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Abstract: In this paper, subspace identification for wind turbines and more generally rotating periodic systems are investigated. Previous works have stressed the difficulty of modeling such systems as Linear Time Invariant and thus to apply classical Stochastic Subspace Identification. Such works plead for periodic or augmented theories. In this paper, the classical SSI can be applied to recover modal information that is related to the eigenstructure of the instrumented system despite the system excitation being modeled as non-stationary.

Keywords: Subspace methods, Vibration and modal analysis, Time series modelling, Signal and identification-based methods, rotating machines, Data-Driven methods

1. INTRODUCTION

Due to the increase in the number of wind farms, it is important to implement reliable monitoring methods based on data collected during operation. The operational modal analysis (OMA) methods are part of the solutions. There are already many OMA methods developed for civil engineering, thus for linear time invariant (LTI) systems, in frequency or time domain, and transposed to the monitoring of those new systems. However, a wind turbine should rather be modelled as a linear time periodic (LTP) system and therefore does not satisfy the assumptions of classical OMA methods, so defining OMA methods specific to LTP systems is needed.

A first method enabling the identification of the eigenstructure of an LTP system, applied to bladed structures, is the multi-blade coordinate (MBC) method (Bir, 2008). This method is based on the reformulation of the LTP system as an LTI system by a change of variable on the data. However, this method is based on the hypothesis of an isotropic rotor, a hypothesis which is not verified in practice (Tcherniak and Larsen, 2013). Other methods are based on the harmonic transfer function (Allen et al., 2011), first defined for control purposes (Wereley and Hall, 1991), and also extended into a subspace method (Tcherniak et al., 2014). The drawback of these methods is that they require to increase the dimension of the observation space, as a consequence of the required formalism. Another problem is that these methods require knowledge of the actual rotational speed of the system. A last group of methods is those adapting the classical subspace methods. One example is the SSI LPTV method (Jhinaoui et al., 2014), which allows to correctly identify the exact eigenmodes of an LTP system. However, the drawback of the method is that it converges in period number, which is more suitable for systems with a high rotational speed as helicopters. All these methods present modifications of classical frequency or time domain identification methods without solving all problems and sometimes introducing constrains and artefacts (Yang et al., 2014).

Stochastic Subspace Identification (SSI) methods have been proved effective for the identification of LTI system, by a now abundant literature (Verhaegen and Dewilde (1992); van Overschee and de Moor (1993); Qin et al. (2005)). They also have the advantage of being applicable in the context of output-only system identification, particularly in mechanical engineering, see Peeters and de Roeck (1999) for example. So, they fit nicely in the context of OMA system identification. Application of the classical SSI method with or without pre- or post-processing has been experimented in civil engineering and wind turbines monitoring (Tcherniak, 2014). The objective of the current paper is to address the validity of such practice.

According to Floquet (1879), the response of an LTP system can be defined as a sum of modes with periodic amplitudes, the so-called Floquet modes. These modes are expanded into Fourier series, with an infinity of components. A system composed of an infinity of Fourier components with constant amplitude (identical to the eigenmodes of an LTI system) would theoretically be equivalent to an LTI system of infinite dimension. However, Acar and Feeny (2016) shows (with an application on the Mathieu oscillator) that only some Fourier components are needed to describe a Floquet mode. Thus, by selecting only some specific components of the modes, an LTI system of finite dimension can be approximated and thus identified. However, the defined approximation is only on the homogeneous part, i.e. the transition and observation matrices. The input and feedthrough matrices remain periodic, giving an LTI system subject to non stationary input, which does not satisfy the assumptions of classical OMA methods.

There are a number of convergence studies on subspace methods in a stationary context in the literature (see Benveniste and Mevel (2007) for some of them). These papers provide deep and technically difficult results including convergence rates. They typically address the problem of identifying the system matrices or the transfer matrix, i.e. both the pole and zero parts of the system. In contrast, the non stationary consistency of the estimation of the eigenstructure (the pole part) has been less studied and we refer to Benveniste and Mevel (2007) for a general consistency proof applied to many subspace methods.

The proposed modeling introduced in this paper leads to an LTI model with a non stationary input. The application of a subspace method to this model warrants the study of its consistency that will be proved and demonstrated on a numerical simulation of a small scale system. The main contribution of the paper is to assess the application of a standard SSI method and evaluate how model and subspace-derived modal estimates are linked through an approximate model.

Section 2 derives the approximate linear time invariant model for the LTP system derived therein. Section 3 recalls the principle of a particular SSI method for identification of modal properties of LTI systems. Section 4 investigates the validity of the proposed model and the impact of the non stationary periodic input. Finally, Section 5 demonstrates the validity of the approximation on a small scale numerical example.

2. LINEAR TIME PERIODIC (LTP) SYSTEM APPROXIMATION

2.1 Dynamic model

The motion of a constant rotating wind turbine can be expressed as a linear time periodic system,

$$\mathcal{M}(t)\ddot{\xi}(t) + \mathcal{C}(t)\dot{\xi}(t) + \mathcal{K}(t)\xi(t) = v(t), \tag{1}$$

where $\xi(t) \in \mathbb{R}^m$ are the displacements of the structure at the degrees of freedom (dof) of the system, and $\mathcal{M}(t+T) = \mathcal{M}(t)$, $\mathcal{C}(t+T) = \mathcal{C}(t)$, $\mathcal{K}(t+T) = \mathcal{K}(t)$, respectively the mass, damping and stiffness matrices. T represents the rotational period. The unknown input $v(t)$ is assumed to be a Gaussian white noise. In the following, the mechanical system is expressed in a state space form, from the definition of the state vector $x(t) \in \mathbb{R}^n$ where $n = 2m$ and the observation $y(t) \in \mathbb{R}^r$.

$$x(t) = \begin{bmatrix} \xi(t) \\ \dot{\xi}(t \end{bmatrix} \quad \text{and} \quad y(t) = C_a\ddot{\xi}(t) + C_v\dot{\xi}(t) + C_d\xi(t), \tag{2}$$

where C_a, C_v and C_d are selection matrices. A noise $w(t)$ can be added to the observation. $w(t)$ is assumed to be a Gaussian white noise. This leads to the following state space expression:

$$\begin{cases} \dot{x}(t) = A_c(t)x(t) + B_c(t)v(t) \\ y(t) = C(t)x(t) + \mathbf{D}(t)v(t) + w(t) \end{cases}, \tag{3}$$

with

$$A_c(t) = \begin{bmatrix} 0 & I \\ -\mathcal{M}(t)^{-1}\mathcal{K}(t) & -\mathcal{M}(t)^{-1}\mathcal{C}(t) \end{bmatrix},$$
$$C(t) = \begin{bmatrix} C_d - C_a\mathcal{M}(t)^{-1}\mathcal{K}(t) & C_v - C_a\mathcal{M}(t)^{-1}\mathcal{C}(t) \end{bmatrix},$$
$$B_c(t) = \begin{bmatrix} 0 \\ -\mathcal{M}(t)^{-1} \end{bmatrix} \quad \text{and} \quad \mathbf{D}(t) = C_a\mathcal{M}^{-1}(t).$$

All matrices are periodic with period T, with $A_c(t) \in \mathbb{R}^{n\times n}$, $C(t) \in \mathbb{R}^{r\times n}$, $B_c(t) \in \mathbb{R}^{n\times m}$ and $\mathbf{D}(t) \in \mathbb{R}^{r\times m}$.

2.2 Modal analysis

The Floquet theory (Floquet, 1879) was initially intended for solving linear differential equations with periodic coefficients. The general solution of the differential equation reads:

$$x(t) = \Phi(t,t_0)x(t_0) + \int_{t_0}^{t} \Phi(t,\tau)B_c(\tau)v(\tau)d\tau, \tag{4}$$

with $\Phi(t,t_0)$ the fundamental matrix. From this theory, it is possible to express the eigenmodes of an LTP system (Skjoldan and Hansen, 2009). Looking first at the homogeneous part of the differential equation:

$$\dot{x}_h(t) = A_c(t)x_h(t) \tag{5}$$

the fundamental matrix $\Phi(t,t_0)$ is the solution of this equation such that

$$x_h(t) = \Phi(t,t_0)x(t_0). \tag{6}$$

To simplify Equation (6), $\Phi(t) := \Phi(t,t_0) = \Phi(t)\Phi(t_0)^{-1}$, with $\Phi(t_0) = \mathbf{I}$ the identity matrix. Also the monodromy matrix Q is defined by $Q = \Phi(T)$, where $\Phi(t+T) = \Phi(t)Q$.

The eigenvalues of Q are called the characteristic multipliers (λ_i), with ψ_i the associated eigenvectors. The characteristic exponents (μ_i) are defined as $\lambda_i = \exp(\mu_i T)$. The fundamental matrix can be factorized into a matrix of n independent periodic vectors $p(t)$ collected into the matrix $P(t)$

$$\Phi(t) = P(t)\exp(Rt), \tag{7}$$

with $R = \frac{1}{T}\log(Q)$. R can then be diagonalized using the characteristic exponents and the eigenvectors of Q,

$$R = \Psi\,[\mu]\,\Psi^{-1}. \tag{8}$$

So the matrix $\Phi(t)$ can be expressed with the characteristic exponents

$$\Phi(t) = P(t)\Psi\exp([\mu]\,t)\,\Psi^{-1}. \tag{9}$$

Finally using Equation (9) and Equation (6), the state vector is expressed as a sum of n Floquet modes

$$x_h(t) = \sum_{j=1}^{n} X_j(t)\exp(\mu_j t)\,q_j(t_0) \tag{10}$$

with $q_j(t_0) = \psi'_j x(t_0)$, ψ'_j the j–th row of the matrix Ψ^{-1} and $X_j(t) = P(t)\psi_j$ the T-periodic amplitude of the j–th mode.

2.3 Approximation of the Floquet modes

The next step is to express the observation vector as a finite sum of eigenmodes to obtain the description of a time invariant system. The Floquet mode decomposition of the observation $y_h(t)$ is

$$y_h(t) = C(t) \sum_{j=1}^{n} X_j(t) \exp(\mu_j t)\, q_j(t_0) \tag{11}$$

$$= \sum_{j=1}^{n} Y_j(t) \exp(\mu_j t)\, q_j(t_0), \tag{12}$$

where $Y_j(t) = C(t)X_j(t)$ is the amplitude of the Floquet modes of the observation and a periodic vector of period $T = \frac{2\pi}{\Omega}$, which can then be expanded into a Fourier series:

$$Y_j(t) = \sum_{l=-\infty}^{\infty} Y_{j,l} \exp(il\Omega t) \tag{13}$$

By combining Equations (12) and (13), the observation vector can be expressed as an infinite sum of terms:

$$y_h(t) = \sum_{j=1}^{n} \sum_{l=-\infty}^{\infty} Y_{j,l} \exp((\mu_j + il\Omega)\, t)\, q_j(t_0) \tag{14}$$

Relevant components of the expansion of $y_h(t)$ are determined by the participation factor (Bottasso and Cacciola, 2015):

$$\phi^y_{j,l} = \frac{\|Y_{j,l}\|}{\sum_{l=-\infty}^{\infty} \|Y_{j,l}\|}. \tag{15}$$

By defining a minimal participation factor (ϕ^y_{min}) an approximation of the observation ($\hat{y}(t)$) is constructed as a finite sum of eigenmodes,

$$\hat{y}_h(t) = \sum_{(j,l),\phi^y_{j,l} \geq \phi^y_{min}} Y_{j,l} \exp((\mu_j + il\Omega)\, t)\, q_j(t_0), \tag{16}$$

$\hat{y}(t)$ can then be expressed as a sum of $\tilde{n}$ eigenmodes

$$\hat{y}_h(t) = \sum_{p=1}^{\tilde{n}} Y_p \exp(\mu_p t)\, q_p(t_0), \tag{17}$$

where each index p corresponds to a pair (j, l) and $\mu_p = \mu_j + il\Omega$. From the approximation of the observation vector, the state space expression, and the associated transition matrices are now formulated.

2.4 Space state expression

From the modal analysis, the transition matrix and the observation matrix of the approximation can be expressed. Once these matrices are defined, they can be inserted in the state space defined for an LTP system (Equation (3)). Equation (17) shows that the shape of the approximation of the observation $\hat{y}(t)$ is the same as the observation of an LTI system (for modal analysis). So there must exist a state-space composed of some state vector $z(t) \in \mathbb{R}^{\tilde{n}}$ and the approximation of the observation $\hat{y}(t)$ such that the homogeneous part reads

$$\begin{cases} \dot{z}_h(t) = \tilde{A} z_h(t) \\ \hat{y}_h(t) = \tilde{\mathbf{C}} z_h(t) \end{cases}, \tag{18}$$

with $\tilde{A}$ and $\tilde{\mathbf{C}}$ the transition and observation matrices of the approximation and

$$z_h(t) = \sum_{p=1}^{\tilde{n}} Z_p \exp(\mu_p t)\, q_p(t_0). \tag{19}$$

With Equations (19) and (18)

$$\sum_{p=1}^{\tilde{n}} Z_p \mu_p \exp(\mu_p t)\, q_p(t_0) = \tilde{A} \sum_{p=1}^{\tilde{n}} Z_p \exp(\mu_p t)\, q_p(t_0), \tag{20}$$

expressing the sums in matrix form

$$\mathcal{Z}\, [\mu_p] \exp([\mu_p]\, t) q(t_0) = \tilde{A} \mathcal{Z} \exp([\mu_p]\, t) q(t_0), \tag{21}$$

with $\mathcal{Z} \in \mathbb{R}^{\tilde{n}\times\tilde{n}}$ regrouping the amplitudes of the state vector Z_p (one vector per column), $[\mu_p]$ a diagonal matrix containing the eigenvalues of the approximation and $q(t_0) \in \mathbb{R}^{\tilde{n}}$ regrouping the scalar $q_p(t_0)$. So, the transition matrix is defined as

$$\tilde{A} = \mathcal{Z}\, [\mu_p]\, \mathcal{Z}^{-1} \in \mathbb{R}^{\tilde{n}\times\tilde{n}}. \tag{22}$$

The observation matrix of the approximation ($\tilde{\mathbf{C}}$) in Equation 18, with Equations (17) and (19)

$$\sum_{p=1}^{\tilde{n}} Y_p \exp(\mu_p t)\, q_p(t_0) = \tilde{\mathbf{C}} \sum_{p=1}^{\tilde{n}} Z_p \exp(\mu_p t)\, q_p(t_0). \tag{23}$$

In a matrix form

$$\tilde{\Phi} \exp([\mu_p]\, t)\, q(t_0) = \tilde{\mathbf{C}} \mathcal{Z} \exp([\mu_p]\, t)\, q(t_0) \tag{24}$$

with $\tilde{\Phi} \in \mathbb{R}^{r\times\tilde{n}}$ regrouping the amplitudes of the observations eigenmodes. The observation matrix is expressed as

$$\tilde{\mathbf{C}} = \tilde{\Phi} \mathcal{Z}^{-1} \in \mathbb{R}^{r\times\tilde{n}}. \tag{25}$$

To express the full discrete state space, the discrete transition matrix and the input matrix are needed. First let us define the observation of the state space using Equation (4)

$$y(t) = C(t)\Phi(t)x(t_0) + C(t)\Phi(t) \int_{t_0}^{t} \Phi(\tau)^{-1} B_c(\tau) v(\tau) d\tau. \tag{26}$$

With the approximation of the observation of the homogeneous part (Equation (17)) and with Equation (6) the approximation of $C(t)\Phi(t)$ is defined in matrix form

$$C(t)\Phi(t) \simeq \tilde{\Phi} \exp([\mu_p]\, t)\, \tilde{\Psi}, \tag{27}$$

with $\tilde{\Psi}$ such that $q(t_0) = \tilde{\Psi} x(t_0)$. To express the approximation of $\Phi(\tau)^{-1}$ the approximation of the homogeneous part of the state space is needed. With the same Fourier components as $\hat{y}_h(t)$

$$\hat{x}_h(t) = \sum_{p=1}^{\tilde{n}} X_p \exp(\mu_p t)\, q_p(t_0), \tag{28}$$

leads to the approximation

$$\Phi(\tau) \simeq \mathcal{X} \exp([\mu_p]\, \tau)\, \tilde{\Psi}, \tag{29}$$

with $\mathcal{X} \in \mathbb{R}^{n\times\tilde{n}}$ regrouping the vectors X_p. For simplicity, assume $t_0 = 0$ and t a multiple of Δt, the approximation of the observation in discrete time is defined as

$$\hat{y}_k = \tilde{\Phi} \exp([\mu_p]\, k\Delta t)\, q(0) + \tilde{\Phi} \exp([\mu_p]\, k\Delta t) \sum_{i=1}^{k} I_i v_{i-1}, \tag{30}$$

with

$$I_i = \int_{(i-1)\Delta t}^{i\Delta t} \exp(-[\mu_p]\, \tau)\, \mathcal{X}^{\dagger} B_c(\tau) d\tau, \tag{31}$$

where $(\cdot)^{\dagger}$ denotes the Moore-Penrose pseudo inverse and with the hypothesis of a zero-order hold on $v(t)$. With

Equation (25) it is possible to express the observation matrix into $\hat{y}_{k+1}$ such that

$$\begin{aligned}\hat{y}_{k+1} &= \tilde{\mathbf{C}}\mathcal{Z}\exp\left([\mu_p](k+1)\Delta t\right)q(0)\\ &+ \tilde{\mathbf{C}}\mathcal{Z}\exp\left([\mu_p](k+1)\Delta t\right)\sum_{i=1}^{k+1} I_i v_{i-1}.\end{aligned} \tag{32}$$

The state vector in discrete time z_k at time index $k+1$ is expressed as

$$\begin{aligned}z_{k+1} &= \mathcal{Z}\exp\left([\mu_p](k+1)\Delta t\right)q(0)\\ &+ \mathcal{Z}\exp\left([\mu_p](k+1)\Delta t\right)\sum_{i=1}^{k+1} I_i v_{i-1}.\end{aligned} \tag{33}$$

Using the definition of z_k

$$z_{k+1} = \tilde{\mathbf{A}} z_k + \mathbf{B}_k v_k. \tag{34}$$

with $\tilde{\mathbf{A}} = \exp\left(\tilde{A}\Delta t\right)$ and $\mathbf{B}_k = \mathcal{Z}\exp\left([\mu_p](k+1)\Delta t\right) I_{k+1}$.

Finally adding the term of the excitation in the observation, the discrete state space of the approximation can be defined as

$$\begin{cases} z_{k+1} = \tilde{\mathbf{A}} z_k + \mathbf{B}_k v_k \\ y_k = \tilde{\mathbf{C}} z_k + \mathbf{D}_k v_k + \tilde{w}_k \end{cases}, \tag{35}$$

with $\tilde{w}_k = w_k + \varepsilon_{y\ k}$ the new observation noise, where $\varepsilon_{y\ k}$ is the approximation error of the observation (suppose to be with zero mean, independent and square-integrable). $\mathbf{B}_k$ and $\mathbf{D}_k = \mathbf{D}(k\Delta t)$ are periodic matrices of period $T_d = \frac{T}{\Delta t}$. Assume $\tilde{\mathbf{A}}$ has all non zero distinct eigenvalues with modulus less than 1.

The final equations in (35) represent a linear system with constant system matrices under a non stationary input forcing, precisely the statistical moments of the input are periodic. In the next section, it will be recalled the classical SSI method and detail how it can still be used for such model.

3. SUBSPACE IDENTIFICATION METHOD

Let us consider an LTI system defined by the quadruplet of matrices ($\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$), evolving under a stationary Gaussian white noise excitation $\mathbf{v}_k$

$$\begin{cases} x_{k+1} = \mathbf{A} x_k + \mathbf{B}\mathbf{v}_k \\ y_k = \mathbf{C} x_k + \mathbf{D}\mathbf{v}_k \end{cases}. \tag{36}$$

The Stochastic Subspace Identification (van Overschee and de Moor, 1993) aims to identify the eigenmodes of the system through the sample correlations. Here the SSI covariance-driven is presented. First, the Hankel matrix filled by correlations must be constructed. It can be done directly from matrices gathering the observations

$$\hat{H} = \mathcal{Y}^+ \left(\mathcal{Y}^-\right)^T. \tag{37}$$

Where $\mathcal{Y}^+ \in \mathbb{R}^{(p+1)r\times N}$ and $\mathcal{Y}^- \in \mathbb{R}^{qr\times N}$ are defined in (van Overschee and de Moor, 1993). $\hat{H}$ can be seen as the Hankel matrix filled with the correlations $\hat{R}_i$, the estimate of the correlation $R_i = \mathbb{E}\left(y_k y_{k-i}^T\right) = \mathbf{C}\mathbf{A}^{i-1}G$, where $G = \mathbb{E}\left(x_{k+1} y_k^T\right)$. The Hankel matrix can be factorized such that $\hat{H} = O_p C_q$, where O_p denotes the observability matrix and C_q the controllability matrix, with $\hat{G}$ a consistent estimate of G.

$$O_p = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^p \end{bmatrix} \text{ and } C_q = \begin{bmatrix}\hat{G}\ \mathbf{A}\hat{G}\ \ldots\ \mathbf{A}^{q-1}\hat{G}\end{bmatrix}. \tag{38}$$

In the LTI case, O_p is full column rank and C_q is full row rank. The observability matrix is obtained from a thin singular value decomposition of $\hat{H}$ and its truncation at the correct model order n

$$\hat{H} = U\Sigma V^T = [U_1\ U_2]\begin{bmatrix}\Sigma_1 & 0 \\ 0 & \Sigma_2\end{bmatrix}\begin{bmatrix}V_1^T \\ V_2^T\end{bmatrix}, \tag{39}$$

$$O_p = U_1 \Sigma_1^{1/2}, \tag{40}$$

where Σ_1 contains the first n singular values and U_1 the n first column of U. The observation matrix $\mathbf{C}$ is identified as the first block row of O_p and the state transition matrix $\mathbf{A}$ is identified in a least-squares sense

$$\mathbf{A} = O_{p\ (2:p+1)}^{\dagger} O_{p\ (1:p)}, \tag{41}$$

with $O_{p\ (1:p)}$ the first p blocks rows of O_p and $O_{p\ (2:p+1)}$ the p last blocks rows. Then, the eigenmodes can be computed with the eigenvalue decomposition of $\mathbf{A}$, namely $\mathbf{A} = \Psi\,[\mu_i]\,\Psi^{-1}$.

The continuous time eigenvalues λ_i are deduced from the discrete time eigenvalues μ_i by $\lambda_i = \log(\mu_i)/\Delta t$. Then the frequency ($f_i$) and the damping ($\zeta_i$) of the associated mode are defined such that $f_i = |\lambda_i|/2\pi$ and $\zeta_i = -100 \times Re\left(\lambda_i\right)/|\lambda_i|$. Finally, t he mode shape matrix is found from $\Phi = \mathbf{C}\Psi$.

The above identification method, also called SSI-cov will be challenged under a non stationary model (Equation (35)) in the next section.

4. APPROXIMATE FACTORISATION

After recalling the SSI approach for the LTI systems, first relax the model (36) to (35). Second, define the submatrices $\hat{H}_{m,n}$ of $\hat{H}$ by restriction to indices (m,n) $m \in [1:p+1]$ and $n \in [1:q]$ by

$$\hat{H}_{m,n} = \mathcal{Y}_m^+ \left(\mathcal{Y}^-\right)_n^T, \tag{42}$$

with $\mathcal{Y}_m^+$ the m–th block line of $\mathcal{Y}^+$ and $\left(\mathcal{Y}^-\right)_n^T$ the n–th block column of $\left(\mathcal{Y}^-\right)^T$.

$$\mathcal{Y}_m^+ = [y_{q+m}\ y_{q+m+1}\ \cdots\ y_{q+N+m-1}]$$
$$\left(\mathcal{Y}^-\right)_n^T = \left[y_{q+1-n}^T\ y_{q+2-n}^T\ \cdots\ y_{q+N+1-n}^T\right]^T$$

Let $k_m = q+m+k$ and $k_n = q+1-n+k$ with $k_m = k_n + m + n - 1$, it yields

$$\begin{aligned}\hat{H}_{m,n} = \frac{1}{N}\sum_{k=0}^{N-1} &\left(\tilde{\mathbf{C}} z_{k_m} + \mathbf{D}_{k_m} v_{k_m} + \tilde{w}_{k_m}\right)\\ &\cdot\left(\tilde{\mathbf{C}} z_{k_n} + \mathbf{D}_{k_n} v_{k_n} + \tilde{w}_{k_n}\right)^T.\end{aligned} \tag{43}$$

Finally

$$\hat{H}_{m,n} = \tilde{\mathbf{C}}\tilde{\mathbf{A}}^{m+n-2}\hat{G}_n + o(1), \tag{44}$$

with $o(1)$ a matrix converging to zero with N, applying Lemma 3 of (Benveniste and Mevel, 2007) under the moment hypotheses cited above, then

$$\hat{G}_n = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{\mathbf{A}} z_{k_n} z_{k_n}^T \tilde{\mathbf{C}}^T + \mathbf{B}_{k_n} v_{k_n} v_{k_n}^T \mathbf{D}_{k_n}^T. \tag{45}$$

Without lack of generality, assume that N is a multiple of T_d,

$$\hat{G}_n = \frac{1}{N}\tilde{\mathbf{A}}\sum_{k=0}^{N-1}\left[z_{k_n}z_{k_n}^T\right]\tilde{\mathbf{C}}^T + \frac{1}{T_d}\sum_{j=1}^{T_d}\mathbf{B}_j\frac{T_d}{N}\sum_{k_j}\left[v_{k_j}v_{k_j}^T\right]\mathbf{D}_j^T, \tag{46}$$

where k_j denote the indices corresponding to the specific instants of the period and where $\mathbf{B}_j$ and $\mathbf{D}_j$ are periodically equal. Notice that $\hat{G}_n$ and $\hat{H}_{m,n}$ have a limit as N tends to infinity due to the periodicity of the input and feedthrough matrices periodic. Also, it can be proven that $\hat{G}_n$ do not depend of the index n and it can be noted G. The matrix $\hat{H}$ can then be expressed similarly to Equation (38). To apply (Benveniste and Mevel, 2007), it is sufficient that the $\tilde{n}$ first singular values of $\tilde{C}_q$ are uniformly lower bounded. This can be assumed considering $\hat{G}$ converges to a constant matrix and the noise is stationary Gaussian. The consistency is then a consequence. Then $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{C}}$ can be estimated up to a change of basis from $\hat{H}$. Also notice that it can be shown that G can be expressed as a sum of statistically equivalent terms due to the periodicity of $\mathbf{B}_j$ and $\mathbf{D}_j$. It implies that the computation of Gaussian confidence intervals is possible similarly to Döhler and Mevel (2013). In the next section, this will be validated numerically on a reduced model of wind turbine.

5. APPLICATION

A theoretical model of wind turbine is used here with an isotropic rotor and a constant rotational speed defined in Skjoldan (2009) (see Figure 1). It is composed of 3 dof of blade bending, and two dof of nacelle bending. The matrices of the system $\mathcal{M}(t)$, $\mathcal{C}(t)$ and $\mathcal{K}(t)$ are periodic matrices of period $T = \frac{2\pi}{\Omega}$, with Ω the rotational speed. The model parameters are identical to those used in Skjoldan (2009). The objective is to validate the identification of the approximation with the SSI method.

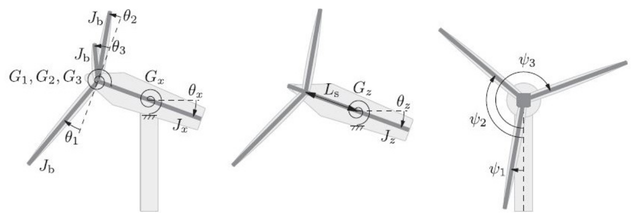

Fig. 1. Wind turbine model (Skjoldan, 2009)

5.1 Model approximation

First, to determine the approximation of the model, the modal analysis of the model is performed. To do this, the transition matrix is computed, then the periodic shape modes are computed, to finally determine the components of the Floquet modes considered in the approximation.

With a rotational speed of 1.4 rad/s and a minimum participation factor of 1%, the Fourier components of the obtained Floquet modes are listed in Table 1. For each Floquet mode the sum of the participation factors is greater than 99%.

Table 1. Fourier components of Floquet modes, with a minimal participation factor of 1%, for a rotational speed of 1.4 rad/s

Floquet mode	Participation factor (%)	Frequency (Hz)	Damping (%)
1	46.04	1.693	0.670
	29.32	1.470	0.794
	23.73	1.248	0.936
2	36.75	1.813	0.599
	31.43	1.590	0.682
	30.97	1.367	0.794
3	48.13	0.642	0.324
	29.24	0.864	0.240
	22.19	1.087	0.191
4	99.34	0.746	0.267
5	90.28	0.670	0.230
	9.15	0.448	0.344

A method to obtain the approximation error is to compare the exact periodic modes shapes with those reconstructed with the approximation. In Figure 2, the periodic mode shape is approximated with a relative approximation error of $\frac{\|Y_j(t)-\hat{Y}_j(t)\|}{\|Y_j(t)\|} = 0.01\%$.

5.2 Identification and comparison

From simulated acceleration data for a model with a constant rotational speed of 1.4 rad/s (sampled at 25 Hz during 600 s), a subspace identification using all degrees of freedom is performed. The SSI-cov is defined in Peeters and de Roeck (1999) coupled by the uncertainty computation method as defined in Döhler and Mevel (2013).

Once the eigenmodes are identified, with their frequency, damping and the estimation of their standard deviations (σ_f and σ_d), they are matched with the eigenmodes defined from the model by comparing the modes shapes with the MAC criterion (Pastor et al., 2012). The MAC criterion between two modes shapes ψ_1 and ψ_2 is

$$MAC\,(\psi_1, \psi_2) = \frac{|\psi_1^H\psi_2|^2}{\psi_1^H\psi_1\psi_2^H\psi_2}. \tag{47}$$

The obtained results are summarized in Table 2.

The eigenmodes determined by the model are clearly identified validating the approximation. Note that no extra modes are identified beside those predicted by the model. A mode with a low participation factor appears as a noise mode during the SSI analysis.

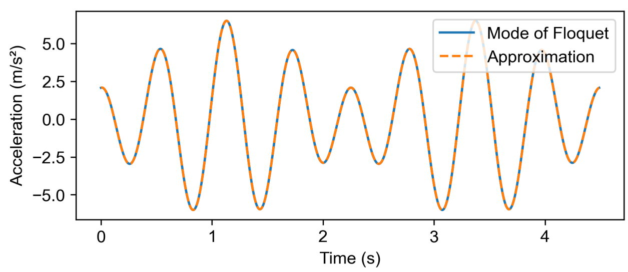

Fig. 2. Comparison model/approximation for the first periodic mode shape over one period at a degree of freedom of the rotor

Table 2. Identification results

Freq. (Hz)	σ_f (Hz)	Damp. (%)	σ_d (%)	MAC
1.698	0.003	0.833	0.173	0.999
1.479	0.003	1.160	0.188	0.994
1.256	0.003	1.295	0.260	0.999
1.825	0.003	0.627	0.142	0.998
1.600	0.003	0.498	0.096	0.999
1.378	0.002	0.585	0.124	0.998
0.643	0.001	0.714	0.179	0.989
0.867	0.002	0.431	0.156	0.986
1.092	0.007	0.578	0.630	0.987
0.747	0.001	0.329	0.098	1.000
0.671	0.001	0.270	0.093	1.000
0.453	0.005	1.399	0.734	0.994

To validate the uncertainty calculation, the estimated uncertainties can be compared with those obtained by Monte Carlo. The system is simulated and the system matrices and variances are identified 2000 times leading to comparable orders of magnitude. In Figure 3, for the first component of the first Floquet mode, the estimated 95% confidence interval is close to the empirical interval, i.e. $\sigma_{f\ MC} = 3.94 \times 10^{-3}$ and $\sigma_{d\ MC} = 2.03 \times 10^{-1}$. While the 10-run means of the estimated standard deviations are $\bar{\sigma}_f = 3.37 \times 10^{-3}$ and $\bar{\sigma}_d = 2.17 \times 10^{-1}$. It can be deduced that the uncertainty computation defined for stationary LTI systems is suitable for our problem.

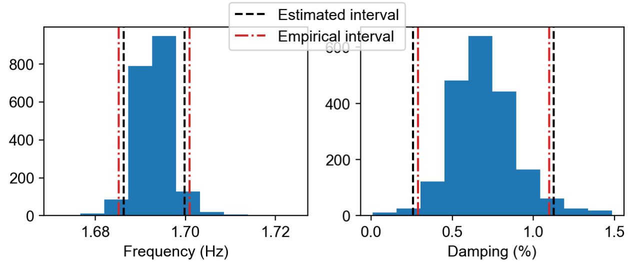

Fig. 3. Perturbation-based uncertainties vs Monte Carlo

6. CONCLUSION

It has been shown that an LTP system could be approximated by an LTI system under a non stationary excitation. For such model, the approximation of the Floquet modes in a finite sum of components has been presented. It has been shown that their components, and not the Floquet modes themselves, could be estimated using a subspace method validating the empirical use of SSI for such rotating structures. A small theoretical wind turbine model has been simulated to illustrate that the model components were correctly identified. Future work will be dedicated to the validation on a more sophisticated wind turbine model and to the comparison with existing LTP methods.

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