A Simple Model of Distribution of Current Over Cathodes of Vacuum Circuit Breakers

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One of the authors passed away prematurely when this work was already at an advanced stage. His coworkers miss very much W. Hartmann, a brilliant scientist and a dear friend.

***Abstract*— There are several hundreds of spots operating simultaneously on cathodes of vacuum arcs in high-power vacuum circuit breakers. In this work, the spot distribution along the contact surface is simulated by means of an approach that is based on the concept of surface density of spots and represents a natural alternative to tracing individual spots. An equation governing the evolution of the surface density of the spots or, equivalently, the distribution of macroscopic (averaged over individual spots) current density over the cathode is obtained by generalizing the concept of random walk of a single cathode spot in low-current vacuum arcs. The model relies on empirical parameters characterizing individual spots (the diffusion coefficient of the random motion of cathode spots and the velocity of drift superimposed over the random motion), which may be taken from experiments with low-current arcs, and does not involve adjustable parameters. The model is simple and physically transparent and correctly reproduces the trends observed in the experiments under conditions where the cathode arc attachment is diffuse. The distribution of the macroscopic current density on the cathode, given by the model, represents the boundary condition that is required for existing numerical models of vacuum arcs in high-power vacuum circuit breakers.**

***Index Terms*— Propagation, vacuum arcs, vacuum circuit breakers.**

I. Introduction

OBSERVATIONS of motion of spots on cathodes of vacuum arcs are reported and analyzed, particularly in [1]–[15]. The lifetime of cathode spots of vacuum discharges is of the order of microseconds or shorter. After the extinction of a spot, a new spot is created in the vicinity of the first one, which is perceived as the motion of a spot. At the macroscopic level, and in the absence of a magnetic field, the motion of a cathode spot can be described as a random walk along the cathode surface. When an external magnetic field is applied parallel to the cathode surface, an ordered motion, directed in the opposite direction of the Amperian force exerted by the magnetic field, is superimposed over the random walk. This motion is commonly referred to as retrograde.

In [16], the random motion of a cathode spot was treated as a Markov process, and a convection–diffusion equation was postulated to describe the evolution of the probability density of the spot position in the presence of external tangential and axial magnetic fields. A description of the motion of a cathode spot in terms of a standard probability distribution assumed for spot displacements has been developed in [17] and [18]. In [19], the Monte Carlo method is used to simulate the random walk of grouped spot cells, with the aim to describe velocity and trail width of arc spots on nanostructured tungsten cathode.

The above-mentioned theoretical works apply to conditions where just one or few spots exist on the cathode surface at any given moment, which means that the arc current is relatively low. A large number of spots, in the order of hundreds, can exist simultaneously on cathodes of high-current arcs, e.g., those in high-power vacuum circuit breakers. A 3-D modeling was performed in [20] to simulate the collective behavior of an ensemble of a large number of spots operating simultaneously, by means of tracing the motion of each individual spot taking into account the effect of external and self-induced magnetic field on the motion of cathode spots. It was found that the behavior of a high-current arc is mainly determined by the interactions between individual cathode spots through the magnetic field generated by the arc. A similar approach was applied in [21].

In the case of high-current arcs, where a large number of cathode spots operate simultaneously, the motion of cathode spots can be described by employing the concept of the surface density of spots. This approach represents a natural alternative to tracing the motion of individual spots and is described in this work. The evolution of the surface density of spots is governed by an equation, which was derived through a generalization of the convection–diffusion equation describing the random walk of a single cathode spot in low-current vacuum arcs. The convective term was estimated from the experimental data, which relate the velocity of the retrograde motion of the spots (drift velocity) to tangential and axial magnetic fields imposed externally and/or induced by the arc.

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The developed model is simple and physically transparent, does not rely on adjustable parameters, and correctly reproduces the trends observed in the experiments under conditions where the cathode arc attachment is diffuse. The distribution of the macroscopic (averaged over individual spots) current density on the cathode, which is given by the model, represents the boundary condition that is required for existing numerical models of vacuum arcs in high-power vacuum circuit breakers (see [22]–[27] and references therein).

This paper is structured as follows. In the next section, the model is introduced. Modeling results for conditions of arc attachment in axial magnetic field are given in Section III. Section IV is concerned with experimental and theoretical results for conditions of a synthetic test circuit with axial and tangential magnetic fields. Conclusions are summarized in Section V.

II. Model

The motion of a spot on a cathode of a low-current vacuum arc can be described as a random walk, representing a sequence of finite displacements. Let us introduce Cartesian coordinates on the cathode surface, (x, y). The evolution of the probability density function $f(x, y, t)$ for the spot to be at a position (x, y) at a time t is governed by the Fokker–Planck equation [16]

$$\frac{\partial f}{\partial t} + \nabla \cdot (-D\nabla f + f\mathbf{v}) = 0 \tag{1}$$

where $\nabla = ((\partial/\partial x), (\partial/\partial y))$, D and $\mathbf{v}$ are the diffusion coefficient and the velocity of drift superimposed over the random motion of a cathode spot. On the microscopic level, both quantities are related to the average distance and time interval between consecutive positions of a spot. Although dealing with (1), D and $\mathbf{v}$ are considered as, respectively, a known parameter and a known function of magnetic field.

Let us now consider an ensemble of a large number $N \gg 1$ of spots existing simultaneously on a cathode of a high-current vacuum arc, and designate by $\boldsymbol{\Omega} = \boldsymbol{\Omega}(x, y, t)$ the surface density (number per unit area) of the spots. Note that $N = \int \boldsymbol{\Omega}\, dS$, where the integral is evaluated over the cathode surface. As shown by the estimates below (the last paragraph of this section), the direct interaction between existing individual spots is weak compared to the effect of the magnetic field induced by the whole ensemble of spots (note that this applies only to adjacent spots, which already exist, and not to the creation of new spots, which is affected by the presence of the spots already existing; see below). Neglecting this interaction, one can relate the surface density of the spots in the ensemble to the probability density function of a single spot; $\boldsymbol{\Omega} = Nf(x, y, t)$. Multiplying (1) by N, we obtain an equation governing the evolution of function $\boldsymbol{\Omega}$

$$\frac{\partial \boldsymbol{\Omega}}{\partial t} + \nabla \cdot \Gamma = 0 \tag{2}$$

where

$$\Gamma = -D\nabla\boldsymbol{\Omega} + \boldsymbol{\Omega}\mathbf{v}. \tag{3}$$

We stress that while (1) applies to a single spot and the unknown function f is the probability of the spot at a given time being localized at a given position, (2) and (3) apply to an ensemble of a large number of spots and the unknown function $\boldsymbol{\Omega}$ is the surface density of spots at a given time at a given position. A further difference appears when we introduce an account of a variation of the total number of spots on the contact with the temporal variation of the total arc current, which is essential in order to apply (2) and (3) to contacts of high-power circuit breakers. This amounts to introducing into (2), w_c the local rate of creation of new spots and w_e the local rate of extinction of existing spots

$$\frac{\partial \boldsymbol{\Omega}}{\partial t} + \nabla \cdot \Gamma = w_c - w_e. \tag{4}$$

Since new spots are ignited in the immediate vicinity of those already existing, one can assume that the value of quantity w_c at a certain position is proportional to the number of spots already existing at this position: $w_c = \chi_c\boldsymbol{\Omega}$, where the proportionality coefficient does not depend on the position of the surface. The probability of ignition of new spots depends on time (it is higher when the arc current and, consequently, the total number of spots are on the rise), hence χ_c depends on time, $\chi_c = \chi_c(t)$. It is legitimate to also assume that the probability of an existing spot to be extinguished does not depend on the position of the spot on the surface or, in other words, the same percentage of spots goes out at all points at the same moment. Then, the value of quantity w_e at a certain position is proportional to the number of spots existing at this position: $w_e = \chi_e\boldsymbol{\Omega}$, where the proportionality coefficient again depends on time, $\chi_e = \chi_e(t)$. Thus, the rhs of (4) is $w_c - w_e = \chi\boldsymbol{\Omega}$, where $\chi = \chi(t) = \chi_c - \chi_e$.

Integrating (4) over the cathode surface, we obtain

$$\frac{dN}{dt} + \oint \Gamma_n\, dl = \chi(t)\, N \tag{5}$$

where Γ_n is the normal component of the flux Γ at the cathode edge and the integral is evaluated along the edge. The number of spots N depends on the current wave $I = I(t)$ and the average current per spot I_0: $N = I(t)/I_0$. Solving (5) for χ, one finds

$$\chi = \frac{1}{I}\frac{dI}{dt} + \frac{I_0}{I}\oint \Gamma_n\, dl. \tag{6}$$

If $I(t)$ and I_0 are known, the term on the rhs of (4) is specified. Although the variation of the total number of spots is governed only by the current wave, the term on the rhs of (4) describes the distribution of this variation over the cathode surface.

Equations (3) and (4) have to be supplemented by a boundary condition at the cathode edge and also by an initial condition. Two boundary conditions have been tested in this work: a Dirichlet boundary condition $\boldsymbol{\Omega} = 0$ and the so-called free boundary condition $\partial\boldsymbol{\Omega}/\partial n = 0$ (here, n is the direction in the plane (x, y) locally normal to the edge of the cathode surface and directed outside the cathode). Modeling results obtained with these two boundary conditions do not differ appreciably, except in the immediate vicinity of the cathode edge. We note, for definiteness, that the modeling results reported in this paper have been computed with the free boundary condition.

The initial condition for (3) and (4) may be specified at a given moment t_0 soon after the arc ignition, when the region of arc attachment to the cathode is still narrow. Since the attachment is narrow, an accurate account of its shape and spot distribution inside the attachment is not needed and one can approximate the attachment by, e.g., a circle of a given (small) radius R_0 and assume that the spots are distributed inside the attachment uniformly. Then, the initial condition at $t = t_0$ reads: $\Omega = I(t_0)/I_0\pi R_0^2$ inside the attachment and $\Omega = 0$ outside the attachment.

Let us designate by $j = j(x, y, t)$ the distribution along the cathode surface of the macroscopic (spots-averaged) current density. Note that this distribution serves as a boundary condition for numerical models of the arc on the whole. Since $j(x, y, t) = I_0\Omega(x, y, t)$, the above mathematical problem may be transformed to the new unknown variable $j(x, y, t)$:

$$\frac{\partial j}{\partial t} + \nabla \cdot (-D\nabla j + j\mathbf{v}) = \chi j \quad (7)$$

$$\chi = \frac{1}{I}\frac{dI}{dt} + \frac{1}{I}\oint j\upsilon_n \, dl. \quad (8)$$

At the cathode edge, $\partial j/\partial n = 0$ (9)

$$t = t_0: \; j = \begin{cases} \dfrac{I(t_0)}{\pi R_0^2} & \text{inside the attachment} \\ 0 & \text{outside the attachment} \end{cases} \quad (10)$$

where υ_n is the normal component of the velocity $\mathbf{v}$ at the cathode edge and the integral is evaluated along the edge.

The problems (7)–(10) do not involve the average current per spot, I_0. It follows that a specific value of I_0 is irrelevant to the modeling: its variations cause only a rescaling of the computed surface spot density Ω and do not affect the distribution of the average current density j.

Due to the dependence of the drift velocity on the magnetic field, the problems (7)–(10) need to be supplemented with information on direction and amplitude of the magnetic field, imposed externally and/or induced by the arc. The latter is governed by Maxwell's equations, which are written in the 2-D approximation [in the plane (x, y)].

The diffusion coefficient D and the speed $\mathbf{v}$ of drift superimposed over the random motion of cathode spots are taken from the experiment. Then, the mathematical model is complete. It should be stressed that the parameters D and $\mathbf{v}$ refer to individual spots and may be taken from experiments with relatively low arc currents, where only one or few spots are present on the cathode surface at each moment. In other words, the model relies on empirical parameters characterizing individual spots and does not involve any empirical parameters characterizing the distribution of spots along the cathode surface, nor adjustable parameters. This is in contrast to the previous work [28], where the diffusion coefficient was treated as a fitting parameter.

The results reported in this work have been computed with the use of the following data on the diffusion coefficient D and the drift velocity $\mathbf{v}$. D was set equal to 10^{-4} $\mathrm{m^2\,s^{-1}}$, which is a characteristic value of the diffusion coefficient for CuCr cathode [6]. The velocity $\mathbf{v}$ of drift is opposite to the direction of the Amperian force and the speed of drift is proportional to the tangential magnetic field B_t, $\upsilon = k_1 B_t$, for low fields and saturates, $\upsilon = k_2$, for high fields. Here, B_t is the net tangential magnetic field (the superposition of the self-induced and external tangential magnetic fields) and k_1 and k_2 are coefficients that depend on the cathode material. Although k_1 and k_2 may also depend on other parameters, in particular, on the state of the cathode surface, the arc current, and the interelectrode gap [11], this dependence was not taken into account in the modeling reported in this work and it was set $k_1 = 230$ $\mathrm{ms^{-1}}$ $\mathrm{T^{-1}}$, $k_2 = 19$ $\mathrm{ms^{-1}}$; values corresponding to CuCr cathodes at an arc current of 30 A [29]. Note that the above-mentioned values of the coefficients D, k_1, and k_2 correspond to CuCr cathodes with the usual surface roughness; fine effects, like the effect of nanostructured layers [19], remain beyond the scope of this work.

When an axial magnetic field B_n is applied, the direction of cathode spot drift is deflected from the retrograde motion direction; the Robson drift. In the modeling reported in this work, it was assumed, in agreement with experiment results [11] for CuCr30 cathode, that the deflection angle θ (the so-called Robson angle) is proportional to φ the angle of inclination of the magnetic field vector to the cathode surface, $\varphi = \arctan B_n/B_t$: $\theta = \eta\varphi$, where $\eta = 0.8$.

Note that it was assumed in the passage from (1) to (2) and (3) that there is no direct interaction between existing individual spots, which implies that the motion of spots is dominated by the magnetic field induced by the whole ensemble of spots rather than their interaction with the adjacent spots. Let us check this assumption. The magnetic field generated by the arc can be estimated as follows. First, considering the arc as a cylinder of radius R carrying current I distributed uniformly over the cross section of the cylinder, the mean value of the magnetic field inside the cylinder may be estimated as $B = \mu_0 I/3\pi R$. Second, let us designate by B_0 the value, at a position occupied by a spot, of the magnetic field induced by a neighboring spot, averaged over the spot ensemble. One can write $B_0 = \mu_0 I_0/2\pi r_0$, where r_0 is the average distance between the spots. The latter may be estimated as $r_0 = (\pi R^2/N)^{1/2}$. One finds $B/B_0 = 2(\pi N)^{1/2}/3 \gg 1$, and the above assumption is justified.

III. Modeling Cathode Attachment in Axial Magnetic Field

Let us first consider results of the application of the model to simulation of distribution of spots over a CuCr cathode during the arc expansion phase in the absence of external tangential magnetic field for different values of the axial magnetic field, reported in [12]. The contacts in these experiments had planar surfaces without slots and a diameter of 40 mm, the current wave was sinusoidal with a frequency of 50 Hz and the current amplitude of 7 kA.

In Fig. 1, the time dependence of the cathode arc attachment radius R_a is plotted as a function of time for different values of the axial magnetic field B_n. The experimental values of R_a have been obtained from the images shown in [12, Fig. 12]. The first image with visible spots in this figure refers to the moment $t = 0.407$ ms, so the modeling was started at this moment with a circular arc attachment with the radius taken

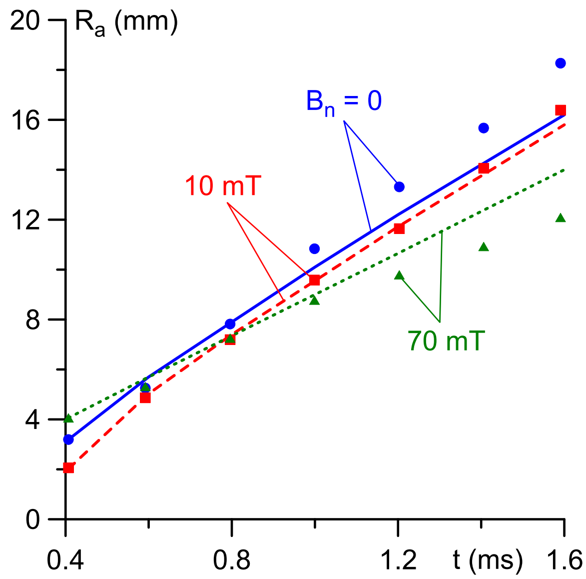

Fig. 1. Time dependence of arc attachment radius for different values of axial magnetic field. Symbols: experiment [12]. Lines: modeling.

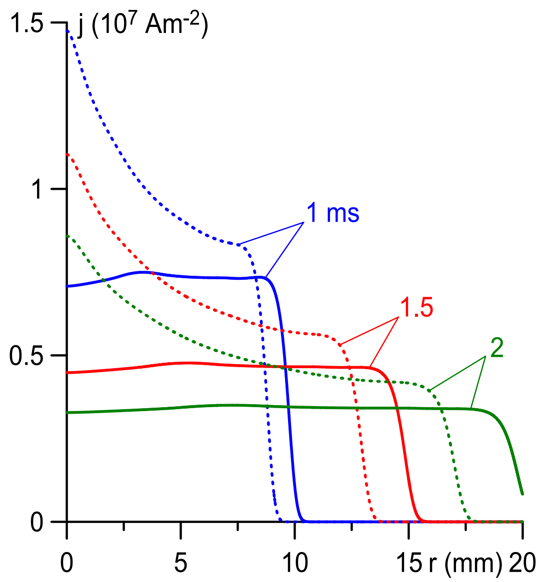

Fig. 2. Calculated radial distributions of the macroscopic (averaged over individual spots) current density at different moments. Solid: $B_n = 0$. Dotted: $B_n = 70$ mT.

from the image. Values of R_a at subsequent moments were deduced from the computed radial distribution of the current density j as a value of r at which j equals 10% of the value at the center. It is seen that the modeling qualitatively correctly reproduces the expansion of the arc attachment observed in the experiment and the quantitative deviations are not big, although the slowing down of the expansion due to the axial magnetic field is somewhat stronger in the experiment than in the modeling.

The computed evolution of the radial distribution of the current density for the case without axial magnetic field and the case $B_n = 70$ mT is shown in Fig. 2. For $B_n = 0$, the cathode spots at all times are virtually uniformly distributed inside the arc attachment region, in agreement with what was observed in [12]. For $B_n = 70$ mT, the current density or, equivalently, the density of spots at the center of the arc attachment region is higher than at the periphery. The physics of this effect appears to be transparent: the tangential magnetic field B_t is lower at the center, hence the Robson angle is bigger, the bending of the spot trajectories is stronger, and the spot drifts in the radial direction slower. However, this effect was not observed in the experiment: the cathode spots remained almost uniformly distributed inside the arc attachment region also in the presence of the axial magnetic field. Thus, this point requires further work.

IV. Investigating Cathode Attachment in Inclined Magnetic Field

This section is concerned with the effect of a superimposed (external) tangential magnetic field. The measurements have been performed in a synthetic test circuit similar to the setup described in [30] and [31]. The current wave had a frequency of 52.6 Hz and an amplitude of 3.68 kA. The arc was drawn by separating the movable anode from the fixed cathode 30 μs after the current onset $t = 0$ at an average speed of 0.54 ms^{-1} up to a gap distance of 4.1 mm. Thus the arcing time was 9.47 ms. Contacts were made of arc-melted CuCr50, had planar surfaces without slots and a diameter of 40 mm. The axial magnetic field was directed from the anode to the cathode, had a bell-shaped radial profile, was proportional to the arc current, and had a magnitude of 40.8 mT near the center of the cathode at the current peak.

The external tangential magnetic field superimposed over the arc was generated by a nearby current return conductor, the current therein flowing antiparallel to and in phase with the current in the arc. Thus, the field was in phase with the arc current. The field had a magnitude of 6.3 mT near the center of the cathode at current peak.

The contact system was mounted inside a continuously pumped demountable ultrahigh vacuum chamber. A quartz window allowed a side-on view on the cathode surface at an angle of approximately 80° (1.4 rad) to the surface normal. A high-speed black-and-white charge-coupled device (CCD) video camera (Phantom V7) was used to observe the cathode spots at a rate of 33 057 frames/ s (interval 30.25 μs between two frames) and an exposure time as low as 2 μs. The image area of the camera chip covered 290×29 pixels corresponding to a recorded object area of 40 mm $\times 4$ mm.

Typical experimental and simulation results are shown in Fig. 3. The return conductor is positioned approximately at 120 mm to the right from the cathode center, i.e., at the point (120 mm, 0) in terms of the coordinate system shown in Fig. 3(a). The tangential magnetic field produced by the return conductor is oriented approximately in the negative y-direction. The spatial distribution of the axial magnetic field, characteristic for the magnetic field system used in the experiment, was used in the modeling.

In the experiment, the first spots were ignited in the region $x > 0$, i.e., in the half-plane facing the return conductor. Note that the average current per spot during the whole half-cycle was approximately 24 A. The initial condition in the modeling was set at $t = 1.36$ ms (which corresponds to the

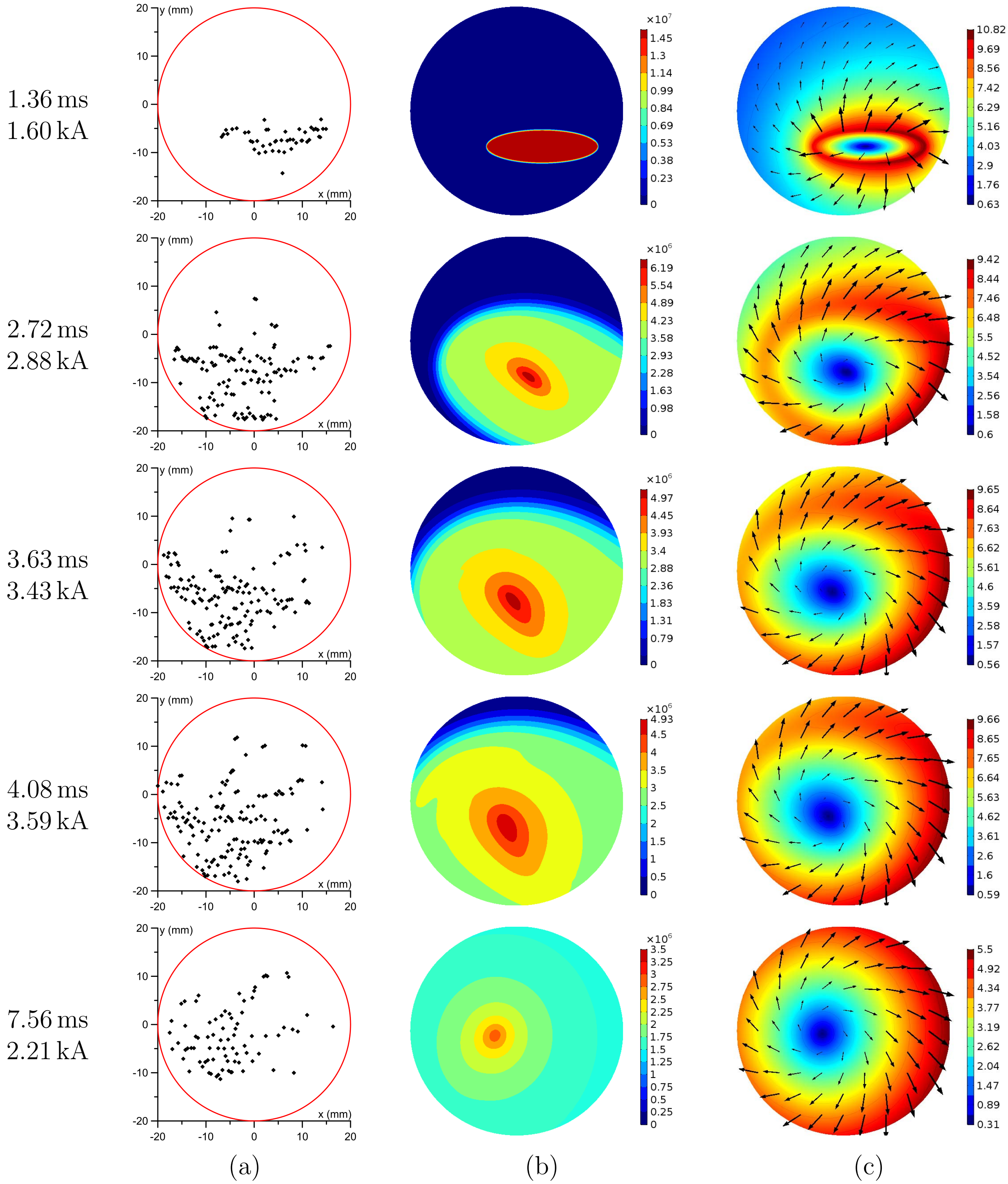

Fig. 3. (a) Distributions of cathode spots observed at different arcing times in the experiment. (b) Computed distributions of macroscopic (averaged over individual spots) current density on the cathode; bars in Am^{-2}. (c) Computed distributions of spot drift velocities; bars in ms^{-1}.

first frame in Fig. 3) and the arc attachment region was taken as an ellipse with the major and minor axes of lengths of 10.5 and 3.2 mm , respectively; the surface density of the spots was constant inside the ellipse.

We are not trying to conduct an accurate statistical analysis of the images shown in Fig. 3(a), and we confine ourselves to the qualitative discussion. As shown in Fig. 3(a), after the ignition, the arc attachment expands over the cathode surface and its "center of mass" is shifted to the left. In other words, the Amperian force associated with the external tangential magnetic field is "repulsing" the arc in a direction opposite to the return conductor. The same tendency is seen in the modeling [Fig. 3(b)]. This result may seem to be somehow counterintuitive: while the force exerted by the external tangential magnetic field over individual spots is directed in the anti-Amperian direction shifted by the Robson angle, the center of mass of the ensemble is displaced in the Amperian direction. In this connection, it is of interest to compare the external tangential magnetic field with the self-induced field.

Let us consider Fig. 3(c), where the computed distributions of the spot drift velocity **v** are shown. One can see that the maximum values of spot drift speed υ are of the order of 10 ms^{-1}. Since this value is below the saturation threshold $k_2 = 19\ \mathrm{ms}^{-1}$ assumed in the modeling (see Section II), υ is proportional to the magnitude B_t of the net tangential magnetic field. Using the proportionality coefficient $k_1 = 230\ \mathrm{ms}^{-1}\,\mathrm{T}^{-1}$, one finds that the maximum values of B_t are of the order of 40 mT. It follows that the external tangential magnetic field, which is 6 mT as mentioned above, is smaller than the self-induced field. The latter is also shown in Fig. 3(c): there is no appreciable angular asymmetry in the distribution of **v**.

It is interesting to note that the experimental distributions of the spots shown in the first to fourth frames are displaced a little downward (below the midplane $y = 0$), while the center of mass of the distribution on the last frame appears to be in the midplane. The same is true for the maximum of the computed macroscopic current density shown in Fig. 3(b).

One can conclude that the computed distribution of the macroscopic (averaged over individual spots) current density shown in Fig. 3(b) correctly reproduces the trends seen in the experimental images of spot distribution in Fig. 3(a). Of course, the correspondence between the modeling and the experiment is not precise; for example, there are no spots in the uppermost and lowermost parts of the contact in the last frame in Fig. 3(a), while the computed macroscopic current density in the last frame in Fig. 3(b) is not zero in these regions.

V. Conclusion

A simple model of the motion of an ensemble of a large number of spots on cathodes of high-current vacuum arcs has been developed by generalizing the concept of random walk of a single cathode spot in low-current vacuum arcs. The model predicts the distribution of macroscopic (spots-averaged) current density over the cathode at different arcing times and relies on empirical parameters characterizing individual spots (the diffusion coefficient of the random motion of cathode spots and the velocity of drift superimposed over the random motion), which may be taken from experiments with low-current arcs. The model does not involve empirical parameters characterizing the distribution of spots along the cathode surface, nor adjustable parameters. This is in contrast to the previous work [28], where the diffusion coefficient was treated as a fitting parameter.

The model was used for the simulation of the arc attachment to CuCr cathodes of high-power vacuum circuit breakers under two sets of experimental conditions: the experiments with the axial magnetic field [12] and the experiments with both axial magnetic field and tangential (external) magnetic field of about 6 mT, performed on a synthetic test circuit. Although there are some differences between the modeling and experimental results, one can say that, in general, the modeling reproduces the experiment with diffuse cathode arc attachment qualitatively correctly, especially taking into account the simplicity of the model and the absence of fitting parameters.

A number of numerical models exist for industrial simulations of vacuum arcs in high-power vacuum circuit breakers (see [22]–[27] and references therein). These models require a distribution of the macroscopic (spots-averaged) current density on the cathode to be specified as a boundary condition. The model of this work gives precisely this distribution and does not require significant computational resources. Therefore, the model can be useful as a module of models of vacuum arcs with diffuse cathode attachment in high-power vacuum circuit breakers.

Numerical models of vacuum arcs with an account of individual cathodic spots and the mixing and interaction of the jets produced by the spots have started to appear recently [32]–[35]. The boundary conditions at the cathode surface mimicked, in 2-D, the uniform distribution of the cathode spots over the cathode surface [32], [35] and described six individual spots positioned in a ring [33], [34]. One could think of using the model of this work in order to improve such boundary conditions.

Two effects that are left beyond the scope of this work are the formation of an expanding ring structure of spots, observed on clean surfaces of degassed pure metal cathodes without an external magnetic field, and the grouping of spots, which occurs if the external transversal magnetic field exceeds 5–10 mT (see [3], [5], [7] and [5], [7], [14], respectively, and references therein). Without discussing these effects in detail, we only note the following. The experiments [3], [5], [7] have been performed with rectangular current pulses. If the model of this work is applied to such a situation, then (8) gives $\chi = 0$ and one can expect that the model would describe an expanding ring structure. On the other hand, the grouping of spots can be introduced in the model in a way similar to the one described in [19].

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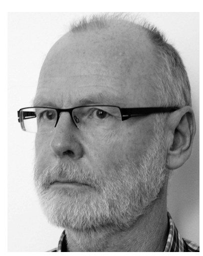

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