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COMMENT

Comment on 'Self-organization in cathode boundary layer discharges in xenon' and 'Self-organization in cathode boundary layer microdischarges'

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Abstract
It is shown that basic mechanisms of glow discharge, which are drift and diffusion of the ions and the electrons, volume ionization and recombination and secondary electron emission, are in principle sufficient to explain patterns on glow cathodes observed in recently published works, so there is no need to introduce special mechanisms to this end.

In the recently published papers [1,2] observations are reported of regularly spaced filaments that appear, when the current is lowered, on molybdenum or tungsten cathodes of dc microdischarges in xenon at pressures in the range from 30 Torr up to 760 Torr. An increasing dependence of the effective secondary emission coefficient on the reduced electric field, which can be associated with photon emission and/or with an increasing influence of neutral collisions at the cathode, and heating of the gas causing an increase in the reduced electric field are indicated as potential mechanisms of the onset of the instability that leads to filamentation. The regular arrangement of the developed filaments is explained assuming a balance of electrostatic forces: the positive charge of the cathode fall in one filament is subject to repulsive forces from other filaments, which are balanced by electrostatic forces due to surface charges deposited on the surface of the surrounding dielectric spacer.

The experiments [1, 2] represent a significant and important step in the investigation of cathode boundary layers in high-pressure glow discharges. There is, on the other hand, an important point to be stressed concerning the interpretation of experimental results offered in these papers.

Questions may be raised on particular mechanisms proposed in [1, 2]; however, neither of these questions is discussed here[1]. Rather, this comment deals with the following question: is there any need to introduce special mechanisms, such as those proposed in [1, 2], in order to explain patterns appearing on cathodes of high-pressure glow discharges? In other words, can self-organization on cathodes of high-pressure glow discharges be explained in terms of basic mechanisms, which are drift and diffusion of the ions and the electrons, volume ionization and recombination and secondary electron emission?

The reasoning that guided the authors [2] to the necessity of introducing new mechanisms seems to be the following. The presence of self-organization is interpreted as an indication that the current density–voltage characteristic (CDVC) $U(j)$ of the near-cathode region possesses two stable (having positive slopes) sections, i.e. it is N-shaped. The $U(j)$ characteristic calculated taking into account only basic mechanisms is U-shaped, i.e. has only one section with a positive slope, namely, the one associated with the abnormal glow mode. This

[1] An example of such a question is as follows. The explanation of self-organization in terms of a balance of electrostatic forces, given in [1], is based on treating each filament as a material point with a given electric charge, subject only to an electrostatic force. A question, of course, arises as to what prevents a filament from breaking apart due to repulsion between positive charges constituting it. In more general terms, a filament represents an ensemble of ions, electrons and atoms, which are moving rather than quiescent and which strongly interact among themselves and/or with the surrounding gas, so the legitimacy of the above-described treatment is hard to justify.

is interpreted as an indication that basic mechanisms are insufficient to describe the self-organization.

However, this reasoning misses another branch of the $U(j)$ characteristic that always exists and is stable: this is the branch that describes the situation where no discharge is present and which coincides with the positive part of the axis of voltages. Indeed, any self-consistent model of the near-cathode region of glow discharge admits a trivial solution in which the ion and electron densities are zero and the applied electric field is unperturbed. In other words, the full $U(j)$ characteristic of the near-cathode region, calculated taking into account only basic glow discharge mechanisms, includes not only the (U-shaped) branch representing the characteristic of the discharge itself but also the branch coinciding with the positive part of the axis of voltages and representing the situation where the discharge has not been ignited. Hence, the full $U(j)$ characteristic is N-shaped, rather than U-shaped, and there are *a priori* no reasons to believe that basic glow discharge mechanisms are insufficient to explain self-organization.

Note that a similar situation occurs in the theory of spots on (hot) cathodes of high-pressure arc discharges: the full $U(j)$ characteristic, calculated in the framework of the model of nonlinear surface heating, includes a U-shaped branch and a branch coinciding with the positive part of the axis of voltages [3], and this model is in fact sufficient to describe modes with spots (e.g. [4–6] and references therein).

Equally, there are *a priori* no reasons to believe that patterns reported in [1, 2] are fundamentally different from well-known normal spots on glow cathodes. Indeed, one can expect that an adequate theoretical model of the near-cathode region of high-pressure dc glow discharges admits solutions describing patterns with different numbers of spots. Under certain conditions, a solution with one spot realizes: the normal mode. Under other conditions, solutions with many spots realize and patterns reported in [1, 2] are observed. Again, this situation is similar to the one occurring in the theory of spots on cathodes of high-pressure arc discharges: the model of nonlinear surface heating admits solutions describing patterns with various numbers of spots [5]; the difference is that under typical conditions of arc cathodes all modes with more than one spot are unstable and do not realize [7].

The question of whether it is possible to describe normal spots on dc glow cathodes in terms of basic glow discharge mechanisms has been extensively explored in the Soviet literature in the 1980s [8–13] and a positive answer has been found. Furthermore, results of the work [12] apply not only to normal spots but also to patterns with more than one spot. Let us show that these results indeed predict the existence of multi-spot patterns under experimental conditions [1, 2].

The approach proposed in [12] was as follows. In the abnormal mode, where the discharge current is high enough, it is distributed over the cathode surface more or less uniformly and the distribution of discharge parameters in the near-cathode region is one dimensional (1D), $f = f(z)$. (The origin of Cartesian coordinates x, y, z is on the cathode surface and the z-axis is perpendicular to the cathode.) Modes with spots are associated with multidimensional distributions of parameters, $f = f(x, y, z)$. Hence, an adequate theoretical model must admit different kinds of steady-state solutions, a 1D one and multidimensional ones. Furthermore, the 1D solution should exist at all current values, although at low currents it is unstable and does not realize. Hence, an adequate theoretical model must admit multiple steady-state solutions to exist for the same discharge current, at least in a certain current range.

It happens frequently in problems with multiple solutions of different symmetries that two solutions become exactly identical at a certain value of a control parameter, a phenomenon called branching of solutions or bifurcation. There are reasons to believe that this also occurs in the problem considered here, i.e. multidimensional solutions describing spot modes branch off from the 1D solution describing the abnormal mode (or, more precisely, the abnormal mode and the unstable mode corresponding to the falling section of the CDVC). Values of the current density at which this branching occurs (bifurcation points) represent points of neutral stability of the 1D solution against multidimensional perturbations, and can be determined by means of the well-known formalism of the linear theory of stability. Thus, in order to answer the question of whether a theoretical model of a glow discharge describes steady-state patterns of a given kind, one can try to find bifurcation points at which multidimensional solutions branch off from the 1D solution. The answer is positive if such points exist and the multidimensional solutions in the vicinity of these points are qualitatively similar to the patterns in question.

In [12], these ideas have been applied to the simplest self-consistent model of a glow discharge, which comprises equations of continuity of the ions of one species and of the electrons written taking into account drift of the charged particles (but not diffusion), ionization and volume losses and the Poisson equation. It was found that the dependence of multidimensional solutions on x and y is described in the vicinity of bifurcation points by eigenfunctions Φ of the 2D Helmholtz equation

$$\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2 + k^2\Phi = 0, \qquad (1)$$

which should be solved in the cross section on the discharge vessel with the homogeneous Neumann boundary conditions. Note that the eigenvalue k may be interpreted as a wave number characterizing variation of the bifurcating spot mode along the cathode.

For a vessel with a circular cross section, which was used in most of the experiments [1, 2], a solution to the Neumann eigenvalue problem for equation (1) is well known:

$$\Phi = J_\nu(kr)\cos\nu\theta, \qquad k = j'_{\nu,s}/R. \qquad (2)$$

Here (r, θ, z) are cylindrical coordinates with the origin at the centre of the cathode, $J_\nu(x)$ is the Bessel function of the first kind of order ν, $\nu = 0, 1, 2, \ldots$, $j'_{\nu,s}$ is a sth zero of the derivative of $J_\nu(x)$, $s = 1, 2, 3, \ldots$, and R is the radius of the vessel. As an example, two of functions (2) are illustrated in figure 1. One can see that distributions described by these functions are not qualitatively different from patterns observed in the experiments [1, 2][2]. Calculations of the bifurcation points have been performed in [12] for the near-cathode space-charge sheath of a long discharge in nitrogen at a pressure of 10 Torr, and it was found that bifurcation points do exist.

[2] Some of the observed patterns comprise a spot at the centre, which is not described by the functions (2). Such patterns can appear through a bifurcation from axially symmetric spot modes; see, e.g. figure 9 in [5].

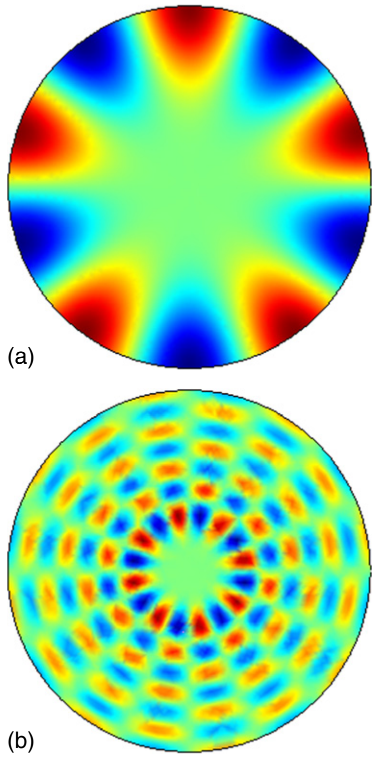

Figure 1. Emerging patterns on a circular cathode, equation (2). (*a*) $\nu = 5, s = 1$. (*b*) $\nu = 8, s = 7$.

(This figure is in colour only in the electronic version)

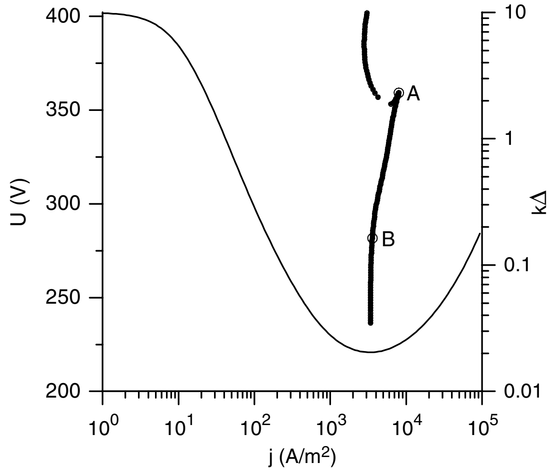

Figure 2. Points: the normalized wave number of the spot mode as a function of the current density at which this mode branches off from the abnormal mode. Line: current density–voltage characteristic. Discharge in xenon at the pressure of 150 Torr, the inter-electrode gap is 0.5 mm.

Thus, results [12] suggest that basic mechanisms of glow discharge may be sufficient to describe patterns observed in [1, 2]. One should check, however, whether bifurcation points exist under conditions of the experiments [1, 2]. In this connection, calculations similar to those reported in [12] have been performed for the conditions [1, 2]. Dissociative recombination was treated as a dominating mechanism of volume losses of charged particles; the parallel-plane discharge configuration was assumed. (The discharge configuration in most of the experiments [1, 2] was more complex; however, an experiment with a parallel-plane configuration reported by the same authors at ICOPS 2006 [14] has revealed similar patterns.) The secondary electron emission coefficient was assumed constant and equal to 0.03.

The calculations have shown that bifurcation points under conditions of the experiments [1, 2] do exist. An example of the calculated relationship between the value of the current density j and the normalized wave number of the spot mode that branches off from the abnormal mode at this value of j is shown in figure 2. The normalization factor Δ is defined as the ratio of the discharge voltage to the electric field on the cathode and has the meaning of a scale of thickness of the near-cathode space-charge sheath. Also shown is the CDVC of the discharge. In the limiting case of long wavelengths, $k\Delta \ll 1$, the bifurcation points tend to the point of minimum of the CDVC as they should. At very short wavelengths, $k\Delta \gtrsim 10$, the model of mobility-dominated space-charge sheath loses its validity. (This model is valid provided that the time of drift of the electrons across the space-charge sheath, $\Delta^2/\mu_e U$, is much smaller than the time of diffusion in the direction along the cathode, $1/D_e k^2$, and this condition holds at $k\Delta \ll \sqrt{U/T_e} \sim 10$, where μ_e, D_e, and T_e are the mobility, diffusion coefficient and temperature of the electrons.) The point designated A, which is the right-most point of the curve $k(j)$, is of particular importance: the abnormal mode is stable at $j > j_A$ and unstable at $j < j_A$. For comparison, also shown in figure 2 is a point B at which the mode with one spot at the edge of the cathode branches off in the case $R = 0.375$ mm. The value of k corresponding to the point A, $k_A \approx 8.7 \times 10^4\ \mathrm{m}^{-1}$, substantially exceeds $k_B \approx 4.9 \times 10^3\ \mathrm{m}^{-1}$, and modes that branch off in the vicinity of A comprise many spots: the mode with 30 spots at the edge of the cathode, $\nu = 30$, $s = 1$, branches off at $k \approx 8.7 \times 10^4\ \mathrm{m}^{-1}$; the mode with three concentric layers of 20 spots each, $\nu = 20$, $s = 3$, branches off at $k \approx 8.5 \times 10^4\ \mathrm{m}^{-1}$; the pattern shown in figure 1(*b*) branches off at $k \approx 8.3 \times 10^4\ \mathrm{m}^{-1}$ etc. One can expect therefore that the mode that occurs at $j < j_A$ is one with many spots, in accord with what is observed in the experiment.

The question of theoretical description of steady-state spot modes on glow cathodes is far from closed: a complete 3D simulation of spot modes in the whole range of their existence and an investigation of their stability are needed. At the moment it seems, however, that basic mechanisms of glow discharges are sufficient to describe spot modes, in accord with what was proposed two decades ago in Soviet works, including modes with more than one spot observed in [1,2]. This includes descriptions of both the instability which causes a disruption of the abnormal mode and development of spots and of a regular arrangement of the developed spots. Therefore, there is no need at present to introduce special mechanisms, such as those proposed in [1, 2]. On the other hand, observations of multi-spot patterns, reported in [1, 2], represent an important contribution and, hopefully, will trigger advances in the theory of spot modes on glow cathodes, which is at present markedly below the level achieved for cathodes of high-pressure arc discharges.

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