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Construction of Single Sampling Plan Indexed Through Average Quality Level

Statistics

KEYWORDS : Average Quality Level, Zero Inflated Poisson (ZIP) distribution, Single Sampling Plan (SSP), Operating Characteristic Curve (OC).

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ABSTRACT

In certain product categories, there will be defective products in virtually every production lot. It is often true even after the manufacturer has checked each individual product and has revamped the defective ones, since visual inspection is not 100% consistent. Consequently, in many supplier or buyer interactions, the supplier is not expected to deliver defect-free commodities. The buyer needs to control the quality of purchased commodities, since the buyer does not want numerous defects. One of the foremost techniques is 'Acceptance Quality Limit' (AQL) which has always stood for the quality level explicitly at the worst tolerability limit. The actual definition of AQL is the percentage of defective parts that is routinely accepted by the sampling plan. In this juncture there is a considerable procedure to deal with this problem using Poisson distribution which is often used as a standard probability model for dealing with this kind of single sampling plan. However many data sets in general are not well fitted by a Poisson model, because they consists of more zero counts than are compatible with the Poisson model. Zero inflated models are used to model count data for which proportion of zero counts is greater than expected. For these situations, a zero inflated Poisson (ZIP) model is generally proposed. It is very logical and encouraging that Zero Inflated Poisson Distribution provided better fit as compared to Poisson distribution in the data sets because of more number of zeros. In this paper the procedure for the construction of Single Sampling Plan (SSP) indexed through Average Quality Level (AQL) using Zero Inflated Poisson (ZIP) distribution as the base line distribution is presented and a table is also presented using Excel packages for the easy selection of the plans.

Introduction

Poisson distribution is often used as a standard probability model for count data. For example, a production engineer may count the number of defects in items randomly selected from a production process. Quite often, however, such datasets are not well fit by a Poisson model because they contain more zero counts than are compatible with the Poisson model. An example is again provided by the production process; indeed, according to Ghosh et al. (2006), when some production processes are in a near perfect state, zero defects will occur with a high probability. However, random changes in the manufacturing environment can lead the process to an imperfect state, producing items with defects. The production process can move randomly back and forth between the perfect and the imperfect states. For this type of production process many items will be produced with zero defects, and this excess might be better modeled by a zero-inflated Poisson distribution than a Poisson distribution. Zero-inflated models have been applied to various kinds of count data. Many attempts can be found, in recent years, which developed procedures of testing zero-inflation and over dispersion.

Ghoshet.al.(2006) has rightly pointed out that when some production processes are in a neat perfect state, zero defects will occur with a high probability. However, random changes in the manufacturing environment can lead the process to an imperfect State, producing items with defects. The production process can move randomly. For this type of production process many items will be produced with zero defects and this excess might be better attributed by a zero inflated Poisson model than a Poisson model. A simple way, to model this zero inflated Poisson model than a Poisson model. A simple way to model this zero inflation is given by Jonson et al(1992).Various authors have considered the zero Inflated Poisson(ZIP)as a possible model for biological count data.

Peach and Littauer (1946) has given a table for determining the single sampling plan for a fixed $\alpha = 0.05$. Burguess (1948) provided graphical method to obtain single sampling plan for a specified (p1, $1-\alpha$) and (p2, β). Grubs (1949) have given a table, which can be used for selecting a single sampling plan at IQL (Indifference Quality Level) and LQL (Limiting Quality Level. Cameron (1952) made extension on the work of Peach and Littauer (1946).

Guenther (1969) developed a procedure for constructing a single sampling plan for a specified p1, p2 and α based on Binomial, hyper geometric and Poisson Models. Golub Abraham (1953) provided a method and tables for finding the acceptance number c of a single sampling plan involving minimum sum of producer and consumer risk with fixed sample size. Soundarajan and Govindaraju (1983) contributed in designing single sampling plan. Suresh and RamKumar (1996) constructed a single sampling plan indexed through MAAOQ (Maximum Allowable Average Outgoing Quality). Radhakrishnan (2002) continued the work of Suresh and RamKumar (1996) and constructed the various sampling plans including continuous sampling plan.

Govindaraju (1989) using a sampling plan with a given Acceptable Quality Level and the producer's risk of 5%, the producer guarantees that if the incoming quality is maintained at or better than AQL, the percentage of production that will be accepted during the periods of sampling is at least 95%. Shankar and Sahu (2002) studied process control plans using AQL, LQL and Average Outgoing Deterioration Limit (AODL). Radhakrishna Rao (1977) suggested the use of Weighted Binomial Distribution in the Construction of Sampling plans. Radhakrishnan and Mohana priya (2008 a,b) constructed Single and Conditional Double Sampling Plans using Weighted Poisson distribution as the base line distribution.

Due to the technological development, production processes are well designed in such a way that the products are in perfect state, so that the number of zero defects will be found more in those cases. However, random fluctuations in the production processes may lead some products to an imperfect state. The appropriate probability distribution to describe such situations is a zero-inflated Poisson (ZIP) distribution. The ZIP distribution can be viewed as a mixture of a distribution which degenerates at zero and a Poisson distribution. ZIP distribution has been used in a wide range of disciplines such as agriculture, epidemiology, econometrics, public health, process control, medicine, manufacturing, etc. Some of the applications of ZIP distribution can be found in Bohning et al. (1999), Lambert (1992), Naya et al. (2008), Ridout et al. (1998), and Yang et al. (2011). Construction of control charts using ZIP distribution are discussed in Sim and Lim (2008) and Xie et al. (2001). Some theoretical aspects of ZIP distributions are mentioned in McLachlan and Peel (2000).

In this Paper a Singe Sampling Plan is constructed by assuming the probability of acceptance of a lot as 0.95, (the proportion defective corresponding to this probability of acceptance in the OC (Operating Characteristic) curve is termed as Average Quality Level (AQL) using Poisson distribution as the base line distribution.

Conditions for Application

The following three points are very important for constructing the AQL table and useful for considering those before accepting based on the same

- Production is steady, so that results of past, present and future lots are broadly indicative of a continuing process.
- Lots are submitted sequentially in the order of their production.
- Inspection is by attributes, with the lot quality defined as the proportion defective.

Glossary of Symbols

ω - Parameter

p - Proportion Defective / Lot Quality

n - Sample Size

α - Producer's Risk

$P_a(p)$ - Probability of acceptance of the lot quality p

c - Number of defectives in Sample

Operating Characteristic Function

The Operating Characteristic (OC) Function of the single sampling plan (SSP) using Inflated Poisson Distribution, inflated at x = 0 is given by

$$P_a(p) = \begin{cases} \omega_i + (1-\omega_i)e^{-\lambda_i}, when x_i = 0 \\ (1-\omega_i)\dfrac{e^{-\lambda_i}\lambda^{x_i}}{x!}, when x_i = 1,2,\ldots \\ 0 \le \omega_i \le 1 \end{cases} \quad (1)$$

$$P_a(p) = \omega_i + (1-\omega_i)e^{-\lambda_i} + \sum_{x=1}^{c}(1-\omega_i)\frac{e^{-\lambda_i}\lambda^{x}}{x!},$$

$$where\ \lambda = np$$

When c=0, the lot acceptance probability becomes as

$$P_a(p) = \omega_i + (1-\omega_i)e^{-\lambda_i}$$

In this distribution, ω may be termed as the mixing proportion. ω and λ are the parameters of the ZIP distribution.

Construction of single sampling plan

Construction of single sampling plan indexed through AQL by fixing the probability of acceptance of the lot, $P_a(p)$ as 0.95 and also based on the evidences given in the earlier work like Peach and Littauer (1946) etc. for determining the single sampling plan for a fixed α = 0.05. The same concept was used for Zero Inflated Poisson Distribution (ZIPD) as the basic distribution and it was applied with the above equation (1). The values of the AQL are obtained from the above equation for various combinations of n and C using Visual Basic program and presented in Table 1.

Table 1: Parameters of SSP for a specified AQL

ω	n	C	AQL	ω	n	C	AQL
0.05	225	6	0.00813	0.05	175	3	0.0001
0.05	225	5	0.00001	0.05	175	2	0.0002
0.05	**225**	**4**	**0.00005**	0.05	150	6	0.0938
0.05	225	3	0.0009	0.05	150	5	0.0008
0.05	225	2	0.0002	0.05	150	4	0.0004
0.05	200	6	0.00915	0.05	150	3	0.0009
0.05	200	5	0.0007	0.05	150	2	0.0004
0.05	200	4	0.0007	0.05	120	6	0.1172
0.05	200	3	0.0001	0.05	120	5	0.0008
0.05	200	2	0.0001	0.05	120	4	0.0007
0.05	175	6	0.01045	0.05	120	3	0.0009
0.05	175	5	0.0008	0.05	120	2	0.0004
0.05	175	4	0.0003				

The parameters of the Single sampling plan, n and C are recorded for various combinations of AQL. By choosing the sample size as n and ϖ value in between 0 to 1, the AQL value will decrease while there is a decrease value of the defects (c). The table 1 gives explanation as for a given AQL is 0.00005, which is the lowest among the group with the lower defectives. Therefore, the value of ω, n and C are treated as optimum results with the arithmetical values as ω =0.05, n = 225 and c = 4. Hence these parameters of Single Sampling Plan are ω =0.05, n = 225 and C = 4 with the specified and best possible AQL is 0.00005. Thus, in this sampling plan for an AQL of 0.00005%, might actually only reject a lot if there are more defectives than the 0.00005%. What the 0.00005% means is that if the true failure rate of this process is very small, it will still, due to the random nature of the sample, gets defective rates over 0.00005% and sometimes, however, 95% of the time, they will be under 1%. Therefore, the actual number of defectives to facilitate this particular plan allows as the lower confidence limit of the AQL value. This also replicates in the operating characteristic (OC) curve for this plan is presented in figure 1 which was constructed based on the table2. This was conspired as a graph using the above parameters and given more details under the heading below.

Operating Characteristic (OC) Curve:

Probability of acceptance is determined with respect to the lot proportion defective. Table 2 indicates the different values. By choosing ω = 0.05, n=225, the proportion of defective p value will increase while there is a decrease value of the probability of acceptance of the lot Pa(p) and same time the probability of acceptance to attain at the minimum value is 0.05 like our parameter ω value.

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Table2 : Probability of acceptance is determined with respect to the lot

ω	n	p	c	$P_a(p)$
0.05	225	0	0	1
0.05	225	0.0005	1	0.994421
0.05	225	0.0007	2	0.871629
0.05	225	0.0009	3	0.826926
0.05	225	0.002	4	0.656782
0.05	225	0.005	5	0.363051
0.05	225	0.008	6	0.214452
0.05	225	0.009	7	0.178868
0.05	225	0.015	8	0.096079
0.05	225	0.078	9	0.059876
0.05	225	0.09	10	0.054872

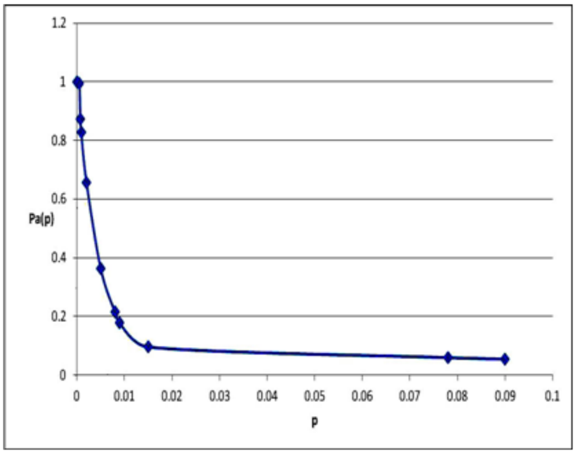

Figure 1: OC curve for the plan ω=0.05, n = 225 and c = 4

Practical Application

Suppose a chemical processing company fixes AQL as 0.00005 (5 Non – confirming units out of 100000 items) then inspect a random sample of 225 units taken from a lot of units produced in a given period of time in any scale and count the number of non – confirming units (d). If d $\leq$ 4, accept the lot of units processed during the period, otherwise reject the lot of units and inform the management for corrective action.

Conclusion:

In this paper, it explores a general procedure for constructing a Single Sampling Plan (SSP) indexed through Average Quality Level (AQL) using Inflated Poisson Distribution. This was inflated with lowest value for a given x (equal to 0) and a table is also provided for the enhanced selection of the plans. These plans are very useful for commercial companies which have at least one defective unit in their lot. Furthermore this is very much useful for companies which are using for subsequent quality lots.

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