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REPLY

Reply to the Comment on 'What is the mathematical meaning of Steenbeck's principle of minimum power in gas discharge physics?'

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There are a number of statements of general character in the comment [1] which are not directly related to the substance of our work [2], for example, *The existence of the variational principle is a consequence of the Onsager symmetry relations of the linear map that transforms between the (stationary) generalized forces and currents* or *Thermodynamic stability is not equivalent to linear stability*. As far as our work [2] is concerned, the following points of the comment [1] need to be addressed.

Benilov and Naidis claim to have a disproof for the commonly accepted statement, that the Steenbeck principle follows from entropy production (rate) principles. In [2] specific arguments were given revealing what we see as indisputable errors invalidating the derivation of Steenbeck's principle from the thermodynamical principle of minimum entropy production, which was given by Peters [3] and referred to by subsequent workers. These arguments have been ignored in [1]. It is correct that *Peters clearly points out that he considers steady states, which satisfy both mass and energy balance*, and this is exactly where an error originates: the principle of minimum entropy production deals with a minimum over all possible states of a given system, including non-stationary states, and does not apply to a set of stationary states of discharges in plasmas with different properties, which represent different systems.

This is explained in detail in [2] for the particular case of the channel model of a cylindrical arc (see end of section 3.1), which is the same model that was treated by Peters [3]. Application of Steenbeck's principle to this model amounts to finding a state with a minimum entropy production among stationary states with different channel radii and with the same wall temperature T_w and arc current I. Since stationary thermal balance of an arc with given T_w and I is normally unique, a question arises: how can stationary states with different channel radii occur for the same T_w and I? The answer is that these states are characterized by different values of the switching temperature, i.e. of the temperature at which the electrical conductivity of the plasma switches from negligible to finite values. In other words, stationary states with different channel radii, which are possible in this approach for the same T_w and I, refer to plasmas with different material properties. Hence, application of Steenbeck's principle to the arc channel model is equivalent to finding a state with a minimum entropy production among stationary states of different systems. This is in contrast to the principle of minimum entropy production in non-equilibrium thermodynamics, where the minimum is sought among all possible states of the same system, including non-stationary states.

The latter can be seen, e.g., from [4, section 3.4] or [5, section V.3]. (Note that the book [4] is cited also in [1].) It can be seen also from the treatment of appendix B [2], where the formalism [4, 5] is applied to a cylindrical arc: the entropy production, given by equation (17), is minimized on a set of all temperature distributions $T(r)$ satisfying the boundary conditions, with the thermal and electrical conductivities being considered as given functions of temperature (and these functions are, of course, the same for all states). One of these temperature distributions is associated with a stationary state, while all the others can be associated only with non-stationary states.

Thus, the same quantity (entropy production) is minimized in Steenbeck's principle and in the principle of minimum entropy production on different sets of functions and under different constraints. This example clearly shows that Steenbeck's principle as it is understood by Peters [3] does not follow from the principle of minimum entropy production as it is understood in non-equilibrium thermodynamics.

States with different channel radii may be considered not only as stationary states of plasmas with different material properties, but also as non-stationary states of a plasma with given properties. However, the Gouy–Stodola theorem, which relates the entropy production and the electrical power consumed, does not apply to non-stationary states. Therefore, such reasoning also cannot be used in order to derive Steenbeck's principle of minimum power from the principle of minimum entropy production. A further discussion of this point can be found in section 2.1 of [2].

Note that there is also a grave problem concerning the principle of minimum entropy production itself: it hardly provides a reasonable approximation in gas discharge physics, as shown by the example of a cylindrical arc treated in appendix B of [2].

The argumentation in section 2.3 is again confusing. This argumentation reflects the way in which Steenbeck's principle was invoked in some preceding works cited in [2].

The interesting examples discussed in section 3 may probably be further analysed in this direction; however, a disprove invoking counterexamples is dispensable in this context. We are glad that our examples are found interesting. These examples clearly illustrate the error which can be inflicted by invoking Steenbeck's principle and scientific objectivity requires that they be considered not less valuable than examples given in [6] and deemed favourable.

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