Adapted Raised Cosine Window Function for Array Factor Control with Dynamic Range Ratio Limitation

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***Abstract* —The use of window functions to improve the side lobe level of antenna arrays is hindered by high value of excitation currents dynamic range ratio. This paper proposes a fast and iterative window function generation strategy aimed at achieving improved side lobe level starting from a preset current dynamic range ratio. Based on this strategy a new window function is developed for standard set of conditions.**

Index Terms—beamforming, dynamic range ratio, side lobe level, array factor, window function, Fourier relation.

I. INTRODUCTION

In a multipath environment or a saturated RF electromagnetic communications environment the receiver antenna can experience fading and interferences. In such cases it is best to point the antenna radiation beam toward the desired direction and to reduce the side lobes to suppress undesired signals. This is important for radar and communication systems [1]. Array antennas offer the possibility to steer the beam. It is well-known that uniformly feed array antennas have a radiation pattern with large secondary lobes.

In literature there are many methods to control the radiation pattern of an antenna array [2]. They can be grouped in deterministic synthesis methods like Chebyshev, Taylor, Villeneauve that offer control of specific characteristics of the radiation pattern, and iterative methods that allow the improvement of some of the desired characteristics at each interaction, in this last group some of them use genetic algorithms to achieve a faster optimization goal [1].

For beamforming it has been already applied window functions like Kaiser, where a graduated attenuation window similar to the process used in digital filters, is applied across the excitation elements of the array to improve side-lobe suppression. These window functions are useful for real time applications since there is only the need to adjust their tuning parameter [3], particularly in systems with small processing capacity.

Window functions can be divided into two categories; Fixed and Adjustable. The most used fixed window functions are: Rectangular, Hanning, Hamming and Blackman window. On the other hand the Kaiser window is a popular kind of adjustable window function [4]. Chebyshev is another very well-known adjustable distribution that can be used as window function to control the radiation pattern. These higher order adjustable window functions offer a trade-off between the main lobe width and the side lobes reduction [5]. However they are of complex formulation and disregard current attenuators limits.

In this paper an iterative method using the Fourier Relation [6] is described to control an array factor using window functions. Next a two parameter window function adapted from the raised cosine function is presented to approximate the performance of other popular window functions with less complexity and including a parameter that defines the maximum attenuation level desired for the array excitation currents, thus limiting the dynamic range ratio (DRR) value.

Finally the paper compares the performances of the proposed window function with the weighting windows of Kaiser, Villeneuve and Ultraspherical when applied to some known radiation patterns examples.

II. ITERATIVE METHOD FOR ARRAY FACTOR CONTROL WITH WINDOW FUNCTIONS USING THE FOURIER RELATION

The Fourier relation is well established in the antenna synthesis process [5]. However in [6] it is presented a variable change to simplify the analysis and synthesis of antenna arrays. The relation between a planar source distribution, *c(x,y)*, and the array factor *F(βx,βy)*, is given by

$$c(x,y) = \frac{1}{(2\pi)^2}\mathrm{F}\left[F(\beta_x,\beta_y)\right] = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(\beta_x,\beta_y)e^{-j(B_x x+B_y y)}d\beta_x d\beta_y \tag{1}$$

$$F(\beta_x,\beta_y) = (2\pi)^2\mathrm{F}^{-1}\left[c(x,y)\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c(x,y)e^{j(B_x x+B_y y)}dxdy \tag{2}$$

where F is the Fourier transform, λ is the wavelength, and the important variables change [6]

$$\begin{aligned}\beta_x &= \beta\sin(\theta)\cos(\phi) = \beta\cos(\theta_x)\\ \beta_y &= \beta\sin(\theta)\cos(\phi) = \beta\cos(\theta_y)\end{aligned} \tag{3}$$

where θ is the angle between the z direction and the point of the far field, φ the angle between the x direction and the point of the far field in the azimuth plan and θx, θy are the angles of θ projections in the plan x and y respectively.

In case of a discrete planar source distribution, the eq. (2) becomes

$$F(\beta_x,\beta_y)=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}c(x_n,y_m)e^{j(B_x x_n+B_y y_m)} \quad (4)$$

With this relation, when the excitation coefficients are known, the array factor is obtained by the Fast Fourier Transform (FFT) and the Fourier Transform properties can also be used to understand what will happen to the radiation pattern of an array when the tapering windows are applied. Considering the focus of this paper on uniform linear arrays, any multiplication in spatial domain corresponds to a convolution in the array factor domain given by

$$F(\beta_x)\otimes W(\beta_x)\Leftrightarrow c(x)\times w(x) \quad (5)$$

To obtain the final radiation pattern, it is only need to reverse the variables βx and βy to θ and φ using (3).

The iterative algorithm is described as follows:

1) Convert θ and φ to the working variables βx and βy using (3) and calculate the excitation currents with (1);

2) Multiply the window function coefficients by the excitation currents obtained, nulling all current outside the number of array elements;

3) Calculate the array factor using (2) and (3);

4) In case the desired performance is not achieved: adapt the window function parameters and repeat steps 2 and 3 until the desired characteristics are inside of an acceptable margin;

III. The Adapted Raised Cosine Window Function

The proposed window function is derived from the Raised Cosine function. The idea was to introduce a parameter that would define the maximum reduction factor for the excitation currents and still be able to control the form of the window. This is made with the use of two control parameters. The Adapted Raised Cosine window is defined by

$$w(nd)=\begin{cases} A+(1-A)\cos\left(\pi B\left(\dfrac{2n+1}{N-1}\right)\right), & -M\le n\le \mathrm{M} \\ A+(1-A)\cos\left(2\pi B\left(\dfrac{n}{N-1}\right)\right), & -M\le n\le \mathrm{M}-1 \end{cases} \quad (6)$$

$$\text{with } A=\left(\frac{R-\cos(\pi B)}{1-\cos(\pi B)}\right)$$

where d is the distance between elements, number of array elements is defined by N=2M+1, R defines the maximum weighting value of the excitation currents, that can be the maximum attenuation level of the attenuators used in array feeding network, ranging from 0 to 1, it can even take values superior higher values than 1 if an amplification is desired in the excitations currents, and B is similar to the control parameter of the Raised Cosine that determines the form as the currents are tapered by the cosine.

Varying the R parameter the result approximates the Kaiser window, reducing the side lobe level and increasing the beam width, like shown in Figure 1 a). The parameter B adds another degree of freedom, giving the possibility to reduce dynamic range ratio of the excitation currents (DRR) by reducing roll of ratio, as shown in Figure 1 b).

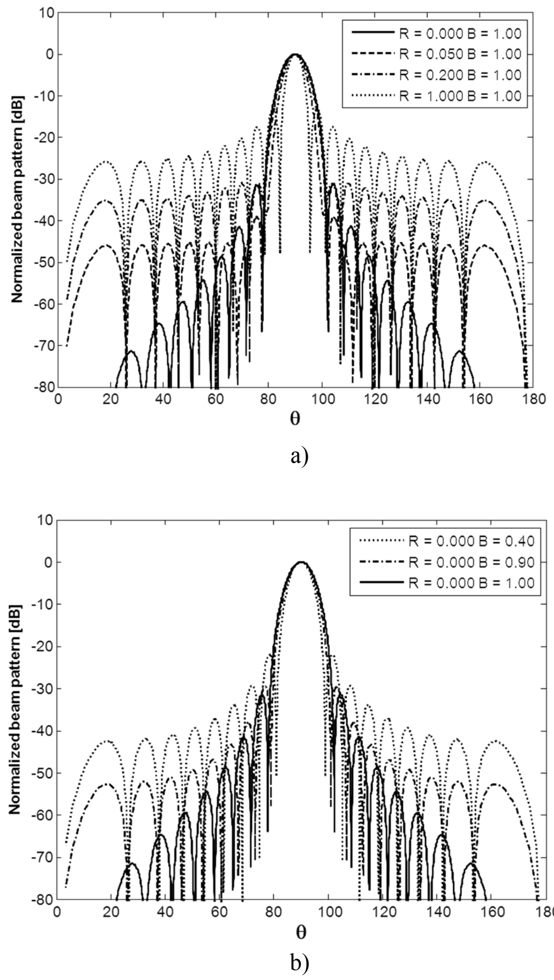


Fig. 1. Different Array factors produced by the Adapted Raised Cosine window function: a) for different R values; b) for different B values.

The R parameter of the proposed window is especially suitable when there is limited attenuation range to apply window functions to the array factor, that is limited DRR. For example considering a 10 dB of attenuator margin the bottom limit of R becomes 0.1. Simulations showed that B = 0.9 is a statistical optimal value to reduce DRR.

For the case of using the window function directly as an array factor, it was derived by curve fitting a relation between R and SLL. The effect of the number of elements of the array was considered. For an error lower than 1.5 dB the relation is defined by

$$R=e^{\left(\frac{SLL-12.9}{\frac{62}{N^{1.37}}-11.817}\right)} \qquad 12.9\le SLL\le 43 \quad (7)$$

The higher value of attenuation of SLL (43 dB) is not reachable for values of N smaller than 20, but the equation remains valid for smaller attenuation values.

IV. Evaluation of the Window Functions

The main characteristics of three adjustable window functions suitable for array factor control are compared to the proposed window function: the Adapted Raised Cosine. These three window functions were the well-known Kaiser-Bessel function with one parameter of control [7], Villeneuve [2] and Ultraspherical [8] windows functions with two parameters of control.

The iterative array factor control method with the Fourier Relation, presented in section 2, was implemented in Matlab to evaluate the different window functions by ripple, side lobe level (SLL), beam with (BW) to measure transition zones, dynamic range ratio (DRR), side lobes level roll of ratio and number of iterations when applied to three different sector beams.

The window parameters are varied one at a time within the useful range much like an iteratively binary search algorithm. The parameter that directly controls SLL reduction is firstly searched. The studied window functions parameters showed to be convergent with the exception of the Ultraspherical window function when manipulating more than one parameter at the same time.

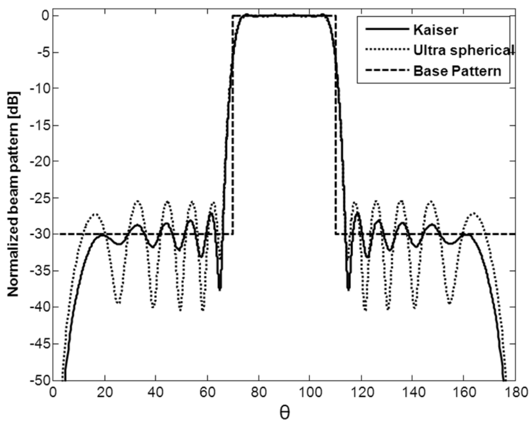
The first situation was to directly apply all window functions to a rectangular sector of BW = 40º and N = 32 elements. For evaluation purposes all window functions parameters that directly control SLL were calculated to 25 dB and the other parameter set to emphasize their individual characteristics. Table 1 compares the performance of the window functions. The result of truncating the base pattern excitation currents with a unitary rectangular window is also presented, to this we call the truncated result. The two most distinct results are presented in figure 2.

TABLE I. Window Functions Performance Applied to a 40º Rectangular Beam Pattern

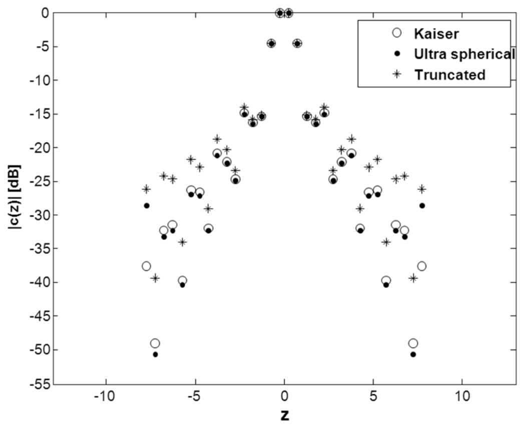
Window function	Ripple (dB)	SLL (dB)	BW (º)	DRR	Roll off Ratio	Parameters
Kaiser	0.14	-27.2	50.3	280	1.11	α=2.665
Adapted Raised Cosine	0.06	-25.8	49.8	213	1.14	R=0.412 B=0.9
Villeneuve	0.04	-26.5	49.8	201	1.09	SLL=25 $\bar{n}$=8
Ultraspherical	0.13	-25.4	49.5	341	1.07	μ=-0.1 x_μ=1.0076
Truncated	0.70	-21.0	45.2	93	1.21	-

From table 1 it is observed that the Adapted Raised Cosine obtained a 24% lower DRR value than the Kaiser Window function with only 1.4 dB higher SLL value.

Villeneuve with $\bar{n}$ = N/4 achieved the lowest DRR by reducing the roll off ratio of the side lobes level. Ultraspherical window with a setup that presents a higher reduction of the inner nearest side lobes equalized the final SLL as demonstrated by the roll off ratio of 1.07 and as shown in figure 2 a). However this result gave the highest DRR of the evaluated set of window functions.



a)



b)

Fig. 2. Kaiser and Ultraspherical window functions applied to a 40º rectangular beam pattern: a) radiation pattern; b) elements excitation currents amplitude.

In the second evaluation situation the window functions were applied to a cosecant sector pattern, as presented in Figure 3 a), using the iterative method described in section 2.

In this case the windows parameters were varied iteratively at first to find the value that gives the SLL < -35 dB and Ripple < -1 dB. Then the second parameter is varied to find the value that reduces DRR and still maintain the first two rules. The number of iterations and performance achieved is registered in Table 2.

Again Villeneuve and Ultraspherical window functions gave the lowest DRR value, but with a 14 degrees higher value of beam width and a lower roll-off ratio.

The Ultraspherical had a higher number of iterations because on every change in the roll-off ratio parameter the previous parameter that directly controls SLL had to be adjusted to maintain the same SLL and ripple. In this case the Ultraspherical brought more complexity and no significant advantage comparing to the Villeneuve window function. The Raised cosine window achieved a 38% lower DRR comparing to Kaiser by a 9% lower roll of ratio.

TABLE 2. WINDOW FUNCTIONS PERFORMANCE WHEN APPLIED TO A COSECANT SECTOR BEAM PATTERN

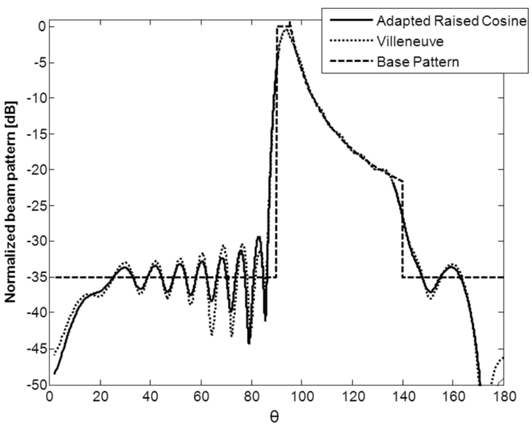
Window function	Ripple (dB)	SLL (dB)	BW (°)	DRR	Roll of Ratio	Iterations
Kaiser	0.5	-34.83	56.8	68	1.52	9
Villeneuve	0.71	-35.35	62.5	25	1.19	10
Ultraspherical	0.95	-34.48	61.5	24	1.07	49
Adapted Raised Cosine	0.55	-34.43	56.6	42	1.39	7
Truncated	1.15	-21.62	56.2	21	1.94	-

The third evaluation case was made over a cosecant sector beam pattern with a deeper null near to the cosecant section as presented in Figure 3 b). The deeper null was also evaluated by width and ripple. The results are presented in Table 3.

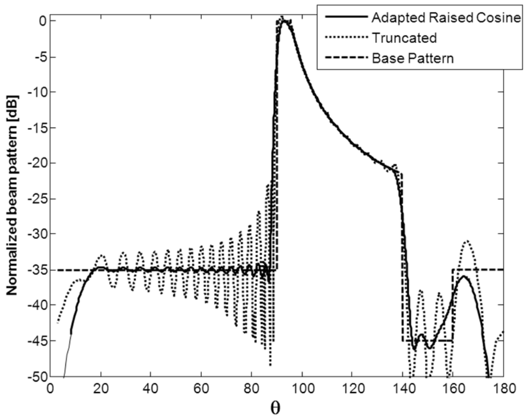
TABLE 3. WINDOW FUNCTIONS PERFORMANCE WHEN APPLIED TO A COSECANT SECTOR BEAM PATTERN WITH A DEEPER NULL

Window function	Ripple (dB)	SLL (dB)	DRR	Null Ripple	Null Width	Iterations
Kaiser	0.05	-34.04	392	0.65	8.5	10
Villeneuve	0.05	-34.32	268	0.98	8.2	14
Ultraspherical	0.03	-34.13	262	0.99	8.8	>100
Adapted Raised Cosine	0.06	-34.12	235	0.97	8.9	11
Truncated	0.83	-22.3	68	7.12	17.5	-

Once more the values registered have shown that the Ultraspherical window function did not surpass Villeneuve. The Adapted Raised Cosine that presented an intermediate result in the first two evaluated cases have shown to be the best result when the deeper null was introduced, offering a 40% lower value of DRR compared to Kaiser for almost the same performance.



a)



b)

Fig. 3. Window functions applied to a cosecant beam pattern: a) 32 elements array; b) 64 elements array with a deeper null.

To confirm this DRR achievement by the Adapted Raised Cosine window various scenarios were calculated using the iterative procedure of section 2. The iteration goal was only to achieve an SLL value of -25, -30, -35 and -40 within a margin of 1 dB. The scenarios were a 20º and 40º rectangular beam pattern, a cosecant sector beam and the same cosecant sector with a deeper null. The linear array number of elements was varied from 16 to 256. The results showed that the SLL target is obtained between 1 to 8 interactions for every window function tested, demonstrating to be independent of the number of elements. The beam width and ripple values obtained for the different window functions were similar. Table 4 presents the DRR results for two SLL targets with different numbers of array elements for one of the cases calculated.

TABLE 4. WINDOW FUNCTIONS DRR VALUES WHEN APPLIED TO A COSECANT SECTOR BEAM PATTERN TO ACHIEVE ITERATIVELY TWO DIFFERENT VALUES OF SLL FOR DIFFERENT NUMBER OF ARRAY ELEMENTS

SLL (dB)	Array elements	Kaiser	Adapted Raised Cosine	Villeneuve	Ultraspherical
-25	**16**	7,6	6,1	6,1	4,9
	32	24,9	25,8	29,5	25,9
	64	52,2	46,7	34,0	40,2
	128	84,5	80,2	82,0	69,4
	256	155,0	136,0	300,0	121,2
-30	**16**	11,1	7,7	8,0	5,9
	32	30,1	29,6	31,7	28,5
	64	86,5	59,9	37,0	61,3
	128	146,5	96,0	93,0	91,0
	256	273,0	176,0	281,8	172,8

From Table 4 it is verified that the proposed window function, the Adapted Raised Cosine, has shown similar or significant lower DRR values when compared to the Kaiser window, particularly for higher number of array elements.

These cases show that every window function evaluated has an advantage depending on the application. The Ultraspherical window function for $\mu < 0$ offers the lowest roll-off ratio thus offering a reduced DRR however with a higher number of iterations. Villeneuve gives almost the same results with a lower number of iterations. Kaiser gives the highest roll-off ratio but with the highest DRR.

The proposed Adapted Raised Cosine window presents a similar performance to the Kaiser window with lower DRR values.

V. CONCLUSIONS

The use of adequate window functions to iteratively achieve the desired radiation pattern characteristics can give speed advantages in real time applications. The technique with the Fourier relation employed in this paper has the advantage of being directly calculated, having the same method for different window functions and the results are obtained quickly using FFT. Fewer than ten iterations are needed to obtain an SLL value between 1 dB margin.

In this paper three window functions known from literature were described in the form for computational use with the Fourier relation. The Adapted Raised Cosine window function was also presented. They were compared in terms of radiation patterns control and speed in order to achieve the desired results, and so depicting their individual characteristics.

The presented Adapted Raised Cosine window function enables the control of the antenna array factor side lobes level by taking into account the current attenuator limits and still be able to obtain generally better results when compared to the Kaiser window function.

ACKNOWLEDGMENT

This work was supported by the CCM research unit of University of Madeira.

REFERENCES

[1] N. Karaboga, K. Guney, and A. Akdagli, Null steering of linear antenna arrays by using modified touring ant colony optimization algorithm. Int J RF Microwave Comput Aided Eng 12 (2002), 375–383.

[2] C. A. Balanis, Antenna Theory, Analysis and Design, 2nd ed., John Wiley & Sons, 1997.

[3] T.B.Lavate, A.M.Sapkal, Capacity enhancement of 3G cellular system using Switched beam smart antenna with windowed beam former, (IJCNS) International journal of computer and network security, Austria, 2010, vol. 2 no. 3, pp. 63-67.

[4] Songxin Qi and Quanrang Yang, Antenna array design using window function, International Journal of Infrared and Millimeter Waves, 1992, vol. 13, no. 10, pp. 1641-1645.

[5] Harris, F. J., On the use of windows for harmonic analysis with the discrete fourier transform, Proceedings of the IEEE, Vol. 66, No. 1, 51-83, Jan. 1978.

[6] Casimiro, A. M. E. S. and J. A. R. Azevedo, A unification procedure to the analysis and synthesis of antenna arrays, Journal of Electromagnetic Waves and Applications, 2005, Vol. 19, No. 14, pp. 1881-1896.

[7] R. W. Hamming, Digital Filters, Prentice-Hall International Editions, 1989.

[8] Antoniou Andreas, Stuart W. A. Bergen, Design of Ultraspherical Window Functions with Prescribed Spectral Characteristics, EURASIP Journal on Advances in Signal Processing, 2004.