ON THE DOUBLE PRIOR SELECTION FOR THE PARAMETER OF MAXWELL DISTRIBUTION

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ABSTRACT

The Maxwell distribution plays an important role in physics, chemistry and other allied sciences. This paper provides a comparison of double informative priors which are assumed for an unknown parameter of Maxwell distribution usually, we use one informative prior to incorporate that prior knowledge and ignoring the other information. So to include two different kind of information in the analysis, two different priors have been selected for a single unknown parameter of Maxwell distribution. Here we have assumed generalized uniform-inverted gamma distribution as double priors.

KEYWORDS: Informative prior,   prior distribution, posterior distribution.

INTRODUCTION:

The Maxwell is the distribution for molecular speeds, and it can also refer to the distribution for velocities , moments, and magnitude of the moments of the molecules, each of which will have different probability distribution function.

In (2005) Bekker and Roux[2] studied empirical Bayes estimation for Maxwell distribution, and have assumed that complete sample information is available. Sanku Dey[6] (2011) studied on Bayes estimators of the parameter of a Maxwell distribution and obtained associated based on conjugate prior under scale invariant symmetric and a symmetric loss function. Double prior selection for discrete case in the case of Poisson distribution is studied by Abdul Haq , Muhammad Aslam.

The object of this paper is to obtain on double prior selection for the parameter of Maxwell distribution.

DESCRIPTION [3], [4], [5]

The Maxwell (or Maxwell-Boltzmann) distribution gives the distribution of speeds of molecules in thermal equilibrium as given by statistical mechanics. Defining $\theta = \frac{2KT}{m}$, where K is the Maxwell constant, T is the temperature, m is the mass of a molecule. The probability density function of Maxwell distribution over the range $x \epsilon [0,\infty)$ is given by:

$$f(x;\theta) = \frac{4}{\sqrt{\pi}} \frac{1}{\theta^{\frac{3}{2}}} \; X^2 \; e^{\frac{-x^2}{\theta}} \quad ; \theta > 0$$

OBJECTIVE:

In this paper, the posterior distribution for the unknown parameter of Maxwell distribution is obtained using general uniform and inverse gamma distribution. The posterior predictive distribution using these informative priors is been developed.

THE POSTERIOR DISTRIBUTION OF THE UNKNOWN PARAMETER OF MAXWELL DISTRIBUTION :

Let $X_1, X_2, \ldots .X_n$ be a random sample drawn from the Maxwell distribution having unknown parameter θ. The likelihood function of the sample observation is given by:

$$L(X_i,\theta) \;=\; \prod_{i=1}^n f(x_i \; ; \theta) = [\frac{4}{\sqrt{\pi}}]^n \frac{1}{\theta^{\frac{3n}{2}}} (\prod_{i=1}^n x_i{}^2) \; e^{\frac{-\Sigma_{i=1}^n x_i{}^2}{\theta}} \qquad \text{-----------(1)}$$

GENERAL UNIFORM DISTRIBUTION [5]

When the prior distribution of θ take general uniform prior distribution, then the p.d.f is given by:

$$f_1(\theta) \;\;=\; \frac{(a-1)(\alpha\beta)^{a-1}}{\beta^{a-1}-\alpha^{a-1}} \;\; \frac{1}{\theta^a} \; ; \;\; 0 < \alpha \le \theta \le \beta, a > 0 \;\; \text{------------(2)}$$

INVERSE GAMMA DISTRIBUTION [5]

Again, when prior distribution of θ take inverse gamma distribution, then the p.d.f. is given by :

$$f_2(\theta) \;\;\;= \frac{\alpha^\beta}{\Gamma\alpha} \frac{1}{\theta^{\beta+1}} e^{-\frac{\alpha}{\theta}} ; \;\; \alpha, \beta, \theta > 0 \qquad \text{----------------(3)}$$

DOUBLE PRIOR: [1]

We define the double prior for θ by combining these two priors as follows:

$$f(\theta) \;\propto\;\; f_1(\theta) \; f_2(\theta)$$

$$\propto \frac{(a-1)(\alpha\beta)^{a-1}}{\beta^{a-1}-\alpha^{a-1}} \frac{1}{\theta^a} \times \frac{\alpha^\beta}{\Gamma\alpha} \frac{1}{\theta^{\beta+1}} e^{-\frac{\alpha}{\theta}}$$

$$\therefore \quad f(\theta) \quad = \quad k \quad \frac{1}{\theta^{a+\beta+1}} \quad e^{-\frac{\alpha}{\theta}} \qquad \text{---------------(4)}$$

$$\text{Where } k \quad = \quad \frac{(a-1)(\alpha\beta)^{a-1}}{\beta^{a-1}-\alpha^{a-1}} \frac{\alpha^\beta}{\Gamma\alpha}$$

THE POSTERIOR DISTRIBUTION OF θ :

Posterior density function of 'θ' for the given random sample X is given by

$$f(\theta/x_1,x_2,\ldots x_n) \quad = \quad \frac{L(x_1,x_2,\ldots x_n/\theta)\, f(\theta)}{\int_0^\infty L(x_1,x_2,\ldots x_n/\theta)f(\theta)d\theta}$$

using equation (1) and (4)

$$f(\theta/\mathbf{x}) \;=\; \frac{\left[\frac{4}{\sqrt{\pi}}\right]n \frac{1}{\theta^{\frac{3n}{2}}}(\prod_{i=1}^n x_i{}^2)e^{\frac{-\sum_{j=1}^n x_i{}^2}{\theta}} k \frac{1}{\theta^{a+\beta+1}}e^{-\frac{\alpha}{\theta}}}{\int_0^\infty \left[\frac{4}{\sqrt{\pi}}\right]n \frac{1}{\theta^{\frac{3n}{2}}}(\prod_{i=1}^n x_i{}^2)e^{\frac{-\sum_{j=1}^n x_i{}^2}{\theta}} k \frac{1}{\theta^{a+\beta+1}}e^{-\frac{\alpha}{\theta}}d\theta}$$

$$(\text{where } \mathbf{x} =(x_1,x_2,\ldots x_n)$$

$$= \quad \frac{e^{-\frac{(\alpha+\sum_i x_i{}^2)}{\theta}}\frac{1}{\theta^{a+\beta+1}}}{\int_0^\infty e^{-\frac{(\alpha+\sum_i x_i{}^2)}{\theta}}\frac{1}{\theta^{a+\beta+1}}d\theta} \qquad \text{----------------(5)}$$

$$\therefore \quad f(\theta/\mathbf{x}) \;\sim\; \text{Inverse Gamma } (\alpha +\textstyle\sum_i x_i{}^2,\, a+\beta+1)$$

DISCUSSION:

On double prior selection for Maxwell distribution we find that posterior distribution is also inverse gamma distribution. As in the case of single prior selection, when prior and posterior come from same family it is conjugate prior, Hence in this case we may conclude that double prior and its posterior distribution belongs to the same family hence the prior is conjugate prior.

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