Stability and accuracy of Runge-Kutta based split-explicit time-stepping algorithms for free-surface ocean models

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Runge-Kutta methods

Advantages of RK methods:

- 2-time-level schemes (no computational mode)
- Good stability properties for advection and diffusion
- Abundant literature (e.g. RK-IMEX, RK-SSP, DIRK, etc)
- Easy to implement (convex combination of Euler steps)

Main questions:

- Many variants of RK schemes
- **RK with mode-splitting ?**

Existing work:

- Many RK-based atmospheric models (e.g. WRF, Cosmo, Homme-NH, etc)
- « *Pseudo-second order Runge-Kutta time stepping scheme* » in HIM/MOM6 [Hallberg, 1997]

Runge-Kutta methods in the oceanic context

Many variants of RK schemes

- Low-storage and « order=stage » method for advection [Wicker & Skamarock, 2002]

$$\partial_t \phi = \mathcal{F}(\phi, t) \quad \longrightarrow \quad \begin{aligned} \phi^{n+1/3} &= \phi^n + \frac{\Delta t}{3}\mathcal{F}(\phi^n) \\ \phi^{n+1/2} &= \phi^n + \frac{\Delta t}{2}\mathcal{F}(\phi^{n+1/3}) \\ \phi^{n+1} &= \phi^n + \Delta t\, \mathcal{F}(\phi^{n+1/2}) \end{aligned}$$

- Introduce Shuman averaging for internal gravity waves (« order=stage-1 »)

Additional difficulty inherent to oceanic models

- 2D barotropic / 3D baroclinic mode splitting to improve computational efficiency (fast and slow dynamics are split into separate subproblems)

- Separation of slow and fast modes is non-orthogonal (depth-independent barotropic mode assumption)

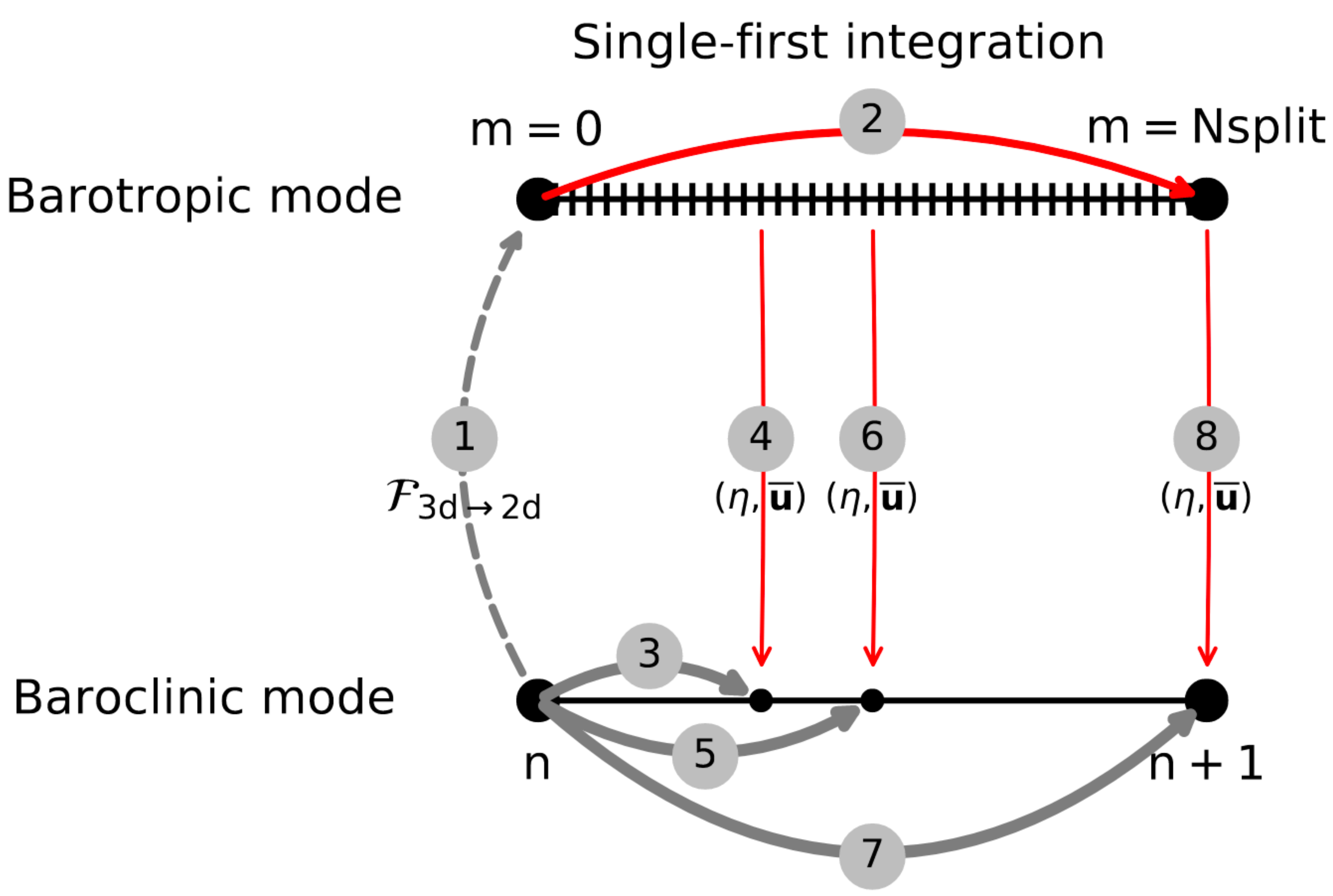
$$\partial_t \overline{u} = -g\partial_x\left(\eta + \frac{1}{\rho_0 g H}\int_{-H}^{0} p_h \,\mathrm{dz}\right)$$

$$\partial_t \eta = -H\partial_x \overline{u}$$

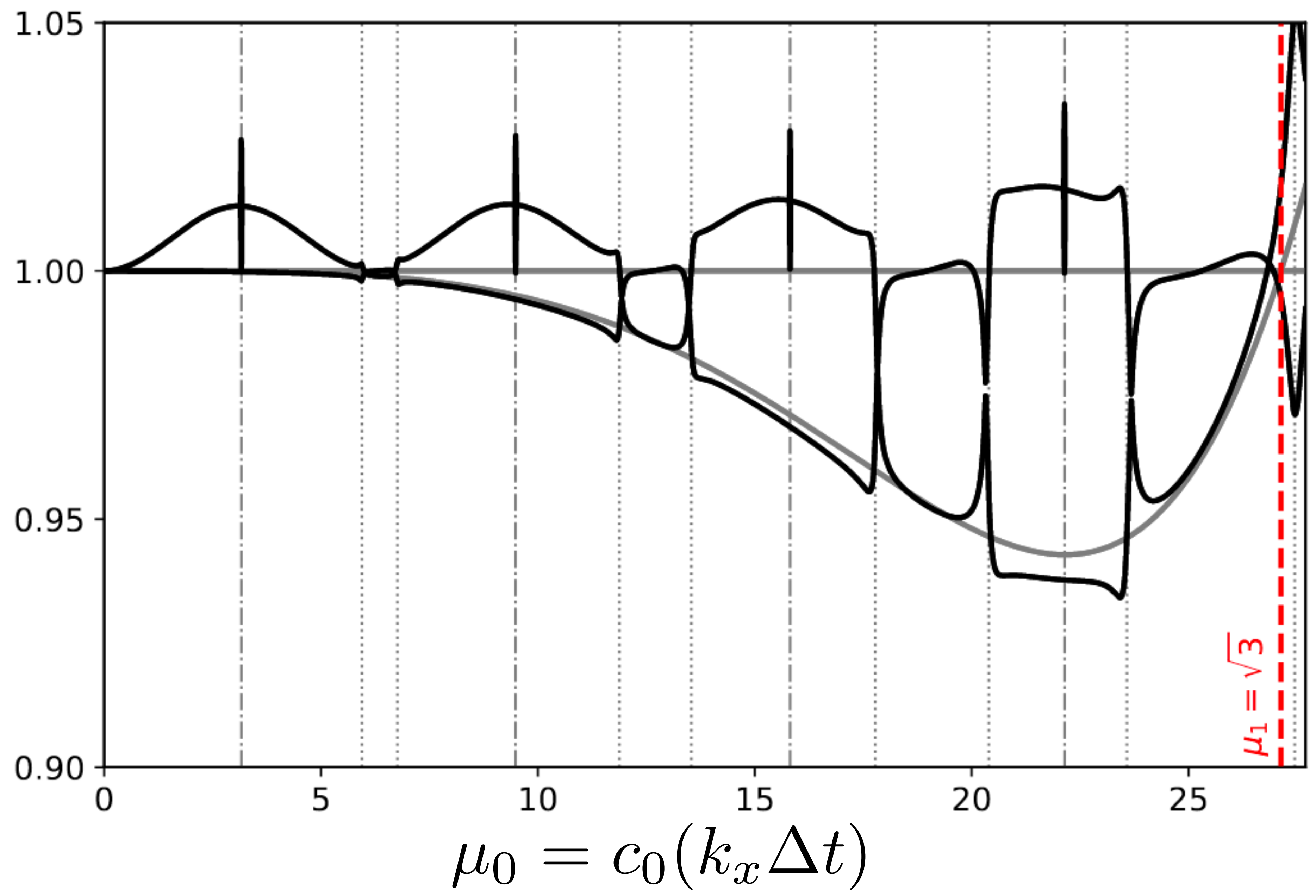
➡ Some form of filtering is required to stabilize the splitting

Robust mode-splitting method based on RK3

Peculiarity of oceanic models: 2D barotropic / 3D baroclinic mode splitting to improve computational efficiency



Gravity waves integration (without dissipation in 2D mode)



Single-first approach:

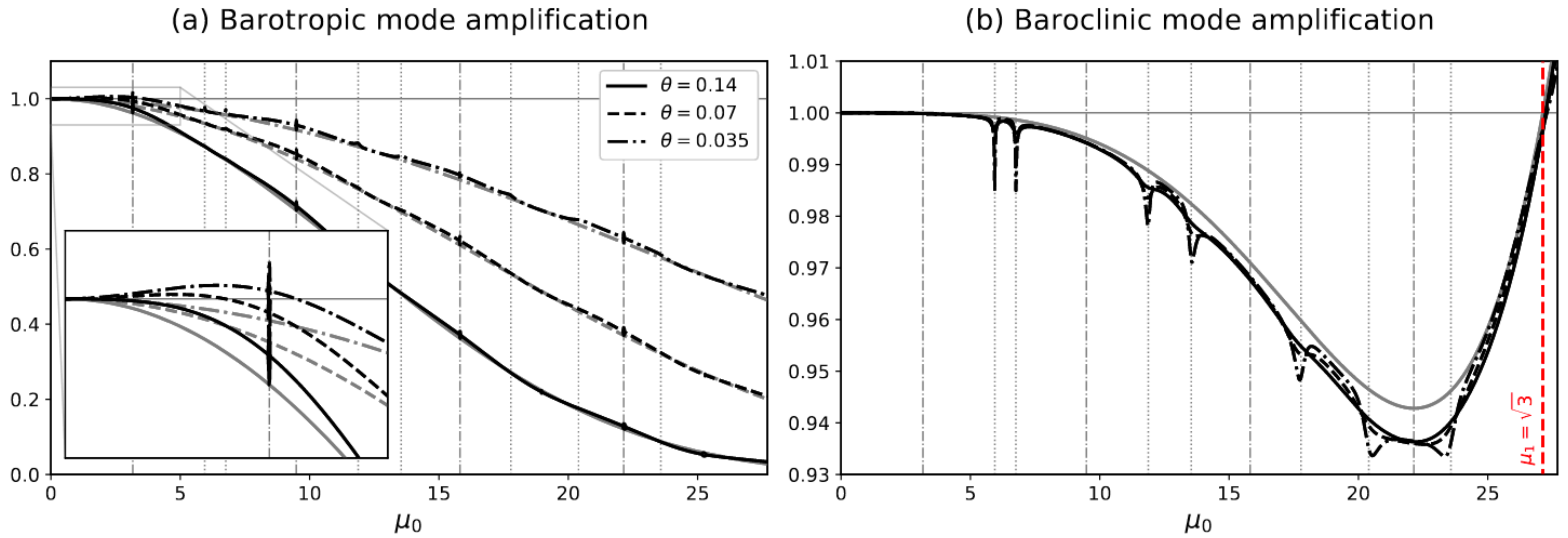
- Only one integration of the 2D within the RK3 time-step
- Split-explicit approach with a dissipative time-stepping for the barotropic mode
- Linear stability analysis can be conducted following Demange et al. (2019)

Robust mode-splitting method based on RK3

Peculiarity of oceanic models: 2D barotropic / 3D baroclinic mode splitting to improve computational efficiency

$$\overline{u}^{n+1} = \overline{u}^n - g\Delta t_0 \partial_x \eta^n$$

$$\eta^{n+1} = \eta^n - H\Delta t_0 \partial_x \left((1+\theta)\overline{u}^{n+1} - \theta \overline{u}^n\right)$$



- Linear stability analysis provides the minimal dissipation to stabilize the splitting
- Stability constrained by barotropic mode and not by baroclinic ones

Conclusion and future work

Summary:

➡ Design of an RK3 based time-stepping algorithm for split-explicit free-surface ocean models

- The single-first strategy appears to be more robust than other alternatives
- Thought for a quasi-Eulerian vertical coordinate
- A non-linear $z^\star$ *2D x-z* model has been developed to check results from stability analysis

Ongoing work:

- Implementation in NEMO (see next talk)
- Accuracy analysis and consequences of mode-splitting

✦ Ducousso, N., Lemarié, F., Debreu, L., Madec, G.: *Stability and accuracy of Runge-Kutta-based time-stepping algorithms for split-explicit free-surface ocean models*. In preparation for J. Adv. Model. Earth Syst.