Conclave: A Collective Stake Pool Protocol

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Abstract. Proof-of-Stake (PoS) distributed ledgers are the most common alternative to Bitcoin's Proof-of-Work (PoW) paradigm, replacing the hardware dependency with stake, i.e., assets that a party controls. Similar to PoW's mining pools, PoS's stake pools, i.e., collaborative entities comprising of multiple stakeholders, allow a party to earn rewards more regularly, compared to participating on an individual basis. However, stake pools tend to increase centralization, since they are typically managed by a single party that acts on behalf of the pool's members. In this work we propose *Conclave*, a formal design of a *Collective Stake Pool*, i.e., a decentralized pool with no single point of authority. We formalize Conclave as an ideal functionality and implement it as a distributed protocol, based on standard cryptographic primitives. Among Conclave's building blocks is a weighted threshold signature scheme (WTSS); to that end, we define a WTSS ideal functionality and propose two constructions based on threshold ECDSA, which enable (1) fast trustless setup and (2) identifiable aborts.

1 Introduction

A major innovation of Bitcoin [31] was combining Proof-of-Work (PoW), to prevent sybil attacks, with financial rewards, to incentivize participation.

Regarding sybil resilience, Bitcoin's PoW depends on the collective network's ability to compute hashes. Thus, PoW limits each party's power and also determines how the distributed ledger is updated, i.e., which blocks can extend its blockchain. However, PoW's deficiencies, particularly its egregious environmental cost,[4] have driven research on alternative designs, most prominently Proof-of-Stake (PoS). PoS removes hardware requirements altogether and internalizes sybil resilience by relying on parties' *stake*, i.e., the assets that they own. These assets are managed by the distributed ledger and serve as both the system's internal currency and consensus participation tokens. PoS systems are almost

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[4] The carbon footprint of: i) a single Bitcoin transaction is equivalent to $1,202,422$ VISA transactions; ii) the total Bitcoin network is comparable to Sweden. (`https://digiconomist.net/bitcoin-energy-consumption`; May 2021)

energy-free, but often rely on complex cryptographic primitives, e.g., secure Multiparty Computation [26], Byzantine Agreement [8,17,27], or Verifiable Random Functions (VRFs) [9,17].

Regarding rewards, blockchain-based financial systems, like Bitcoin, aim to incentivize participation in the consensus mechanism. The rewards comprise of newly-issued assets and of transaction fees, i.e., assets paid by parties for using the system. Interestingly, both PoW and PoS ledgers are economies of scale, who favor parties with large amounts of participating power. One reason is poorly-designed incentives, resulting in disproportionate power accumulation [23,13]. Another is temporal discounting, i.e., the tendency to disfavor rare or delayed rewards [35]. Specifically, in Bitcoin, a party is rewarded for every block it produces, so parties with insignificant amounts of power are rarely rewarded. In contrast, accumulating the power of multiple small parties in "pools" yields a steadier reward. As a result, PoW systems see the formation of mining pools,[5] while PoS systems usually favor delegation to stake pools [7,21] over "pure" PoS, where parties act independently. Finally, the ledger's performance and security are often better under fewer participants. For instance, PoS systems require participants to be constantly online, since abstaining is a security hazard; this requirement is more easily guaranteed within a small set of dedicated delegates.

A major drawback of existing stake pools is that they are typically managed by a single party, the *operator*. This party participates in consensus, claims the rewards offered by the system, and then distributes them among the pool's members (after subtracting a fee). However, the operator is a single point of failure. In this work, we explore a more desirable design, which allows players to jointly form a *collective pool*, i.e., a conclave. This design assumes no single operator, minimizing excess fees, and trust and security concerns, altogether. Collective stake pools also promote a more fair and decentralized environment. In existing incentive schemes [3], operators who can pledge large amounts of stake to the pool are preferred. Consequently, the system favors a few major pool operators and, in the long run, its wealth is concentrated around them, resulting in a "rich get richer" situation. Although this problem is inherent in all decentralized financial systems [23], a well-designed collective pool may offset the stakeholder imbalance and slightly decelerate this tendency.

Desiderata. Our design assumes a group of stakeholders who jointly create a stake pool without a single operator. Since large stakeholders typically form pools on their own, our protocol concerns smaller stakeholders, who could otherwise not participate directly. Therefore, our design could e.g., be appealing to a group of friends or colleagues, who aim for a more steady reward ratio without relying on a third party. Importantly, it should operate in a trustless environment as, unfortunately, even in these scenarios, trust is not a given. Notably, our targeted audience is parties who wish to actively participate, i.e., always be online to

[5] 86% of Bitcoin's hashing power and 83% of Ethereum's hashing power are controlled by 5 entities each. (`https://miningpools.com`; May 2021)

perform the required consensus actions; parties who wish to remain offline may instead opt for delegation schemes [7,21].

In the absence of a central party, the responsibility of running the pool is shared among all pool's members, requiring some level of coordination which may be cumbersome. For instance, if the protocol requires unanimous actions, a single member could halt the pool's operation. To ensure good performance, the pool should allow a subset (of a carefully chosen size) to act on behalf of the whole group. The choice of such subsets depends on each party's "weight", which is in proportion to their stake. In summary, we have the following initial assumptions, which form the basis for outlining our work's desiderata:

- **small number of parties**: a collective pool is operated by a small group of players;
- **small stake disparity**: the profiles of the collective pool's members are similar, i.e., they contribute a similar amount of stake to the pool;
- **stake proportion as "weight"**: each party is assigned a weight for participating in the pool's actions, relative to their part of the pool's total stake.

Next, we provide an exhaustive list of basic requirements of a collective stake pool. We note that an *admissible party set* is a set of parties with enough stake, i.e., above a threshold of the total pool's stake which is agreed upon during the pool's initialization. To the extent that some desiderata are conflicting, our design will aim to satisfy as many requirements as possible:

- **Proportional Rewards**: the claim of each member on the entire pool's protocol rewards should be proportional to their individual contribution.
- **Joint Control of Rewards**: the members of a pool should jointly control the access to its funds.
- **Unilateral Reward Withdrawal**: at any point in time, a stakeholder should be able to claim their reward, accumulated up to that point, without necessarily interacting with other members of the pool.
- **Permissioned Access**: new users can join the pool following agreement by an admissible set of pool members.
- **Robustness against Aborting**: the pool should not fail to participate in consensus, unless an admissible set of members aborts or is corrupted.
- **Public Verifiability**: stake pool formation and operation should be publicly verifiable (s.t. consensus could take into account the aggregate pool's stake).
- **Stake Reallocation**: users should freely change their personal stake allocated to the pool, without interacting with other members of the pool.
- **Parameter Updates**: an admissible set of parties should be able to update the stake pool's parameters.
- **Force Removal**: an admissible set of parties should be able to remove a member from the pool.
- **Pool Closing**: an admissible set of parties should be able to permanently close the stake pool.
- **Prevention of Double Stake Allocation**: a party should not simultaneously commit the same stake to two different stake pools.

Our Contributions and Roadmap. We propose *Conclave*, a collectively managed stake pool protocol that aims to satisfy the listed desiderata. Our first contribution is the ideal functionality $\mathcal{F}_{pool}$, a simulation-based security definition of collective stake pools, which captures the core security properties that our collective pool scheme should possess. We then describe π_{pool}, a distributed protocol executed by a set of n parties $\mathbb{P}$ which realizes $\mathcal{F}_{pool}$. π_{pool} employs certificates, which are published on the ledger, to announce its formation and closing. A major consideration and performance enhancement of our design is load balancing of transaction verification. Each transaction is verified by a (deterministically elected) committee of parties, whose size is a tradeoff between balancing workload, i.e., not requiring each party to verify every transaction, and reducing trust on the chosen validator(s). We thus construct a *distributed mempool*, i.e., a collectively managed set of unpublished transactions, s.t. if a majority of the committee's members are honest, transaction verification is secure. Our scheme uses a weighted threshold signature scheme (WTSS), to share the pool's key among its members, and a smart contract to manage the rewards. To that end, we provide a WTSS Universally Composable ideal functionality (Section 2), which may be of independent interest, and construct an ECDSA WTSS, based on [15,16], s.t. each party has as many shares as "units" of weight.

Related Work. In the past years a multitude of PoS protocols have been proposed. The Ouroboros family [2,9,25,26] offers, like Bitcoin, eventual guarantees of liveness and persistence. Subselection has been employed in systems like Algorand [17], which employs Byzantine Agreement to achieve transaction finality in (expected) constant time, and Snow White [8,32], which uses the notion of "robustly reconfigurable consensus" to address potential lack of participation. Our work is complementary to these protocols and can be composed with them, as it is agnostic to the underlying PoS ledger's consensus mechanism.

Real-world PoS implementations often opt for stake representation and delegation. Systems like Cardano[6], EOS [7], and (to some extent) Tezos [19], employ different consensus protocols, but all enforce that a (relatively small) subset of representatives is elected to participate. Decred [10] takes a somewhat different approach, where stakeholders buy a ticket for participation, akin to PoS with optional participation. However, these systems typically assume single parties that act as delegates, either individually or as pool operators; our design directly aims at relaxing this restriction without requiring changes to the consensus protocol.

In cryptographic literature, pools are mostly treated from an engineering perspective. In PoW systems, SmartPool [30] is a notable design of a distributed mining pool for Ethereum, which, similar to our work, utilizes smart contracts for reward distribution. On the PoS domain, Ouroboros [26] offers a brief description of how delegation can be used within the protocol. This idea is expanded in [21], which provides a formal definition of PoS wallets and includes stake pool formation method via certificates. However, the pool's management is again centralized around the operator; our work extends this line of work by enabling the

[6] `https://cardano.org`

formation of a collective pool. Another work, orthogonal to ours, by Brünjes *et al.* [3] considers the incentives of distributing rewards among stake pools and aims to incentivize the creation of a (pre-defined) number of pools. However, it assumes that the pool operator commits part of their stake to make the pool more appealing, thus favoring larger pool operators. Our work eases such wealth concentration tendencies by enabling a collective pool to be equally competitive to a centralized one.

2 UC Weighted Threshold Signature

In this section, we present the weighted threshold signature ideal functionality $\mathcal{F}_{wtss}$ (Figure 1). This functionality is used in the Collective Pool Protocol π_{pool}, which employs weighted threshold signatures for collectively signing certificates and new blocks. The functionality $\mathcal{F}_{wtss}$ is inspired by Almansa *et al.* [1], which is in turn inspired by Canetti [5]. However, unlike Almansa *et al.* and similar to Canetti, during signature verification we consider the case of a corrupted signer, i.e., a set of parties such that the majority (of weights) is corrupted.

$\mathcal{F}_{wtss}$ interacts with a set of n parties. Each party P_i is associated with an integer w_i, i.e., its weight. $\mathcal{F}_{wtss}$ also keeps the following, initially empty, tables: i) pubkeys: tuples $\langle sid, vk \rangle$ of sid and a public key vk; ii) sigs: tuples (m, σ, vk, f) of message m, a signature σ, a public key vk, and a verification bit f. The mapping $\omega[p] \rightarrow w_p$ denotes the weight of a party p, while the term ω also denotes the set of keys the participating parties.

As highlighted in the definition, *completeness, consistency*, and *unforgeability* are enforced upon verification, whereas *threshold completeness* is enforced upon signature generation. Hence, it should be infeasible to issue a signature unless using keys with enough weight, i.e., above the threshold, say, a value T.

3 The Collective Stake Pool

Our analysis is based on the UC Framework, following Canetti's formulation of the "real world" [4]. Specifically, we define the collective pool ideal functionality $\mathcal{F}_{pool}$, which distills the required (operational and security) properties; for readability, $\mathcal{F}_{pool}$ is divided into two parts, *management* and *consensus participation*. The ideal functionality is realized – in the "real world" – by the distributed protocol π_{pool}, which employs various established cryptographic primitives, and, therefore, π_{pool} can be described with auxiliary functionalities. Before proceeding with the functionality's definition, we first describe the hybrid execution of π_{pool} and its building blocks.

3.1 Hybrid Protocol Execution

The protocol π_{pool} is performed by n parties, where each party p_i holds two pairs of keys: $(\mathsf{vk}_{p_i}, \mathsf{sk}_{p_i})$ for issuing transactions, and $(\mathsf{vk}_{s_i}, \mathsf{sk}_{s_i})$ for staking

Weighted Threshold Signature Functionality $\mathcal{F}_{wtss}$

Each message is associated with $sid = \langle \mathcal{P}, \omega, T, sid' \rangle$, where $\mathcal{P}$ is the set of parties, ω is a mapping of parties to weights, T is the collective signature weight threshold, and sid' is a unique identifier.

Key Generation: Upon receiving (KeyGen, sid) from every honest party $P \in \mathcal{P}$, send (KeyGen, sid, P) to $\mathcal{S}$. Upon receiving a response $(\text{KeyGen}, sid, \mathsf{vk})$ from $\mathcal{S}$, record $\langle sid, \mathsf{vk} \rangle$ to $\mathsf{pubkeys}$ and send $(\text{KeyGen}, sid, \mathsf{vk})$ to every party in $\mathcal{P}$. Following, all messages that do not contain the established sid are ignored.

Signature Generation: Upon receiving (Sign, sid, m) from a party p, forward it to $\mathcal{S}$. After a subset of parties $\mathcal{P}' \subseteq \mathcal{P}$ has submitted a Sign message for the same m, and upon receiving $(\text{Sign}, sid, m, \sigma)$ from $\mathcal{S}$, check that $\sum_{p \in \mathcal{P}'} \omega[p] > T$ (*Note: This condition guarantees threshold completeness.*) Next, if $(m, \sigma, \mathsf{vk}, 0) \notin \mathsf{sigs}$ (for the key vk that corresponds to sid in $\mathsf{pubkeys}$), record $(m, \sigma, \mathsf{vk}, 1)$ to sigs and reply with $(\text{Sign}, sid, m, \sigma)$.

Signature Verification: Upon receiving $(\text{Verify}, sid, m, \sigma, \mathsf{vk}')$ from P, forward it to $\mathcal{S}$. Upon receiving $(\text{Verified}, sid, m, \sigma, \phi)$ from $\mathcal{S}$, set f as next:

1. If $\mathsf{vk}' = \mathsf{vk}$ and $(m, \sigma, \mathsf{vk}, 1) \in \mathsf{sigs}$, $f = 1$. (*This guarantees completeness.*)
2. Else, if $\mathsf{vk}' = \mathsf{vk}$, the aggregate weight of the corrupted parties in $\mathcal{P}$ is strictly less than T, and $(m, \sigma, \mathsf{vk}, 1) \notin \mathsf{sigs}$, $f = 0$ and record $(m, \sigma, \mathsf{vk}, 0)$ to sigs. (*This guarantees unforgeability, if the aggregate weight of the corrupted parties is below the threshold.*)
3. Else, if $(m, \sigma, \mathsf{vk}', b) \in \mathsf{sigs}$, $f = b$. (*This guarantees consistency.*)
4. Else, $f = \phi$ and record $(m, \sigma, \mathsf{vk}', f)$ to sigs.

Finally, send $(\text{Verified}, sid, m, \sigma, \mathsf{vk}', f)$ to P.

Fig. 1. Weighted Threshold Signature Ideal Functionality

operations, e.g., issuing delegation certificates (cf. [21]). The public key vk_i is also used to generate an address α_i. Each pool member p_i pledges the funds of an address α_i (which it owns) to the pool. These funds are the player's stake in the pool and form the player's weight in the weight distribution mapping ω.

We assume the members' stake, i.e., their weight w_i in the pool, is public. Therefore, the weight distribution mapping ω is also public. Furthermore, each member of the pool has its own signature key, and can issue standard signatures through a standard signature scheme. A weighted version for a threshold signature scheme follows by having each party holding as many shares, of the original threshold scheme, as its weight. This approach has the extra advantage that security guarantees of the original scheme are carried straightforwardly into the weighted version. The full description of the WTSS Σ_{thresh} based on ECDSA is presented in Section 4.

Additionally, our construction relies on the consensus sub-protocol $\pi_{consensus}$ to validate a transaction by the elected committee. Specifically, the collective stake pool protocol is parameterized by: i) the validation predicate $\mathsf{Validate}$, ii) the permutation algorithm π_{perm}, and iii) a consensus sub-protocol $\pi_{consensus}$.

Finally, our (modular) protocol is described in a hybrid world whith auxiliary functionalities for established primitives. The functionality $\mathcal{F}_{BC}$ [20] provides a broadcast channel to all parties; $\mathcal{F}_{corewallet}$ [21] enables delegation to the pool; $\mathcal{F}_{wtss}$ (cf. Section 2) is used for weighted threshold signature operations; the Smart Contract Functionality Γ_{reward} realizes the reward distribution mechanism; $\overline{\mathcal{G}}_{simpleLedger}$ is a global Ledger Functionality [24]. Let $\mathsf{HYBRID}^{pool}_{\pi_{pool},\mathcal{A},\mathcal{Z}}$ denote the $\{\overline{\mathcal{G}}_{simpleLedger}, \mathcal{F}_{BC}, \mathcal{F}_{corewallet}, \mathcal{F}_{wtss}, \Gamma_{reward}\}$-hybrid execution of π_{pool} in the (global) UC Framework.

3.2 Part 1: Stake Pool Management

The functionality's first part (Figure 3) includes all operations that are not consensus-oriented. First, establishing a stake pool consists of two parts, defined as corresponding interfaces in the ideal functionality. The pool's members *gather* and jointly decide to create a staking pool; they contact each other, e.g., via off-chain direct channels, agree on the pool's parameters, and generate its key. Importantly, the participants are aware of the total number of participants in the pool, as well as their weights. Then, the members of the pool perform a setup protocol and *register* the new pool via a *registration certificate*, which is signed by the pool's key and published on the ledger. Following, the pool receives rewards for participating in the consensus protocol. The rewards are managed by a smart contract and, at any point, each each party can withdraw their part, which is proportional to the internal stake distribution. Finally, to close the pool, the members sign and publish a revocation certificate.

In more detail, the functionality $\mathcal{F}_{pool}$ interacts with n parties $p_1, \ldots, p_n$ and is parameterized by:

- the validation predicate $\mathsf{Validate}(\cdot,\cdot)$ which, given a transaction τ and a chain $\mathcal{C}$, defines whether τ can be appended to $\mathcal{C}$ (as part of a block);

- the algorithm $\mathsf{blockify}$ which, given a set of transactions, serializes them (deterministically) in a block;
- the probability $\Pi^{\theta,t,n}$ that the elected committee, responsible for a transaction's verification, is corrupted, dependent on the subselection parameter θ and the number of corrupted parties t out of n total parties.

It also keeps the (initially empty) variables: i) the signature threshold T; ii) the public key vk_{pool}; iii) the reward address α_{reward}; iv) the set of valid and unpublished transactions $\mathsf{mempool}$; v) a mapping of parties to weights W; vi) a table of signatures sigs.

Gathering and Registration. The first step in creating a pool is the gathering of parties, in order to collectively create the pool's public key vk_{pool}. Following, the parties create and publish on the ledger the registration certificate cert_{reg}, which contains the following:

- ω: a mapping identifying each member's weight;
- α_{reward}: the address which accumulates the pool's rewards;
- vk_{pool}: the pool's threshold public key;
- σ_{pool}: the signature of $\langle \omega, \alpha_{reward} \rangle$ created by vk_{pool}.

Reward Withdrawal. During the life cycle of the pool, a member may want to withdraw the rewards received up to that point. As per the desiderata of Section 1, any party should be able to do so, without the explicit permission of the other pool's members. Additionally, the rewards that each party receives should be proportional to its stake, i.e., its weight within the collective pool. Reward withdrawal is implemented as the smart contract functionality Γ_{reward}. The contract is initialized with the weight distribution of the pool's members and each member's public key. We assume that the contract is associated with an address and can receive funds, similar to real-world smart contract systems like Ethereum [36]. The state transition functionality Γ_{reward} is defined in Figure 2.

Reward Smart Contract Functionality Γ_{reward}

Γ_{reward} maintains a mapping ω, of parties to weights, and a variable b.
Initialization: Upon receiving $(\text{INIT}, sid, \omega')$, forward it to $\mathcal{S}$. Upon receiving a response $(\text{INIT-OK}, sid, \alpha_{sc})$, set $\omega \leftarrow \omega'$ and return $(\text{INIT-OK}, sid, \alpha_{sc})$.
Balance Update: On receiving $(\text{TRANSACTION}, sid, \tau)$ from $\mathcal{U}$, such that $\tau = \langle \alpha_s, \alpha_r, v, f \rangle$, if $\alpha_s = \alpha_{sc}$ set $b := b - v$, else if $\alpha_r = \alpha_{sc}$ set $b := b + v$.
Withdrawal: Upon receiving $(\text{WITHDRAW}, sid, \alpha, f)$ from the party p, set $r = \frac{w_p}{\sum_{p' \in \omega} w_{p'}} \cdot b$ and return $(\text{TRANSACTION}, sid, \langle \alpha_{sc}, \alpha, r, f \rangle)$.

Fig. 2. The pool's Reward Smart Contract Functionality.

Closing. Eventually, the members halt the operation of the pool. In order to do so, they revoke the pool's registration by jointly producing a revocation certificate cert_{rev}. The certificate is relatively simple, containing a timestamp x announcing the end of the pool and signed by the pool's public key vk_{pool}.

The first part of our functionality definition is given by Figure 3, whereas the management routines, i.e., the first part of the description, of our protocol construction is given by Figure 4.

Collective Pool Functionality $\mathcal{F}_{pool}^{T,\omega}$ (first part)

Gathering: Upon receiving (GATHER, sid) from p, forward it to $\mathcal{S}$. After every party $p_i, i \in [1,n]$ has submitted gather, upon receiving from $\mathcal{S}$ $(\text{GATHER-OK}, sid, \mathsf{vk}_{pool})$, store T and vk_{pool}, add all party-weight pairs (p_i, ω_i) to W, and reply with $(\text{GATHER-OK}, sid, \mathsf{vk}_{pool})$ to all parties.
Pool Registration: Upon receiving $(\text{REGISTER}, sid, W)$ from p, forward it to $\mathcal{S}$. After all parties $p_i, i \in [1,n]$ have submitted $\mathsf{register}$, upon receiving from $\mathcal{S}$ $(\text{REGISTER-OK}, sid, \alpha_{reward}, \sigma_{pool})$, set $\mathsf{cert}_{reg} = \langle (W, \alpha_{reward}, \mathsf{vk}_{pool}, \sigma_{pool}) \rangle$. Then check if $\forall (m, \sigma, b') \in \mathsf{sigs} : \sigma \neq \sigma_{pool}, (\mathsf{cert}_{reg}, \sigma_{pool}, 0) \notin \mathsf{sigs}$; if the checks hold, insert $(\mathsf{cert}_{reg}, \sigma_{pool}, 1)$ to sigs. Finally, store α_{reward} and reply with $(\text{REGISTER-OK}, sid, \mathsf{cert}_{reg})$.
Reward Withdrawal: Upon receiving the message $(\text{WITHDRAW}, sid, \alpha, f)$ from p_i, forward it to $\mathcal{S}$. Then, compute $r = \frac{w_{p_i}}{\sum_{j=1}^{n} w_{p_j}} \cdot r_{pool}$, where r_{pool} is the funds of address α_{sc} as defined in $\overline{\mathcal{G}}_{simpleLedger}$. Finally, return $(\text{TRANSACTION}, sid, \langle \alpha_{sc}, \alpha, r, f \rangle)$.
Closing: Upon receiving (CLOSE, sid, x) from p, forward it to $\mathcal{S}$. After a set of parties B has submitted close for the same x, if $\sum_{p \in B} w_p > T$, upon receiving $(\text{CLOSE-OK}, sid, \sigma_{pool})$ from $\mathcal{S}$, check if $\forall (m, \sigma, b') \in \mathsf{sigs} : \sigma \neq \sigma_{pool}, (x, \sigma_{pool}, 0) \notin \mathsf{sigs}$; if the checks hold, insert $(x, \sigma_{pool}, 1)$ to sigs. Finally, return to all parties $(\text{CLOSE-OK}, sid, \mathsf{cert}_{rev})$, with $\mathsf{cert}_{rev} = \langle x, \sigma_{pool} \rangle$.

Fig. 3. The first part of the Collective Pool Functionality, parameterized with threshold T and weight mapping ω, refers to the creation and management of the pool (the second part is given by Figure 5).

3.3 Part 2: Participation in Consensus

After a pool is set up, the functionality's second part (Figure 5) considers participation in the system, i.e., *validating transactions* and *issuing blocks*. The pool members continuously monitor the network for new transactions, which they collect, validate, and organize in a *mempool*. As mentioned in the introduction, the pool members *remain online* for the entirety of the execution to perform the pool's operations. Specifically, when the pool is elected to participate, the

Collective Pool Protocol $\pi_{pool}^{T,\omega}$ (first part)

Gathering: Upon receiving (GATHER, sid), send (KEYGEN, sid) to $\mathcal{F}_{wtss}$, with sid containing the weight mapping ω and the threshold T. Upon receiving the reply (KEYGEN, sid, vk_{pool}), return (GATHER-OK, sid, vk_{pool}).
Pool Registration: Upon receiving (REGISTER, sid, W), send (INIT, sid, W) to Γ_{reward} and wait for the reply (INIT-OK, sid, α_{reward}). Then, set $m = (W, \alpha_{reward})$ and send (SIGN, sid, m) to $\mathcal{F}_{wtss}$. Upon receiving a reply (SIGN, sid, m, σ_{pool}), return (REGISTER-OK, sid, cert_{reg}), where $\mathsf{cert}_{reg} = \langle (W, \alpha_{reward}, \mathsf{vk}_{pool}, \sigma_{pool}) \rangle$.
Reward Withdrawal: Upon receiving (WITHDRAW, sid, α, f), forward it to Γ_{reward}. Upon receiving a response (TRANSACTION, $sid, \langle \alpha_{sc}, \alpha, r, f \rangle$) return it.
Closing: Upon receiving (CLOSE, sid, x), send (SIGN, sid, x) to $\mathcal{F}_{wtss}$. Upon receiving a reply (SIGN, sid, x, σ_{pool}), return (CLOSE-OK, sid, cert_{rev}) with $\mathsf{cert}_{rev} = \langle x, \sigma_{pool} \rangle$.

Fig. 4. The first part of the Collective Pool Protocol, which describes the set of management operations (the second part is given by Figure 6).

mempool's transactions are serialized and published in a block. Under PoS, the pool participates proportionally to its aggregated member and delegated stake.

To improve performance, we define a distributed mechanism for transaction verification, i.e., a *distributed mempool*. Such a load balancing mechanism increases efficiency by requiring only a subset of the pool's members to verify each transaction. Notably, this is in contrast to the standard practice of Bitcoin mining pools, where the pool's operator decides the transactions to be mined by its members; instead, our approach further reduces these trust requirements.

To construct a distributed mempool, we consider a subselection mechanism to identify the parties that verify each transaction. This mechanism should be: a) *non-interactive* b) *deterministic*, c) *balanced*, i.e., every party should be chosen with the same probability. Subselection is secure if a majority of the elected committee is honest. However, since the adversary may corrupt some pool members, this may not always be the case. We model this uncertainty via the probability $\Pi^{\theta,t,n}$, which depends on the size of the committee and the power of the adversary among the pool's members.

A straightforward way to implement subselection is to assume that the pool's members are ordered in a well-defined manner, e.g., lexicographically. Given the ordered list $L = [p_1, p_2, \ldots, p_n]$ of the pool's members, we use a permutation algorithm $\pi_{perm}(\cdot, \cdot, \cdot)$, which takes i) a transaction τ, ii) a chain $\mathcal{C}$, and iii) the ordered list of pool members L, and outputs a pseudorandom permuted list L_τ. For every transaction τ and a given chain $\mathcal{C}$, the committee responsible for verification consists of the θ first members in L_τ. Naturally, this proposal is rather simple, so alternative, e.g., VRF-based, mechanisms could be proposed to improve performance.

We note that using $\mathcal{C}$ during the subselection mechanism is important to avoid adaptive attacks. Specifically, the chain $\mathcal{C}$ simulates a randomness beacon, such that at least one of its last u blocks is honest, for some parameter u. If $\mathcal{C}$ was not used, the adversary could construct a malicious transaction in such a way that the subselected committee would also be malicious. By using $\mathcal{C}$ as a seed to the pseudorandom permutation, the adversary's ability to construct such malicious transactions is limited. Alternatively, cryptographic sortition [17] could be employed to fully handle adaptive adversaries.

The (honest) members need to always have the same view of the distributed mempool; this is achieved via authenticated broadcast. Assuming a Public Key Infrastructure, as is our setting, it is possible to achieve deterministic authenticated broadcast in $t+1$ rounds for t adversarial parties [28,33,12]. Each time a party adds a transaction to its mempool, it broadcasts it, such that, at any point in time, the honest members of the pool have the same view of the network w.r.t. the canonical chain and the mempool of unconfirmed transactions. We remind that, as shown by Garay *et al.* [14], $\mathcal{F}_{BC}$ can be implemented to ensure adaptive corruptions using commitments. We note that, in existing distributed ledgers, the order with which transactions are added to the mempool does not affect the choice when creating a new block; for instance, transactions of a new block are typically chosen based on a fee-per-byte score. If the order of transactions is pertinent, a stronger primitive like Atomic Broadcast [11] could be employed.

Following, the committee employs a consensus sub-protocol to agree on the transaction's validity. When a party p retrieves a new transaction τ from the network, it broadcasts it as above. Then, each party computes the permuted list L_τ. Each party, which is in the validation committee for τ, computes locally the validation predicate and submits its output to the consensus protocol. The consensus protocol should offer *strong validity*, i.e., if all honest parties should have the same input bit, they should output this bit. Finally, the output of the consensus protocol is broadcast to the rest of the pool. To verify the committee's actions, a party may request the transcript of the consensus sub-protocol.

Finally, to compute the probability of electing an honest committee, we have a hypergeometric distribution, with population size n and $n-t$ honest parties, where a sample of parties of size θ is chosen *without replacement*. Thus, the probability of honest committee majority is: $\mathit{\Pi}^{\theta,t,n} = 1 - \sum_{v=\lfloor \frac{\theta+1}{2} \rfloor}^{\min(\theta,t)} \frac{\binom{t}{v}\cdot\binom{n-t}{\theta-v}}{\binom{n}{\theta}}$.

Following, Figure 5 defines the second part of our functionality, while Figure 6 presents the second part of our protocol.

3.4 The Security of the Conclave Collective Stake Pool

Theorem 1 formalizes the security of π_{pool}; due to space constraints, the full proof is available at the paper's full version [22].

Theorem 1. *The protocol π_{pool}, parameterized by a validation predicate* Validate*, a permutation algorithm π_{perm}, and a consensus protocol $\pi_{consensus}$ securely realizes $\mathcal{F}_{pool}$ with the hybrid execution* $\mathsf{HYBRID}^{pool}_{\pi_{pool},\mathcal{A},\mathcal{Z}}$ *in the global* $\overline{\mathcal{G}}_{simpleLedger}$

Collective Pool Functionality $\mathcal{F}_{pool}^{T,\omega}$ (second part)

Transaction Verification: Upon receiving (TRANSACTION, sid, τ, θ) from p_i, forward it to $\mathcal{S}$. Then send READ to $\overline{\mathcal{G}}_{simpleLedger}$ on behalf of p_i and wait for the reply $\mathcal{C}$. Following, set t as the number of corrupted parties; with probability $\Pi^{\theta,t,n}$ set $b := \mathsf{Validate}(\tau, \mathcal{C})$, otherwise (with probability $1 - \Pi^{\theta,t,n}$), send (TRANSACTION-VER, sid, τ) to $\mathcal{S}$, wait for a reply (TRANSACTION-OK, $sid, \mathcal{C}, \tau, f$), and set $b := f$. Finally, if $b = 1$, insert τ to mempool and send (TRANSACTION, $sid, \mathcal{C}, \tau, b$) to all parties.
Mempool Update: Upon receiving (TRANSACTION, $sid, \mathcal{C}', \tau, 1$) from p_i, forward it to $\mathcal{S}$. Then send READ to $\overline{\mathcal{G}}_{simpleLedger}$ on behalf of p_i and wait for the reply $\mathcal{C}$. If $\mathcal{C}' \prec \mathcal{C}$ and p_i is honest, insert τ to mempool and return (MEMPOOL-UPDATED, sid, τ).
Block issuing: Upon receiving (ISSUE-BLOCK, sid) from a party p, forward it to $\mathcal{S}$. When a set of parties $\mathbb{P}$ has submitted (ISSUE-BLOCK, sid), if $\sum_{j\in[1,m]} W[p_j] > T$, then for every party $p_i \in \mathbb{P}$, send READ to $\overline{\mathcal{G}}_{simpleLedger}$ on behalf of p_i and wait for the reply $\mathcal{C}_i$. If all received chains equal, i.e., are the same chain $\mathcal{C}$, remove every τ in mempool that also exists in $\mathcal{C}$. Then, set $b = \mathsf{blockify}(\mathsf{mempool})$, send (ISSUE-BLOCK, sid, b) to $\mathcal{S}$, and wait for the reply (ISSUE-BLOCK, sid, b, σ_{pool}). Following, check if $\forall(m, \sigma, b') \in T : \sigma \neq \sigma_{pool}, (b, \sigma_{pool}, 0) \notin T$; if the checks hold, insert $(b, \sigma_{pool}, 1)$ to T. Finally, reply with (BLOCK, sid, b, σ_{pool}).

Fig. 5. The second part of the proposed Pool Functionality, which defines the consensus participation operations.

Collective Pool Protocol $\pi_{pool}^{T,\omega}$ (second part)

Transaction Verification: Upon receiving (TRANSACTION, sid, τ, θ), send READ to $\overline{\mathcal{G}}_{simpleLedger}$ and wait for the reply $\mathcal{C}$. Then, set $b = \mathsf{Validate}(\mathcal{C}, \tau)$, compute $L' = \pi_{perm}(\tau, \mathcal{C}, L)$ and initiate protocol $\pi_{consensus}$ with the θ first parties in L' with input b. Upon computing the output of $\pi_{consensus}$, β, send (TRANSACTION, $sid, \mathcal{C}, \tau, \beta$) to $\mathcal{F}_{BC}$ and return it.
Mempool Update: Upon receiving (TRANSACTION, $sid, \mathcal{C}', \tau, 1$), p_i, send READ to $\overline{\mathcal{G}}_{simpleLedger}$ and wait for the reply $\mathcal{C}$. If $\mathcal{C}' \prec \mathcal{C}$, insert τ to mempool and return (MEMPOOL-UPDATED, sid, τ).
Block Issuing: Upon receiving (ISSUE-BLOCK, sid), send READ to $\overline{\mathcal{G}}_{simpleLedger}$ and wait for the reply $\mathcal{C}$. For every τ in mempool, if τ is also in $\mathcal{C}$, then remove τ from mempool. Next, set $b = \mathsf{blockify}(\mathsf{mempool})$ and send (SIGN, sid, b) to $\mathcal{F}_{wtss}$. Upon receiving a reply (SIGN, sid, b, σ_{pool}), return (BLOCK, sid, b, σ_{pool}).

Fig. 6. The second part of our protocol, which describes the set of operations for consensus participation.

model, and $\Pi^{\theta,t,n} = 1 - \sum_{v=\lfloor \frac{\theta+1}{2} \rfloor}^{\min(\theta,t)} \frac{\binom{t}{v} \cdot \binom{n-t}{\theta-v}}{\binom{n}{\theta}}$*, assuming* $\sum_{p \in P_{\mathcal{A}}} w_p < T$*, where* θ *is the subselection parameter for transaction verification,* $P_{\mathcal{A}}$ *is the set of* t *corrupted parties out of* n *total parties,* ω *is the weight distribution of the* n *parties, and* T *is the signature threshold.*

Proof (Sketch). There are multiple points of interest in proving the security of π_{pool}. First, when $\mathcal{A}$ advances a party in the real world, the simulator (i.e., the ideal adversary) $\mathcal{S}$ follows suit in the ideal world; importantly, $\mathcal{A}$ should advance parties correctly, s.t. the security of $\overline{\mathcal{G}}_{simpleLedger}$ is guaranteed and the (honest) pool members are synchronized w.r.t. the ledger state and mempool. Second, regarding the consensus sub-protocol $\pi_{consensus}$, if π_{pool} does not securely realize $\mathcal{F}_{pool}$, a transcript of an execution of $\pi_{consensus}$ exists s.t. the *validity* property of $\pi_{consensus}$ is violated. Finally, security of the *Reward Withdrawal* interface depends on the security guarantees of Kachina [24], which formalizes smart contracts and is the basis for Γ_{reward}, security of *Mempool Issuing* relies on $\mathcal{F}_{BC}$ to ensure that all members are synchronized w.r.t. the mempool and produce the same block without further coordination, while the security of the other interfaces relies on the Weighted Threshold Signature Functionality $\mathcal{F}_{wtss}$.

4 Weighted Threshold ECDSA

Our final contribution is a weighted threshold signature construction, which can be used in the implementation of π_{pool}. Our scheme is based on [15]; specifically, we introduce weights, with each party having as many shares as "units" of weight.

Our construction is a (t, n, ω)-weighted threshold ECDSA. We assume that each player p_i has a associated a weight w_i, identified by the (publicly available) weight function ω such that $\omega[p_i] = w_i$; ω is a parameter in the following two algorithms. Furthermore, we assume an index function $\mathcal{I}(i, w)$ in the secret sharing scheme, which assigns a unique index to each pair (p_i, w_i).

Following, we instantiate the algorithms Thresh-Key-Gen and Thresh-Sign. We outline the changes of our constructions to obtain identifiable abort capability based on [16], to make it suitable for an incentive-compatible pool. We note that some PoS protocols employ a Verifiable Random Function (VRF) [9,17]. Thus, this section's secret sharing techniques can also be used to distribute the VRF key in a weighted manner.

Due to space constraints, we refer to the paper's full version [22] for the computation and communication complexities of both schemes.

4.1 Key Generation Protocol Thresh-Key-Gen$_\omega$

Each party p_i is associated with a public key for the homomorphic encryption E_i and the weight w_i.

- Phase 1: Each party p_i picks its share proportionally to its weight, i.e., w_i shares. Then it commits to them and broadcast them together with its homomorphic encryption key E_i.

- Pick uniformly random local values $u_i^{(1)}, \ldots, u_i^{(w_i)} \in \mathbb{Z}_p$
- Compute $y_i^{(w)} = \mathsf{com}(g^{u_i^{(w)}}) = [C_i^{(w)}, D_i^{(w)}]$, for $\forall w = \{1, \ldots, w_i\}$
- Broadcast $C_i^{(1)}, \ldots, C_i^{(w_i)}$
- Broadcast E_i

- Phase 2: The confirmation of the values is done through opening of commitments, and each value for each weight is secretly shared among all the players. Therefore each player executes as many secret-sharing instance as weight "units" it has, resulting in its combined shares for the secret key $(x_i^{(1)}, \ldots, x_i^{(w_i)})$ proportionally to its weight w_i.
  - Broadcast $D_i^{(1)}, \ldots, D_i^{(w_i)}$
  - Receive the decommitments for $(y_j^{(1)}, \ldots, y_j^{(w_j)})$, $\forall j \in \{1, \ldots, n\}, j \neq i$
  - Perform secret-sharing for each share $u_i^{(1)}, \ldots, u_i^{(w_i)}$, s.t. for each value $u_i^{(w)}$ compute the shares $u_{i,\mathcal{I}(j,w')}^{(w)}$ and secretly send to p_j, with respect to weight $1 \leq w' \leq w_j$ and index $\mathcal{I}(j, w')$, receiving back the share $u_{\mathcal{I}(j,w'),i}^{(w)}$
  - Each player p_i compute its respective set of shares

$$x_i^{(1)} = \sum_{\substack{1 \leq j \leq n \\ 1 \leq w' \leq w_j}} u_{\mathcal{I}(j,w'),i}^{(1)} \ , \ \cdots \ , \ x_i^{(w_i)} = \sum_{\substack{1 \leq j \leq n \\ 1 \leq w' \leq w_j}} u_{\mathcal{I}(j,w'),i}^{(w_i)}$$

    with the values received from other parties p_j.
- Phase 3: For the public key E_i, the module $N_i = p_i \cdot q_i$ for primes p_i and q_i provide zero-knowledge proof for:
  - for p_i and q_i (Proof of knowledge for factoring [34])
  - and $x_i^{(1)}, \ldots, x_i^{(w_i)}$ (Schnorr based)

Note that the joint public-key is $\mathsf{vk} = \prod_{i=1}^{i=n} \prod_{w=1}^{w=w_i} y_i^{(w)}$, whereas the joint secret-key is $\mathsf{tsk} = \sum_{i=1}^{i=n} \sum_{w=1}^{w=w_i} x_i^{(w)}$.

4.2 Signing Protocol Thresh-Sign$_\omega$

We assume a set B of parties p_i that jointly compute a signature.

- Phase 1: Each party selects two tuples of values, each with w_i values, and broadcasts w_i commitments to one of the sets.
  - Pick random values $k_i^{(1)}, \ldots, k_i^{(w_i)} \in_R \mathbb{Z}_p$
  - Pick random values $\gamma_i^{(1)}, \ldots, \gamma_i^{(w_i)} \in_R \mathbb{Z}_p$
  - Define $k = \sum_{i \in B} \sum_{w=1}^{w=w_i} k_i^{(w)}$ and $\gamma = \sum_{i \in B} \sum_{w=1}^{w=w_i} \gamma_i^{(w)}$
  - Compute w_i commitments $\mathsf{com}(g^{\gamma_i^{(w)}}) = [C_i^{(w)}, D_i^{(w)}]$ for $\forall w = \{1, \ldots, w_i\}$
  - Broadcast $C_i^{(1)}, \ldots, C_i^{(w_I)}$
- Phase 2: Each party computes the interpolation coefficients $\lambda_i^{(w)}$ for each share it keeps, that is the shares for weights $w = \{1, \ldots, w_i\}$, taking into account its indexes $\mathcal{I}(i, w)$.

– For $w = \{1, \ldots, w_i\}$ and $w' = \{1, \ldots, w_j\}$, compute the Lagrangian coefficients $\lambda_{i,B}^{(w)} = \prod_{j \in B, w'=1}^{w'=w_j} \frac{-\mathcal{I}(j,w')}{\mathcal{I}(i,w)-\mathcal{I}(j,w')}$
– Compute the values

$$\mathbf{x}_i^{(1)} = (\lambda_i^{(1)}) \cdot (x_i^{(1)}) \ , \ \ldots \ , \ \mathbf{x}_i^{(w_i)} = (\lambda_i^{(w_i)}) \cdot (x_i^{(w_i)}).$$

- Phase 2A - Local Shares: The party p_i executes locally the MtA protocol with the local shares, which are $(k_i^{(1)}, \ldots, k_i^{(w_i)})$ and $(\gamma_i^{(1)}, \ldots, \gamma_i^{(w_i)})$ to compute α and β such that $k_i^{(w)}\gamma_i^{(w')} = \alpha_{i,i}^{(w)(w')} + \beta_{i,i}^{(w)(w')}$ for p_i and $1 \leq w, w' \leq w_i$. Note that both values of the pair $k_i^{(w)}$ and $\gamma_i^{(w)}$ are used which means MtA is executed twice for a given party p_i and weight w.
- Phase 2B - Online Shares: Party p_i executes MtA protocol between its local shares $(k_i^{(1)}, \ldots, k_i^{(w_i)})$ and shares of the remaining parties, other than p_i:

$p_1, \ldots, p_{(i-1)}$	$p_{(i+1)}, \ldots, p_n$
$(\gamma_1^{(1)}, \ldots, \gamma_1^{(w_1)})$	$(\gamma_{i+1}^{(1)}, \ldots, \gamma_{i+1}^{(w_{i+1})})$
$\vdots$	$\vdots$
$(\gamma_{i-1}^{(1)}, \ldots, \gamma_{i-1}^{(w_{i-1})})$	$(\gamma_n^{(1)}, \ldots, \gamma_n^{(w_n)})$

  Like Local Shares, there will be two MtA executions for each pair $k_i^{(w)}$ and $\gamma_i^{(w)}$, i.e., $k_i^{(w)}\gamma_j^{(w')} = \alpha_{i,j}^{(w)(w')} + \beta_{j,i}^{(w)(w')}$.
- Phase 2C - Compute $\delta_i^{(w)}$, for $1 \leq w \leq w_i$ and $1 \leq i \leq n$ the following values by summing the produced values from steps 2A and 2B. Second and third terms from 2A, and the remaining terms from 2B:

$$\delta_i^{(w)} = k_i^{(w)}\gamma_i^{(w)} + \sum_{\substack{w'=1 \\ w \neq w'}}^{w'=w_i} \alpha_{i,i}^{(w)(w')} + \sum_{\substack{w'=1 \\ w \neq w'}}^{w'=w_i} \beta_{i,i}^{(w)(w')}$$

$$+ \sum_{\substack{1 \leq \ell \leq i-1 \\ 1 \leq w' \leq w_\ell \\ j \in B}} \left(\alpha_{i,j}^{(w)(w')} + \beta_{j,i}^{(w)(w')}\right) + \sum_{\substack{i+1 \leq \ell \leq n \\ 1 \leq w' \leq w_\ell \\ j \in B}} \left(\alpha_{i,j}^{(w)(w')} + \beta_{j,i}^{(w)(w')}\right).$$

- Phase 2D - Local Shares: Party p_i executes locally the MtA protocol wi the local shares which are $(k_i^{(1)}, \ldots, k_i^{(w_i)})$ and $(\mathbf{x}_i^{(1)}, \ldots, \mathbf{x}_i^{(w_i)})$ to compute μ and ν such that $k_i^{(w)}\mathbf{x}_i^{(w')} = \mu_{i,i}^{(w)(w')} + \nu_{i,i}^{(w)(w')}$ for p_i and $1 \leq w, w' \leq w_i$.
- Phase 2E - Online Shares: Party p_i executes MtAwc protocol between its local shares $(k_i^{(1)}, \ldots, k_i^{(w_i)})$ and shares of the remaining parties except p_i:

$p_1, \ldots, p_{(i-1)}$	$p_{(i+1)}, \ldots, p_n$
$(\mathbf{x}_1^{(1)}, \ldots, \mathbf{x}_1^{(w_1)})$	$(\mathbf{x}_{i+1}^{(1)}, \ldots, \mathbf{x}_{i+1}^{(w_{i+1})})$
$\vdots$	$\vdots$
$(\mathbf{x}_{i-1}^{(1)}, \ldots, \mathbf{x}_{i-1}^{(w_{i-1})})$	$(\mathbf{x}_n^{(1)}, \ldots, \mathbf{x}_n^{(w_n)})$

Likewise the Local Shares, there will be two executions of the MtAwc protocol for each pair $k_i^{(w)}$ and $\mathbf{x}_i^{(w)}$, that is $k_i^{(w)}\mathbf{x}_j^{(w')} = \mu_{i,j}^{(w)(w')} + \nu_{j,i}^{(w)(w')}$.

- Phase 2F: Compute $\sigma_i^{(w)}$, for $1 \leq w \leq w_i$ and $1 \leq i \leq n$ the following values by summing the produced values from Steps 2D and 2E. Second and third terms from 2D, and the remaining terms from 2E:

$$\sigma_i^{(w)} = k_i^{(w)}\mathbf{x}_i^{(w)} + \sum_{\substack{w'=1 \\ w \neq w'}}^{w'=w_i} \mu_{i,i}^{(w)(w')} + \sum_{\substack{w'=1 \\ w \neq w'}}^{w'=w_i} \nu_{i,i}^{(w)(w')}$$

$$+ \sum_{\substack{1 \leq \ell \leq i-1 \\ 1 \leq w' \leq w_\ell \\ j \in B}} \left(\mu_{i,j}^{(w)(w')} + \nu_{j,i}^{(w)(w')}\right) + \sum_{\substack{i+1 \leq \ell \leq n \\ 1 \leq w' \leq w_\ell \\ j \in B}} \left(\mu_{i,j}^{(w)(w')} + \nu_{j,i}^{(w)(w')}\right).$$

- Phase 3: At this point each party p_i has two sets of values $(\delta_i^{(1)}, \ldots, \delta_i^{(w_i)})$ and $(\sigma_i^{(1)}, \ldots, \sigma_i^{(w_i)})$ from, respectively, Steps 2C and 2F. The party p_i broadcasts the former set, and all parties reconstruct the value $\delta = \sum_{\substack{w=1 \\ i \in B}}^{w=w_i} \delta_i^{(w)} = k \cdot \gamma$ (as defined in Step 1).
- Phase 4: Release w_i commitments computed in Step 1, and use them to compute the r as the first part of the signature.
  - Broadcast the values $D_i^{(w)}$ which open the commitments for $\Gamma_i^{(w)} = g^{\gamma_i^{(w)}}$
  - p_i proves in ZK the knowledge of $\gamma_i^{(w)}$ for $1 \leq w \leq w_i$
  - All compute

$$R = \left(\prod_{\substack{i \in B \\ 1 \leq w \leq w_i}} \Gamma_i^{(w)}\right)^{\delta^{-1}} = g^{\left(\sum_{\substack{i \in B \\ 1 \leq w \leq w_i}} \gamma_i^{(w)}\right) k^{-1}\gamma^{-1}} = g^{\gamma k^{-1}\gamma^{-1}} = g^{k^{-1}}$$

  - Compute the first half of the signature as r=$R \mod p$
- Phase 5: Each player p_i computes $s_i^{(w)} = mk_i^{(w)} + r\sigma_i^{(w)}$, so each player p_i holds the set $(s_i^{(1)}, \ldots, s_i^{(w_i)})$ of shares of the second part of the signature.
- Phase 5A: To build the second half of the signature it is necessary to randomly sample and commit to two value sets:
  - Choose two sets of random values $(\ell_i^{(1)}, \ldots, \ell_i^{(w_i)})$ and $(\rho_i^{(1)}, \ldots, \rho_i^{(w_i)})$ such that $\ell_i^{(w)} \in \mathbb{Z}_p$ and $\rho_i^{(w)} \in \mathbb{Z}_p$.
  - Compute the set $(V_i^{(1)}, \ldots, V_i^{(w)})$ such that $V_i^{(w)} = r^{s_i^{(w)}} g^{\ell_i^{(w)}}$
  - Compute $(A_i^{(1)}, \ldots, A^{(w)_i})$ such that $A_i^{(w)} = g^{\rho_i^{(w)}}$
  - Compute the commitments $([\widehat{C}_i^{(1)}, \widehat{D}_i^{(1)}], \ldots, [\widehat{C}_i^{(w_i)}, \widehat{D}_i^{(w_i)}])$, such that $\mathsf{com}(V_i^{(w)}, A_i^{(w)}) = [\widehat{C}_i^{(w)}, \widehat{D}_i^{(w)}]$
  - Broadcast $(\widehat{C}_i^{(1)}, \ldots, \widehat{C}_i^{(w_i)})$
- Phase 5B: Once all committed values were received, open the commits in order to joint compute V and A:

- Broadcast $(\widehat{D}_i^{(1)}, \ldots, \widehat{D}_i^{(w_i)})$
- Prove in ZK, for each value w, such that $1 \leq w \leq w_i$, the knowledge of $\ell_i^{(w)}$, $\rho_i^{(w)}$ and $s_i^{(w)}$ such that $V_i^{(w)} = R^{s_i^{(w)}} g^{\ell_i^{(w)}}$ and $A_i^{(w)} = g^{\rho_i^{(w)}}$
- Compute:

$$V = g^{-m} \cdot (\mathsf{vk})^{-r} \cdot \prod_{\substack{i \in B \\ 1 \leq w \leq w_i}} V_i^{(w)}, A = \prod_{\substack{i \in B \\ 1 \leq w \leq w_i}} V_i^{(w)}$$

- Phase 5C: Like Step 5A, compute two sets of values $U_i^{(w)}$ and $T_i^{(w)}$ and prove the knowledge of them via ZK proofs. These values are used to guarantee consistency of the shares:
  - Compute the set $(U_i^{(1)}, \ldots, U_i^{(w_i)})$ such that $U_i^{(w)} = V^{\rho_i^w}$
  - Compute the set $(T_i^{(1)}, \ldots, T_i^{(w_i)})$ such that $T_i^{(w)} = A^{\ell_i^w}$
  - Compute the commitments $([\widetilde{C}_i^{(1)}, \widetilde{D}_i^{(1)}], \ldots, [\widetilde{C}_i^{(w_i)}, \widetilde{D}_i^{(w_i)}])$, such that $\mathsf{com}(U_i^{(w)}, T_i^{(w)}) = [\widetilde{C}_i^{(w)}, \widetilde{D}_i^{(w)}]$
  - Broadcast $(\widetilde{C}_i^{(1)}, \ldots, \widetilde{C}_i^{(w_i)})$
- Phase 5D: Once the commitments are received, broadcasts their openings and verify the consistency of the shares:
  - Broadcast $(\widetilde{D}_i^{(1)}, \ldots, \widetilde{D}_i^{(w_i)})$
  - If $\prod_{\substack{i \in B \\ 1 \leq w \leq w_i}} T_i^{(w)} \neq \prod_{\substack{i \in B \\ 1 \leq w \leq w_i}} U_i^{(w)}$, then abort
- Phase 5E: Broadcast the shares of the second half of the signature, and reconstruct it:
  - Broadcast the set $(s_i^{(1)}, \ldots, s_i^{(w_i)})$
  - Compute the second signature share as $s = \sum_{\substack{i \in B \\ 1 \leq w \leq w_i}} s_i$. If (r, s) is not a valid signature, then abort.

4.3 Identifiable Abort

Here we describe the changes required to provide identifiable abort capability considering weights as it is used in our proposed construction. As mentioned earlier, weights can be also introduced in the extended version of [16]; we note that weights can be similarly applied to the scheme of [6], which extends [16]. The changes yield a similar construction as the one presented earlier, and affect only Phase 3, and the substitution of the Phases 5, 5A, 5B, 5C, 5D and 5E, to new Phases 5, 6 and 7. Identification follows similarly to [16], therefore we refer the reader to that work for a full description of the procedure.

Concretely, for the new phases with weights below, consider $w \in \{1, \ldots, w_i\}$:

- Phase 3:
  - All parties reconstruct $\delta = \sum_{\substack{w=1 \\ i \in B}}^{w=w_i} \delta_i^{(w)} = k \cdot \gamma$ and compute $\delta^{-1} \mod p$
  - Compute $(T_i^{(1)}, \ldots, T_i^{(w_i)})$ such that $T_i^{(w)} = g^{\sigma_i^{(w)}} h^{\ell_i^{(w)}}$, and provide a ZK proof of knowledge of $(\ell_i^{(1)}, \ldots, \ell_i^{(w_i)})$ and $(\sigma_i^{(1)}, \ldots, \sigma_i^{(w_i)})$

- Phase 5: All players broadcast $\widetilde{R}_i^{(w)} = R^{k_i^{(w)}}$ and a ZK proof of range (as the ones sent in the MtA on Phase 2) between $R_i^{(w)}$ and $E_i(k_i^{(w)})$. If $g \neq \prod_{\substack{i \in B \\ 1 \leq w \leq w_i}} \widetilde{R}_i^{(w)}$, the protocol aborts.
- Phase 6: All parties broadcast $S_i^{(w)} = R^{\sigma_i^{(w)}}$ and a ZK knowledge proof (as in Phase 3) between $S_i^{(w)}$ and $T_i^{(w)}$. If $y \neq \prod_{\substack{i \in B \\ 1 \leq w \leq w_i}} S_i^{(w)}$, the protocol aborts.
- Phase 7: Each player broadcasts $s_i^{(w)} = mk_i^{(w)} + r\sigma_i^{(w)}$ and sets $s = \sum_{\substack{i \in B \\ 1 \leq w \leq w_i}} s_i$. If (r, s) is not a valid signature, abort.

5 Conclusion

Our work explores a novel design for collective stake pools for Proof-of-Stake ledgers, i.e., pools without a central operator. Our first contribution is a security definition for collective stake pools, which takes the form of the ideal functionality $\mathcal{F}_{pool}$ that articulates the security properties and functions that a collective pool should offer. Following, we propose the concrete protocol *Conclave* which UC-realizes $\mathcal{F}_{pool}$. Our construction incorporates a load balancing mechanism for transaction verification, to boost performance, as well as a Weighted Threshold Signature Scheme (WTSS). Regarding the latter, we present the ideal functionality $\mathcal{F}_{wtss}$ (Section 2) that formalizes this new definition and might be of independent interest, and propose two constructions based on threshold ECDSA. We stress that the collective pool is modular and agnostic to the WTSS implementation, so any scheme that securely realizes $\mathcal{F}_{wtss}$ suffices.

Our design satisfies most of the desiderata outlined in Section 1. Some (e.g., pool proportional rewards or stake reallocation) are dependent on the underlying ledger system's details, therefore are outside of our scope; nevertheless, our design does not pose restrictions in capturing them. The reward functionality Γ_{reward} handles the reward-specific desiderata, while $\mathcal{F}_{pool}$'s first part (Figure 3) covers the requirements for permissioned access and closing of the pool. However, $\mathcal{F}_{pool}$'s handling of stake reallocation and updating of the pool's parameters could be more dynamic, as it currently requires closing and re-creating a pool with the new parameters; a more efficient design is an interesting direction for future research. Additionally, an improvement to the WTSS scheme of Section 4, which would be directly applicable by π_{pool}, could assign a single weighted share to each party, instead of using multiple shares depending on each party's weight.

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