Can Return Forecasts enhance International Asset Allocation? Evidence from the Sum-of-Parts Approach

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Abstract

We examine whether real-time return forecasts are valuable to an investor looking to allocate their portfolio across a wide selection of countries. We expand the Sum-of-Parts (SoP) method for forecasting stock returns to an international setup by adding FX returns as an additional component. We use two different methods to calculate the forecasts. The first method (Empirical Mode Decomposition) uses wavelets to frequency decompose each part into locally independent sub-signals, while the second method combines historical averages and predictive regressions. We then compare the performance of various types of portfolia under the SoP and historical average forecasts, with rebalancing taking place every period. We find that SoP forecasts deliver economic gains over the historical average, especially when the EMD method is implemented. We further demonstrate that economic gains can be generated for investors based in various different countries.

Keywords:

Return forecasting, Sum of Parts, Global asset allocation, EMD, International portfolio optimisation

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1. Introduction

This paper investigates the crucial question of whether real-time equity index return forecasts can help investors to improve their portfolio allocation internationally across countries. Generally, the return forecasting literature examines whether it is possible to beat a benchmark within that country and whether the forecast can help improve portfolio allocation between the domestic risk-free rate and the domestic equity index (e.g. Welch and Goyal (2008); Campbell and Thompson (2008); Ferreira and Santa-Clara (2011); Jordan et al. (2017), Jordan et al. (2014)). A separate important literature focuses on the issue of portfolio allocation internationally; this body of work tends to conduct portfolio allocation in-sample and using historical mean and variance as inputs into the decision-making problem (Solnik (1974); Solnik and Noetzlin (1982); Solnik (1993); Errunza et al. (1999)). However, an important under-researched question is how valuable is predictability to a real-time investor who can allocate funds globally? To our knowledge this question is yet to be fully addressed. The goal of this paper is to quantify the extent to which predictability can enhance the economic value to international investors.

In contrast, our approach is to conduct out-of-sample analysis using forecasted returns in the international portfolio allocation problem. We build on the work of Ferreira and Santa-Clara (2011), who demonstrate that decomposing the equity return into separate components and then forecasting each one separately can lead to substantial improvements in forecast performance in the US. We extend their framework to a global allocation setting by introducing the change in exchange rates as an additional component; thus, all equity returns can be quoted in the same currency. This allows for cross-country portfolio allocation where returns are in the reference currency of the investor (e.g. the US dollar for a US-based investor). This enables us to address our key research question: how valuable are return forecasts to an agent with a global investment mandate?

We produce Sum-of-Parts forecasts with two different methods. The first relies on the original paper (Ferreira and Santa-Clara (2011)) and involves a combination of predictive regressions and historical averages. The second relies on frequency decomposition of the different SoP components via wavelets (Faria and Verona (2018)) which we replace with Ensemble Mode Decomposition (EMD) based on Wu and Huang (2009). We opt for EMD compared to EEMD because mode mixing does not appear for our low frequency dataset and therefore implementing EEMD is unnecessary. We then use these real-time forecasts as expected returns in the mean-variance optimisation of an international portfolio where the investor allocates wealth to the indices of 44 countries, and compare portfolio performance with the standard case where the forecast is the historical average.

Overall, we find that model forecasts are considerably more accurate than the (historical average) benchmark forecast for the large majority of countries. Encompassing tests reveal that, when these three forecasts are considered, the optimal weight on the historical average forecast is never statistically different from 0 for any country but in some cases the optimal weight on the model forecast is statistically different from 0. Furthermore, we find that using the model forecasts for international portfolio allocation in real time increases portfolio performance relative to using the benchmark forecasts. We demonstrate this is the case using a range of different specifications.

2. Literature review

We build on the important work of Ferreira and Santa-Clara (2011) who demonstrate that decomposing the equity return into three components (price multiple growth, earnings growth and dividend-price ratio) and then forecasting each part can lead to better forecast accuracy than a historical average benchmark. They apply shrinkage to the estimates which help to reduce estimation error. Overall, they find substantial gains could be made by a US investor who applied this approach in a (domestic) two-asset portfolio allocation exercise. Sum-of-Parts leads to significant economic and statistical gains out-of-sample over the historical mean that range between 1.3% on a monthly and 13.4% on an annual basis. On a monthly basis, the gains can be increased to 2.6% if the forecast relies on the sum of parts obtained via a wavelet frequency decomposition of the returns time series (Faria and Verona (2018)).

In domestic settings there is no need to model the exchange rate; however, in an international setting the currency component will need to be incorporated. Conventionally, it is thought that exchange rates follow random walks (Meese and Rogoff (1983)) and therefore the numeraire might not matter for forecasting the mean; however it could still affect the standard error of the coefficient estimate potentially due to noise (Jordan et al., 2015). The random walk of exchange rates has been challenged in recent times by studies reporting some predictive power for currency returns (Lustig and Verdelhan (2007), Ang and Chen (2010), Burnside et al. (2011), Barroso and Santa-Clara (2015), Menkhoff et al. (2012)). If this is the case, then forecasting the currency return could improve forecast power overall.

3. Data and Methodology

3.1. Expansion of the Sum-of-Parts methodology

We begin this section by outlining the Sum-of-Parts methodology of Ferreira and Santa-Clara (2011) and demonstrating how it can be extended to be utilised by an investor looking to invest in multiple countries. We expand the scope from a single asset representing a stock, a portfolio or an index, to an international portfolio, with the aim to conduct a cross-country portfolio optimisation exercise and examine if Sum-of-Parts provides an advantage in such an environment. Technically, we introduce exchange rates as an additional factor and use the forecast of each country as its expected return in the optimisation of a portfolio that allocates wealth across different countries. We then examine whether Sum-of-Parts provides material gains to international investors which are exposed to currency exchange risk. The original Sum-of-Parts approach is based on aggregating the separate forecasts of three components of stock market returns into a single return forecast. The three components are the dividend-price ratio, earnings growth and the growth of the price-earnings ratio. The total (gross) return of the stock market index consists of capital gains and the dividend yield. For return R, dividend D and earnings per share E

$$1 + R_{t+1} = \frac{P_{t+1}}{P_t} + \frac{D_{t+1}}{P_t} = \frac{P_{t+1}/E_{t+1}}{P_t/E_t} \times \frac{E_{t+1}}{E_t} + \frac{D_{t+1}}{P_{t+1}} \times \frac{P_{t+1}}{P_t} \tag{1}$$

"The fraction of the price-earnings ratio at t and $t+1$, $\frac{P_{t+1}/E_{t+1}}{P_t/E_t}$, can be written as $1 + GM_{t+1}$ where GM is the growth rate of the PE ratio. Similarly, the earnings fraction E_{t+1}/E_t can be written as $1 + GE_{t+1}$ where GE is the earnings growth rate. This allows the first term to be written as $(1 + GM_{t+1})(1 + GE_{t+1})$. The second term, using the dividend-price ratio DP and similar rewriting, allows the dividend yield to be expressed as $DP_{t+1}(1 + GM_{t+1})(1 + GE_{t+1})$. Equation (1) then becomes

$$
\begin{aligned}
1 + R_{t+1} &= (1 + DP_{t+1})(1 + GM_{t+1})(1 + GE_{t+1}) \\
r_{t+1} &= gm_{t+1} + ge_{t+1} + dp_{t+1}
\end{aligned}
\tag{2}
$$

The bottom row is the expression in the top row in logs, where dp is the log of the dividend-price ratio, ge the log of the earnings growth rate and gm the log of the price-earnings growth rate. Sum-of-Parts is markedly superior to historical mean forecasts, providing out-of-sample R^2 of 1.3% with monthly data and 13.4% with annual data.

We expand the model to an international setup by introducing a currency return component, since a change in the exchange rate of a country where portfolio wealth has been allocated with the reference currency of the investor would affect total returns. We thus include the domestic currency-to-US dollar exchange rate as a fourth component of returns. For an international investor who uses a currency different than the domestic one, the change of the spot exchange rate S between t and $t + 1$ generates further returns and the price ratio can be written as $P_{t+1}S_{t+1}/P_tS_t$. Since the capital gains need to be measured in the investor's home currency (or reference currency, such as US dollars), the stock price is multiplied by the spot exchange rate. Following similar formulations, Equation (1) becomes

$$
1 + R_{t+1} = \frac{P_{t+1}S_{t+1}/E_{t+1}}{P_tS_t/E_t} \times \frac{E_{t+1}}{E_t} + DP_{t+1} \times \frac{P_{t+1}S_{t+1}}{P_tS_t}
$$

$$
= (1 + GM_{t+1})(1 + GE_{t+1})(1 + FX_{t+1}) + DP_{t+1}(1 + GM_{t+1})(1 + GE_{t+1})(1 + FX_{t+1}) \Leftrightarrow
$$

$$
\begin{aligned}
1 + R_{t+1} &= (1 + GM_{t+1})(1 + GE_{t+1})(1 + FX_{t+1})(1 + DP_{t+1}) \\
r_{t+1} &= fx_{t+1} + gm_{t+1} + ge_{t+1} + dp_{t+1}
\end{aligned}
\tag{3}
$$

where FX (fx)is the growth rate, or return, of the (log) exchange rate of the domestic currency with the US dollar, equivalently to Equation (2).

3.2. Data

The data frequency is quarterly and the sample period ranges from June 1973 to November 2018 containing end-of-quarter values. Due to differences in data availability between countries, we consider two different approaches. In the first case (all countries present), we limit our sample based on the country with the shortest time series (Poland) and use a

5-year (20 observations) window for the predictive regression. This creates a sample of 44 countries and quarterly time series of 99 observations, starting on April 1994. In the second case (sequential), new countries are introduced to the portfolio as sufficient data becomes available with a 10-year (40 observations) window for the predictive regression. There are 16 countries with fully available data on June 1973[1], which amounts to 183 observations. Norway is introduced in January 1980, Sweden in January 1982, Italy and Malaysia in January 1986, and after that a new country is introduced generally every six months until Poland, where all are available. Descriptive statistics and the order of country introduction follow Table 1. All data comes from Datastream apart from the risk-free rate, which is proxied by the US 3-month Treasury bill available on FRED.

Table 2 presents two arithmetic examples where decomposing the return to its four constituents yields almost identical results to calculating the return directly. The first example uses artificial values and demonstrates perfect equality between total returns and the sum of the decomposed constituents. For a domestic investor, total return is exactly equal to the sum of ge, gp and dp. For an international investor, total return in USD is equal to the sum of ge, gp, dp and fx, and the difference between the two returns is exactly equal to the percentage change of the exchange rate. In the second example, UK values on January and February 1973 are used. There is a marginal difference of 0.0003 between total return and the sum of the three parts, but the difference between returns is again exactly equal to the return on FX. This demonstrates that measuring the constituents of returns separately and the returns themselves is virtually the same in terms of accuracy.

3.3. Optimisation of the international portfolio

The international investor allocates portfolio wealth to each country index i and a risk-free asset according to mean-standard deviation optimisation. The vector of stock market Sum-of-Parts forecasts is used as the expected returns in a typical Markowitz mean – standard deviation minimization exercise with a risk-free asset where optimal weights are can be

[1]These countries are Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Ireland, Japan, the Netherlands, Singapore, Switzerland, the UK, the USA and South Africa. Brazil and Russia are not included in the sample due to lack of data.

restricted between zero and one or unrestricted. For a vector of optimal portfolio weights w, a vector of forecasted (expected) returns $r_{fc,i}$,a risk-free rate r_f , risk aversion $\gamma = 2$, covariance matrix Σ and portfolio return r_p, the investor seeks to minimise portfolio variance, $\min_w \frac{1}{\gamma}\boldsymbol{w'\Sigma_p w}$ subject to the following constraints, depending on whether short-selling is allowed or not:

$\boldsymbol{w'1} + (1 - \boldsymbol{w'1}) = 1$ (all weights, risky and risk-free, sum to 1)

$E(r_p) = (1 - \boldsymbol{w'1})r_f + \boldsymbol{w'}r_{fc,i}$ where $\boldsymbol{w'1}$ is the sum of all elements in $\boldsymbol{w}$

$0 \leq w_i \leq 1$ (short-selling is forbidden, restriction omitted if allowed)

Solving this problem for each period leads to a vector of optimal weights $\mathbf{w}$ based on the Sum-of-Parts forecasts, which the investor uses in the allocation of portfolio wealth over the next period. When the next period arrives, the realised portfolio returns are observed. We compare the performance of the Sum-of-Parts approach with the standard CAPM case where the forecast is the historical average. The introduction of the third constraint changes the quadratic programming optimisation problem from allowing negative weights and thus short-selling, which has a known closed-form solution available in Pennacchi (2008), to prohibiting short-selling. In that case the problem can only be solved using numerical methods. We provide results on both cases with and without a risk-free asset. The covariance matrix is calculated on the same rolling window as the predictive regression (i.e. 20 or 40 last observations).

Since the solution for the optimal portfolio weights under a linear constraint without short-selling can only be found using numerical methods, we apply the direct and the iterative methods to maximise the Sharpe ratio. The direct method relies on turning the function of the Sharpe ratio into a quadratic expression and using a numerical algorithm to approximate the solution. Two possible candidates are the interior-point-convex and trust-region-reflective algorithms. The interior-point-convex algorithm proposes predictor-corrector steps that fall strictly within the constrains, after simplifying the problem if possible, and stops when an optimal solution has been found. The trust-region-reflective algorithm relies on the interior-reflective Newton method, which uses proposed consecutive neighbourhood regions of a function (trust regions) to gradually lower its value after a number of iterations. A similar alternative is the active-set algorithm. The iterative method relies on producing iterations of the efficient frontier in order to find the portfolio that maximises the Sharpe ratio. The consecutive interpolations gradually lead to an optimal solution, but the method is able to produce only local solutions and is relatively slower.

A point of note is the fact that for the sequential case the covariance matrix for each step is not guaranteed to be positive semi-definite. Although there are no missing observations in the sample, the length of each return series is different. To construct the covariance matrix for series of unequal lengths, the correct statistical process is to calculate each pairwise covariance based on the data length of the shortest series. This may create numerical or precision errors and may lead to the first eigenvalue of the covariance matrix to be almost equal to zero but negative. When that issue appears, the nearest symmetric positive semi-definite covariance matrix in the Frobenius norm to an arbitrary real matrix A is shown to be $(B+H)/2$, where H is the symmetric polar factor of $B = (A + A')/2$ (Higham and Higham (1998)). In our case, the resulting differences in both the eigenvalues and the elements of the approximate matrix are miniscule. In practice, the results under the direct case are left unchanged if the issue is not treated but the results for the iterated method, which produces local solutions and can still function numerically with a non-positive semi-definite matrix, are very slightly altered. We consider this point to be of use to the interested reader, although it does not lead to a material change in our results or statistical approach.

4. Performance tests

We use Theil's U to measure whether the Sum-of-Parts forecasted returns are an improvement compared to forecasts based on the historical average (HA) of all past returns. The statistic for country i is defined as

$$U_{i,T} = \sqrt{\frac{\sum_{t=1}^{T}(R_{it} - R_{it,SoP})^2}{\sum_{t=1}^{T}(R_{it} - R_{it,HA})^2}} \tag{4}$$

Where R is realised returns, R_{SoP} the Sum-of-Parts forecast and R_{HA} the historical average forecast. A Theil's U lower than 1 means that the Sum-of-Parts method performs better than the historical average, while the opposite means that HA provides a better forecast. The difference between Theil's U and 1 represents an improvement in percentage terms.

To determine independence of information we use the Harvey, Leybourne and Newbold HLN forecast encompassing test (Harvey et al. (1998).

Unlike the Clark and West (2007) test, which has equal forecasting accuracy, the HLN test can be used to compare forecasts from non-nested models as well. It examines two competing non-nested models and gives the optimal weight on the forecast (lambda) as well as enabling the testing the null hypothesis that the optimal weight is 0. HLN is also preferable to the Diebold and Mariano (DM) test due to our relatively small sample, although both tests produce a statistic compared to a t-statistic. The test examines whether one of the forecasts encompasses all relevant information from the other. The idea is that the forecast with the worse performance may contain some information that is not fully incorporated in the better performing forecast, then a combination is preferable. However, if no such information is contained, the better performing forecast "encompasses" the worse performing one and can be used independently. The HLN encompassing test (Harvey et al. (1998)) is an evolution of the DM test (Diebold and Mariano (2002)). The DM statistic is defined as

$$DM = \frac{\bar{d}}{\sqrt{Var(\bar{d})}}$$

Where $d_t = L(e_{i,t})–L(e_{j,t})$, t = 1,. . . , T is a loss differential series, L(.) a loss function (e.g. mean square error) and $e_{i,t}, e_{j,t}$ are two forecast error series. Some common definitions for the loss differential d_t are $d_t = e_{i,t}^2 - e_{j,t}^2$ and $d_t = |e_{i,t}| - |e_{j,t}|$. The HLN statistic modifies d_t to $d_t = (e_{i,t}–e_{j,t})e_{i,t}$ and the DM statistic as

$$HLN = T^{-1/2}\left(T + 1 - 2k + T^{-1}k(k-1)\right)^{1/2} DM \tag{5}$$

for k-step ahead forecasts and dependence between them up to lag k-1. The null hypothesis $E(d_t) = 0$ (equiv. MDM=0) is that the forecast of model i encompasses the forecast of model j. Rejecting H0 implies that forecast j stays in the forecast set.

We apply the HLN test on the collected portfolio weights for each country during the forecast period under Sum-of-Parts and Historical Average (HA). In our context, we use the test to assess country specific performance and the importance and contribution of a country to portfolio returns. The null hypothesis is that a country's portfolio weight is zero, i.e. nothing is invested in that country's stock index by examining whether a country's portfolio weight is statistically different than zero. The alternative hypothesis is that it is positive, i.e. this country generates a fraction of the portfolio's return. Specifically, the null hypothesis of the HLN test is for the left section that the SoP forecast has a weight of zero when combined with the historical average forecast. The alternative hypothesis in the left section is that the SoP forecast is not encompassed by the historical average forecast, i.e., the SoP forecast contains information above and beyond that in the historical average forecast. l_1 gives the optimal estimated weight on SoP and p_1 is the p-value associated with the HLN test. A p-value of less than 0.10 indicates that the weight on SoP is statistically different from 0 at the 10% significance level or better when this forecast is added as a second explanatory variable to a single regression model of the historical average return.

4.1. Forecasting methodologies

Sum-of-Parts decompositions allow separate forecasting methodologies for each part based on their individual characteristics and empirical facts. The base case SoP method uses a combination of predictive regressions and historical averages. We opt for using the last observed value of dp as the forecast for $t+1$, fx and ge are forecasted as the historical average of all past available observations at time t and gm is forecasted by a predictive regression on the log of the price-earnings ratio specified as $gm_{t+1} = \alpha + \beta \times log(PE_t) + \epsilon_{t+1}$. However, the forecasting accuracy of Sum-of-Parts can be improved by decomposing the individual components. Specifically, the predictive power of the price-earnings growth rate, one of the most important components, is low (Dai and Zhu (2020)). Faria and Verona (2018) apply wavelet decomposition and sum only some of the frequency decomposed parts, achieving significant statistical and economical gains over historical mean forecasts and a monthly out-of-sample R^2 of 2.60%. Further out-of-sample improvement can be achieved by Empirical Mode Decomposition (EMD), first introduced by Huang et al. (1998), and Ensemble Empirical Mode Decomposition (EEMD), which analyse the original time series (signal) to a small number of independent (locally orthogonal), zero-mean amplitude and frequency modulated components called intrinsic mode functions (IMFs), plus any residuals. The two methods are designed to extract signals from non-stationary and non-linear data, rely on the local timescale and extrema and are thus adaptive and highly efficient. The basic idea is that such data can be analysed to different intrinsic mode functions (IMFs), which are in essence oscillators. These intrinsic oscillatory modes can be identified according to their distinct features over time (time stamps) and decomposed. The key property is whether an oscillator crosses zero between two extrema, which allows for its separation from the main signal. The algorithm treats each IMF as a sub-signal at a local level and separates them into locally non-overlapping scale components. It breaks down a signal $x(t)$ into its component IMFs obeying two properties: 1) and IMF has only one extremum between two subsequent zero crossings 2) a mean value of zero. , which implies stationarity but does not prevent amplitude modulation or changing frequency. In more detail,

- assume a time series (signal) $x(t)$ which needs to be decomposed to n IMFs $x_n(t)$ and a residual $r(t)$. Define an input signal $h(t)$ to be analysed. Initialise $h(t) = x(t), n = 1$ and the sifting step $k = 1$.

- for $h(t)$ identify local minima/ maxima, create the upper envelopes $s_u(t)$, $s_d(t)$ and subtract their mean $m(t) = (s_u(t) - s_d(t))/2$ from $h(t)$.

- if $h(t) - m(t)$ does not fulfil the requirements of an IMF, then set $h(t) - m(t)$ as input signal and repeat the process (increase k by 1). This process is often called "sifting".

- if $h(t) - m(t)$ fulfils the requirements of an IMF then store it as $x_n(t)$ and calculate $r(t) = h(t) - x_n(t)$. If $r(t)$ is not a residual then set $h(t) - x_n(t)$ as input and repeat from (ii), increasing n by 1. If $r(t)$ is a residual then the process ends. The original signal can be reconstructed as $x(t) = \sum x_n(t) + r(t)$

The stoppage criteria for sifting and for identifying residuals may vary. A residual typically contains only one extremum, is a constant or a monotonic slope. Sifting is calibrated to stop at a threshold, e.g. if the input signal variance falls below a level, or according to the signal's energy ratio (the ratio of the energy of the signal at the beginning of sifting and the average envelope energy). A concise discussion of EMD and its practical features can be found in Zeiler et al (2010).

Dai and Zhu (2020) combine SoP and EEMD to find a monthly out-of-sample R2 above 20%. However, they use the frequency-decomposed parts that improve the stock return forecast and leave out those that reduce predictability. To avoid look-ahead bias, we combine SoP with EMD without removing any components. We select EMD over EEMD because, when EMD is applied to our low frequency dataset, mode mixing does not appear and therefore EEMD is not particularly advantageous. We apply EMD on each component series and get between 3 and 5 IMFs, plus the residuals. We then sum IMF2 with IMF3 and IMF4 with IMF5, and conduct (AR) predictive regressions on IMF1 and the two sums over a rolling window of 40 observations. We then aggregate the results of the predictive regressions with the last known residual for the corresponding time stamp of the original time series[2]. This produces a set of forecasts for that component, and the process is applied to all four parts. The regression windows for EMD forecasts are 20 observations when all countries are present and 40 when they are introduced sequentially.

5. Empirical results

5.1. Forecast accuracy and performance

Thus far, the forecast accuracy of Sum-of-the-Parts (SoP) and EMD methods have primarily focused on the US market. An open question is how well does these methods perform for other equity markets? In particular, firstly which method performs best internationally and secondly how well do these methods perform in emerging markets?

Consequently, our empirical analysis begins by examining the forecast accuracy of the SoP method for each country in our sample denominated in US dollars. We comprehensively cover this by examining two datasets and two estimation methods. The results are affirmative. Table 1 reports Theil's U, out-of-sample R^2 and mean squared errors for logged returns. We find that for quarterly data the SoP method performs better than the historical average for the majority of the countries. However, the most striking result is that EMD produces

[2]The IMF and residual series have the same length (T) as the component series. For a regression window of 40 observations, the residuals between 41 and T are used, since the series produced by the AR estimation has length T-40.

much better forecasts than either the original SoP approach of predictive regressions or the historical average. In the sequential case, EMD- based Sum-of-Parts outperforms the historical average in 38 countries, while the original SoP outperforms the historical average in 33 out of 44 countries (Table 1 (a)). The greatest improvements can be found in Pakistan (28.95%) for EMD Sum-of-Parts and Greece for the original approach (2.64%), while the average improvements are 12.46% for EMD Sum-of-Parts and 0.96% for original SoP respectively. When all countries are present, EMD Sum-of-Parts performs better for 40 countries, with Pakistan improving the most in both cases (32.65% for EMD Sum-of-Parts, 6.25% for original SoP) (Table 1 (b)).

Out-of-sample (OOS) R^2 is vastly in favour of EMD Sum-of-Parts, with Pakistan reporting the highest OOS R^2 (54.46%) and 7 countries being above 30%, compared to a maximum OOS R2 of 5.72% (UK) and 5 countries above 3% for original Sum-of-Parts in the sequential case. When all countries are present, the results improve further. The maximum OOS R^2 for EMD Sum-of-Parts is 56.60% (Pakistan) with 6 countries having values above 40%, while for original Sum-of-Parts the maximum is 10.66% (Pakistan) with three countries above 10%. This implies substantial gains for both forecasting methods which are, however, vastly greater when EMD is used. In the sequential case, EMD Sum-of-Parts leads to 38 countries with positive OOS R^2 compared to 25 for the original case, demonstrating an improvement in forecasting performance for the vast majority, while with all countries present the respective numbers are 40 and 35.

Forecast accuracy is tested via a one-sided t-test for mean squared errors (MSE-t test), similar to the MSE-F tests used in Vivian et al (2013). The test assesses whether the forecast error from Sum-of-Parts is smaller than the historical average for each country in the sample. In terms of statistical significance, the MSE-t tests reveals that for EMD (original) Sum-of-Parts there is statistical outperformance in 26 (5) countries at the 10% significance level for the sequential case and 30 (12) for all countries present. However, this, at least partly reflects the well-known lack of power for this test which is unfortunately an issue that has not yet been resolved in the context of non-nested models. Differences in development or geographical location do not seem to play a role.

Table 2, panels (a) and (b) contain the results for the HLN encompassing tests. The results for the portfolio weights under SoP forecasts are designated by $(l_1/\lambda_1, p_1)$, and for the portfolio weights under the historical average benchmark by $(l_2/\lambda_2, p_2)$. For the sequential case under EMD Sum-of-Parts (Panel a), the optimal weight λ_1 is positive for all countries. Further, the optimal weight is greater than 0.5 in 38 of the 44 countries, which signifies more weight on the SoP forecast than the benchmark. The encompassing test results report that the weight on the SoP forecast is statistically different from 0 at the 10% level for all 44 countries (p_1). By contrast, for the historical average forecast, the optimal weight λ_2 is statistically significant only for 20 countries ($p_2 < 0.1$) and greater than 0.5 for 6. Three countries report negative weights, indicating that for those countries there is no value to following the historical average forecast at all. Original Sum-of-Parts for the sequential case still performs better than the historical average. 17 countries report statistically significant weights at the 10% level and 25 report weights above 0.5, while for the historical average there are 19 countries with weights above 0.5, 8 with statistically significant and 9 with negative weights.

The results are stronger in the all countries present case (Panel b), where the improvement for original Sum-of-Parts is considerable. 35 countries now report weights above 0.5, 20 are statistically significant and only 2 have negative weights, while for the historical average 9 countries are above 0.5, 7 are statistically significant and 22 have negative weights. Thus, overall, our results suggest that the SoP forecasts are greatly preferred to the historical average forecasts when EMD is used. However, for many countries the difference is not statistically significant. Consequently, how well these models perform in portfolio allocation tests will be of great interest to see how these competing approaches compare from the perspective of a real-world investor looking to allocate their portfolio across countries.

5.2. Portfolio performance for a US-based investor

We now begin to answer to the key question of this paper how valuable are return forecasts to an investor looking to allocate funds globally. The empirical results are presented in in Table 3, Panels a, b and c, which present the economic value to an investor based in the US (base case) and a selection of developed and emerging economies, based on Sharpe ratios and certainty equivalents. The constrained (no short-selling) case is reported in Panel a, the unconstrained (short-selling allowed) case in Panel b and the all equity case (no risk-free asset) in Panel c. Panel c reports constrained (C) and unconstrained (UC) portfolia jointly Each panel contains 9 countries (US, UK, Germany, Japan, Switzerland, South Africa, India, China, Chile) and for each country annualised Certainty Equivalents (CE), Sharpe ratios (SR), portfolio returns (R) and standard deviations (SD) are reported for the forecasts being the historical average (HA), the original Sum-of-parts method (OR SoP) and EMD Sum-of-Parts (EMD SoP) over the 73-18 (sequential) and 93-18 (all countries present) periods as well as the direct and iterative methods for calculating optimal portfolio weights.

For the US (Panel a), it is clear that the SoP method leads to increases in the Sharpe ratio over the full sample period. The Sharpe ratios from the EMD SoP method are huge in comparison to the other approaches; for EMD SoP using iterative weights the Sharpe ratio is 1.24 which is much higher than the original SoP, 0.49, and the historical average benchmark, 0.29. For the unconstrained portfolio (Panel b), the results are again greatly in favour of EMD SoP. Both the iterative and the direct portfolios are slightly higher but the direct method now produces a higher Sharpe ratio of 1.56 compared to 0.22 for the original SoP and a negative ratio of -0.07 for HA. When the allocation is purely amongst risk assets, i.e. the country equity indices (Panel c), we have a similar picture. The Constrained results for the Sharpe Ratios are 1.24 for EMD SoP, which is still considerably higher than 0.49 for original SoP and 0.39 for HA, while the Unconstrained portfolia produce a substantially increased Sharpe ratio of 2.15 compared to 0.6 and 0.31 respectively.

On certainty equivalents, there are modest gains from implementing the original SoP method of about 2.0% p.a. whereas for the sequential EMD approach the gains are 8.80% using the iterative approach for the constrained portfolio (Panel a) and 23.20% for the all-equity approach (Panel c); the magnitude of the gains for the fully approach reflect the high returns and Sharpe Ratios generated. The unconstrained portfolio shows even an greater improvement of almost 3% for OR Sop and 25% for EMD SoP (Panel b), while the all-equity unconstrained case the gains are almost 7% and 76% (Panel c). The results for the 93-18 period are qualitatively similar. Specifically, the EMD SoP maintains Sharpe ratios above 1.2 for the iterative (Panel a) and the all-equity approach (Panel b). The original SoP has slightly higher Sharpe ratio at around 0.57. The HA increase to 0.4 but the HA fully remains about the same. The economic gains are also of similar magnitude for the EMD SoP method at 8.57% for the iterative approach and 21.45% for the fully approach. The gains for the original SoP method remain positive and for the fully approach increase to 4.0%.

The Sum-of-Part forecasts lead to a vast increase in portfolio performance compared to historical average in almost all cases for both SoP methods. The increase is most pronounced when a risk-free asset is not used (Panel C) and when short-selling is allowed. However, the portfolia that use EMD rather than the original approach for Sum-of-Parts forecasts report Sharpe ratios that are often twice as high, or higher, reaching values of 2.91 (Panel b 93-18) and 3.1 (Panel c UC). When compared to the Sharpe ratios generated by portfolia using historical averages, the difference can be more than four times higher. Certainty Equivalents follow the same pattern as the Sharpe ratios across all panels, with EMD SoP often reporting gains of 20-25%. The increased performance is attributed to the forecast improvement of the FX and GE components under EMD SoP. The parts forecasted by predictive regressions or using the last known value performe quite worse by comparison, leading to a worse SoP forecast.

Although there is not a definite pattern on whether the direct or iterative method for calculating optimal portfolio weights is preferable, the iterative method appears to have an advantage in the absence of short-selling. Across the entire Panel a (constrained portfolia) the iterative method produces higher Sharpe Ratios, some times substantial, for both EMD SoP and OR SoP forecasts. In Panel b (unconstrained portfolia), the direct method outperforms the iterative method for EMD but there is no clear pattern for OR SoP. On the other hand, a comparison between including a risk-free asset and having an all-equity portfolio is revealing. For constrained portfolia, investing in a risk-free asset or not does not affect the Sharpe Ratio in the US - the respective values for the iterative case in Panels a and c are very similar for both samples. On the other hand, if short-selling is allowed, the differences between Panels b and c are much more pronounced. For sequential EMD SoP, the Sharpe ratios are 1.56 (direct), 1.33 (iterative) and 2.14 (all equity) while in the 93-18 sample the values are 2.91 (direct), 2.13 (iter) and 3.12 (all equity). The same holds for OR SoP, although with considerably lower Sharpe Ratios. This demonstrates that the more aggressive the allocation of a portfolio the more important forecast accuracy becomes, as it leads to considerably increased performance. A better performing method can lead to exponential financial and economic gains.

5.3. Portfolio performance for different home countries

To control for country bias in our results and our data, we conduct the same estimations using 8 alternative home countries. The results show that the method performs comparably regardless of the domicile of the investor. Broadly, the results are qualitatively similar, although there is moderate variation in some of the magnitudes. Firstly, in terms of the HA there is substantial variation in performance depending on the domicile of the investor. In Panel a, for both sample periods EMD SoP Sharpe Ratios are typically between 1 and 1.3 (iterative), OR SoP values range between 0.3 to 0.65 and HA values range between 0.2 and 0.4. The respective Certainty Equivalents range between 10 and 12% with some upwards exceptions, 0.5 to 2.5% (OR SoP) and close to zero, if not negative (HA). This is a markedly homogeneous pattern that does not change in Panel b, but more variation and higher values appear. EMD Certainty equivalents are around 25% while OR SoP CE are 5-10%. However, in the 93-18 sample, the original Sum-of-Parts method is sometimes outperformed by the historical average. Us, UK, Germany, Japan, South Africa, India and Chile report negative Certainty Equivalents and Sharpe Ratios that are below those of the Historical Average. Notably, EMD vastly outperforms both, with many Sharpe Ratios being close to 3. Panel c reports the greatest divergence between EMD SoP and OR SoP. While, as noted earlier, the Constrained results are very similar to Panel a, the Unconstrained case reports staggering differences in Certainty Equivalents. All sequential EMD Certainty Equivalents range between 50 and 75%, and Sharpe Ratios of 1.5 to 2, while OR SoP reports CE of 4-8% and SR around 0.5. The 93-18 sample reports even higher respective values. EMD Certainty Equivalents are now between 75-100%, with China having a 282% value, while Sharpe ratios are often 2.5-3. OR SoP results are also increased but remain incomparable.

Apart from acting as a successful robustness test, our results show that the Sum-of-Parts method is applicable to countries outside the US. Although investors located in both developed and developing economies experience economic and performance gains, it is no-table that non-Western home countries perform comparably, if not better, to their Western counterparts. This observation becomes more apparent in Panels b and c, where CE and SR values are quite similar and often outperform the US, the UK, Germany, Japan and Switzerland.

5.4. The effect of data frequency

Ferreira and Santa-Clara (2011) report an out-of-sample R^2 improvement of 13.4% with quarterly data and 1.3% with monthly data. Although this is to be expected, it is natural to check the robustness of our results with monthly data and see if the qualitative patterns we identify change. Table A.7 in Appendix A contains the collected results on the US for Theil's U, Sharpe Ratios and Certainty Equivalents for monthly data. There is not a significant difference for EMD Sum-of-Parts. 37 countries beat the historical average, with a maximum improvement of 17.6% and a mean improvement of $8,91\%$. The CE and SR values are also comparable to the earlier quarterly results for both the sequential and the all countries present portfolia. Thus, data frequency does not materially affect forecasting accuracy and performance. The results are weaker for original Sum-of-Parts, where only 15 countries perform better than the historical average in the 73-18 period. However, Sharpe ratios and Certainty Equivalents are quite higher than those under HA forecasts, implying that an improvement even in a small cluster of countries can be beneficial. As earlier, the all-equity (fully) portfolio performs best.

6. Conclusion

This paper primarily investigates whether forecasting of international stock returns is beneficial to an investor with a global mandate. Firstly, we demonstrate the sum of the parts method can be (easily) augmented to suit an international investment setting. Specifically returns are decomposed into four components rather than three with the foreign exchange rate return added (to earnings growth, the dividend yield and the change in price-earnings ratio). Secondly, we examine whether stock returns can be forecast in each of the 44 countries in our sample. We find in general that the sum of the parts method can lead to improved forecasts especially when empirical mode decomposition (EMD) is used. Thirdly, we examine our key question of whether return forecasts can be used in real-time portfolio allocation by an investor with a global remit and whether this improves performance over using the historical average benchmark. We demonstrate that substantial gains are possible both in terms of the economic value and in terms of portfolio performance metrics and the individual forecasting of each component provides a substantial improvement in the performance of an international portfolio under mean-variance optimisation. Specifically, EMD Sum-of-Parts forecasting performs much better than the historical average forecast. Our main finding is that by using a Sum-of-Parts approach that substantial gains are feasible to a global investor regardless of which country they are domiciled in.

Tables and Figures

	OR SoP	73-18			EMD SoP		73-18	
	TU	OOS R2	MSE-t	p MSE-t (one-sided)	TU	OOS R2	MSE-t	p MSE-t (1-sided)
Australia	0.9997	1.52%	0.5290	0.2980	0.8822	25.97%	1.5530	0.0600
Austria	0.9978	1.04%	0.4580	0.3230	0.8925	17.97%	0.9770	0.1640
Belgium	0.9883	1.86%	0.6010	0.2740	0.9795	9.13%	0.5840	0.2800
Canada	0.9999	1.24%	0.4120	0.3400	0.8730	25.15%	1.9020	0.0290
Denmark	1.0086	-1.22%	-0.3090	0.6210	0.9835	8.59%	0.4650	0.3210
France	0.9936	2.19%	0.8800	0.1890	0.8456	28.50%	2.3510	0.0090
Germany	0.9926	1.72%	0.7220	0.2350	0.8933	20.37%	1.4950	0.0670
Hong Kong	0.9844	3.45%	1.4480	0.0740	0.8762	24.29%	1.6530	0.0490
Ireland	0.9979	2.19%	0.6400	0.2610	0.8961	24.11%	1.5190	0.0640
Japan	1.0006	1.74%	0.4540	0.3250	0.8254	31.42%	2.3670	0.0090
Netherlands	1.0027	0.89%	0.2480	0.4020	0.8826	26.22%	1.3930	0.0820
Singapore	0.9941	0.98%	0.3130	0.3770	0.9850	4.92%	0.2850	0.3880
Switzerland	1.0265	-3.69%	-0.9390	0.8260	0.8445	28.97%	2.4110	0.0080
UK	0.9765	5.72%	1.9890	0.0230	0.8683	27.12%	1.9190	0.0270
US	0.9894	2.94%	1.0080	0.1570	0.9101	18.33%	1.1730	0.1200
South Africa	0.9882	4.44%	1.4410	0.0750	0.8441	30.00%	3.1100	0.0010
Norway	1.0150	-2.15%	-0.6360	0.7380	1.0028	2.53%	0.1460	0.4420
Sweden	0.9952	-0.30%	-0.1240	0.5490	1.0024	-4.42%	-0.2200	0.5870
Italy	0.9938	-0.41%	-0.1150	0.5460	0.9189	16.25%	1.0910	0.1380
Malaysia	0.9897	-2.45%	-1.1060	0.8660	0.8996	21.37%	1.1620	0.1230
Thailand	0.9854	0.10%	0.0290	0.4880	1.3717	-7.62%	-0.2690	0.6060
Spain	0.9985	-1.13%	-0.4740	0.6820	0.8045	34.28%	2.5510	0.0050
Sri Lanka	0.9942	0.78%	0.2760	0.3910	0.8618	25.64%	1.7460	0.0400
Korea	0.9973	-0.32%	-0.1090	0.5430	0.8146	24.20%	1.4120	0.0790
New Zealand	1.0291	-4.97%	-1.8120	0.9650	0.8446	31.78%	2.6580	0.0040
Finland	0.9988	0.10%	0.0520	0.4790	0.8761	21.97%	1.3100	0.0950
Taiwan	0.9804	-1.75%	-0.3410	0.6340	0.8654	20.72%	1.2340	0.1090
Philippines	0.9924	1.40%	0.5740	0.2830	0.8588	28.23%	1.9860	0.0240
Chile	0.9864	-0.56%	-0.1380	0.5550	0.8049	32.37%	2.0690	0.0190
India	0.9779	2.57%	1.6420	0.0500	0.7775	39.70%	3.4310	0.0000
Greece	0.9736	-2.54%	-0.9000	0.8160	0.7468	44.82%	3.1190	0.0010
Portugal	0.9990	-0.15%	-0.0540	0.5220	0.8877	21.45%	1.2920	0.0980
Turkey	0.9789	2.39%	0.7240	0.2350	0.9047	22.12%	1.4500	0.0740
Mexico	0.9961	-2.05%	-0.7340	0.7680	0.8844	22.79%	1.5480	0.0610
Hungary	0.9925	0.91%	0.2700	0.3940	0.8814	29.20%	1.7020	0.0440
Luxembourg	1.0188	-5.87%	-0.8800	0.8110	1.6211	-120.10%	-3.1380	0.9990
Pakistan	0.9740	5.13%	1.8750	0.0300	0.7105	54.46%	2.3280	0.0100
Cyprus	0.9998	-5.36%	-1.0650	0.8570	0.9879	-1.18%	-0.0480	0.5190
Israel	0.9830	3.55%	0.9080	0.1820	1.1010	-17.61%	-0.6970	0.7570
Colombia	0.9929	1.79%	0.9420	0.1730	0.9031	11.39%	0.5920	0.2770
China	1.0114	-1.24%	-0.5450	0.7070	0.9115	15.86%	1.0690	0.1430
Czech	1.0009	-0.77%	-0.1720	0.5680	0.8881	19.61%	1.6410	0.0500
Peru	1.0092	0.73%	0.1700	0.4330	1.0480	-10.39%	-0.4200	0.6630
Poland	1.0125	-3.89%	-0.8610	0.8050	0.9512	13.13%	0.6490	0.2580

Table 1a: US Theil U, Out-of-sample R^2 and MSE tests for Original and EMD Sum-of-Parts, 73 – 18

	OR SoP		94-18		EMD SoP		94-18	
	TU	OOS R2	MSE-t	p MSE-t (one-sided)	TU	OOS R2	MSE-t	p MSE-t (one-sided)
Australia	0.9825	2.86%	1.6130	0.0530	0.8159	39.38%	2.0210	0.0220
Austria	0.9903	0.73%	0.2730	0.3920	0.9339	19.08%	0.7210	0.2360
Belgium	0.9895	1.33%	0.3820	0.3510	0.9702	14.32%	0.6290	0.2650
Canada	0.9852	2.03%	0.9230	0.1780	0.7969	40.31%	1.9850	0.0240
Denmark	0.9840	2.94%	1.2600	0.1040	0.9030	25.08%	1.0080	0.1570
France	0.9847	2.92%	1.2480	0.1060	0.8770	25.34%	1.7640	0.0390
Germany	0.9921	0.60%	0.2130	0.4160	0.8186	33.52%	2.2890	0.0110
Hong Kong	0.9950	1.65%	0.5320	0.2970	0.8383	30.92%	1.7370	0.0410
Ireland	0.9865	2.77%	0.8150	0.2070	0.9187	23.63%	1.0370	0.1500
Japan	1.1057	-22.56%	-2.3050	0.9890	0.8289	28.83%	1.6760	0.0470
Netherlands	0.9944	1.07%	0.2830	0.3890	0.8566	32.79%	1.4960	0.0670
Singapore	0.9818	3.09%	0.9360	0.1750	0.9452	10.69%	0.4880	0.3130
Switzerland	1.0050	-1.40%	-0.5120	0.6960	0.9017	21.93%	1.3740	0.0850
UK	0.9533	8.57%	3.3070	0.0000	0.8023	40.31%	2.0570	0.0200
US	0.9446	10.24%	2.7630	0.0030	0.8617	28.77%	1.4300	0.0760
South Africa	0.9697	5.54%	1.8170	0.0350	0.8472	29.80%	2.4080	0.0080
Norway	0.9978	0.40%	0.2040	0.4190	0.9464	14.75%	0.6710	0.2510
Sweden	0.9894	1.07%	0.4250	0.3350	0.7920	39.27%	2.5210	0.0060
Italy	0.9928	0.28%	0.0910	0.4640	0.8876	24.69%	1.6750	0.0470
Malaysia	0.9852	3.16%	0.9680	0.1660	0.8793	21.57%	1.0290	0.1520
Thailand	0.9727	6.21%	1.8430	0.0330	1.6883	-35.47%	-0.9850	0.8380
Spain	0.9883	2.11%	0.8720	0.1920	0.7747	41.69%	3.2660	0.0010
Sri Lanka	0.9813	3.51%	0.6730	0.2500	0.8250	31.62%	1.8700	0.0310
Korea	1.0005	1.63%	0.3530	0.3620	0.7757	37.80%	2.3680	0.0090
New Zealand	1.0255	-4.77%	-1.5320	0.9370	0.8495	30.70%	2.0930	0.0180
Finland	0.9741	7.89%	1.3690	0.0850	0.8200	34.84%	2.0170	0.0220
Taiwan	0.9773	3.71%	1.0170	0.1550	0.8246	28.10%	1.5990	0.0550
Philippines	0.9775	4.19%	1.3730	0.0850	0.8172	33.43%	2.1810	0.0150
Chile	0.9933	1.65%	0.9120	0.1810	0.7903	38.08%	2.2250	0.0130
India	0.9682	5.70%	1.9600	0.0250	0.7358	46.75%	4.1550	0.0000
Greece	0.9780	4.59%	1.4060	0.0800	0.7384	51.24%	3.6660	0.0000
Portugal	0.9897	1.65%	0.7720	0.2200	0.8460	30.56%	1.9160	0.0280
Turkey	0.9630	10.44%	2.3770	0.0090	0.8654	34.22%	2.2540	0.0120
Mexico	1.0090	-0.19%	-0.0490	0.5200	0.8613	27.91%	1.5630	0.0590
Hungary	0.9787	3.43%	1.6660	0.0480	0.8720	32.53%	1.7740	0.0380
Luxembourg	1.0158	-3.82%	-1.0300	0.8480	1.4379	-71.84%	-2.4410	0.9930
Pakistan	0.9375	10.66%	2.2880	0.0110	0.6735	56.60%	2.5870	0.0050
Cyprus	1.0141	0.08%	0.0140	0.4940	1.0863	-12.03%	-0.5470	0.7080
Israel	1.0112	-2.04%	-0.2940	0.6160	0.9325	10.61%	0.5400	0.2950
Colombia	1.0319	-5.72%	-1.1580	0.8770	0.9278	12.44%	0.8080	0.2100
China	0.9847	3.63%	0.7460	0.2280	0.8249	26.47%	1.2920	0.0980
Czech	0.9878	1.14%	0.5050	0.3070	0.7683	37.99%	2.3860	0.0090
Peru	1.2148	-46.62%	-2.4570	0.9930	1.0429	-9.16%	-0.4200	0.6630
Poland	1.0086	-1.23%	-0.4710	0.6810	0.9940	4.69%	0.2120	0.4160

Table 1b: US Theil U, Out-of-sample R^2 and MSE tests for Original and EMD Sum-of-Parts, 93 – 18

	OR SoP				EMD SoP			
	l1	p1	l2	p2	l1	p1	l2	p2
Australia	0.7510	0.0620	0.2490	0.2980	0.7950	0.0010	0.2060	0.1240
Austria	0.7460	0.0880	0.2550	0.3180	0.6560	0.0000	0.3440	0.0200
Belgium	0.7440	0.0400	0.2570	0.2630	0.5750	0.0000	0.4250	0.0030
Canada	0.6660	0.0540	0.3340	0.2040	0.8340	0.0000	0.1660	0.1810
Denmark	0.4070	0.0890	0.5930	0.0310	0.5700	0.0010	0.4300	0.0030
France	0.8440	0.0200	0.1560	0.3420	0.8130	0.0000	0.1870	0.0560
Germany	0.8430	0.0440	0.1570	0.3700	0.6840	0.0000	0.3160	0.0070
Hong Kong	1.4870	0.0180	-0.4870	0.7650	0.7840	0.0000	0.2170	0.0860
Ireland	0.6640	0.0070	0.3360	0.0920	0.7160	0.0000	0.2840	0.0240
Japan	0.6450	0.0240	0.3550	0.1370	0.7840	0.0000	0.2160	0.0330
Netherlands	0.5820	0.0490	0.4180	0.1000	0.7430	0.0000	0.2570	0.0750
Singapore	0.7120	0.1560	0.2880	0.3320	0.5500	0.0020	0.4500	0.0090
Switzerland	0.2310	0.2090	0.7690	0.0050	0.8600	0.0000	0.1400	0.1510
UK	1.2610	0.0010	-0.2610	0.7570	0.7930	0.0000	0.2070	0.0570
US	0.8940	0.0140	0.1060	0.3920	0.7390	0.0000	0.2610	0.1230
South Africa	1.0260	0.0030	-0.0260	0.5290	0.9380	0.0000	0.0620	0.3240
Norway	0.2680	0.2380	0.7320	0.0230	0.5270	0.0050	0.4730	0.0070
Sweden	0.3690	0.3650	0.6310	0.2770	0.4680	0.0000	0.5320	0.0050
Italy	0.3980	0.3320	0.6020	0.2470	0.6670	0.0010	0.3330	0.0150
Malaysia	-0.8940	0.7650	1.8940	0.0760	0.9040	0.0140	0.0960	0.3800
Thailand	0.5220	0.2460	0.4780	0.2730	0.4710	0.0070	0.5290	0.0120
Spain	-0.3250	0.5730	1.3250	0.2240	0.8700	0.0000	0.1300	0.1520
Sri Lanka	0.7600	0.2180	0.2400	0.3990	0.8080	0.0000	0.1920	0.1610
Korea	0.3830	0.3650	0.6170	0.2830	0.7900	0.0020	0.2100	0.1470
New Zealand	-0.8950	0.8800	1.8950	0.0100	0.8620	0.0000	0.1380	0.1180
Finland	0.5520	0.2910	0.4480	0.3290	0.8010	0.0040	0.1990	0.1790
Taiwan	0.3010	0.3090	0.6990	0.1150	0.7660	0.0070	0.2340	0.0870
Philippines	1.0490	0.1420	-0.0490	0.5210	0.8450	0.0000	0.1550	0.1960
Chile	0.4140	0.2520	0.5860	0.1800	0.8360	0.0000	0.1640	0.1410
India	2.8980	0.0280	-1.8980	0.9020	1.0390	0.0000	-0.0390	0.6230
Greece	-0.5420	0.6820	1.5420	0.1020	1.0190	0.0000	-0.0190	0.5520
Portugal	0.4640	0.2510	0.5360	0.2080	0.8030	0.0010	0.1970	0.2150
Turkey	1.2100	0.1150	-0.2110	0.5840	0.8950	0.0110	0.1060	0.3120
Mexico	-0.4530	0.6380	1.4530	0.1410	1.0070	0.0040	-0.0070	0.5090
Hungary	0.7560	0.2220	0.2440	0.3990	0.9770	0.0030	0.0230	0.4620
Luxembourg	-0.1600	0.5830	1.1600	0.0650	0.2010	0.0270	0.7990	0.0000
Pakistan	2.6050	0.0170	-1.6050	0.9290	0.9480	0.0000	0.0520	0.3740
Cyprus	0.0110	0.4910	0.9890	0.0180	0.4950	0.0000	0.5050	0.0020
Israel	1.1970	0.0670	-0.1970	0.6020	0.3900	0.0280	0.6100	0.0020
Colombia	2.6670	0.1290	-1.6670	0.7630	0.6550	0.0130	0.3450	0.1110
China	-0.1670	0.5530	1.1670	0.1730	0.7250	0.0040	0.2750	0.1180
Czech	0.3340	0.3670	0.6660	0.2470	0.8850	0.0010	0.1150	0.3150
Peru	0.6060	0.1700	0.3950	0.2730	0.3940	0.0580	0.6060	0.0210
Poland	-1.0150	0.7200	2.0150	0.1370	0.7160	0.0210	0.2840	0.2100

Table 2a: US HLN Encompassing tests for Original and EMD Sum-of-Parts forecasts, 73–18

	OR SoP				EMD SoP			
	l1	p1	l2	p2	l1	p1	l2	p2
Australia	2.8940	3.10%	-1.8940	0.9000	0.9150	0.30%	0.0850	0.2430
Austria	0.6660	14.20%	0.3340	0.2960	0.6570	0.60%	0.3430	0.0650
Belgium	0.7090	11.10%	0.2910	0.2950	0.5920	0.10%	0.4080	0.0060
Canada	1.6430	10.20%	-0.6430	0.7010	1.0580	0.30%	-0.0580	0.6050
Denmark	1.3540	2.70%	-0.3540	0.6980	0.6970	0.60%	0.3030	0.0310
France	1.3830	3.20%	-0.3830	0.7090	0.7600	0.10%	0.2400	0.0310
Germany	0.7920	28.40%	0.2080	0.4410	0.8760	0.00%	0.1240	0.2050
Hong Kong	0.9110	13.00%	0.0890	0.4540	0.7330	0.10%	0.2670	0.0120
Ireland	0.7560	1.10%	0.2440	0.2360	0.6930	0.40%	0.3070	0.0630
Japan	0.1620	13.30%	0.8380	0.0000	0.7450	0.00%	0.2550	0.0620
Netherlands	0.7040	14.80%	0.2960	0.3550	0.7940	0.20%	0.2060	0.1210
Singapore	1.2950	7.30%	-0.2950	0.6380	0.5800	1.40%	0.4200	0.0010
Switzerland	0.0210	49.10%	0.9790	0.1510	0.7500	0.10%	0.2510	0.0610
UK	4.1200	0.00%	-3.1200	0.9980	0.8640	0.30%	0.1370	0.1580
US	2.9610	0.10%	-1.9610	0.9880	0.8020	0.10%	0.1980	0.1690
South Africa	1.9460	1.00%	-0.9460	0.8840	0.8550	0.00%	0.1450	0.1410
Norway	0.6530	20.10%	0.3470	0.3190	0.6860	1.70%	0.3140	0.1210
Sweden	0.9880	20.10%	0.0130	0.4960	0.9100	0.00%	0.0900	0.2590
Italy	0.5960	29.70%	0.4040	0.3480	0.7480	0.00%	0.2520	0.0410
Malaysia	2.0170	10.60%	-1.0170	0.7420	0.7930	1.20%	0.2080	0.2110
Thailand	1.6980	0.70%	-0.6980	0.8600	0.3980	1.90%	0.6020	0.0490
Spain	1.1860	7.10%	-0.1860	0.5930	0.8730	0.00%	0.1270	0.1130
Sri Lanka	0.9780	9.40%	0.0220	0.4880	0.8600	0.00%	0.1400	0.2230
Korea	0.7910	17.50%	0.2090	0.4010	0.9430	0.00%	0.0570	0.3730
New Zealand	-0.7130	81.90%	1.7130	0.0200	0.8290	0.00%	0.1710	0.0830
Finland	1.3100	1.90%	-0.3100	0.7040	0.9130	0.10%	0.0870	0.3070
Taiwan	1.6150	8.00%	-0.6150	0.7160	0.8280	0.40%	0.1720	0.1270
Philippines	1.4430	2.40%	-0.4430	0.7450	0.8260	0.00%	0.1740	0.1060
Chile	1.7250	10.50%	-0.7250	0.7040	0.8920	0.00%	0.1080	0.2280
India	2.0610	0.70%	-1.0610	0.9090	1.0520	0.00%	-0.0520	0.7040
Greece	1.5990	2.10%	-0.5990	0.7670	0.9930	0.00%	0.0070	0.4760
Portugal	1.4190	12.30%	-0.4190	0.6380	0.8250	0.00%	0.1750	0.1600
Turkey	1.9760	0.20%	-0.9760	0.9470	0.9830	0.10%	0.0170	0.4580
Mexico	0.4760	17.20%	0.5240	0.1440	0.8660	0.20%	0.1340	0.2910
Hungary	1.5450	1.20%	-0.5450	0.8090	0.9090	0.20%	0.0910	0.3130
Luxembourg	-0.0840	55.70%	1.0840	0.0290	0.2730	0.50%	0.7270	0.0000
Pakistan	2.7210	0.60%	-1.7210	0.9690	0.9240	0.00%	0.0760	0.2650
Cyprus	0.5040	3.40%	0.4960	0.0360	0.4490	0.00%	0.5510	0.0000
Israel	0.3840	17.10%	0.6160	0.0630	0.5590	0.10%	0.4410	0.0000
Colombia	0.0250	47.60%	0.9750	0.0120	0.6230	0.10%	0.3770	0.0220
China	1.1780	10.80%	-0.1780	0.5790	0.8050	0.60%	0.1950	0.1600
Czech	0.9030	13.50%	0.0970	0.4520	0.8990	0.00%	0.1010	0.2360
Peru	0.0960	26.80%	0.9040	0.0000	0.4070	3.60%	0.5930	0.0120
Poland	0.0290	48.90%	0.9710	0.1670	0.5430	0.80%	0.4570	0.0250

Table 2b: US HLN Encompassing tests for Original and EMD Sum-of-Parts forecasts, 93–18

	Countries above HA	Median TU	Max Improvement	Avg Improvement
	US			
EMD Seq	38	0.8861	0.2895	0.1246
EMD all	40	0.8530	0.3265	0.1515
OR SoP Seq	33	0.9947	0.0264	0.0096
OR SoP all	33	0.9881	0.0625	0.0197
	GERMANY			
EMD Seq	40	0.8981	0.2596	0.1118
EMD all	39	0.8482	0.2818	0.1534
OR SoP Seq	27	0.9966	0.0356	0.0120
OR SoP all	28	0.9901	0.0555	0.0203
	SWITZERLAND			
EMD Seq	40	0.8923	0.2696	0.1155
EMD all	39	0.8515	0.2956	0.1558
OR SoP Seq	28	0.9951	0.0312	0.0131
OR SoP all	32	0.9884	0.0556	0.0193
	CHILE			
EMD all	39	0.8777	0.3048	0.1322
OR SoP all	24	0.9979	0.0688	0.0214
	CHINA			
EMD all	41	0.7166	0.3891	0.2691
OR SoP all	39	0.9685	0.0892	0.0372

	Countries above HA	Median TU	Max Improvement	Avg Improvement
	UK			
EMD Seq	35	0.9073	0.2755	0.1174
EMD all	39	0.8677	0.2945	0.1399
OR SoP Seq	29	0.9950	0.0386	0.0127
OR SoP all	27	0.9927	0.0601	0.0201
	JAPAN			
EMD Seq	41	0.8966	0.2733	0.1165
EMD all	41	0.8526	0.3088	0.1504
OR SoP Seq	23	0.9984	0.0229	0.0096
OR SoP all	34	0.9904	0.0479	0.0172
	SOUTH AFRICA			
EMD Seq	38	0.8891	0.2386	0.1188
EMD all	39	0.8571	0.2699	0.1501
OR SoP Seq	29	0.9954	0.0332	0.0132
OR SoP all	32	0.9920	0.0498	0.0169
	INDIA			
EMD all	38	0.8713	0.2965	0.1333
OR SoP all	34	0.9873	0.0677	0.0219

Table 3: Theil's U statistics for different home countries

US

73-18	CE	R	SD	Sharpe
HA		4.51%	3.15%	0.2945
OR SoP Dir	0.49%	5.12%	4.66%	0.3301
OR SoP Iter	1.96%	6.80%	6.56%	0.4911
EMD SoP Dir	3.95%	8.57%	4.56%	1.0925
EMD SoP Iter	8.80%	13.90%	8.31%	1.2423

93-18	CE	R	SD	Sharpe
HA		3.32%	4.04%	0.4005
OR SoP Dir	0.65%	3.95%	3.75%	0.5976
OR SoP Iter	1.71%	5.26%	6.23%	0.5709
EMD SoP Dir	1.72%	5.00%	3.55%	0.9286
EMD SoP Iter	8.57%	12.54%	9.00%	1.2044

UK

73-18	CE	R	SD	Sharpe
HA		1.54%	3.62%	0.2076
OR SoP Dir	0.88%	1.78%	4.60%	0.3726
OR SoP Iter	0.91%	1.83%	6.09%	0.3113
EMD SoP Dir	4.01%	2.60%	5.74%	0.8644
EMD SoP Iter	10.86%	4.50%	10.54%	1.1947

93-18	CE	R	SD	Sharpe
HA		3.94%	5.41%	0.2644
OR SoP Dir	0.96%	4.76%	3.93%	0.5728
OR SoP Iter	2.38%	6.37%	5.87%	0.6579
EMD SoP Dir	3.40%	7.18%	3.76%	1.2448
EMD SoP Iter	9.55%	14.00%	8.96%	1.2824

GER

73-18	CE	R	SD	Sharpe
HA		4.48%	3.69%	0.2485
OR SoP Dir	0.20%	4.90%	5.91%	0.2252
OR SoP Iter	2.30%	7.31%	8.18%	0.4580
EMD SoP Dir	3.81%	8.43%	5.18%	0.9393
EMD SoP Iter	9.20%	14.51%	9.79%	1.1175

93-18	CE	R	SD	Sharpe
HA		2.98%	3.94%	0.2892
OR SoP Dir	0.16%	3.11%	3.71%	0.3442
OR SoP Iter	1.36%	4.53%	5.89%	0.4574
EMD SoP Dir	3.12%	6.08%	3.72%	1.1402
EMD SoP Iter	7.54%	11.09%	8.56%	1.0814

JAP

73-18	CE	R	SD	Sharpe
HA		2.49%	3.19%	0.3812
OR SoP Dir	0.14%	2.81%	5.33%	0.2882
OR SoP Iter	1.59%	4.76%	8.81%	0.3948
EMD SoP Dir	3.82%	6.48%	5.18%	1.0043
EMD SoP Iter	10.42%	13.97%	10.77%	1.1787

93-18	CE	R	SD	Sharpe
HA		3.47%	8.96%	0.3756
OR SoP Dir	-0.90%	2.36%	7.72%	0.2922
OR SoP Iter	-1.05%	2.01%	6.29%	0.3025
EMD SoP Dir	19.95%	23.25%	7.96%	2.9062
EMD SoP Iter	13.67%	17.05%	8.50%	1.9948

SWI

73-18	CE	R	SD	Sharpe
HA		3.38%	3.84%	0.2527
OR SoP Dir	0.88%	4.40%	5.34%	0.3727
OR SoP Iter	1.03%	4.87%	7.77%	0.3161
EMD SoP Dir	4.47%	8.14%	6.52%	0.8778
EMD SoP Iter	9.62%	13.68%	9.08%	1.2416

93-18	CE	R	SD	Sharpe
HA		2.10%	5.21%	0.2653
OR SoP Dir	0.33%	2.35%	4.25%	0.3818
OR SoP Iter	3.15%	5.56%	7.58%	0.6383
EMD SoP Dir	4.47%	8.14%	6.52%	0.8778
EMD SoP Iter	9.62%	13.68%	9.08%	1.2416

SAF

73-18	CE	R	SD	Sharpe
HA		11.70%	3.50%	0.2519
OR SoP Dir	0.59%	12.50%	5.79%	0.2908
OR SoP Iter	1.80%	14.05%	8.18%	0.3948
EMD SoP Dir	4.83%	16.69%	5.34%	1.0999
EMD SoP Iter	10.47%	23.11%	10.30%	1.1929

93-18	CE	R	SD	Sharpe
HA		9.07%	4.22%	0.2825
OR SoP Dir	0.07%	9.16%	4.51%	0.2850
OR SoP Iter	1.62%	10.94%	6.54%	0.4674
EMD SoP Dir	2.69%	11.70%	3.50%	1.0929
EMD SoP Iter	8.36%	18.00%	8.68%	1.1661

Table 4a: Constrained portfolio metrics: annualised certainty equivalents (CE), returns (R), standard devi-ations (SD), Sharpe Ratios for US, UK, Switzerland, Japan, Germany, S.Africa, India, China, Chile

IND				
93-18	CE	R	SD	Sharpe
HA		7.86%	4.55%	0.2292
OR SoP Dir	0.58%	8.36%	3.62%	0.4271
OR SoP Iter	1.77%	9.66%	4.92%	0.5778
EMD SoP Dir	3.35%	11.13%	3.60%	1.1974
EMD SoP Iter	8.43%	16.62%	7.29%	1.3437

CHN				
93-18	CE	R	SD	Sharpe
HA		3.78%	5.93%	-0.3100
OR SoP Dir	1.78%	5.56%	5.92%	-0.0107
OR SoP Iter	-0.03%	4.67%	11.27%	-0.0846
EMD SoP Dir	3.82%	7.88%	7.94%	0.2847
EMD SoP Iter	11.20%	16.57%	13.94%	0.7856

CHL				
93-18	CE	R	SD	Sharpe
HA		4.79%	4.29%	0.2084
OR SoP Dir	0.99%	5.67%	2.68%	0.6614
OR SoP Iter	1.78%	6.67%	5.23%	0.5294
EMD SoP Dir	3.31%	8.02%	3.19%	1.2923
EMD SoP Iter	8.07%	13.23%	7.43%	1.2564

Table 4a continued

US				
73-18	CE	R	SD	Sharpe
HA		2.88%	10.39%	-0.0670
OR SoP Dir	2.99%	5.73%	9.64%	0.2227
OR SoP Iter	2.91%	5.34%	7.91%	0.2223
EMD SoP Dir	24.91%	29.47%	16.59%	1.5603
EMD SoP Iter	19.87%	24.06%	15.45%	1.3255
93-18	CE	R	SD	Sharpe
HA		6.47%	9.12%	0.5231
OR SoP Dir	-2.78%	3.48%	7.87%	0.2263
OR SoP Iter	-2.43%	3.60%	6.26%	0.3032
EMD SoP Dir	20.47%	26.86%	8.64%	2.9101
EMD SoP Iter	15.38%	21.92%	9.49%	2.1305

UK				
73-18	CE	R	SD	Sharpe
HA		3.56%	11.47%	-0.1623
OR SoP Dir	5.35%	8.63%	10.14%	0.3159
OR SoP Iter	5.02%	8.13%	9.30%	0.2914
EMD SoP Dir	26.58%	31.19%	15.38%	1.6756
EMD SoP Iter	21.57%	25.70%	13.71%	1.4793
93-18	CE	R	SD	Sharpe
HA		6.41%	9.27%	0.4207
OR SoP Dir	-1.31%	4.83%	7.67%	0.3028
OR SoP Iter	-0.69%	5.28%	6.48%	0.4281
EMD SoP Dir	20.93%	27.31%	9.16%	2.7085
EMD SoP Iter	15.60%	22.17%	10.14%	1.9389

GER				
73-18	CE	R	SD	Sharpe
HA		-0.67%	11.34%	-0.3740
OR SoP Dir	7.17%	6.38%	10.80%	0.2603
OR SoP Iter	7.12%	6.02%	9.27%	0.2641
EMD SoP Dir	26.16%	26.10%	13.77%	1.6364
EMD SoP Iter	9.20%	14.51%	9.79%	1.1175
93-18	CE	R	SD	Sharpe
HA		5.88%	8.61%	0.4689
OR SoP Dir	-1.77%	3.98%	7.86%	0.2732
OR SoP Iter	-1.75%	3.82%	6.62%	0.2992
EMD SoP Dir	21.01%	26.97%	9.08%	2.7691
EMD SoP Iter	14.66%	20.66%	9.31%	2.0211

JAP				
73-18	CE	R	SD	Sharpe
HA		0.68%	13.17%	-0.0457
OR SoP Dir	5.50%	6.22%	13.33%	0.3706
OR SoP Iter	5.00%	5.23%	11.33%	0.3485
EMD SoP Dir	24.65%	15.29%	13.45%	1.6124
EMD SoP Iter	19.62%	20.37%	13.45%	1.4200
93-18	CE	R	SD	Sharpe
HA		3.47%	8.96%	0.3756
OR SoP Dir	-0.90%	2.36%	7.72%	0.2922
OR SoP Iter	-1.05%	2.01%	6.29%	0.3025
EMD SoP Dir	19.95%	23.25%	7.96%	2.9062
EMD SoP Iter	13.67%	17.05%	8.50%	1.9948

SWI				
73-18	CE	R	SD	Sharpe
HA		-0.11%	12.09%	-0.2086
OR SoP Dir	5.15%	4.80%	11.06%	0.2160
OR SoP Iter	6.41%	5.60%	8.75%	0.3642
EMD SoP Dir	25.19%	25.72%	14.52%	1.6056
EMD SoP Iter	19.16%	19.25%	12.90%	1.3057
93-18	CE	R	SD	Sharpe
HA		1.07%	10.95%	0.0322
OR SoP Dir	2.48%	3.26%	9.51%	0.2672
OR SoP Iter	3.50%	3.96%	7.64%	0.4243
EMD SoP Dir	29.15%	30.36%	11.57%	2.5620
EMD SoP Iter	23.95%	25.08%	11.17%	2.1810

SAF				
73-18	CE	R	SD	Sharpe
HA		9.96%	10.73%	-0.0794
OR SoP Dir	4.64%	14.45%	9.99%	0.3638
OR SoP Iter	4.87%	14.37%	8.29%	0.4286
EMD SoP Dir	28.50%	39.57%	15.01%	1.9153
EMD SoP Iter	23.19%	34.20%	14.80%	1.5801
93-18	CE	R	SD	Sharpe
HA		11.34%	9.56%	0.3627
OR SoP Dir	-2.15%	8.91%	7.90%	0.1302
OR SoP Iter	-2.03%	8.86%	6.78%	0.1451
EMD SoP Dir	18.59%	29.74%	8.51%	2.5703
EMD SoP Iter	13.46%	24.62%	8.56%	1.9557

Table 4b: Unconstrained portfolio metrics: annualised certainty equivalents (CE), returns (R), standard deviations (SD), Sharpe Ratios for US, UK, Switzerland, Japan, Germany, S.Africa, India, China, Chile

IND				
93-18	CE	R	SD	Sharpe
HA		9.71%	9.44%	0.3058
OR SoP Dir	-1.25%	8.19%	7.92%	0.1732
OR SoP Iter	-1.08%	8.17%	6.65%	0.2036
EMD SoP Dir	20.79%	30.36%	8.66%	2.7177
EMD SoP Iter	16.78%	26.42%	9.07%	2.1616

CHN				
93-18	CE	R	SD	Sharpe
HA		-0.02%	18.60%	-0.3030
OR SoP Dir	9.43%	7.51%	12.48%	0.1515
OR SoP Iter	9.64%	7.31%	10.71%	0.1577
EMD SoP Dir	37.38%	35.22%	11.45%	2.5847
EMD SoP Iter	34.62%	32.91%	13.30%	2.0520

CHL				
93-18	CE	R	SD	Sharpe
HA		7.59%	8.76%	0.4220
OR SoP Dir	-1.33%	6.09%	7.70%	0.2842
OR SoP Iter	-2.40%	4.87%	6.65%	0.1456
EMD SoP Dir	19.28%	26.77%	8.14%	2.8090
EMD SoP Iter	12.33%	19.82%	8.13%	1.9572

Table 4b continued

US				
73-18	CE	R	SD	Sharpe
HA Con		10.78%	18.53%	0.3886
OR SoP C	1.97%	13.13%	19.54%	0.4888
EMD SoP C	23.20%	38.50%	28.21%	1.2378
HA UnCon		10.39%	21.45%	0.3177
OR SoP UC	6.90%	21.19%	29.15%	0.6041
EMD SoP UC	75.69%	102.95%	46.34%	2.1446
93-18	CE	R	SD	Sharpe
HA Con		9.61%	21.83%	0.3623
OR SoP Con	3.97%	12.02%	17.87%	0.5770
EMD SoP Con	21.45%	31.63%	23.07%	1.2969
HA UC		14.11%	23.70%	0.5234
OR SoP UC	-1.01%	12.49%	22.37%	0.4821
EMD SoP UC	77.03%	94.29%	29.62%	3.1264

UK				
73-18	CE	R	SD	Sharpe
HA Con		2.60%	16.20%	0.3073
OR SoP C	2.25%	3.29%	17.66%	0.4370
EMD SoP C	24.79%	9.91%	26.61%	1.2864
HA UnCon		14.24%	24.08%	0.3660
OR SoP UC	4.93%	20.64%	26.96%	0.5643
EMD SoP UC	69.32%	94.96%	41.48%	2.1587
93-18	CE	R	SD	Sharpe
HA Con		9.56%	21.17%	0.3332
OR SoP C	4.84%	12.71%	16.72%	0.6105
EMD SoP C	23.88%	34.36%	23.23%	1.3710
HA UC		11.03%	26.94%	0.3162
OR SoP UC	3.75%	12.57%	22.46%	0.4479
EMD SoP UC	82.01%	95.25%	30.77%	3.0145

GER				
73-18	CE	R	SD	Sharpe
HA Con		8.95%	18.51%	0.2908
OR SoP C	2.68%	12.77%	21.38%	0.4305
EMD SoP C	23.59%	36.35%	26.89%	1.2190
HA UC		5.94%	25.91%	0.0916
OR SoP UC	6.85%	13.37%	27.00%	0.3630
EMD SoP UC	54.84%	69.87%	39.75%	1.6681
93-18	CE	R	SD	Sharpe
HA Con		7.59%	21.14%	0.2719
OR SoP C	4.37%	10.56%	17.55%	0.4972
EMD SoP C	23.59%	32.14%	25.67%	1.1801
HA UC		12.00%	23.76%	0.4278
OR SoP UC	-0.67%	10.28%	21.42%	0.3942
EMD SoP UC	82.51%	99.83%	33.11%	2.9593

JAP				
73-18	CE	R	SD	Sharpe
HA Con		8.30%	20.31%	0.3457
OR SoP C	2.79%	12.22%	22.94%	0.4772
EMD SoP C	25.49%	37.87%	28.64%	1.2776
HA UC		6.77%	30.21%	0.1818
OR SoP UC	9.98%	17.69%	31.73%	0.5172
EMD SoP UC	55.00%	71.98%	43.97%	1.6078
93-18	CE	R	SD	Sharpe
HA C		9.25%	25.44%	0.3596
OR SoP C	3.54%	10.45%	20.32%	0.5093
EMD SoP C	23.93%	34.18%	27.35%	1.2463
HA UC		10.83%	30.31%	0.3538
OR SoP UC	3.32%	11.21%	25.02%	0.4442
EMD SoP UC	89.39%	102.56%	33.95%	3.0177

SWI				
73-18	CE	R	SD	Sharpe
HA C		8.12%	20.49%	0.2786
OR SoP C	1.30%	10.91%	23.84%	0.3564
EMD SoP C	24.99%	37.15%	28.70%	1.2106
HA UnCon		2.66%	29.85%	0.0082
OR SoP UC	12.24%	15.59%	30.98%	0.4252
EMD SoP UC	61.32%	72.30%	41.50%	1.6838
93-18	CE	R	SD	Sharpe
HA Con		8.41%	28.95%	0.2655
OR SoP C	6.47%	10.81%	20.77%	0.4857
EMD SoP C	24.54%	33.90%	30.54%	1.0862
HA UC		1.15%	38.95%	0.0109
OR SoP UC	17.80%	11.26%	27.35%	0.3853
EMD SoP UC	116.18%	123.63%	46.33%	2.6526

SAF				
73-18	CE	R	SD	Sharpe
HA C		16.83%	19.48%	0.3084
OR SoP C	2.50%	21.02%	23.44%	0.4354
EMD SoP C	26.28%	48.15%	29.73%	1.2558
HA UC		17.24%	18.24%	0.3525
OR SoP UC	4.68%	26.37%	27.87%	0.5581
EMD SoP UC	74.60%	114.29%	50.76%	2.0384
93-18	CE	R	SD	Sharpe
HA Con		12.08%	20.75%	0.2028
OR SoP C	5.14%	16.24%	18.21%	0.4590
EMD SoP C	25.98%	38.92%	22.73%	1.3659
HA UC		15.66%	19.35%	0.4024
OR SoP UC	-2.10%	14.00%	20.46%	0.2994
EMD SoP UC	83.31%	105.57%	32.17%	3.0369

Table 4c: All-equity portfolio metrics: annualised certainty equivalents (CE), returns (R), standard deviations (SD), Sharpe Ratios for US, UK, Switzerland, Japan, Germany, S.Africa, India, China, Chile

IND				
93-18	CE	R	SD	Sharpe
HA C		10.23%	19.69%	0.1730
OR SoP C	4.95%	13.99%	16.38%	0.4374
EMD SoP C	23.92%	34.81%	21.31%	1.3134
HA UnCon		13.64%	20.65%	0.3301
OR SoP UC	1.01%	14.47%	20.22%	0.3782
EMD SoP UC	74.48%	94.60%	32.78%	2.6779

CHN				
93-18	CE	R	SD	Sharpe
HA C		-2.79%	39.34%	-0.2138
OR SoP C	10.01%	2.17%	32.28%	-0.1070
EMD SoP C	38.31%	32.71%	35.59%	0.7612
HA UC		-22.23%	98.12%	-0.2838
OR SoP UC	66.03%	10.19%	79.16%	0.0577
EMD SoP UC	282.85%	256.49%	95.99%	2.6135

CHL				
93-18	CE	R	SD	Sharpe
HA C		9.33%	17.29%	0.3139
OR SoP C	4.54%	12.97%	14.48%	0.6267
EMD SoP C	23.43%	33.74%	19.93%	1.4969
HA UC		13.04%	20.44%	0.4474
OR SoP UC	0.35%	12.97%	19.37%	0.4682
EMD SoP UC	79.44%	100.02%	34.22%	2.8091

Table 4c continued

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Appendix A. Additional Graphs

	Mean	Standard deviation	Minimum	Maximum	Observations
Stock return	0.0210	0.1418	-0.9808	0.9333	6223
FX return	-0.0041	0.0539	-0.5891	0.2082	6223
Dividend-price ratio	0.0077	0.0049	0	0.0952	6223
Earnings growth rate	0.0184	0.1631	-2.4258	2.7988	6223
Price-earnings ratio growth rate	-0.0010	0.2041	-2.7629	2.4036	6223
Price-earnings ratio	2.6694	0.4443	-0.5108	4.7095	6223

Country	DS Code	Country	DS Code
AUSTRALIA	TOTMKAU	SRI LANKA	TOTMKCY
AUSTRIA	TOTMKOE	KOREA	TOTMKKO
BELGIUM	TOTMKBG	NEW ZEALAND	TOTMKNZ
CANADA	TOTMKCN	FINLAND	TOTMKFN
DENMARK	TOTMKDK	TAIWAN	TOTMKTA
FRANCE	TOTMKFR	PHILIPPINES	TOTMKPH
GERMANY	TOTMKBD	CHILE	TOTMKCL
HONG KONG	TOTMKHK	INDIA	TOTMKIN
IRELAND	TOTMKIR	GREECE	TOTMKGR
JAPAN	TOTMKJP	PORTUGAL	TOTMKPT
NETHERLANDS	TOTMKNL	TURKEY	TOTMKTK
SINGAPORE	TOTMKSG	MEXICO	TOTMKMX
SWITZERLAND	TOTMKSW	HUNGARY	TOTMKHN
UK	TOTMKUK	LUXEMBURG	TOTMKLX
US	TOTMKUS	PAKISTAN	TOTMKPK
SOUTH AFRICA	TOTMKSA	CYPRUS	TOTMKCP
NORWAY	TOTMKNW	ISRAEL	TOTMKIS
SWEDEN	TOTMKSD	COLOMBIA	TOTMKCB
ITALY	TOTMKIT	CHINA	TOTMKCH
MALAYSIA	TOTMKMY	CZECH REP.	TOTMKCZ
THAILAND	TOTMKTH	PERU	TOTMKPE
SPAIN	TOTMKES	POLAND	TOTMKPO

Figure A.5: Descriptive statistics and Datastream codes

Note: All values are reported in logs with quarterly frequency between June 1973 and November 2018 (where available). Country observations range between 183 for the whole period and 99 for Poland, which has the shortest sample. The codes refer to the price index in local currency. For other series adapt as in the example for Australia: TOTMAU\$(RI)∼U\$ for the price index in USD, TOTMKAU(DY) for the dividend yield, TOTMKAU(PE) for the price-earnings ratio.

	Numerical		UK Jan-Feb 73	
	Values	**Logs**	**Values**	**Logs**
Index t	100		310.74	
Index t+1	110		305.2	
Dividend	5		3.63	
Earnings t	8		11.9690608	
Earnings t+1	10		13.0184049	
USD spot FX t	1.5		0.85138701	
USD spot FX t+1	1.75		0.8878768	
PE ratio t	12.5		18.1	
PE ratio t+1	11		16.3	
Return t+1 local	1.15	0.139762	0.98217159	-0.0179893
E growth rate	1.25	0.223144	1.08767138	0.0840391
PE growth rate	0.88	-0.12783	0.90055249	-0.1047468
DP ratio (div yield)	1.045455	0.044452	1.003025	0.0030204
Egrowth + PE growth +DP		0.139762		-0.0176873
Index return USD	1.341667	0.293913	1.02426671	0.0239769
FX ret	0.857143	-0.15415	1.04285923	0.0419662
USD ret - FX ret		0.448063		-0.0179893
R-GE-GM-DP t+1		0		-0.0003019

Figure A.6: Sum-of-Parts examples

Note: The index in the numerical example does not include dividends. In the real data example, the index is the total return index which includes dividends, and the dividend value is the annualized value.

	EMD Seq	EMD all	OR SoP Seq	OR SoP all		EMD Seq	EMD all	OR SoP Seq	OR SoP all
Australia	0.8239	0.8210	1.0046	0.9949	Sri Lanka	0.9022	0.8089	0.9994	0.9930
Austria	0.8398	0.7600	1.0062	0.9960	Korea	0.9253	0.8838	0.9980	0.9986
Belgium	0.9247	0.9173	0.9991	0.9959	New Zealand	0.9467	0.9159	1.0205	1.0216
Canada	0.8491	0.8480	1.0066	0.9916	Finland	0.9874	0.9414	1.0069	0.9941
Denmark	1.1445	0.8994	1.0174	0.9969	Taiwan	0.9652	0.8932	1.0006	0.9956
France	0.8521	0.8198	1.0019	0.9898	Philipinnes	0.9818	0.9231	0.9984	0.9894
Germany	0.9104	0.8851	1.0026	1.0066	Chile	0.9272	0.8624	1.0035	0.9983
Hong Kong	0.8945	0.8634	0.9924	0.9981	India	0.9009	0.7961	0.9948	0.9840
Ireland	0.9241	0.9551	1.0137	1.0009	Greece	1.0473	1.0705	0.9926	0.9804
Japan	0.8707	0.8319	1.0062	1.1746	Portugal	0.9021	0.8436	1.0003	0.9972
Netherlands	0.8961	0.8576	1.0051	0.9957	Turkey	0.9375	0.8439	0.9953	0.9803
Singapore	0.8734	0.8269	0.9964	0.9886	Mexico	0.9534	0.8442	1.0071	1.0121
Switzerland	0.8807	0.8591	1.0179	1.0088	Hungary	1.0318	1.0217	1.0020	0.9924
UK	0.8733	0.8307	0.9936	0.9771	Luxemburg	1.0418	1.0199	1.0214	1.0283
US	0.8572	0.8148	0.9972	0.9692	Pakistan	0.9539	0.8905	0.9955	0.9684
South Africa	0.8569	0.8479	0.9964	0.9898	Cyprus	1.0951	1.1129	1.0104	1.0415
Norway	0.9611	0.9024	1.0121	1.0054	Israel	1.0287	1.1975	1.0056	1.0298
Sweden	0.9689	0.9206	1.0006	1.0008	Colombia	0.8845	0.8553	1.0048	1.0537
Italy	0.9347	0.8594	0.9987	0.9940	China (H)	0.9535	0.8665	1.0025	1.0014
Malaysia	0.8263	0.7882	1.0010	1.0016	Czech	0.9299	0.8382	1.0038	1.0031
Thailand	0.9803	1.0313	0.9991	0.9886	Peru	1.3099	1.6768	1.0295	1.4175
Spain	0.8997	0.8540	1.0003	0.9938	Poland	0.9524	0.8863	1.0034	1.0054

	EMD Seq	EMD all	OR SoP Sec	OR SoP all
No of countries beating the HA	37	37	15	27
Min Improvement	0.0126	0.0449	0.0006	0.0014
Max Improvement	0.1761	0.2400	0.0076	0.0316
Avg Improvement	0.0891	0.1390	0.0035	0.0099

	SoP Direct	SoP Iter	SoP Fully	HA	HA Fully	SoP Direct	SoP Iter	SoP Fully	HA	HA Fully
	EMD Sequential 73-18, Constrained					EMD Sequential 73-18, Unconstrained				
SR	0.9032	1.1004	1.1595	0.1905	0.2435	1.7568	1.5384	2.2114	-0.0329	0.1704
CE	0.0255	0.0466	0.1194			0.1392	0.1038	0.4731		
	Original SoP - Sequential 73-18, Constrained					Original SoP - Sequential 73-18, Constrained				
SR	0.2214	0.3178	0.3151	0.1905	0.2435	0.2327	0.1236	0.4111	-0.0329	0.1704
CE	0.0019	0.0077	0.0087			0.0164	0.0093	0.0368		
	EMD All countries present 94-18, Constrained					EMD All countries present 94-18, Constrained				
SR	1.0544	1.2710	1.2251	0.2716	0.2444	1.0544	1.2710	1.2251	0.2716	0.2444
CE	0.0227	0.0389	0.1108			0.0227	0.0389	0.1108		
	Original SoP All countries present 94-18, Constrained					Original SoP All countries present 94-18, Constrained				
SR	0.3679	0.3969	0.4235	0.2716	0.2444	0.3679	0.3969	0.4235	0.2716	0.2444
CE	0.0023	0.0052	0.0157			0.0023	0.0052	0.0157		

Figure A.7: Country Theil's U, Sharpe Ratios (SR) and Certainty Equivalents (CE) for the US, monthly data