numerical evaluation of integral (8) must employ an adaptive choice of the numerical grid. A suitable method is Romberg integration.¹⁷ First, let us transform the integral to the integration variable y ,

$$I_2 = \int_0^{\sqrt{2}} \frac{r_1 r_2}{y^2} dy, \quad (19)$$

where

$$r_1 = \ln \left[1 + \exp \left(\frac{g}{y} - b \right) \right], \quad r_2 = \left[1 + \exp \frac{av(y)}{y^{3/2}} \right]^{-1}. \quad (20)$$

In order to avoid overflow which may occur in evaluation of the exponential functions for small y , it is advisable to rewrite Eq. (20) as

$$r_1 = \ln \left[1 + \exp \left(b - \frac{g}{y} \right) \right] - \left(b - \frac{g}{y} \right),$$

$$r_2 = \frac{\exp \left[-\frac{av(y)}{y^{3/2}} \right]}{\exp \left[-\frac{av(y)}{y^{3/2}} \right] + 1}. \quad (21)$$

In cases where $\exp \left(\frac{g}{y} - b \right)$ is very small, the use of the first expression in Eq. (21) causes accumulation of errors and the Romberg integration (or, more precisely, Richardson's deferred approach to the limit) may fail. The same happens if the first expression in Eq. (20) is used; in particular, for $\exp \left(\frac{g}{y} - b \right)$ sufficiently small while still above the underflow limit the computer-evaluated argument of the logarithm will be exactly 1 and the logarithm exactly 0. Therefore, in cases where $\exp \left(\frac{g}{y} - b \right)$ is small, say, smaller than 0.01, the quantity r_1 should be evaluated by means of a series in powers of $\exp \left(\frac{g}{y} - b \right)$, which is obtained by expanding the logarithm in the first expression in Eq. (20).

In this framework, the Romberg integration as implemented in the subroutine given in Sec. 4.3 of Ref. 17 and the above-described method on the whole work smoothly for all the variants tested ($\phi = 4.5$ eV, $10 \text{ V m}^{-1} \leq E_w \leq 10^{11} \text{ V m}^{-1}$ and $300 \text{ K} \leq T_w \leq 6000 \text{ K}$).

III. ELECTRON EMISSION FROM CATHODES OF ARC DISCHARGES

The method of evaluation of electron emission described above is fast and robust and can be used in all conditions where the Murphy-Good theory is applicable. Let us now perform, with the use of this method, a comparative study of electron emission from cathodes of arcs in ambient gas and vacuum arcs. The consideration is limited to cases where the size of non-uniformities of the cathode surface exceeds significantly the thickness of the near-cathode plasma layer and, as far as cathodes of vacuum arcs are concerned, the cathode is hot enough so that supply of cathode vapor into the discharge gap is dominated by vaporization and not explosive emission. Then, the current transfer through the near-cathode plasma layer may be analyzed in the framework of a 1D model. There are several such models in the literature (e.g., Refs. 18–20 for arcs in ambient gas and Refs. 21–23 for vacuum arcs; further references can be found in reviews 24–26). Calculations reported in this work have been performed by means of the model described in Refs. 27–29 and summarized in Ref. 19 for the case of arcs in ambient gas and with the use of the model²³ for the case of vacuum arcs. In both cases, the electron emission current density is evaluated as described above.

Calculations are reported for the following conditions: an arc burning in Ar with the argon pressure in the discharge gap being 1 bar; arcs in Hg and Xe for different values of the

gas pressure, $p = 1, 15,$ and 200 bar; and a vacuum arc with a Cu cathode. Note that a 1 bar Ar arc represents a standard example of an atmospheric arc, while very-high pressure Hg and Xe arcs are of interest in connection with projection and car headlight arc lamps, where pressures of the order of 100 or 200 bar are rather a rule than an exception. The work function of the cathode material is 4.5 eV, a value appropriate for both Cu and W, which is the most frequently used material for electrodes of arcs in ambient gas.

In the framework of a 1D model, the physics of near-cathode arc plasma layers is governed by two control parameters, one of them being T_w the temperature of the cathode surface at the point of the arc attachment being considered and the other being an electric parameter, e.g., the near-cathode voltage drop U . (Since U does not vary much from one point of arc attachment to another, it makes a more convenient control parameter than the local current density.) Calculations reported in this work refer to $3000 \text{ K} \leq T_w \leq 5000 \text{ K}$ and $U = 20 \text{ V}$.

The pressure of the Cu vapor equals 1 bar for $T_w = 2835 \text{ K}$ and 200 bar for $T_w = 4620 \text{ K}$; therefore, physical conditions in the attachments of vacuum arcs and arcs in ambient gas to the cathode are not very different. Accordingly, the models of near-cathode layers in the vacuum arc and in the arcs in ambient gas are similar in many respects. The most important difference between the models is in the mechanism of formation of the ion flux to the cathode surface. In the model,^{27–29} the ion flux is assumed to be formed

in the quasi-neutral ionization layer, which is adjacent to the near-cathode space-charge sheath; see discussion in Sec. 3.2 of Ref. 24 for details. In the model,²³ the ion flux is assumed to be formed inside the sheath as a result of ionization of neutral metal vapor emitted by the cathode; see Ref. 30 for details.

Computed characteristics of the near-cathode arc plasma layers are shown in Fig. 1. Here, T_e is the temperature of thermalized electrons in the layer, j_{iw} is the density of current of ions coming from the plasma to the cathode surface, j_{em} and E_w are, as before, electron emission current density and the electric field at the surface of the emitter (cathode).

The main source of electron energy in the near-cathode layer is acceleration of the emitted electrons by the sheath electric field. As T_w increases, the emission current also increases and so does the electron temperature, and the latter is what can be seen in Fig. 1(a). The ionization degree of the plasma in the near-cathode layer increases as well.

One can see from Fig. 1(b) that with increasing T_w the ion current density increases at first approximately exponentially. This is due to the increase of the ionization degree and, in the case of vacuum arc, of the vapor pressure. Values of j_{iw} for different arcs are not very different in this T_w range. As the ionization degree approaches unity, which happens at T_w values which vary from one arc to another but are generally between approximately 3500 K and 4000 K, the situation changes. In the case of arcs in ambient gas, the ion current gets saturated. (In fact, it even weakly decreases; cf. Fig. 5

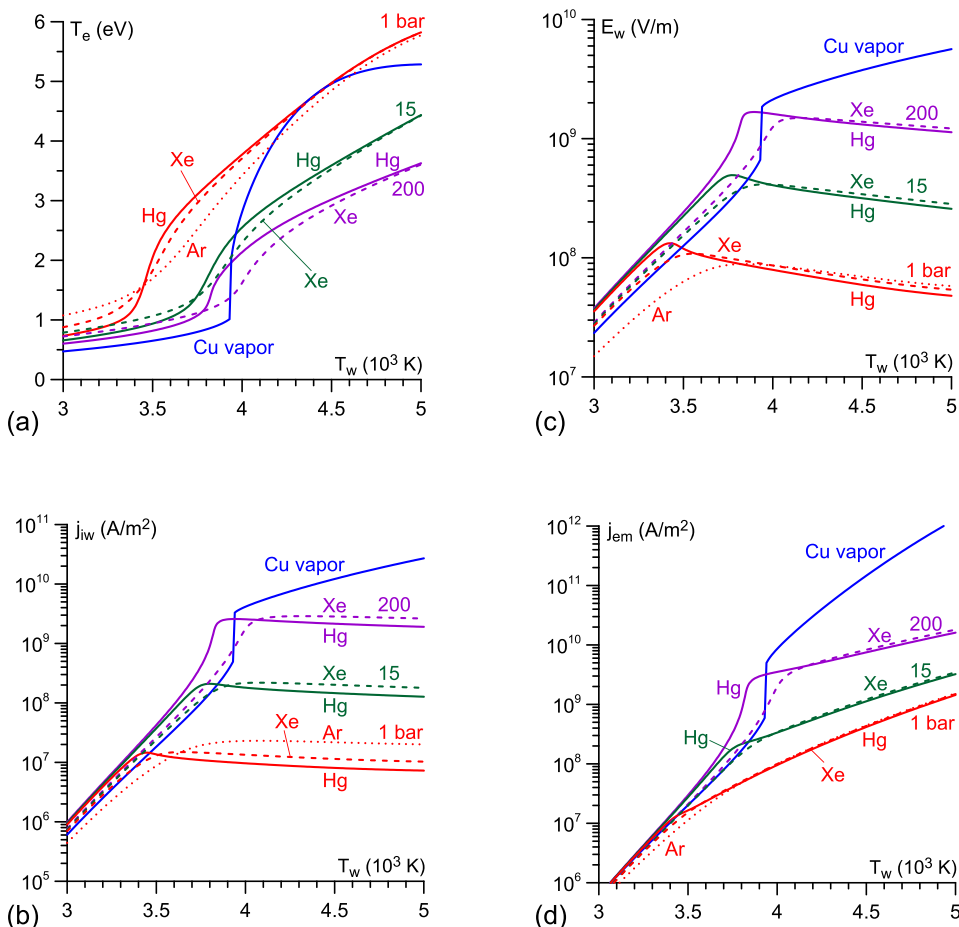


FIG. 1. Computed characteristics of near-cathode plasma layers in arcs in different plasma-producing gases.

of Ref. 24.) In the case of vacuum arc, the ion current continues to increase due to continuing increase in the vapor pressure and due to a rapid increase in the ion backflow coefficient up to values very close to unity, which happens at T_w slightly below 4000 K (cf. Fig. 2(b) of Ref. 23). As a consequence, the ion current to cathodes of ambient-gas arcs in the range $T_w \geq 4000$ K is the greater, the higher is the pressure; however, even for the 200 bar arcs the ion current is significantly lower than that to the vacuum arc cathode, at least in the framework of the models of near-cathode layers being used.

The dependence of computed electric field at the cathode surface on the surface temperature is shown in Fig. 1(c), which is quite similar qualitatively to Fig. 1(b). The latter may be understood as follows: analysis of computed values of E_w and j_{iw} has shown that these values for all arcs obey the Mackeown equation to the accuracy of a factor of 2 or 3, hence E_w is roughly proportional to $\sqrt{j_{iw}}$ and the proportionality coefficient does not vary much from one arc to another, given that U is the same and the dependence on the ion mass is weak. The computed electron emission current is shown in Fig. 1(d), which is qualitatively similar to Fig. 1(c) and, consequently, Fig. 1(b) except that the dependence $j_{em}(T_w)$ for obvious reasons is monotonically increasing also for arcs in ambient gas.

Analyzing the data shown in Fig. 1(c) in view of those shown in Fig. 2 (where j_{RS} is the electron emission current density given by the Richardson-Schottky formula and j_{em} is the one given by the Murphy-Good formalism evaluated numerically as described in Sec. II), one concludes that the Richardson-Schottky formula represents a good approximation for the arcs in ambient gas in cases $p = 1$ and 15 bar but may represent a poor approximation for the vacuum arc and maybe also for the arcs in ambient gas in the case $p = 200$ bar. Therefore, the applicability of the Richardson-Schottky formula for the arcs in ambient gas with $p = 200$ bar and for the vacuum arcs requires a more detailed

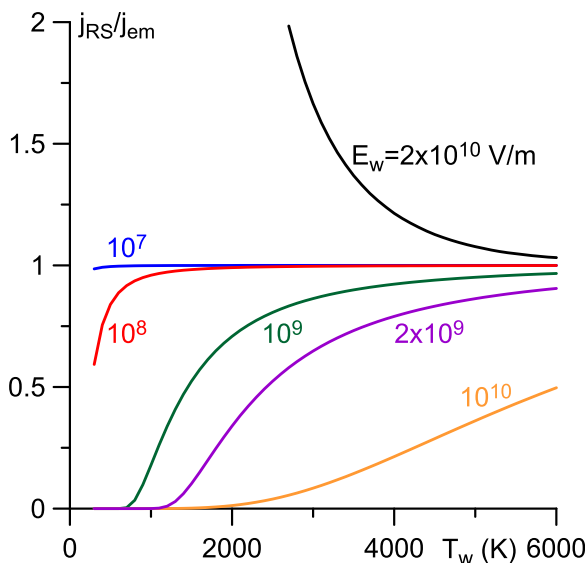


FIG. 2. Electron emission current density given by the Richardson-Schottky formula normalized by the Murphy-Good value.

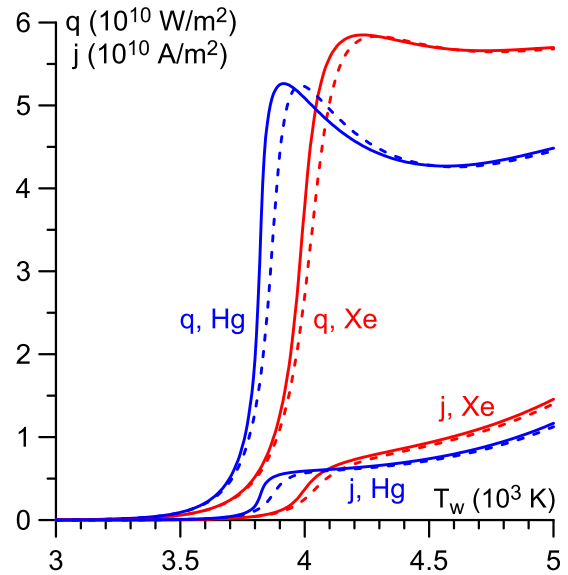


FIG. 3. Densities of energy flux and electric current from 200 bar arc plasmas to the cathode evaluated using the Murphy-Good formalism (solid) and the Richardson-Schottky formula (dashed).

investigation. In this connection, the most important parameters of the near-cathode plasma layer, which are densities of energy flux and electric current from the plasma to the cathode, evaluated with the use of the Murphy-Good formalism and the Richardson-Schottky formula are shown in Fig. 3 for Hg and Xe arcs in the case $p = 200$ bar. One can see that the Richardson-Schottky formula represents a reasonably good approximation. The situation is different as far as the vacuum arcs are concerned, which is seen from Fig. 4: the usage of the Richardson-Schottky formula introduces a significant error.

Line 3 in Fig. 4 represents computations with the electron emission current determined by means of a straightforward numerical evaluation of Eq. (1) with a fixed step of

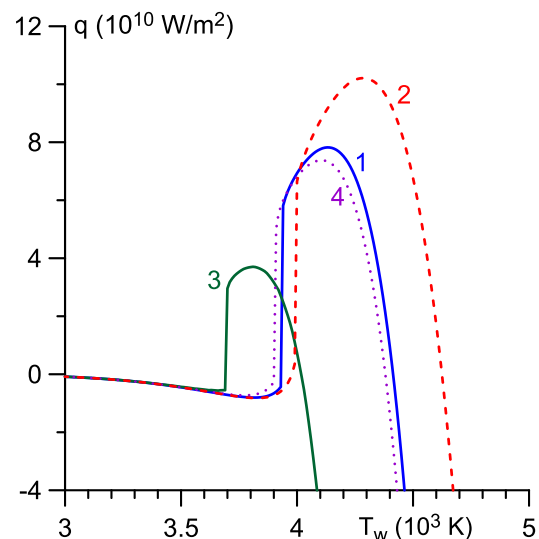


FIG. 4. Density of energy flux from the vacuum arc plasma to the Cu cathode computed using different techniques of evaluation of electron emission. 1: Murphy-Good formalism, evaluation method described in Sec. II. 2: Richardson-Schottky formula. 3: Murphy-Good formalism, straightforward evaluation. 4: Formula (22).

integration (over the variable W) equal to 10^{-20} J ≈ 0.0624 eV and the lower limit of integration equal to -4ϕ . There is a significant difference between this line and the line 1; an indication of relevance of the use of the integration variable $1/W$ and an adaptive choice of the numerical grid.

Line 4 in Fig. 4 represents the density of energy flux computed with the use of the Hantzsche fit formula for electron emission from a metal with the work function of 4.5 eV (j_{em} in $A m^{-2}$, E_w in $V cm^{-1}$, T_w in K)

$$j = K_1(120 T_w^2 + 406 E_w^{9/8}) \times \exp\left[-\left(\frac{T_w^2}{2.727 \times 10^9} + \frac{E_w^2}{4.252 \times 10^{17}}\right)^{-1/2}\right], \quad (22)$$

where

$$K_1 = 1.45 \times 10^4 \frac{8100 T_w^3 + 0.35 E_w^2}{9100 T_w^3 - 5300 E_w T_w + 0.75 E_w^2}. \quad (23)$$

Note that Eq. (22) represents the combined equations (22) and (24) of Ref. 2 and Eq. (23) represents Eq. (23) of Ref. 2 with corrections given by Eq. (9) of Ref. 31. According to Ref. 2, the error of this formula relative to the corresponding Murphy-Good value is between -14% and 23% in the range of T_w between 2500 and 5000 K and E_w between 5×10^7 and 2×10^{10} V m $^{-1}$. Evaluation of this work has given a higher error for $E_w = 2 \times 10^{10}$ V/m (see Fig. 5), but for lower E_w this formula is reasonably accurate in the above-mentioned temperature range. Therefore, the fact that line 4 in Fig. 4 is close to line 1 is not surprising.

It is hardly possible to devise an analytical formula uniformly valid in a wide range of conditions from field to thermo-field to thermionic emission; Fig. 5 represents a good example. Therefore, a numerical evaluation of the Murphy-Good formalism is inevitable in cases where a wide range of conditions may occur, as in the general case of plasma-cathode interaction in vacuum arcs. The technique proposed in this work may be the method of choice to this end.

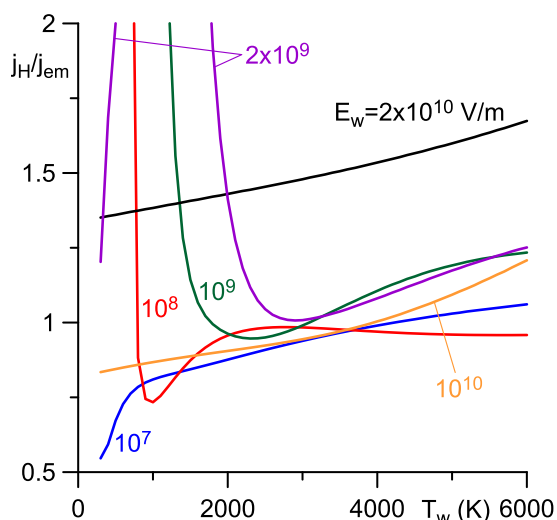


FIG. 5. Electron emission current density given by the Hantzsche fit formula normalized by the Murphy-Good value.

IV. CONCLUSIONS

A simple, accurate, and computationally efficient method of evaluation of field to thermo-field to thermionic electron emission current density in the framework of the Murphy-Good formalism is devised with the use of Padé approximants. Unsurprisingly, the method is not fully analytical and still involves a numerical evaluation of one integral. Since the integrand represents a multi-scale function, an efficient numerical evaluation of the integral must employ an adaptive choice of the numerical grid. A suitable method is Romberg integration.

Calculations for conditions of cathodes of arcs in ambient gas and vacuum arcs are performed for the case where the size of non-uniformities of the cathode surface exceeds significantly the thickness of the near-cathode plasma layer and, as far as cathodes of vacuum arcs are concerned, the cathode is hot enough so that supply of cathode vapor into the discharge gap is dominated by vaporization and not explosive emission. It is found that electron emission from cathodes of arcs in ambient gas is of thermionic nature and adequately described by the Richardson-Schottky formula even for extremely high gas pressures (up to 200 bar) typical for automotive and projection arc lamps. Emission from hot cathodes of vacuum arcs is of thermo-field nature and can be adequately described by the fit formula proposed by Hantzsche. If a method is needed which would be uniformly valid in the full range of conditions from field to thermo-field to thermionic electron emission, as in the general case of plasma-cathode interaction in vacuum arcs, then a numerical evaluation of the Murphy-Good formalism is inevitable and the approach proposed in this work may be the method of choice.

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