Efficient derivatives evaluation under a jump-diffusion process

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Extended abstract

Merton (1976) provides a jump-diffusion model, where the dynamics of the price of the underlying are subject to variations due to a Brownian process and also to possible jumps. Under the Black and Scholes (1973) assumptions on the Brownian component and considering a compound Poisson process for the jump part, the model admits a series solution for the European option pricing. Amin (1993) proposes a procedure for derivative pricing by discretising the distribution of the underlying allowing the jumps to have a random amplitude which must be a multiple of the Brownian move. Hilliard and Schwartz (HS, 2005) considering the independency of the two factors develop a multinomial lattice: one variable mimicking the diffusion process and the second one the log-normal jumps in the compound Poisson process. HS procedure provides more accurate results than Amin and the weak convergence of the discrete price is ensured in the special case of deterministic jump amplitude and numerically justified otherwise. HS bivariate tree can be applied to the evaluation of American options. The time complexity of the HS backward procedure is $O(n^3)$. Dai *et al.* (2010) build on the HS procedure reducing complexity to $O(n^{2.5})$ by dissolving the intermediate nodes on the tree introduced by the jumps in the nearest diffusion node, therefore providing a one-dimensional tree. Here, also starting from the HS technique we introduce a procedure which further reduces the complexity both in the

European and in the American case. We prove this theoretically when the log-normal distribution is discretised by a variable with tree possible states (up, down and no jump). In the general case, our procedure is numerically justified as in the Hilliard and Schwartz paper.

Keywords
Derivatives pricing; Jump processes; Trees; Complexity and estimated error.

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