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A Hybrid Dual Jacobian Approach for Autonomous Control of Concentric Tube Robots in Unknown Constrained Environments

Balint Thamo, Farshid Alambeigi*, Kev Dhaliwal*, and Mohsen Khadem*

***Abstract*—Concentric Tube Robots (CTR) have been gaining ground in minimally-invasive robotic surgeries due to their small footprint, compliance, and high dexterity. CTRs can assure safe interaction with soft tissue, provided that precise and effective motion control is achieved. Controlling the motion of CTRs is still challenging. In this paper, we present a hybrid approach to overcome the aforementioned difficulties. This hybrid solution uses the solution of a kinematic model of the robot to estimate initial values for a model-free data-driven method. The proposed algorithm combines both model-based and data-driven algorithms to provide real-time motion control of CTRs interacting with an unknown external environment. Three different simulations studies were performed to thoroughly evaluate the efficacy of the proposed hybrid control approach as compared to two common model-based and data-driven control techniques. The results demonstrate superior performance of the proposed method. The root-mean-square error of the proposed hybrid approach is less than 1.1 mm, which is 9 times less than a common model-based controller.**

I. Introduction

Concentric tube robots (CTRs) are a type of continuum robot (CR) comprised of several concentrically arranged pre-curved elastic tubes. Tubes are able to make translational and rotational movements, which changes the robot's shape and tip pose, enabling complex movements in constrained environments. Accurate control of the robot motion is essential for safe deployment of CTRs. The kinematic model that is widely used to control CTRs was derived in [1], [2]. This model provides an approximation of the shape of the robot as a function of joint inputs, *i.e.*, rotation and translation of the tubes. The model is composed of a set of nonlinear differential equations accompanied with boundary conditions that must be solved to find the shape of the robot.

The most common model-based approach for controlling the motion of the CTRs is based on estimating the Jacobian of the robot. The Jacobian can be estimated numerically [3] or analytically [4] using the aforementioned kinematic model. Several researchers have used the Jacobian for open-loop [1], [5], [6] and closed-loop control [3], [7], [8] for the CTR's motion. These methods assume the robot is moving in free space and do not consider the effect of external forces acting on the robot in a realistic constrained environment. Therefore, these methods can potentially cause damage as the robot traverses the anatomy, or be unable to exercise the forces required for tissue manipulation.

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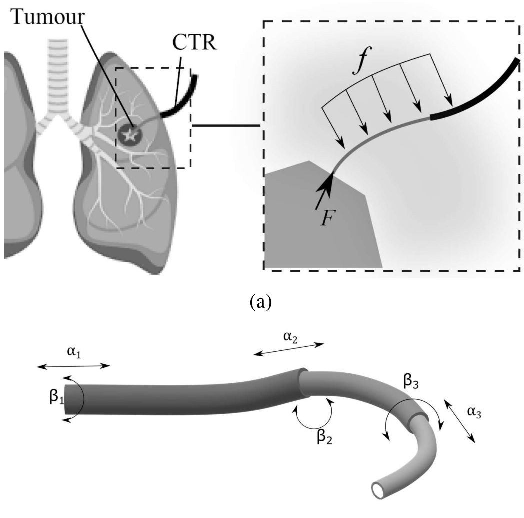


(b)

Figure 1. (a) A schematic of lung ablation using a concentric tube robot (CTR). Unknown external concentrated point force F and distributed forces f may act on the robot during its motion. (b) Illustration of a concentric tube robot. The robot is comprised of several concentrically arranged precurved elastic tubes. By rotating (α_i) and translating (β_i) the tubes and the shape of the robot can be controlled.

Emerging learning-based and data-driven controllers tend to overcome the aforementioned difficulties of controlling continuum robots by deep learning or using model-free data-driven approaches that can learn the complex dynamics of the robot as it interacts with its surrounding environment. For example, Iyengar *et al.* [9] used deep reinforcement learning to control a CTR with two tubes. They used a multi-layer perceptron network, which was trained on a large data-set. The training data-set was gathered offline using a CTR moving in free space. The trained model was later used for open-loop control of the robot motion. Chattopadhyay *et al.* [10] compared application of reinforcement learning and neural networks in controlling a cable driven CR. Lai *et al.* [11] used a simplified model with a feed-forward neural network to learn the inverse kinematics of a cable-driven CR with two bending sections. The trained network was later used for open-loop control of the robot in simulations. Goharimanesh *et al.* [12] applied a fuzzy reinforcement learning framework for the control of CRs in free space. Lee *et al.* [13] used a generic control framework with locally weighted projection regression. The controller was trained offline and later combined with an online learning strategy to control a fluidic CR. Yip *et al.* [14] proposed a model-less closed-loop feedback controller that estimates the robot Jacobian in real-time based on the measurements of the robot movement. Alambeigi at al. [15] generalized this approach to control the motion of a CR manipulating soft-tissue.

The application of data-driven methods on CRs is still at an early stage. Most of the aforementioned algorithms are trained offline using a large experimental [9], [13] or simulated [10], [11] data-set. These methods are used for control of the robot in free space without any contact with the environment or external forces. This is mainly due to the fact that gathering a large data-set of robot motion with variety of forces is very cumbersome and infeasible. Some researchers proposed online learning paradigms [14], [15] that require considerable amount of time to let the algorithm converge and learn before it can reach a desired accuracy. Therefore, the proposed method can potentially cause damage before the robot learns to move.

To address the aforementioned challenges, in this paper, we propose a hybrid control approach that takes advantage of utilizing a closed-loop data-driven controller initialized by the nominal model of the robot. The proposed method has a great potential to overcome the difficulties of the model-based approaches including predicting the effects of unknown external forces, robot's dynamics, and unexpected disturbances that might happen in a real surgical setting during the robot's motion. Moreover, to address the challenges associated with current data-driven techniques, it employs the predictions of the kinematic nominal model of the robot to guarantee fast and accurate convergence of the controller. We study the performance of the controller in several scenarios including the robot with inaccurately identified parameters, the robot under external forces, and the robot manipulating a soft-tissue. Our algorithm will be available online[1].

The remainder of the paper is organised as follows. Section II gives a concise description of the problem to be addressed in the paper. In sections III details of the proposed hybrid controller is provided including: a brief overview of CTR's kinematic model, numerical methods to estimate the model-based and data-driven Jacobians of the robot, and the proposed hybrid dual-Jacobian control strategy. In sections IV and V, simulations are performed to verify the performance of the proposed method and results are discussed, respectively. Concluding remarks appear in Section VI.

II. Problem Statement

Commonly used model-based control approaches often employ simplified geometric/dynamic assumptions, which could be very inaccurate in the presence of unmodelled disturbances and external interaction forces. Additionally, application of emerging data-driven algorithms in real-time control of CTRs is limited due to the fact that these controllers require considerable amount of time to let the algorithm develop enough to reach a desired accuracy and relevancy.

The goal of this paper is to present a way to control the motion of a CTR with unknown dynamics in contact with an unknown environment using only measurements of the robot's tip position. A schematic of the CTR is shown in Fig. 1(a), where a CTR is used for the ablation of lung tumours. A CTR carrying an ablation probe is inserted through the chest cavity towards the tumour. The goal is to press the ablation probe on the surface of the tumour without having prior knowledge about the deformation behavior of the tumour or the tissue surrounding the robot. In such scenarios, generally, there are three sources of disturbance that can lead to an inaccurate motion control of the robot:

1) Unknown external forces applied to the robot from the surrounding tissue.
2) Interaction forces between the robot and the soft tissue during tissue manipulation.
3) Inaccuracies in the the mathematical model of the robot.

We assume that during the operation, we can track the position of the tip of the robot via commonly used electromagnetic trackers (EMT). It is assumed that the position feedback has a random Gaussian noise. In addition, we assume that we have the nominal kinematic model of the CTR robot, which may cause $\pm 10\%$ error in predicting the position of the robot's tip [3]. We propose an approach to control the CTR's motion in presence of the aforementioned disturbances. This novel approach includes three main modules: (i) a model-based estimation of robot's kinematics in free space, (ii) a data-driven estimation of robot's motion in the presence of unknown disturbances, and (iii) a hybrid control procedure that fuses these approaches together to accurately and quickly control the robot tip's position in real-time. The following sections describe these modules in detail.

Of note, in the reminder of the paper, scalars are denoted as x, vectors as $\boldsymbol{x}$, and matrices as $\mathbf{x}$. A prime denotes the derivative with respect to the spatial coordinate s, and a dot derivative is the derivative with respect to time t.

III. Methodology

A. Review of the CTR Model

We begin by a brief review of the CTR kinematics first presented in [1], [2]. Fig. 1(b) shows a schematic of a CTR composed of 3 tubes. Subscript indices $i = 1, 2..N$ denote the i^{th} individual tube, where 1 is the innermost and N is the outermost one, so it is the total number of tubes. The configuration of the robot can be defined along its arc length s using $\boldsymbol{r}(s) : [0, \ell] \rightarrow \mathbb{R}^3$ describing the shape of the robot, and $\mathbf{R}(s) : [0, \ell] \rightarrow \mathrm{SO}(3)$ representing the orientation change and twisting along s. The arc length is zero ($s = 0$) at the proximal end of the tubes. The length of the i^{th} tube is denoted as ℓ_i while α_i and β_i are actuation input values. α_i is the proximal rotation of the base of the i^{th} tube and β_i is the translation of the i^{th} tube. $\boldsymbol{U}_i(s)$ denotes the i^{th} tube's curvature vector in its initial state and $\boldsymbol{u}_i(s)$ is the curvature vector of the i^{th} tube in its deformed state. The robot's backbone's shape can be obtained from the position vector and rotation matrix:

$$\boldsymbol{r}'(s) = \mathbf{R}(s)\boldsymbol{e}_3, \tag{1a}$$

$$\mathbf{R}'(s) = \mathbf{R}(s)\hat{\boldsymbol{u}}, \tag{1b}$$

where $\boldsymbol{e}_3 = [0, 0, 1]^T$ is a unit vector aligned with the z axis of the global coordinate frame, $\hat{}$ operator is the isomorphism between a vector in $\mathbb{R}^3$ and its skew-symmetric cross product matrix.

[1]https://github.com/SIRGLab/Hybrid_Control_CTR

Assuming that each tube conforms to the equilibrium shape of the combined tubes and using the balance of forces and moments, the curvature of the i^{th} tube can be obtained by

$$\begin{aligned} \boldsymbol{u}_i'\big|_{x,y} = &-\left(\sum_{i=1}^{N}\mathbf{K}_i\right)^{-1}\sum_{i=1}^{N}\mathbf{R}_{\theta_i} \\ &\Big[\mathbf{K}_i(\mathbf{u}_i'-\mathbf{U}_i')+\hat{\mathbf{u}}\mathbf{K}_i(\boldsymbol{u}_i-\boldsymbol{U}_i)\Big]- \\ &\left(\sum_{i=1}^{N}\mathbf{K}_i\right)^{-1}\left[e_3\times\mathbf{R}^T(\boldsymbol{F}+\int_0^s\boldsymbol{f}(\epsilon)d\epsilon)\right]\Bigg|_{x,y}, \end{aligned} \tag{2a}$$

$$\mathbf{u}_{iz}' = \frac{E_iI_i}{G_iJ_i}(u_{ix}U_{iy}-u_{iy}U_{ix}), \tag{2b}$$

$$\theta_i' = u_{iz}-u_{1z}, \tag{2c}$$

where θ_i denotes the angle of twist of the i^{th} tube around the local z-axis with respect to the global frame; $\mathbf{R}_{\theta_i}$ is a rotation around the z axis by a magnitude of θ_i; $\boldsymbol{f}$ is the external distributed force applied to the robot; $\boldsymbol{F}$ is the external point load on the robot tip; and $\mathbf{K}_i$ is defined as $\text{diag}(E_iI_i,\ E_iI_i,\ G_iJ_i)$. E is Young's modulus, I is the second moment of inertia of the tube's cross section, G is the shear modulus and J is the polar moment of inertia of the tube's cross section.

The boundary conditions of the robot are defined as follows:

$$\begin{aligned} &\boldsymbol{r}(s)|_{s=0} = \begin{bmatrix}0, & 0, & 0\end{bmatrix}^T, \quad \mathbf{R}(s)|_{s=0} = \mathbf{R}_{z(\alpha_1-\beta_1u_{1z})}, \\ &\theta_i'(s)|_{s=0} = \alpha_i-\beta_iu_{iz}\ , \quad \boldsymbol{u}_i(s)|_{s=l_i+\beta_i} = \boldsymbol{U_i}(s). \end{aligned} \tag{3}$$

Solving (1), and (2) with boundary conditions (3) gives the robot's backbone curvature and shape.

B. Model-Based Jacobian in Free Space

Without loss of generality, we assume that the CTR is composed of three tubes, the final deformed curve of all tubes at a given time t must be equal to the curve of the innermost tube, and that the end-effector is the tip of the inner most tube, denoted by $\boldsymbol{x}=\boldsymbol{r}\big|_{s=\ell_1+\beta_1}$, where $\boldsymbol{x}$ is a 3×1 vector denoting the the Cartesian coordinates of the robot tip.

For achieving motion control, the Jacobian of the robot is required. It can be numerically estimated using

$$\mathbf{J}_M = \frac{\Delta\boldsymbol{x}}{\Delta\boldsymbol{q}} = \begin{bmatrix} \frac{\boldsymbol{x}^T\left(\boldsymbol{q}+\frac{\Delta q_1}{2}e_1\right)-\boldsymbol{x}^T\left(\boldsymbol{q}-\frac{\Delta q_1}{2}e_1\right)}{\Delta q_1} \\ \vdots \\ \frac{\boldsymbol{x}^T\left(\boldsymbol{q}+\frac{\Delta q_6}{2}e_6\right)-\boldsymbol{x}^T\left(\boldsymbol{q}-\frac{\Delta q_6}{2}e_6\right)}{\Delta q_6} \end{bmatrix}^T, \tag{4}$$

where $\boldsymbol{e}_i$ is the i^{th} unit vector of canonical basis of joint space and $\boldsymbol{q}=\begin{bmatrix}\beta_1, & \beta_2, & \beta_3, & \alpha_1, & \alpha_2, & \alpha_3\end{bmatrix}^T$ is the vector of joint inputs.

We select the above formulation as it gives rise to parallelisable computations without sacrifices in the kinematics model's accuracy. For a three-tubed CTR, the Jacobian is a 3×6 matrix and it maps the joint velocities $\dot{\boldsymbol{q}}\in\mathbb{R}^6$ to the end-effector velocities $\dot{\boldsymbol{x}}\in\mathbb{R}^3$.

The Jacobian given in (4), is calculated by numerically linearising the solution of the robot's model given by (1) and (2). The approximation uses the quasi-static model thus neglecting the dynamics of the robot. The nonlinearities in the equations are removed as well. Moreover, it requires a knowledge of the interaction forces between the robot and the surrounding environment, *i.e*, $\boldsymbol{F}$ and $\boldsymbol{f}$ in (2a), which are not always available. Additionally, the Jacobian is susceptible to uncertainties in the identified value of the model parameters, *e.g,* the tubes' stiffness E. It has been shown that the accuracy of the model is equal to 8% of the robot's length due to these uncertainties [2]. In the next section we propose a data-driven Jacobian that can overcome the aforementioned difficulties.

C. Data-Driven Jacobian for Free and Constrained Motions

To learn the robot's dynamic behaviour on-the-fly and compensate for the effects of disturbances and external forces, we require an algorithm that updates the elements of the Jacobian in real-time. To this end, we have chosen the Broyden's Jacobian update approach that can safely and incrementally learn a CR's Jacobian on-the-fly [15]–[17]. This algorithm employs the current measurements of the CTR's tip position $\boldsymbol{x}$, joint inputs $\boldsymbol{q}$, and updates the Jacobian accordingly as follows:

$$\widehat{\mathbf{J}}_D^{k+1} = \widehat{\mathbf{J}}_D^{k} + \chi\frac{\Delta\boldsymbol{x}^k-\widehat{\mathbf{J}}_D^{k}\Delta\boldsymbol{q}^k}{(\Delta\boldsymbol{q}^k)^T(\Delta\boldsymbol{q}^k)}(\Delta\boldsymbol{q}^k)^T, \tag{5}$$

where $\widehat{\mathbf{J}}_D^{k+1}$ is the estimated Jacobian matrix at sample time $k+1$, $\Delta\boldsymbol{x}^k$ is the displacement of the robot's tip at sample time k, $\Delta\boldsymbol{q}^k$ represents a vector of actuator input change, and χ is the learning rate ($0<\chi<1$).

The advantage of this approach is the ability to adapt to parameter uncertainties and unknown perturbations in the environment due to its online learning behaviour [17]–[19]. The main drawback of this method, however, is the long learning time required for estimating the robot Jacobian, particularly when the robot is interacting with an unknown environment [17]. Of note, this is mainly due to random initialization of the estimated Jacobian matrix and the learning rate parameter χ. As shown in [16], bad initialization of these parameters may cause the algorithm to never converge or need many iterations to estimate a Jacobian matrix.

D. Hybrid Dual Jacobian

Here, we propose a solution for calculating a novel hybrid Jacobian by combining both the model-based and the data-driven Jacobians (Fig. 2). First, the model-based Jacobian is calculated using (4). Next, it is used as a weighted initial guess in Broyden's recursive method given in (5). The updated equation for estimating the hybrid Jacobian J_H is given by

$$\begin{aligned} \widehat{\mathbf{J}}_H^{k+1} = &e^{-\lambda_1k}\mathbf{J}_M^k + \frac{1-e^{-\lambda_2k}}{1+e^{-\lambda_2k}}\Bigg[\widehat{\mathbf{J}}_H^{k}+ \\ &\chi\frac{\Delta\boldsymbol{x}^k-\widehat{\mathbf{J}}_H^{k}\Delta\boldsymbol{q}^k}{(\Delta\boldsymbol{q}^k)^T(\Delta\boldsymbol{q}^k)}(\Delta\boldsymbol{q}^k)^T\Bigg], \end{aligned} \tag{6}$$

where $\widehat{J}_H$ and J_M denote the data-driven and the model-based Jacobians, respectively. Also, λ_1 and λ_2 are constant positive scalars working as the weighting factors of the considered Jacobians.

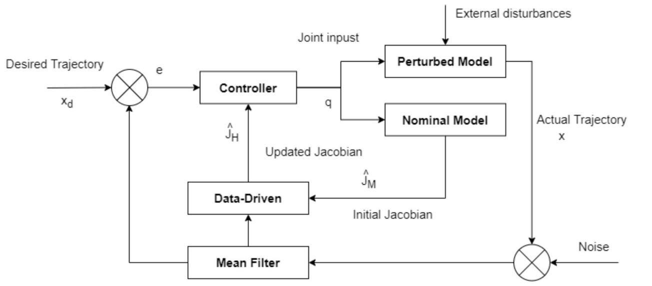


Figure 2. Block diagram of the proposed hybrid approach for motion control of CTRs.

Table I
NOMINAL PARAMETERS OF THE CTR.

	Tube 1	Tube 2	Tube 3
Inner Diameter[mm]	0.7	1.4	2
Outer Diameter[mm]	1.1	1.8	2.2
Length[mm]	431	332	174
Curved Part Length[mm]	103	113	134
Curvature[1/m]	21.3	13.1	3.5
Young's Modulus, E[GPa]	64.3	52.5	47.1
Shear Modulus, G[GPa]	25	21.4	29.7

In (6), at time zero (i.e., $k = 0$), the data-driven Jacobian $\widehat{\mathbf{J}}_H^k$ is initially set to zero and the model-based Jacobian $\mathbf{J}_M^k$ is used to update the hybrid Jacobian estimation $\mathbf{J}_H^{k+1}$. Then, in the next sample times, the contribution of the model-based Jacobian exponentially decays toward zero at the rate of λ_1, while the weight of the data-driven Jacobian (i.e., $\dfrac{1-e^{-\lambda_2 k}}{1+e^{-\lambda_2 k}}$) converges toward 1 at an exponential rate of λ_2.

Of note, the proposed hybrid Jacobian is capable of adapting to the unknown perturbations and robot dynamics at a low computational cost, while benefiting from the model-based kinematics to shorten the learning time of the data-driven approach.

E. Motion Control of CTR

Using the proposed hybrid Jacobian, we design a controller to steer the tip of the robot to a desired Cartesian position $\boldsymbol{x}_d \in \mathbb{R}^3$. For the CTR robot, this means to find a combination of joint input, *i.e.*, tubes rotation α_i and translation β_i that results in an end-effector position $\boldsymbol{x} = \boldsymbol{x}_d$. To this end, we calculate the forward differential kinematics of the robot as

$$\dot{\boldsymbol{x}} = \mathbf{J}_H(\boldsymbol{q})\dot{\boldsymbol{q}}. \tag{7}$$

By introducing the pseudo-inverse of the hybrid Jacobian [20],

$$\widehat{\mathbf{J}}_H^{\dagger} = \mathbf{J}_H^T(\mathbf{J}_H\mathbf{J}_H^T)^{-1}, \tag{8}$$

equation (7) can then be modified to obtain the inverse kinematics of a CTR:

$$\dot{\boldsymbol{q}} = \widehat{\mathbf{J}}_H^{\dagger}\dot{\boldsymbol{x}}. \tag{9}$$

To follow a desired trajectory $\boldsymbol{x}_d$, an error between the desired and actual error is introduced: $\boldsymbol{e} = \boldsymbol{x}_d - \boldsymbol{x}$. Equation (9) can be converted to the following control law to minimize this error:

$$\dot{\boldsymbol{q}}_d = \widehat{\mathbf{J}}_H^{\dagger}\left[\dot{\boldsymbol{x}}_d + \mathbf{K}_P\boldsymbol{e}\right], \tag{10}$$

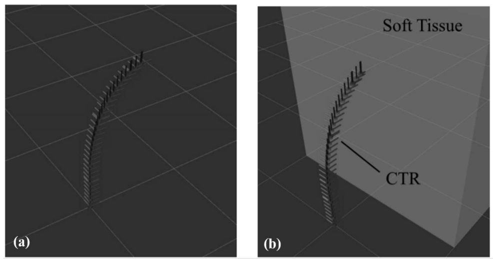


Figure 3. CTR simulated in ROS environment. (a) CTR in free space, (b) CTR in contact with soft tissue.

where $\mathbf{K}_P$ is a 3x3 symmetric positive definite matrices. Fig. 2 shows a block diagram of the proposed motion control approach.

IV. SIMULATION STUDIES

To evaluate the performance of the proposed approach, we compared the proposed hybrid dual-Jacobian controller with two commonly used controllers: (i) a model-based controller proposed in [3], and (ii) a data-driven controller employing the Broyden's regression algorithm [15], [17] via three different types of simulations. In the performed simulations, a nominal model of the CTR was first used to calculate the model-based Jacobian in free space via (4). Of note, this nominal model was derived based on an existing robot used in [3]. The model parameters of this simulated CTR have been summarized in Table I. Next, the model-based Jacobian was used in (6) to calculate the hybrid Jacobian during the performed simulations and on-the-fly. Finally, the control law given in (10) employed the hybrid Jacobian to steer the robot on a pre-defined desired trajectory. The algorithm was tested on a perturbed CTR model in an unknown environment. This perturbed model simulates the effects of unknown disturbances including unknown external distributed/point loads. To make the simulation more realistic, a normally distributed random noise is added to the robot's tip position:

$$n = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(\frac{-x^2}{2\sigma^2}\right), \tag{11}$$

with $\sigma = 2$ mm. Later, a mean filter is used to denoise the feedback signal. The filter takes the average of 5 consecutive samples to reduce the noise.

To evaluate the proposed hybrid approach, the following different simulation scenarios were considered while using three different perturbed models:

(S_1) *Robot with inaccurately identified parameters:* The controller is tested on a perturbed CTR model with $\pm 20\%$ inaccuracy in initial pre-curvature of the robot $\boldsymbol{U}$, tube's Young's Modulus (E), and Shear Modulus (G) of the tubes. These inaccuracies result in maximum of 10% error in estimating the robot's tip position [3].

(S_2) *Robot under unknown external forces:* The controller was tested on a CTR model that includes external forces. A point load equal to $[0.5, 0.5, 0.5]^T$ N was applied to the tip of the robot. The controller has no prior knowledge of this force.

(S_3) *Robot manipulating a soft tissue:* The CTR robot is in contact with a soft-tissue without any *a priori* knowledge

Table II

COMPARISON OF ERROR BETWEEN DESIRED AND ACTUAL TRAJECTORIES IN THREE SCENARIOS. RESULTS OF MODEL-BASED, DATA-DRIVEN, AND HYBRID CONTROLLER ARE PRESENTED. e_{MEAN} IS THE AVERAGE ERROR, σ_e IS THE STANDARD DEVIATION OF ERROR, e_{max} IS THE MAXIMUM ERROR, AND RMSE IS THE ROOT MEAN SQUARE ERROR. THE VALUES ARE ALL IN MILLIMETER.

	$\mathbf{S_1}$				$\mathbf{S_2}$				$\mathbf{S_3}$			
	e_{mean}	σ_e	e_{max}	RMSE	e_{mean}	σ_e	e_{max}	RMSE	e_{mean}	σ_e	e_{max}	RMSE
Data-Driven	14.530	25.239	120.237	25.239	18.410	33.052	114.045	37.856	37.992	48.242	108.476	61.404
Model-Based	18.410	2.724	11.715	7.206	14.089	6.781	26.699	15.739	5.982	3.180	14.420	6.792
Hybrid	0.686	0.518	1.945	0.861	0.719	0.535	1.908	0.896	1.105	0.829	4.772	1.382

of the tissue's mechanical characteristics. This study simulates the tissue ablation procedure, where a probe capable of applying intensive heat/cold is pressed and moved on the surface of tissue to destroy abnormal tissue. In the performed simulation, the robot is commanded to deform the surface of the tissue by a magnitude of 1 mm and move along a rectangle while keep pushing the tissue by the same amount. To simulate this scenario, deformation behavior of the tissue was modelled using the soft tissue model presented in [21]:

$$F = a_1 x + a_2 x^2 \tag{12}$$

where F is the contact force between the robot and the tissue and x is the relative surface deformation of the tissue. Also, $a_1 = 0.03\ \frac{\text{N}}{\text{mm}}$ and $a_2 = 0.003\ \frac{\text{N}}{\text{mm}^2}$ are constants representing the mechanical characteristics of kidney and liver tissue [21]. In addition to the mentioned contact force F, a time-varying distributed force f was also applied along the whole shape of the robot:

$$f = sin\Big(\frac{2\pi t}{2}\Big), \tag{13}$$

where t is time. Of note, this distributed force only has a y component with a maximum value of 1 $\frac{\text{N}}{\text{m}}$. The controller has no prior knowledge of this force.

For all three scenarios, the nominal model of the robot was fixed and no external forces were considered during the calculation of the model-based Jacobian. Also, the robot's tip was steered to follow a square trajectory with 35 mm base length in the first two cases and a 60×30 mm rectangle trajectory in the third case, which simulates the size of a lung tumor at stage 2 or 3 [22]. A square trajectory was selected since the CTRs have more difficulties in traversing straight lines and sharp corners as opposed to circular paths.

Using C++, we simulated these scenarios in Robot Operating System (ROS). Simulation sampling time and the robot's tip desired velocity were selected as 15 millisecond and 5 $\frac{\text{mm}}{\text{sec}}$, respectively. In addition to these, the following controller parameters were chosen: $\mathbf{K}_p = 3\mathbf{I}$, where $\mathbf{I}$ is a 3×3 identity matrix. Also, we selected λ_1, λ_2. and χ in (5) as 10, 10, and 0.1, respectively. These values were tuned via trial and error to achieve the minimum tracking error. Fig. 3 shows the robot in free space and in touch with the tissue in the ROS simulated environment.

V. RESULTS AND DISCUSSION

Results of the the three scenarios are summarized in Table II. The average tracking error, the standard deviation of error, the maximum tracking error, and root-mean-square-error (RMSE) are reported. The root-mean-square error is calculated as

$$\text{RMSE} = \sqrt{\frac{\sum_{j=1}^{m}(\|\boldsymbol{x} - \boldsymbol{x}_d\|_j)^2}{m}}, \tag{14}$$

and is used as a measure of the differences between the actual trajectory of the robot's end-effector, $\boldsymbol{x}$, and the desired trajectory, *i.e.* $\boldsymbol{x}_d$, for m data points along the robot's tip trajectory.

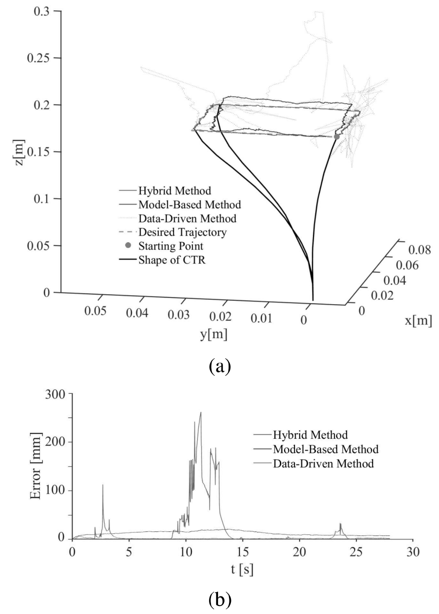
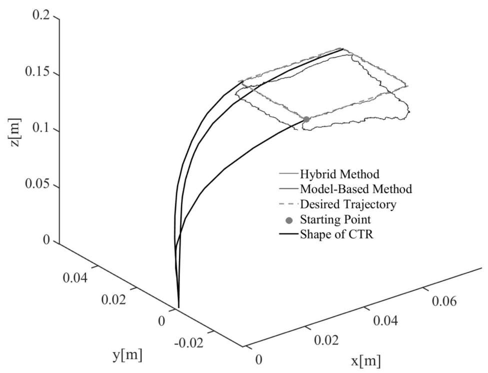


Figure 4. (a) Following a square trajectory using a model with inaccurate parameters. (b) Mean error of the three method while following a square trajectory using a model with inaccurate parameters.

Fig. 4(a) shows the actual trajectories of the three compared methods (i.e., data-driven, model-based, and hybrid dual Jacobian) in the 1st scenario (S_1), while Fig. 4(b) presents the tracking error versus time. Of note, for these simulations, the pure data-driven approach without any initial training failed to learn the Jacobian and was unable to follow the desired trajectory. The model-based method struggled to follow the desired trajectory with the unknown changes in the model parameters, while the proposed hybrid method quickly adapted to these changes. The largest errors in this approach appeared when the trajectory changed its directions. The errors with the latter approach are an order of magnitude smaller than with the former ones (as shown in Table II).

In the 2nd scenario (S_2), similar to the previous simulation, the robot was asked to follow a square trajectory with 35 mm base length. Fig. 5(a) shows a comparison of the model-based and hybrid methods. Similar to S_1 scenario, the pure



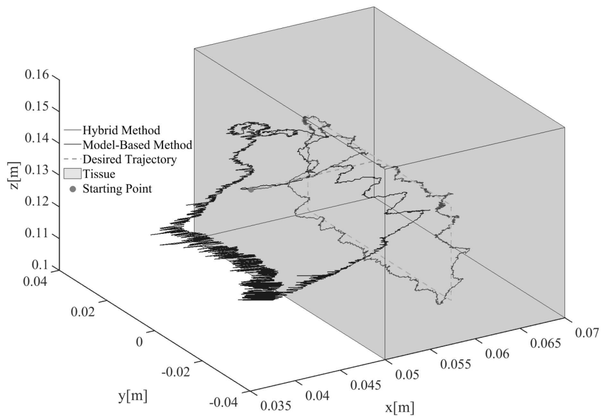


Figure 5. (a) Following a square trajectory with a $[0.5, 0.5, 0.5]^T$ N external force on the tip of the robot. (b) The robot is steered to follow a rectangle trajectory, while pushing a soft tissue by 1mm in the presence of distributed force along the shape of the robot..

data-driven approach without any initial training failed to learn the Jacobian and was not able to follow the desired trajectory. Therefore, due to its large errors, we did not take it into account in the figure. The model-based approach could not also accurately follow the trajectory and obtained RMSE= 15.7 mm. As mentioned, this is mainly due to not considering the unknown external forces during deriving the model-based Jacobian. Nevertheless, similar to previous scenario, the hybrid dual Jacobian approach is not only able to follow the rectangular trajectory but it is much more accurate as well (i.e., RMSE= 0.9 mm). This is mainly because of quickly learning the effect of external forces on the deformation behavior of the robot. Table II shows that the average error for our proposed dual Jacobian approach is less than 0.72 mm, while it is more than 6.89 mm with the model-based approach, which is not able to adapt to the unknown disturbances.

In the 3rd scenario (S_3), the CTR indirectly manipulates particular points on the tissue surface without having prior knowledge about the external contact forces during manipulation. In this simulation, the tissue was placed parallel to the y-z plane at $x = 5$ cm. The external point force was approximately 0.3 N when the tissue was pressed by 1 mm . The results summarized in Table II demonstrate the superior performance of the proposed hybrid approach. The hybrid approach has some difficulties at the beginning when it reaches the soft tissue and deviates from the desired trajectory when the desired trajectory changes its direction. However, it is able to recover quickly and follow the trajectory with more accuracy compared to the model-based method. As shown in Fig. 5(b), the model-based controller struggles to follow the desired trajectory and remains close to the surface of the tissue.

The proposed hybrid approach is able to adapt to uncertainties in the model and in the environment. The hybrid Jacobian proposed in (6) always requires some time to learn the effects of uncertainties and disturbances. To investigate the time response of the controller, we performed 16 trials; in each trial the robot was tasked to follow a trajectory at a velocity between 0.1 to 10 mm/sec with a sampling times varying between 15 to 100 milliseconds, in presence of model uncertainties. Results are summarised in Fig. 6. As it can be seen the controller's error linearly increases with respect to tracking velocity. This shows that the Hybrid controller requires enough time to adapt to the changes in the feedback signal and is most suitable for autonomous control of the CTR at velocities below 10 mm/sec, which is appropriate for needle-based interventions [23] such as lung biopsy and ablation.

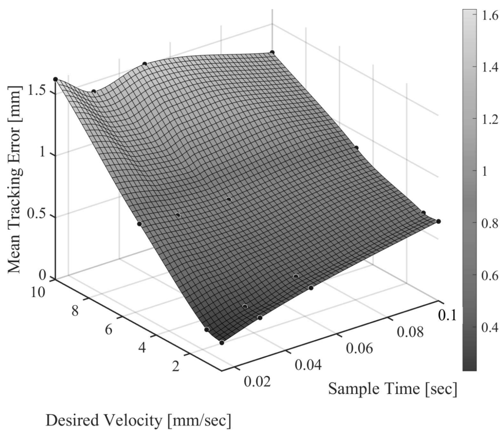


Figure 6. Hybrid controller's error with respect to sampling time and desired velocity.

VI. Concluding Remarks

In this paper, we demonstrated a hybrid approach to control CTRs experiencing unknown perturbations and external forces. This hybrid approach used an initial Jacobian obtained from the solution of the CTR model and then efficiently combined it with a data-driven approach to update the Jacobian in real-time and estimate the variations in the Jacobian caused by unknown external forces and perturbations. With this approach the CTR was able to adapt to uncertainties such as unknown external forces (with up to 0.5 N magnitude) and up to 20% error in estimating the model parameters. Various simulation studies demonstrated that the RMSE of the proposed hybrid approach in tracking a rectangular trajectory is about 9 times less than a common model-based controller (i.e., < 1.1 mm). In the future, experiments will be performed using a real CTR to verify the accuracy of the proposed approach in controlling the robot's motion in an unknown constrained environment.

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