Rana Shahout Technion ranas@cs.technion.ac.il Roy Friedman Technion roy@cs.technion.ac.il Ran Ben Basat University College London r.benbasat@cs.ucl.ac.uk

# Abstract

Stream monitoring is fundamental in many data stream applications, such as financial data trackers, security, anomaly detection, and load balancing. In that respect, quantiles are of particular interest, as they often capture the user's utility. For example, if a video connection has high tail latency, the perceived quality will suffer, even if the average and median latencies are low.

In this work, we consider the problem of approximating the *per-item* quantiles. Elements in our stream are (ID, latency) tuples, and we wish to track the latency quantiles for each ID. Existing quantile sketches are designed for a single number stream (e.g., containing just the latency). While one could allocate a separate sketch instance for each ID, this may require an infeasible amount of memory. Instead, we consider tracking the quantiles for the heavy hitters (most frequent items), which are often considered particularly important, without knowing them beforehand.

We first present a simple sampling algorithm that serves as a benchmark. Then, we design an algorithm that augments a quantile sketch within each entry of a heavy hitter algorithm, resulting in similar space complexity but with a deterministic error guarantee. Finally, we present SQUAD, a method that combines sampling and sketching while improving the asymptotic space complexity. Intuitively, SQUAD uses a background sampling process to capture the behaviour of the latencies of an item before it is allocated with a sketch, thereby allowing us to use fewer samples and sketches. Our solutions are rigorously analyzed, and we demonstrate the superiority of our approach using extensive simulations.

#### 1 Introduction

Maintaining statistics about network traffic is important for supporting various functionalities such as security and anomaly detection, traffic engineering, and load balancing [20, 23, 29, 38]. Latency is an important metric in assessing a network's health and in debugging various networking middle-boxes and smart data-planes [40].

In particular, latency distribution is a fundamental task of data monitoring and analysis. Yet, latency distribution is often very heavy tailed, implying that tail latency is usually more significant than average latency. Generally, quantiles are the most commonly used for data distribution representation. They are equivalent to the cumulative distribution function (cdf), from which the probability distribution function is derived (pdf). Thus, quantile computation is undoubtedly one of the most basic data analysis challenges. For example, consider a large e-commerce web site that offers short average response times, but with a tail latency of several seconds, beyond what Internet users are willing to tolerate. A customer is likely to ditch this website due to a single long latency, so having a short average response time is not enough here. To support fast and efficient tail latency tracking, several quantile sketches have been developed [3, 22, 25, 30, 33, 34, 43]. Other research focuses on the problem's variants and extensions, such as calculating quantiles across sliding windows [7], over distributed data [4, 26, 28, 43], quantile computations using GPUs [24], continuous monitoring of quantiles [15, 47] and biased quantiles [18].

Such sketches return an approximation of a q-quantile's latency up to a  $\varepsilon$  error guarantee. Existing sketches of this type track the tail latency of an entire stream.

Further, the ability to perform drill-down queries, in which we examine the behavior of the system at finer and finer granularity, may also be beneficial. One may distinguish between different *item identifiers* (or simply *items*) in the stream of elements. For example, in the case of datastores, an item identifier is typically the object's key, whereas in e-commerce sites, the item identifier might be a username or an item's SKU. In networking applications, an item identifier may be a 5-tuple consisting of the corresponding packet's source IP, source port, destination IP, destination port, and protocol; in this case, it is common to refer to items as *flows*.

Tracking tail latencies for all items can provide much richer insight about the system than looking at the aggregated tail latency, and is therefore desirable. In the e-commerce example, a user would quit the website based on its own tail latency experience, regardless of the overall tail latency, which might be much better. However, tracking tail latencies for all items is often impractical, given that the number of items can be extremely large and the fact that the space overhead of known quantile sketches is non-trivial. Hence, we may instead focus on tracking tail latencies for a subset of significant items. In particular, we are interested in the subset of *heavy-hitters*, which consists of all items whose associated elements consume more than a threshold  $\theta$  of the overall stream.

There are two complementary reasons why focusing on heavyhitters makes sense in the context of tail latency monitoring. First, since each heavy-hitter accounts for a significant fraction of the overall system load, it is important to ensure good quality of service for them. Second, when there are only a few elements associated with a given item, e.g., the item only appears in one or two transactions, it is enough that a single transaction suffers from a longer than usual delay in order for the tail-latency of that item to be very large. Such one-time events can be caused by, e.g., caching initialization, storage warm-up, route discovery overheads, and "bad luck" in terms of temporal overloads on intermediate components and devices. On the other hand, a large tail latency for a heavy hitter points to a repetitive problem, which hopefully is easier to discover and fix, and one that is also very important to resolve.

In this work, we focus on the problem of reporting the tail latencies of heavy hitter items. Our goal is to figure out how to find all heavy hitters' *q*-tail latencies with a maximum accuracy error of  $\varepsilon$  for given parameters *q*,  $\theta$  and  $\varepsilon$ .

#### **Contributions:**

Our first contribution is the formal definition of the heavy hitters q-tail latencies problem, nicknamed  $(\theta, \epsilon, \delta)$ -HH-latencies problem, where given a stream of elements and parameters q,  $\theta$ , and  $\varepsilon$ , we report the q-tail latency of every item x which is larger than  $\theta N$ , denoted by  $\hat{\ell_x}$ , such that  $|\hat{\ell_x} - \ell_x| < \varepsilon$ , where  $\ell_x$  is the true q-tail latency of x.

Our second contribution is a pure sampling based solution to the heavy hitters *q*-tail latencies problem called SQUARE. This algorithm is memory wasteful but it serves as a baseline for comparing our more sophisticated solutions. We formally analyze this solution and show that it takes  $O(\theta^{-1}\epsilon^{-2}\log\delta^{-1})$  space.

Our third contribution in a solution, nicknamed QUASI, for the heavy hitters q-tail latencies problem  $((\theta, \epsilon, \delta)$ -HH-latencies problem) which is based on combining the space saving algorithm for heavy hitters detection [37] with a q-quantile sketch. Space saving [37] is considered the best approximate solution for the heavy hitters problem, as it requires  $O(\epsilon^{-1})$  space for solving the  $\theta\epsilon$  heavy hitters problem. Formally, given user specified  $\theta$ , where  $0 \le \theta \le 1$ , a heavy hitter element is one with a frequency greater than  $\theta N$  in a stream of size N. In QUASI, we take the space saving data structure and add to each counter a q-quantile sketch, GKsketch [25]. We formally analyze this solution and show that it takes  $O(\theta^{-1}\epsilon^{-2} \cdot \log(N\epsilon^2\theta))$  space which results in similar space complexity as SQUARE but with a deterministic error guarantee rather than probabilistic one.

Our fourth contribution is the SQUAD algorithm, which is a combination of our first two solutions. Figure 1 illustrates the algorithms presented in this paper. We formally analyze this solution and show that it takes  $O(\theta^{-1}\epsilon^{-1.5} \cdot \log \epsilon^{-1})$  space.

We present formal correctness proofs as well as space analysis. The asymptotic space requirements of our solutions are summarized in Table 1. Following that, we discuss several enhancements that help our algorithms process elements more efficiently.

Our next contribution is a performance evaluation study of the above three solutions. To our knowledge, this is the first research to solve quantiles on a per-element level. Thus, we compare our algorithms along with (*i*) GK-algorithm and Random [33], that serve as a best case reference point since it solves a more straightforward problem : tail latency of an entire stream (*ii*) the state-of-the-art Space Saving (SS) [37], which solves the heavy hitters problem which is a building block in QUASI and SQUAD. We evaluate our algorithms using large-scale NS3 simulations [2] and a FatTree topology. The traffic is produced using the flow size distribution in web search from Microsoft [5] and Hadoop from Facebook [41].

The results show that given the same error guarantees  $\varepsilon$ ,  $\theta$ , SQUAD is the most space-efficient algorithm. While SQUARE is the fastest algorithm in terms of update runtime, it has a large memory cost for solving the ( $\theta$ ,  $\varepsilon$ ,  $\delta$ )-HH-latencies problem. When SQUAD is compared against QUASI's update runtime, SQUAD performs better. Yet, optimizing SQUAD enhances its update speed, making it excellent for both the performance and memory consumption

Table 1: A comparison of the algorithms presented in this work, in terms of their space complexity. The  $\widetilde{O}$  notation hides polylogarithmic factors.

Algorithm	Space	Deterministic	Reference	
SQUARE	$\widetilde{O}(\theta^{-1}\epsilon^{-2})$	×	Section 4	
QUASI	$\widetilde{O}(\theta^{-1}\epsilon^{-2})$	1	Section 5	
SQUAD	$\widetilde{O}(\theta^{-1}\epsilon^{-1.5})$	X	Section 6	

metrics. Last, we extend our results to support tail latencies for traffic volume. All our code is open sourced [1].

**Paper roadmap:** We briefly survey related work in Section 2. We state the formal model and problem statement in Section 3. Our first algorithm SQUARE is described in Section 4. QUASI is described and analyzed in Section 5. The improved algorithm, SQUAD is then described in Section 6. We present the optimizations that enable our algorithms to process elements faster in Section 7. The performance evaluation of our algorithms and their comparison to GK-algorithm, Random and SS is detailed in Section 8. Section 9 discusses extensions of our work. Finally, we conclude with a discussion in Section 10.

#### 2 Related Work

To the best of our knowledge, this is the first work that deals with the problem of per-element quantile estimation. Several earlier studies on streaming quantiles consider queries to be ranks, where the algorithm must associate an item y in the stream with a rank close to its true rank, defined as the number of stream elements that are smaller than or equal to y. In contrast, in our study, we focus on the quantile of individual elements in streams composed of identifiers and latencies, as described in Section 3. Below, we discuss prior work that has been done on solving streaming quantiles that guarantee an additive error with a constant failure probability  $\delta$ .

Munro and Paterson included a p-pass algorithm for obtaining accurate quantiles in their classic study [39]. Although not explicitly studied, the method's initial run results in a streaming approach for producing approximate quantiles using  $O(\epsilon^{-1} \log^2(N\epsilon))$  space. Manku, Rajagopalan, and Lindsay [34] extended this work by proposing a deterministic solution that stores no more than  $O(\epsilon^{-1} \log N\epsilon)$ objects, assuming previous knowledge of N. Though [35] has the same worst-case space bound, the algorithm is empirically better. In 2001, Greenwald and Khanna [25] developed a complex deterministic streaming algorithm, referred to as the GK-algorithm below, that stores  $O(\epsilon^{-1} \log(N\epsilon))$  objects in the worst case. However, their experimental work used a simplified approach for which it is not clear if the  $O(\epsilon^{-1} \log(N\epsilon))$  space limit still holds. Nonetheless, they demonstrated that their method beats Manku et al [34]'s approach practically. Each of these methods is deterministic and relies on comparisons. The GK-algorithm is often considered the best in this area, both theoretically and experimentally. Section 3.2 goes over it in detail.

Shrivastava et al [43] created q-digest in 2004, which is a deterministic, fixed universe method that consumes  $O(\epsilon^{-1} \log \mathcal{U})$  space, where  $\mathcal{U}$  is the universe. This approach was developed to compute quantiles in sensor networks and is a mergeable summary [3], a more flexible model than streaming. However, no further efficient

Input:	$(y, \ell_1)$	$(z, \ell_2)$	$(a, \ell_3)  (x, \ell_4)$	(а,	$\ell_5)$	(b, l	6)	$(x, \ell_7)$	$(c, \ell_8)$	$(x, \ell_9)$
$(y, \ell_1)$	ID	Count	Quantile Sketch		ID	Count	Incr.	Timestam	o Quanti	ile Sketch
$(a, \ell_3)$	x	4	$\{\ell_4, \ell_9\}$		x	5	2	7	$\{\ell_7$	,ℓ <sub>9</sub> }
$(x, \ell_4)$ $(b, \ell_c)$	С	3	$\{\ell_8\}$		С	4	1	8	{1	l <sub>8</sub> }
$(x, \ell_9)$	а	2	$\{\ell_3, \ell_5\}$			(a,ℓ <sub>3</sub> ,	3)	$(x, \ell_4, 4)$	$(x, \ell)$	9,9)
(a) SQUARE		(b) Ç	QUASI					(c) SQUAD		

Figure 1: An illustration of our algorithms. SQUARE simply selects a uniform random element subset from the input stream and uses the sample to infer frequencies and quantiles. QUASI uses a heavy hitters algorithm that has a quantile sketch embedded in each counter. SQUAD combines the two approaches to asymptotically reduce the space complexity. Crucially, SQUAD adds timestamps to both samples and sketches so that it can combine an item's sketch only with the samples that were not inserted into it. It also adds an increments counter, which is the increase in count since the item became monitored, thus reducing the frequency estimation error. SQUAD continues to sample elements of monitored items ( $(x, \ell_9, 9)$  in this example) as the item may stop being monitored, i.e., the sampling process is independent of the sketching.

fixed-universe method exists in the streaming model. Note that the  $\log \mathcal{U}$  and  $\log N$  terms are not theoretically equivalent, and [43] omitted an experimental comparison with the GK-algorithm.

Randomized methods have also been considered in the past. The seminal results of [44] show that a random sample of size  $O(\epsilon^{-2} \log \epsilon^{-1})$  contains all quantiles with at least a constant probability within the  $\epsilon$  error. This fact was shown in [34] and was used to compute quantiles using a random sample fed to a deterministic algorithm. However, since this method needs knowledge of N in advance, it is not a true streaming algorithm.

Manku et al. [35] developed a randomized approach that does not require knowledge of N and demonstrated that the space required is  $O(\epsilon^{-1} \log^2 \epsilon^{-1})$  factor, which may be greater or less than GK's log N factor, although neither of these algorithms has been empirically tested.

Agarwal, Cormode, Huang, Phillips, Wei, and Yi [3] proposed a mergeable sketch with the size  $O(\epsilon^{-1} \log^{1.5} \epsilon^{-1})$ . For this new, simpler approach, called Random, Luo et al [33] were able to provide an improved  $O(\epsilon^{-1} \log^{1.5} \epsilon^{-1})$  bound. We refer to this algorithm as "Random" and overview it in detail below. Felber and Ostrovsky [22] reduced the space complexity by using a combination of sampling and the GK-algorithm to  $O(\epsilon^{-1} \log \epsilon^{-1})$ .

Finally, Karnin, Lang, and Liberty [30] solved the problem by developing the KLL sketch, which is an optimal  $O(\epsilon^{-1})$ -space solution. The KLL sketch achieves optimal accuracy in space. The algorithm's fundamental building component is a buffer called a compactor, which accepts an input stream of N items and generates a stream of no more than  $\frac{N}{2}$  items that "approximates" the input stream. The overall KLL sketch is constructed as a series of at most log N such compactors, with each compactor's output stream acting as the input stream for the next compactor.

Several studies have attempted to provide more accurate quantile estimates for low and high rankings. Only a few provide answers to the relative error quantiles problem (also known as the biased quantiles problem). Gupta and Zane [27] presented an approach for computing relative error quantiles that saves  $O(\epsilon^{-3} \log^2(N\epsilon))$  items and uses this to estimate the number of inversions in a list; their technique needs knowledge of the stream length, *N*. Zhang et al. [49] previously described an approach for storing

 $O(\epsilon^{-2} \log(N\epsilon^2))$  items. Cormode et al. [17] devised a deterministic sketch that stores  $O(\epsilon^{-1} \log(N\epsilon \log(|\mathcal{U}|)))$  elements and necessitates previous knowledge of the data universe  $\mathcal{U}$ . Shrivastava et al [43]'s work on additive error has influenced their approach. Zhang and Wang [48] proposed a deterministic merge-andprune method that stores  $O(\epsilon^{-1} \log^3(N\epsilon))$  items and is capable of performing arbitrary merges with an upper constraint on n as well as streaming updates for unknown N. However, it does not address the most general case of merging without prior knowledge of N. Cormode and Vesely [19] have shown that every deterministic comparison-based technique has a space constraint of  $\Omega(\epsilon^{-1} \log(N\epsilon))$  items.

Cormode et al. [16] presented a relative error variation of the KLL sketch. They achieve relative error  $\epsilon$  in the randomized environment using  $O(\epsilon^{-1} \log^{1.5}(N\epsilon))$  with constant failure probability by varying the sampling technique throughout the distribution and employing a hierarchy modeled after [30].

# 3 Preliminaries

#### 3.1 Model

Given a *universe*  $\mathcal{U}$ , we consider a 2-tuple *stream* (sequence of elements)  $S = \langle (x_1, \ell_1), (x_2, \ell_2) \dots \rangle \in (\mathcal{U} \times \mathbb{R})^+$ . Here, each element  $(x_i, \ell_i)$  has an *identifier*  $x_i \in \mathcal{U}$ , and *latency*  $\ell_i \in \mathbb{R}$ .

We denote by  $f_x = |\{(x_i, \ell_i) \in S : x_i = x\}|$  the frequency (size) of *x*. Its (multi-) set of latencies is denoted by  $L_x = \{\ell_i : (x, \ell_i) \in S\}$ .

Given a quantile  $q \in [0, 1]$ , let  $\mathcal{L}_{x,q}$  represent the  $q^{th}$  quantile (i.e., the  $[q \cdot f_x]^{th}$  largest value) of  $L_x$ . The inverse operation is normalized rank, denoted by  $\operatorname{rank}_x(\ell)$ , which returns the quantile of  $\ell$  in  $L_x$  (that is,  $\operatorname{rank}_x(\mathcal{L}_{x,q}) = q$ ).

Any item x with frequency  $f_x \ge N\theta$  is called a heavy hitter, where N = |S| is the overall number of elements, and  $\theta \in [0, 1]$  is a given *threshold*.

Let  $\epsilon, \delta \in [0, 1)$  be additional error parameters: given the parameters  $\theta, \epsilon, \delta$ , we consider the  $(\theta, \epsilon, \delta)$ -HH-latencies problem that tracks the latency quantiles of heavy hitters. Specifically, we seek algorithms that support the following operations:

- INSERT $(x, \ell)$  process a new element  $(x, \ell)$ .
- QUERY(x, q) return a tuple  $(\widehat{f_x}, \widehat{\mathcal{L}_{x,q}})$  satisfying:

#### **Table 2: List of Symbols**

Symbol	Meaning		
S	The data stream		
U	The universe of elements		
R	The universe of latencies		
N	The number of elements in the stream		
q	The quantile q i.e. the $q^{th}$ largest value		
L <sub>x</sub>	The set of latencies of $S$ with identifier $x$		
$\mathcal{L}_{x,q}$	The $q^{th}$ quantile of $L_x$		
$\widehat{\mathcal{L}_{x,q}}$	an estimation of $\mathcal{L}_{x,q}$		
$\operatorname{rank}_{\boldsymbol{X}}(\ell)$	rank <sub><i>x</i></sub> ( $\ell$ ) The quantile of $\ell$ in $S_x$		
f <sub>x</sub>	$f_x$ The frequency of an element <i>x</i> in <i>S</i>		
$\widehat{f}_x$	$\widehat{f_x}$ An estimate of $f_x$		
e	$\epsilon$ An estimate accuracy parameter		
δ	A bound on the failure probability		
θ	$\theta$ The heavy hitters threshold		

- (1)  $\Pr[|f_x \hat{f}_x| > N\epsilon] \le \delta$ . As standard in heavy hitter algorithms, we return an estimate of the item's frequency.
- (2) If f<sub>x</sub> ≥ θN, Pr[|rank(L<sub>x,q</sub>) q|) > ε] ≤ δ. That is, if x is a heavy hitter the algorithm is likely to return an estimate whose quantile is off by no more than ε.

We note that part (1) of our query response is designed to help the user understand whether the quantile estimate is reliable and a similar guarantee can be obtained by running a separate heavy hitters algorithm. Specifically, if  $\widehat{f_x} > N\theta(1+\epsilon)$ , then *x* is likely to be a heavy hitter and therefore  $\widehat{\mathcal{L}_{x,q}}$  is a credible approximation of  $\mathcal{L}_{x,q}$ .

For ease of reference, Table 2 includes a summary of basic notations used in this work.

#### 3.2 Useful Streaming Algorithms

In this work, we utilize the Reservoir Sampling (RS) algorithm [45] in Section 4 as well as the Space Saving (SS) algorithm [37] and the GK-algorithm [25] in Section 5. We overview them here.

**Reservoir sampling (RS) [45]:** is a randomized algorithm for selecting a uniform random sample of a given size from an input stream of an unknown size without replacement in a single pass through the objects.

The algorithm keeps a *k*-sized reservoir, which initially holds the first *k* items of the input. On the arrival of the *n*'th item, RS selects a uniform random integer  $i \in \{0, ..., n-1\}$ ; the item overrides slot *i* of the reservoir if i < k and is otherwise discarded.

**Space Saving (SS) [37]:** is a counter-base algorithm for (approximately) finding the most frequent items in a data stream, a.k.a. heavy hitters. SS processes a stream of identifiers with the goal of estimating the size (frequency) of each. SS maintains a set of  $1/\epsilon$  integer counters, each with an associated ID. When an item arrives, SS increments its counter if one exists. Otherwise, SS allocates the item with a minimal-valued counter before incrementing it (disassociating the previous ID). For example, assume that the smallest counter was associated with ID *x* and had a value of 4; if *y* arrives and has no counter, it will take over *x*'s counter and increment its value to 5 (leaving *x* without a counter). When queried for the frequency of an item, we return the value of its counter if it has one, or the minimal counter's value otherwise.

If we denote the overall number of insertions processed by the algorithm by Z, then we have that the sum of counters equals Z, and thus the minimal counter is at most  $Z\epsilon$ . This ensures that the error in the SS estimate is at most  $Z\epsilon$ .

The GK-algorithm (GK) [25]: is a deterministic algorithm for supporting single-pass quantile summaries of a data stream. A quantile summary is a subset of the input data sequence that uses quantile estimations to provide approximate answers to any arbitrary quantile query.

The GK technique is based on the idea that if a sorted subset of the input stream of size N can be kept so that the ranks of  $v_i$  and  $v_{i+1}$  are within  $2N\epsilon$  of each other, then any quantile query can be answered with an error no larger than  $N\epsilon$ . That is, given a quantile q, GK can produce an estimate that satisfies  $\hat{q} \in [q - \epsilon, q + \epsilon]$  by finding the closest ranked element in the subset. GK allows maintaining such a subset using  $O(\frac{1}{\epsilon} \log N\epsilon)$  elements, which has recently been shown to be optimal for comparison-based deterministic algorithms [19].

**The Random algorithm [33]:** is an algorithm that reports all quantiles within the specified error with constant probability.

Random separates the stream into fixed-size buffers, each of which is assigned a level. Whenever there are two buffers at the same level, Random merges them into a buffer at one level higher, such that at any time, there is at most one buffer at any level. Random aggregates the ranks of *x* in all buffers to report the rank of an element *x*. Overall, it requires keeping  $O(\frac{1}{\epsilon} \log^{1.5} \frac{1}{\epsilon})$  elements to guarantee that for any *q*, its estimate would satisfy  $\Pr\left[\widehat{q} \in [q - \epsilon, q + \epsilon]\right] \ge 2/3$  (and this can be amplified to  $1 - \delta$  using the median

of  $O(\log \delta^{-1})$  independent repetitions). If we aim for a specific quantile, then the space reduces to  $O(\frac{1}{\epsilon} \log \frac{1}{\epsilon})$  per repetition.

# 4 The SQUARE Algorithm

Here, we present the Sampled QUAntile REconstruction (SQUARE) algorithm for solving the  $(\theta, \epsilon, \delta)$ -HH-latencies problem that employs RS (see section 3.2) as a blackbox.

Intuitively, sampling is a common technique that is useful for many applications, including quantile estimation, and will serve as a baseline for our more complex algorithms. It is long known that for approximating a quantile from a uniformly selected subset of a number stream to within an additive- $\epsilon$  error with probability  $1 - \delta$ , one needs to sample  $\theta$  ( $\epsilon^{-2} \log \delta^{-1}$ ) elements [35].

As a result, any heavy hitter x (an ID that appears at least  $N\theta$  times) must be sampled  $\Omega(\epsilon^{-2} \log \delta^{-1})$  times to solve the  $(\theta, \epsilon, \delta)$ -HH-latencies problem using sampling. Notice that there can be at most  $\theta^{-1}$  heavy hitters. As a result,

Notice that there can be at most  $\theta^{-1}$  heavy hitters. As a result, we employ RS to sample  $M = \Theta(\theta^{-1}\epsilon^{-2}\log\delta^{-1})$  elements from S. This way, a given heavy hitter is sampled  $\Omega(\epsilon^{-2}\log\delta^{-1})$  times with probability  $1 - \delta/2$ , and its samples allow us to produce an appropriate quantile estimate with probability  $1 - \delta/2$ ; using the union bound, we get that the overall estimate is accurate with probability  $1 - \delta$ .

Furthermore, by selecting  $\hat{f}_x$  to be N/M times the number of samples (e.g., see the analysis of [8]), we can estimate the frequency of an item to within an  $N \cdot \epsilon \sqrt{\theta}$  factor with probability  $1 - \delta$ .

The following theorem states the memory consumption of the SQUARE algorithm.

THEOREM 1. SQUARE solves the  $(\theta, \epsilon, \delta)$ -HH-latencies problem while requiring  $O(\theta^{-1}\epsilon^{-2}\log \delta^{-1})$  space.

# 5 The QUASI Algorithm

Next, we offer a **deterministic** algorithm called QUAntile Sketches for heavy Items (QUASI). Intuitively, QUASI allocates a separate GK sketch [25] to track the latency quantiles of each potential heavy hitter. Because we don't know the IDs of the heavy hitters ahead of time, QUASI uses a space-saving instance with  $k = 2\epsilon^{-1}\theta^{-1}$  entries, where each entry has a GK sketch instance configured for error  $\epsilon_{GK} = \epsilon/2$  in addition to its counter and ID fields. This way, QUASI can use the space saving counter value to estimate the frequency, and use the GK sketch to approximate the latency quantile.

Whenever an item  $(x, \ell)$  arrives, if x has an allocated counter, QUASI increments x's counter and inserts  $\ell$  to the associated GK instance. Otherwise, we replace the item that has the minimal counter value with x and reset its corresponding GK instance. Then, we insert  $\ell$  to this GK instance.

Using the SS variant mentioned above, we can compute any QUERY(x, q) as follows. If x has an allocated SS entry, we estimate its frequency using its counter value. Its GK instance is then queried to estimate the  $q^{th}$  quantile of  $L_x$ . Otherwise, if x has no allocated entry, we estimate the minimal SS counter value as (an upper bound on) its frequency and do not report the latency. Since SS deterministically guarantees that every element with a frequency larger than  $N/k \leq N\theta$  (i.e., in particular, every heavy hitter) will have an entry, we can satisfy the accuracy guarantees.

Algorithm 1 provides a high level pseudo code of QUASI using the pseudo code of Space Saving as shown in [37] without implementation details. The additions to manipulate the GK sketch instances are highlighted in blue. Table 3 contains a list of the used variables.

Algorithm 1 QUASI				
1: <b>function</b> INSERT $(x, \ell)$				
<b>if</b> <i>x</i> is monitored <b>then</b>				
3: Increment $count_x$ , the counter of $x$				
4: Insert $\ell$ to $GK_x$ , the GK sketch of $x$				
5: <b>else</b>				
6: <b>if</b> Less than $k$ items are monitored <b>then</b>				
7: $count_x \leftarrow 1$				
8: Initialize a GK sketch for $x$				
9: else				
10: Let $x'$ be the element with smallest $count_x$				
11: Start monitoring $x$ instead of $x'$ ;				
12: $count_x \leftarrow count_{x'} + 1$				
13: Reset the GK sketch for $x$				
14: Insert $\ell$ to the GK sketch of $x$				
15 franction company(1, a)				

15: **function** QUERY(x, q)

17:	return (	(count <sub>x</sub> , GK <sub>x</sub> .Quantile(	<b>q</b> ))

```
18: else
```

```
19: return (count_{min}, undefined)
```

Table 3: Variables used by QUASI (Algorithm 1)

k	number of entries in the SS
$count_x$	counter of <i>x</i> in the SS
count <sub>min</sub>	minimal counter value in the SS
$GK_{x}$	the GK sketch instance of <i>x</i>

Accuracy Guarantees. Using the standard analysis for an SS instance with k entries, we have every heavy hitter receive a spacesaving instance no later than its N/k arrival (because the counters sum to at most N, the minimal cannot be greater than N/k).

Therefore, if the queried element has no counter, it cannot be a heavy hitter. Otherwise, the GK sketch of the queried heavy hitter x processes all but at most  $N/k = N\theta\epsilon$  latencies from  $L_x$ . Let  $L'_x \subseteq L_x$  denote the subset of latencies processed by  $GK_x$ . Due to  $GK_x$ 's guarantees, our output  $\hat{q} = GK_x$ . Quantile(q) is within  $\epsilon$  from the true quantile of  $L'_x$ . That is,  $\hat{q}$  deviates from the true quantile by at most  $|L'_x| \cdot \epsilon_{GK} = |L'_x| \cdot \epsilon/2$  values. Together with the missing latencies of  $L_x \setminus L'_x$ , we have that  $\hat{q}$  deviates by at most  $|L'_x| \cdot \epsilon/2 + N\theta\epsilon/2$ . In terms of quantiles, this means that our error is

$$\frac{|L'_{x}| \cdot \epsilon + N\theta\epsilon}{|L_{x}|} \leq \epsilon \cdot \left(1 + \frac{N\theta}{|L_{x}|}\right) \leq \epsilon.$$

Let us analyze the space next. We use  $k = O(\epsilon^{-1}\theta^{-1})$  entries, each with a GK sketch configured for  $\epsilon_{GK} = \epsilon/2$ . Let  $a_i$  be the number of times the *i*'th *GK* instance was incremented; therefore,  $\sum_{i=1}^{k} a_i \leq N$ . By the space complexity of the GK algorithm we have that QUASI's overall space requirement is

$$\begin{split} \sum_{i=1}^{k} O(\epsilon^{-1}(1 + \log(a_i\epsilon))) &= O\left(k \cdot \epsilon^{-1} \cdot \left(1 + \log\frac{N \cdot \epsilon}{k}\right)\right) \\ &= O\left(\theta^{-1}\epsilon^{-2} \cdot \left(1 + \log(N\epsilon^2\theta)\right)\right). \end{split}$$

Here, we used Jansen's inequality and the concaveness of the logarithm function. We summarize the analysis in the following theorem.

THEOREM 2. QUASI solves  $(\theta, \epsilon, 0)$ -HH-latencies problem (i.e., deterministically) while requiring  $O\left(\theta^{-1}\epsilon^{-2} \cdot (1 + \log(N\epsilon^2\theta))\right)$  space.

#### 6 The SQUAD Algorithm

The quadratic dependency on  $1/\epsilon$  of the previous algorithms is sometimes prohibitively costly. Interestingly, while both the sampling (SQUARE) and sketching (QUASI) approaches require  $\tilde{\Omega}(1/\epsilon^2)$ space<sup>1</sup>, applying them in tandem we can significantly improve the space complexity. Specifically, we present Sketching/sampling QUAntiles Duo (SQUAD), a hybrid algorithm that requires only  $\tilde{O}(\epsilon^{-1.5})$  space<sup>1</sup>. Intuitively, the sampling helps us capture the behavior of the latencies experienced by an item before it was allocated with an SS entry and a quantile sketch.

SQUAD keeps  $z = O(e^{-1.5}\theta^{-1}\log\delta^{-1})$  samples chosen by RS (see section 3.2). Each sample is a *triplet* (*ID*, *latency*, *timestamp*). That is, when sampling an element  $(x_i, \ell_i)$  we store the triplet  $(x_i, \ell_i, i)$ . Additionally, SQUAD employs an enhanced Space Saving [37] (SS) as described in QUASI (Section 5), but instead of using

<sup>&</sup>lt;sup>1</sup>The  $\tilde{\Omega}, \tilde{\Theta}$  and  $\tilde{O}$  notations assume that the heavy hitters parameter  $\theta$  is constant and hide polylogarithmic factors.

instances of the GK algorithm, it employs instances of the Random algorithm (configured for  $\epsilon/2$  error similarly to QUASI)^2

In contrast to QUASI, which requires  $k = \Theta(\epsilon^{-1}\theta^{-1})$  entries, SQUAD only uses  $m = 4\epsilon^{-0.5}\theta^{-1}$ . Intuitively, we can avoid sketching additional latencies from  $L_x$  because the ones processed before *x* is allocated to a sketch, are well approximated by the sample.

In addition to the counter and the Random instance, an entry for ID *x* in the SS structure has a timestamp  $(t_x)$  that indicates when *x* was last allocated with an entry.

If RS decides to consider the *i*'th element  $(x_i, \ell_i)$ , we store  $(x_i, \ell_i, i)$ in the samples array. After that, we update the augmented SS as follows: If  $x_i$  has an allocated counter, SQUAD increments it and inserts  $\ell_i$  into the associated random instance. Otherwise, we reallocate the counter with the lowest value for *x*, flush its Random instance, and set its  $t_{x_i}$  to *i*. After that, we add  $\ell$  to this Random instance. Notice that *x* continues to participate in the RS process *regardless of whether it has a counter in SS or not*. Intuitively, *x*'s entry could become minimal and it could be evicted from the SS, so we keep tracking it by sampling.

For answering QUERY(x, q), we search for x in the augmented SS. If x does not have an entry, our algorithms cannot promise anything about its  $q^{th}$  quantile (similarly to QUASI, this means that x is not a heavy hitter). To estimate the frequency of an item x, we use both the sample and the SS counter. Specifically, let  $t_x$  denote the timestamp of x in the SS, and let  $S_x$  denote the number of samples that belong to x with a timestamp smaller than  $t_x$ . We estimate the number of times that x arrived before  $t_x$  as  $N/z \cdot S_x$  because the probability that RS samples a specific element is z/N. As a result, we estimate the frequency as  $\hat{f_x} = N/z \cdot S_x + I_x$ .

The latencies are estimated in a similar way: we take the samples collected before  $t_x$  as representing the latencies before  $t_x$  and merge them with the entries stored in  $Random_x$  (that represent entries between  $t_x$  and N).

To merge the samples and the sketch, we duplicate  $Random_x$  and then insert the  $S_x$  samples, each with a weight of N/z. Our approximation of the *q*'th quantile is the quantile of the combined array.

An alternative approach is to merge the samples of x,  $Samples_x$ , with the buffers of the Random algorithm in side buffers in the function QUERY(x, q), and then report the rank of x using the merged buffers. This approach requires an additional modification in the implementation of the Random QUANTILE(q) function.

The variables of the SQUAD algorithm are described in Table 4 and its pseudocode appears in Algorithm 2.

Accuracy Guarantees. Intuitively, our analysis relies on the observation that if the sample approximates *x*'s frequency before  $t_x$  to within an  $\alpha$  additive error, and the space saving approximates its frequency since  $t_x$  to within  $\beta$  elements, then the error of the merging process cannot exceed  $\alpha + \beta$ ; a similar logic applies to the latency quantiles (e.g., see [26]).

Let us start by analyzing the sample. Let  $f_{x,1}$  denote x's frequency before (not including)  $t_x$  and let  $f_{x,2}$  denote its frequency starting with  $t_x$  (i.e.,  $f_x = f_{x,1} + f_{x,2}$ ). As before, we denote by

# Algorithm 2 SQUAD

116		
1: 1	<b>function</b> INSERT $(x, \ell)$	
2:	$n \leftarrow n + 1$	The current timestamp
3:	$RS.Add(x, \ell, n)$	
4:	<b>if</b> <i>x</i> is monitored <b>then</b>	
5:	Increment $count_X$ , the	counter of <i>x</i>
6:	Increment $I_X$ , the cour	nt since $x$ became monitored
7:	Insert $\ell$ to $Rnd_x$ , the R	andom sketch of <i>x</i>
8:	else	
9:	if Less than <i>m</i> items a	re monitored <b>then</b>
0:	Initialize a Randor	n sketch for <i>x</i>
1:	$count_x \leftarrow 1$	
2:	else	
3:	Let $x'$ be the element	ent with smallest $count_{X'}$
4:	Start monitoring <i>x</i>	instead of $x'$ ;
5:	$count_x \leftarrow count_{x'}$	+1
16:	Reset the Random	sketch for <i>x</i>
17:	$I_x \leftarrow 1$	
8:	$t_x \leftarrow n$	
9:	Insert $\ell$ to the Random	n sketch of <i>x</i>
:0: 1	<b>function</b> QUERY $(x, q)$	
21:	$S_x \leftarrow 0$	
2:	$SList_x \leftarrow empty \ list$	
3:	<b>for</b> $j \in 0, 1,, z$ <b>do</b>	
4:	<b>if</b> $RS[j].ID = x$ and $L$	$RS[j].ts < t_x$ then
5:	$S_x \leftarrow S_x + 1$	
6:	Insert $RS[j].\ell$ to $S$	$SList_x$
27:	$sf_x = \frac{n}{z} \cdot S_x$	
28:	if <i>x</i> is monitored then	
9:	$RndNew_x = Rnd_x$	
0:	Insert samples from SL	$ist_x$ with weight $\frac{n}{z}$ to RandNew <sub>x</sub>
1:	<b>return</b> $(sf_x + I_x, Rand$	$lNew_{x}.Quantile(\tilde{q}))$
2:	else	
3:	<b>return</b> $(sf_x, undefine$	<i>d</i> )

Table 4: Variables used by SQUAD (Algorithm 2)

number of arrived elements		
samples size used by RS		
a Reservoir Sampling instance with maximal <i>z</i> sam-		
ples.		
number of entries in the SS		
counter of <i>x</i> in the SS		
the count since <i>x</i> became monitored		
timestamp of <i>x</i> in the SS		
number of samples that belongs to <i>x</i>		
estimation of <i>x</i> 's frequency before $t_x$		
the Random instance of <i>x</i>		

 $S_x$  the number of x's samples collected before  $t_x$  (observe that  $S_x \sim$  Hypergeometric $(N, f_{x,1}, z)$ ). Thus, we use the approximation  $\widehat{f_{x,1}} = S_x \cdot N/z$ . Denoting  $p = f_{x,1}/N$ , we can use standard concentration bounds (e.g., [42]) on the hypergeometric distribution to

 $<sup>^2</sup>$ We chose Random as it is the fastest algorithm we are aware of, and our guarantee is random anyhow due to the sampling. One can replace it with a state-of-the-art sketch such as KLL [30], slightly improving the accuracy at a potential loss of speed. In any case, the complexity remains  $\widetilde{\Theta}(\epsilon^{-1.5})$ .

bound the sampling error as, for any  $\Delta \in (0, z \cdot p]$ 

$$\Pr\left[|S_{\chi} - \mathbb{E}[S_{\chi}]| \ge \Delta\right] < 2e^{-\frac{\Delta^2}{3z \cdot p}}.$$
(1)

Notice that once an item that reaches a frequency of N/m it cannot have the minimum SS entry. Therefore, we have that  $f_{x,1} \leq$ N/m. As the sampling error is monotonically increasing in  $f_{x,1}$  (for  $f_{x,1} < N/2$ ), we bound the error by analyzing the error of an item with  $f_{x,1} = N/m$ . In our context, we sample  $z = O(e^{-1.5}\theta^{-1}\log \delta^{-1})$ elements from the stream; that is, the probability for each of the zsamples to belong to the first  $f_{x,1}$  insertions of x is  $p = \frac{f_{x,1}}{N} = 1/m$ . Next, let  $\Delta = \sqrt{\frac{3z \log(2/\delta)}{m}} = \Theta\left(\sqrt{\frac{z \log \delta^{-1}}{m}}\right)^3$ . Our goal in what follows is to show that the error in estimating  $f_{x,1}$  is likely to be lower than  $N \cdot \sqrt{\frac{3\log(2/\delta)}{z \cdot m}} = \Theta(N \cdot \epsilon \cdot \theta).$ Using (1), we have that:

$$\Pr\left[|\widehat{f_{x,1}} - f_{x,1}| \ge N \cdot \sqrt{\frac{3\log(2/\delta)}{z \cdot m}}\right]$$
$$= \Pr\left[|S_x \cdot N/z - \mathbb{E}[S_x] \cdot N/z| \ge \frac{N}{z} \cdot \Delta\right]$$
$$= \Pr\left[|S_x - \mathbb{E}[S_x]| \ge \Delta\right] \le 2e^{-\frac{\Delta^2}{3z \cdot p}} = \delta.$$

Next, recall that  $f_{2,x}$  is calculated accurately using  $I_x$ , and therefore  $|\widehat{f_x} - f_x| = |\widehat{f_{x,1}} - f_{x,1}|$ . Therefore, we established that the frequency estimation error is bounded by  $N \cdot \epsilon \cdot \theta$ , with probability  $1 - \delta$ , using  $z = c \cdot \epsilon^{-1.5} \theta^{-1} \log \delta^{-1}$  samples, for an appropriate constant c > 0.

To analyze the quantile estimation error, we consider the error of the sampling phase (before  $t_x$ ) separately from the error once x is allocated with a sketch (starting with  $t_x$ ). An analysis similar to the above (with different constants) yields that the error in the sampling phase is bounded by  $\epsilon/2$  except with probability  $\delta/2$ . Specifically, we can get  $S_x = \Omega(z \cdot f_{x,1}/N) = \Omega(\epsilon^{-1} \log \delta^{-1})$  samples except with probability  $\delta/4$ , and have these approximate the quantile within an additive  $\Theta(\sqrt{\epsilon})$  error except with further  $\delta/4$  error probability. This means that the rank of the latency is off by at most  $\Theta(\sqrt{\epsilon})$  $f_{x,1}$  =  $\Theta(\sqrt{\epsilon} \cdot N/m) = \Theta(N\epsilon\theta)$  from the true quantile. Therefore, by configuring the quantile sketch to have an  $\epsilon/2$  error with probability  $1 - \delta/2$ , we can get that the overall estimate error, which results from the combination of the sample and the sketch, is bounded by  $f_x \cdot \epsilon + \Theta(N\epsilon\theta) = O(f_x\epsilon)$ , as  $f_x \ge N\theta$  per our problem definition. We summarize the analysis in the following theorem.

THEOREM 3. SQUAD solves  $(\theta, \epsilon, \delta)$ -HH-latencies problem deterministically while requiring  $O(\theta^{-1}\epsilon^{-1.5} \cdot \log \epsilon^{-1})$  space.

#### 7 **Optimizing the Processing Speed**

We now detail several optimizations that enable our algorithms to process elements faster. First, we use the Algorithm L [31], which provides a fast simulation of RS. Intuitively, instead of drawing a

<sup>3</sup>Observe that  $z \cdot p = z/m = \Theta(\epsilon^{-1} \log \delta^{-1})$ , and therefore:

$$\Delta = \Theta\left(\sqrt{\frac{z\log\delta^{-1}}{m}}\right) = \Theta\left(\sqrt{\epsilon^{-1}}\cdot\log\delta^{-1}\right) = o\left(z\cdot p\right).$$

random integer per item, it generates geometric random variables that represent how many items to skip before the next one is admitted into the reservoir. Once an item is chosen, it replaces a uniform slot in  $\{0, \ldots, k-1\}$ . As a result, the total number of updates falls to  $O(k(1 + \log(N/k)))$ , implying that it takes o(1) computation per element because the majority are skipped.

While we can use the above to optimize the RS process, QUASI's and SQUAD's processing speed is limited as arrivals of elements not tracked by the SS require initializing a new sketch. To speed up the processing of both, we propose using an initial probabilistic filtering stage. Intuitively, as both quantiles and frequencies can be accurately estimated from sampled streams for heavy hitters, we can process a small (e.g., 10%) of the input and obtain rather precise results. Namely, consider a wrapper that with probability p calls the INSERT function of SQUAD (or QUASI<sup>4</sup>) and otherwise ignores the packet. This means that the algorithms look at a sampled stream  $\mathcal{S}' \subset \mathcal{S}$ such that each element in S appears with probability  $\mathfrak{p}$  in S' i.i.d.

Intuitively, for a constant error probability, the sampling error would be of size  $\Theta(\sqrt{N\theta/\mathfrak{p}})$ ; if this is comparable or smaller than the  $\Theta(N\epsilon\theta)$  error of SQUAD, we can compensate for the error resulting from analyzing S' (rather than S) without asymptotically increasing the space requirements.

There are several approaches to selecting p. One option is to dynamically change p as N grows, inserting elements with a weight of  $1/\mathfrak{p}$ , e.g., as suggested by [11, 32]. For simplicity, here we consider using a fixed probability, which means that the accuracy guarantees of the algorithms only hold after a short convergence time (as common in some sampling algorithms [10, 13, 36]). Namely, consider setting SQUAD to solve the  $(\theta, \alpha \cdot \epsilon, \alpha \cdot \delta)$ -HH-latencies problem for some  $\alpha \in (0, 1)$  (e.g.,  $\alpha = 0.9$ ). Then, if the frequencies and latency quantiles are maintained in S' (the frequency, after scaling by  $1/\mathfrak{p}$ ) to within error  $(1 - \alpha)\epsilon$ , except with probability  $(1 - \alpha)\delta$ , then the overall scheme solves  $(\theta, \epsilon, \delta)$ -HH-latencies problem. Here,  $\alpha$ is a tradeoff parameter: the larger  $\alpha$  is, the less space the algorithm requires, but also the higher the sampling probability needs to be.

As analyzed above, a sample of size  $|S'| = \Omega(\theta^{-1}\epsilon_s^{-2}\log\delta_s^{-1})$ is enough for S' to be an  $\epsilon_s$  approximation of the quantiles and frequency (the s subscript represents sampling) of an element except with probability  $\delta_s$ . In our case, we have  $|S'| = N\mathfrak{p}$ , i.e.,  $N\mathfrak{p} =$  $\Omega(\theta^{-1}\epsilon_s^{-2}\log\delta_s^{-1})$ , and thus we need a convergence time of at least  $N = \Omega(\theta^{-1}((1-\alpha)\epsilon)^{-2}\mathfrak{p}^{-1}\log((1-\alpha)\delta)^{-1})$  elements before the algorithm solves the  $(\theta, \epsilon, \delta)$ -HH-latencies problem.

Intuitively, since in practical applications  $N \gg \theta^{-1} \epsilon^{-2} \log \delta^{-1}$ , we can set a large  $\alpha$  value (e.g.,  $\alpha = 0.9$ ). We can then use an intermediate value for  $\mathfrak{p}$  (e.g.,  $\mathfrak{p} \in [0.1, 0.01]$ ) as this gives a large speed boost and lowering the sampling probability further is not as beneficial. This way, we do not require significantly more space (about 20% increase for  $\alpha = 0.9$ ) nor compromise the accuracy guarantees (following the short convergence time) while significantly accelerating the solution.

<sup>&</sup>lt;sup>4</sup>Note that this makes the algorithm randomized.

# 8 Evaluation

# 8.1 Setup

We developed a C++ prototype for each of the algorithms mentioned in this paper: SQUARE, QUASI and SQUAD. The QUASI and SQUAD are implemented here using Quantile Sketch [46] as a building block. Additionally, we compared our results to the GK-algorithm [25] and the Random algorithm [33] as a general baseline, since these are the state of the art for the more basic problem of quantiles across whole data streams, rather than per-element quantiles. To to the best of our knowledge, this is the first study that solves quantiles on a per-element level. Furthermore, we compared to Space Saving (SS) [37], since this is a building element in QUASI and SQUAD.

*8.1.1 Dataset:* We evaluate our algorithms using NS3 simulations [2] for a FatTree topology comprised of 16 Core switches, 20 Agg switches, 20 ToRs, and 320 servers (16 in each rack). Each server has a single 100Gbps NIC and the default load is 60%. Each connection between Core and Agg switches, as well as between Agg switches and ToRs, has a capacity of 400Gbps. The switch buffer size is 32MB. The traffic follows the flow size distribution in web search from Microsoft [5] or Hadoop from Facebook [41].

The evaluation was performed on an Intel(R) 3.20GHz Xeon(R) CPU E5-2667 v4 running Linux with kernel 4.4.0-71. Each data point in all runtime measurements is shown as a 95% confidence interval of 10 runs. Our evaluation includes only the web search trace as the Hadoop trace exhibits very similar results.

# 8.2 Accuracy Comparison

We measure accuracy in this experiment as a function of used memory. Specifically, given quantile q, we measure  $|\operatorname{rank}(\widehat{\mathcal{L}_{x,q}}) - q|$ ), a.k.a *percentage error*, as a function of consumed memory for each x that satisfies  $f_x \ge N\theta$ . Additionally, we present the theoretical error which demonstrates that the empirical error is constrained by the theoretical error.

Figure 2 illustrates the percentage error in terms of quantiles: 50%, 90% and 95% for each algorithm: SQUARE, QUASI, and SQUAD as a function of memory use with a constant value of  $\theta = 0.01$  using NS3-simulated online search trace. Note that all graphs have the same amount of points, but some of them overlap in several graphs. Additionally, Figure 3 illustrates the percentage error for SQUAD in terms of quantiles: 50%, 90% and 95% using an NS3-simulated trace following the Hadoop flow size distribution.

Throughout, as memory use increases, our algorithms get more precise, resulting in a decrease in empirical error. As can be seen, SQUAD is the most compact algorithm among SQUARE and QUASI, whereas SQUARE is the most resource-intensive. As previously stated, SQUARE stores  $\Theta(\theta^{-1}\epsilon^{-2}\log\delta^{-1})$  elements from the stream. To ensure a small error of *epsilon*, SQUARE should keep a high number of samples, which results in saving the whole stream size in small values of  $\epsilon$  and  $\theta$ , as seen in Figure 3c.

For the QUASI algorithms, keeping the heavy hitters in the SS instance together with their GK-algorithm sketch results in a smaller footprint than SQUARE. SQUAD, on the other hand, is the most efficient algorithm for solving the ( $\theta, \epsilon, \delta$ )-HH-latencies problem due to its compact data structure, as seen in Table 1. In general, a lower space consumption required for a specific  $\epsilon$  and  $\theta$  values translates into better empirical error. For example, QUASI consumes more memory than SQUAD for the same  $\epsilon$  and  $\theta$ . Thus, for a given memory budget, QUASI is more accurate than SQUARE and SQUAD is more accurate than both.

### 8.3 Performance Comparison

Figures 4 and 5 compare the update speed. We explore the trade-off of  $\epsilon$  with a fixed  $\theta = 0.01$ .

8.3.1 QUASI Update Time: Figure 4a illustrates the performance of QUASI in terms of update time when compared to its building blocks: Space Saving (SS) [37] and the GK-algorithm [25]. Bear in mind that, although SS is the quickest, neither it nor the GK-algorithm solve the  $(\theta, \epsilon, \delta)$ -HH-latencies problem and rather serve as a best-case reference point.

As can be observed, QUASI's update performance is comparable to that of the GK algorithm. Recall that the QUASI update operation is equivalent to an update operation in SS and an update operation on the corresponding GK instance. Due to the high effectiveness of SS updates, the run time of QUASI updates is limited by the run time of GK. That is, while GK-algorithm solves the quantile problem for the full stream, the QUASI algorithm solves per-element quantiles without adding any extra update time cost.

Specifically, we may replace the GK instances in QUASI with any sketch that solves quantiles, such as Random [33], which has a higher update speed, as seen in Figure 4b. However, QUASI will no longer be a deterministic solution in this case. As a result, there is a trade-off between update speed and determinism. Additionally, it was shown that the GK-algorithm is an optimal **deterministic** comparison based algorithm.

*8.3.2* SQUAD Update Time: Figure 4b compares SQUAD's update speed to that of its building blocks: Space Saving (SS) [37] and the Random-algorithms [33].

Recall that the Random algorithm reports quantiles over the entire stream, thus it does not solve the  $(\theta, \epsilon, \delta)$ -HH-latencies problem and only serves as a best case reference point. SQUAD update operation is translated to sampling operation, SS update and Random update. As a result, the run time of its update is impacted by the run time of all of them.  $SQUAD_p = 1$  in the graph indicates that the implementation excludes the optimizations detailed in Section 7.

8.3.3 Comparing SQUARE, QUASI and SQUAD Update Time: As seen in Figure 5a, SQUARE is the fastest algorithm since each update is converted to a sampling update. SQUAD, on the other hand, performs better than QUASI in terms of update performance since it is based on the Random algorithm, which has a faster update time than the GK- algorithm, the building block of QUASI. While SQUARE is the quickest, it solves the  $(\theta, \epsilon, \delta)$ -HH-latencies problem with a high memory cost, as seen in Figure 2. When  $\epsilon$  is small, SQUARE keeps a significant number of samples more than the stream size. In this scenario, SQUARE just saves all streams, which results in superior performance than larger values of  $\epsilon$ , which allows items to override earlier samples.

Figure 5b shows how the filtering optimization discussed in Section 7 improves the update performance of SQUAD. In this scenario,



Figure 2: Accuracy as a function of used memory using NS3-simulated online search trace. Each marker corresponds with one heavy hitter; i.e., we show the error ( $|rank(\widehat{L_{x,q}}) - q|$ )) for each x that satisfies  $f_x \ge N\theta$  for a fixed value of  $\theta = 0.01$ , as a function of memory consumed. We examine the quantiles 50%, 90%, 99% of each algorithm: SQUARE, QUASI, and SQUAD. Notice the different x/y-axis ranges.

when the filter sampling probability  $\mathfrak{p}$  is 0.1, its performance comparable or better than SQUARE, and with  $\mathfrak{p} = 0.01$ , SQUAD becomes the clear winner. We explore these optimizations further below.

8.3.4 Query Speed Comparison: For comparing the query speed, we used the quantiles 50%, 90%, 99%. We investigated the effect of the  $\epsilon$  parameter using a fixed  $\theta$  of 0.01, and the experiment includes quantile queries for items *x* that satisfy the condition  $f_x \ge N\theta$ . For decreasing  $\epsilon$  values, more latencies access the quantile sketches. Consequently, we got slower query operations in all algorithm. As seen in Figure 6, QUASI performs better than SQUAD because

SQUAD relies on Random queries, which perform worst than the GK. Additionally, SQUAD checks its samples part to figure out the samples that were taken before the time the given identifier enters the SS. This becomes extremely expensive when the sample size is large, i.e. when the  $\epsilon$  value is small.

Particularly, as seen in Figure 4b, we may replace the Random instances in SQUAD with a GK sketch that has a faster query performance. However, as seen in Figure 4, GK is slower in update performance. Indeed, there is a trade-off between update and query



Figure 3: Accuracy as a function of used memory using NS3-simulated trace following the flow size distribution in Hadoop. Each marker corresponds with one heavy hitter; i.e., we show the percentage error  $(|\operatorname{rank}(\overline{\mathcal{I}_{x,q}}) - q|))$  for each x that satisfies  $f_x \ge N\theta$  with fixed value of  $\theta = 0.01$ , as a function of memory consumed. We examine the quantile 50%, 90%, 99% of SQUAD.



Figure 4: Update runtime as function of the accuracy guarantee ( $\epsilon$ ) with fixed  $\theta = 0.01$  (a) QUASI update performance compared to its building blocks: SS and GK (b) SQUAD update performance compared to its building blocks: SS and Random (RND in the graph).  $SQUAD_p = 1$  indicates that the implementation excludes the optimizations detailed in Section 7.



Figure 5: Update runtime as function of the accuracy guarantee ( $\epsilon$ ) with fixed  $\theta = 0.01$  (a) Comparing all three of our algorithms: SQUARE, QUASI and SQUAD (b) The effect of the optimization on the performance of SQUAD update.

performance. However, since is most streaming applications updates occur more often than query operations, Random would usually be the preferred choice.

# 8.4 Optimizations Comparison

In this section we evaluate the optimizations described in Section 7. We examine the influence of the optimizations on the update runtime, memory usage, and empirical error.



Figure 6: Query runtime as function of the accuracy guarantee ( $\epsilon$ ) with fixed  $\theta = 0.01$  when asked for the 90% quantile (a) QUASI query performance compared to Random (RND).



Figure 7: (a) Memory consumption as function of  $\alpha$  with  $\epsilon = 0.0025$  and  $\theta = 0.01$  (b) Mean Accuracy as a function of the measurement length size using an NS3-simulated trace that follows the Hadoop flow size distribution. Each marker represents the mean of the heavy hitters' percentage error, with fixed values of  $\theta = 0.01$  and  $\epsilon = 0.025$ . We examine the sampling probability  $\mathfrak{p} = 0.01, 0.001$  in SQUAD (c) Accuracy as a function of the measurement length till the stream's end when sampling with  $\mathfrak{p} = 0.01$  in SQUAD with  $\theta = 0.01$  and  $\epsilon = 0.025$ . [Ran: TODO: add p = 0.1, switch to log scale on the x-axis, merge (b) and (c), add a horizontal line for  $\epsilon$ .]

8.4.1 Effect of Optimization on SQUAD Update Time: We implement the optimizations in SQUAD since it is the most space-efficient algorithm. The update runtime is shown in Figure 5b as a function of  $\epsilon$  with a fixed  $\theta = 0.01$ . The three SQUAD implementations differ in the probability of the wrapper calling the INSERT function of SQUAD. We consider three probabilities:  $\mathfrak{p} = 1$  (indicating that the implementation does not include optimizations),  $\mathfrak{p} = 0.1$ , and  $\mathfrak{p} = 0.01$ . That is, each element in *S* occurs with probability  $\mathfrak{p}$  in the sampled stream i.i.d. As expected, decreasing the value of  $\mathfrak{p}$  results in improved update speed, as the algorithm invokes the SQUAD INSERT function infrequently. As can be observed, the optimizations considerably improve the speed of the update.

8.4.2 Effect of  $\alpha$  on Memory Consumption: Figure 7a shows the space consumed by our algorithms as function of  $\alpha$  with fixed values of  $\epsilon = 0.025$  and  $\theta = 0.01$ . As seen in Figure 7a, the larger  $\alpha$  is the less space the algorithm requires, but also the higher the sampling probability needs to be. For  $\alpha = 0.9$ , with  $\epsilon = 0.025$  and  $\theta = 0.01$  we get an increase of 18% in the space requirement of SQUAD. Parameter  $\alpha$  has less impact on the memory of SQUAD than its impact on SQUARE and QUASI since SQUAD space complexity is better than the others for the same values of  $\epsilon$  and  $\theta$ .

As seen in Figure 7a, the greater  $\alpha$  is, the less space is required for the algorithm, but the sampling probability must be increased. With  $\alpha = 0.9$ ,  $\epsilon = 0.025$ , and  $\theta = 0.01$ , the space needed for SQUAD increases by 18%. The parameter  $\alpha$  has a smaller effect on the memory of SQUAD than it does on SQUARE and QUASI, since SQUAD has a better space complexity than the others for the same values of  $\epsilon$  and  $\theta$ .

8.4.3 The Effect of the Length of the Measurement on the Error: We study the trade-off between geo-sampling rate  $\mathfrak{p}$  and the convergence time (in terms of the number of packets) and report the results in Figure 7b. We use an NS3-simulated trace that follows the Hadoop flow size distribution with fixed values of  $\theta = 0.01$  and  $\epsilon = 0.025$ . Since SQUAD with optimizations uses sampling to select packets, it requires a convergence time to produce a guaranteed accurate result (analyzed in Section 7). As expected, larger  $\mathfrak{p}$  value leads to faster convergence time as we sample elements in higher probability. In addition, we examine the mean error of SQUAD during the whole trace and show it in Figure 7c.

# 9 Extensions of Supporting Tail Latencies for Traffic Volume Heavy-Hitters

It is often desirable to find the tail latencies for heavy hitters in terms of traffic volume. That is, consider a stream in which each element has a *size* and our goal is to find the tail latency for items that use the majority of the bandwidth. Formally, we look at a *weighted stream*  $S = \langle (w_1, x_1, \ell_1), (w_2, x_2, \ell_2) \dots \rangle \in (\{1, 2, \dots, M\} \times \mathcal{U} \times \mathbb{R})^+$  and define item's volume as the sum of sizes for elements that belong to it. It is worth noting that the weight  $w_i$  refers to the element *i*, which is composed of both  $x_i$  and  $\ell_i$ .

Sampling may be performed on the basis of the number of elements or, more broadly, on the basis of some weight (e.g., the size of the associated data). Weighted sampling provides a more precise view of the underlying byte-traffic. This is desirable for applications such as traffic engineering and load balancing that aim to stay within the bandwidth constraints of a network, as well as writes to SSDs and corresponding write-amplification etc. Priority Sampling [21] is a weighted sampling technique that is optimal. That is, when compared to other sampling techniques, Priority Sampling has a lower or equivalent variance. The goal is to produce a sample of *k* keys with a probability proportional to their weight. Priority Sampling accomplishes this by assigning the value  $\frac{W_i}{r}$  to each key, with *r* randomly selected from the range [0, 1]. Priority sampling includes the *k* keys with the highest values.

The Space Saving algorithm [37] can find weighted heavy hitters in a stream with an update time of  $O(\log \epsilon^{-1})$  [14]. Recent advancements [6, 9, 12] reduce this runtime to a constant. Thus, the tail latency problem for weighted heavy hitters may be solved with the same asymptotic complexity as the unweighted versions and with an error of up to  $M\epsilon$ .

Intuitively, to address this problem, we can use Priority Sampling instead of RS sampling and modify the Space Saving method to get the weighted heavy hitters. Furthermore, because the weight  $w_i$  corresponds to the latencies ( $\ell_i$ ), we must ensure that the reporting quantile of weighted latencies is met. KLL sketch [30] can also handle weighted items. Its size remains  $O(\epsilon^{-1}\sqrt{\log \epsilon^{-1}})$  but the update time becomes  $O(\log \epsilon^{-1})$ . That is, we need to replace the quantile-sketch in QUASI and SQUAD with a KLL sketch to support weighted items.

Putting it all together, in SQUARE we employ Priority Sampling instead of RS sampling, and in QUASI we use the Space Saving version for weighted streams with KLL sketch as quantile-sketch. In SQUAD, we employ Priority Sampling for the sampling part, as well as a Space Saving weighted variant with KLL sketch.

Thus, our algorithms are capable of solving the  $(\theta, \epsilon, \delta)$ -HHlatencies problem for weighted streams with the same space complexity as the unweighted version and with an error of at most  $M\epsilon$ .

#### 10 Discussion

In this paper, we studied the problem of reporting the tail latencies of heavy hitter items. To our knowledge, this is the first research to solve quantiles on a per-element level rather than reporting quantiles of an entire stream. Such capabilities can be useful when one wishes to assess a network's health and to debug various networking middle-boxes and smart data-planes. We presented a formal definition of the generalized problem and explored three solutions: a sample approach (SQUARE) and more sophisticated solutions called QUASI and SQUAD. QUASI is a deterministic solution that assigns a unique quantile-sketch (GK) to each potential heavy hitter that is obtained from a Space Saving instance. SQUAD combines the SQUARE and QUASI algorithms, resulting in superior memory reduction.

SQUAD is the most memory-efficient algorithm. Both SQUAD and SQUARE use about the same amount of memory. QUASI, on the other hand, is deterministic, but SQUARE has an error probability. This is true both asymptotically and in measurements throughout a large-scale NS3 simulation, where we observed orders of magnitude memory reductions for similar estimation errors in the SQUAD algorithm.

While SQUARE has a faster update rate than QUASI and SQUAD, it consumes a lot of memory. To that end, we suggested several efficiency enhancements for the update operation of our algorithms in Section 7. In fact, the update performance of QUASI is comparable to that of the state-of-the-art method, which can only handle quantiles throughout the whole stream, not per-element quantiles. Our approach can be applied to the case of volume traffic, where each element in the stream has a size and our algorithms determine the tail latency for the elements that use the majority of the available bandwidth.

Code Availability: All code is available online [1].

**Acknowledgements:** This work was partially funded by the Technion-HPI research school and the Israel Science Foundation grant #3119/21.

### References

- [1] Open source code. https://github.com/r4n4sh/squad.git.
- [2] The Network Simulator ns-3. https://www.nsnam.org/research/wns3/wns3-2015/.
- [3] P. K. Agarwal, G. Cormode, Z. Huang, J. M. Phillips, Z. Wei, and K. Yi. Mergeable summaries. ACM Transactions on Database Systems (TODS), 38(4):1–28, 2013.
- [4] P. K. Agarwal, G. Cormode, Z. Huang, J. M. Phillips, Z. Wei, and K. Yi. Mergeable summaries. ACM Transactions on Database Systems (TODS), 38(4):26, 2013.
- [5] M. Alizadeh, A. Greenberg, D. A. Maltz, J. Padhye, P. Patel, B. Prabhakar, S. Sengupta, and M. Sridharan. Data center tcp (dctcp). In *Proceedings of the ACM SIGCOMM 2010 Conference*, pages 63–74, 2010.
- [6] D. Anderson, P. Bevan, K. Lang, E. Liberty, L. Rhodes, and J. Thaler. A highperformance algorithm for identifying frequent items in data streams. In Proceedings of the 2017 Internet Measurement Conference, pages 268–282. ACM, 2017.
- [7] A. Arasu and G. S. Manku. Approximate counts and quantiles over sliding windows. In Proceedings of the twenty-third ACM SIGMOD-SIGACT-SIGART symposium on Principles of database systems, pages 286–296, 2004.
- [8] R. B. Basat, G. Einziger, S. L. Feibish, J. Moraney, and D. Raz. Network-wide routing-oblivious heavy hitters. In Proceedings of the 2018 Symposium on Architectures for Networking and Communications Systems, pages 66–73, 2018.
- [9] R. B. Basat, G. Einziger, and R. Friedman. Fast flow volume estimation. *Pervasive and Mobile Computing*, 2018.
- [10] R. B. Basat, G. Einziger, R. Friedman, M. C. Luizelli, and E. Waisbard. Volumetric hierarchical heavy hitters. In 2018 IEEE 26th International Symposium on Modeling, Analysis, and Simulation of Computer and Telecommunication Systems (MASCOTS), pages 381–392. IEEE, 2018.
- [11] R. B. Basat, G. Einziger, M. C. Luizelli, and E. Waisbard. A black-box method for accelerating measurement algorithms with accuracy guarantees. In 2019 IFIP Networking Conference (IFIP Networking), pages 1–9. IEEE, 2019.
- [12] R. Ben-Basat, G. Einziger, R. Friedman, and Y. Kassner. Optimal elephant flow detection. In *IEEE INFOCOM*, 2017.
- [13] R. Ben Basat, G. Einziger, R. Friedman, M. C. Luizelli, and E. Waisbard. Constant time updates in hierarchical heavy hitters. In *Proceedings of the Conference of the* ACM Special Interest Group on Data Communication, pages 127–140, 2017.
- [14] R. Berinde, P. Indyk, G. Cormode, and M. J. Strauss. Space-optimal heavy hitters with strong error bounds. ACM Transactions on Database Systems (TODS), 35(4):26, 2010.
- [15] G. Cormode, M. Garofalakis, S. Muthukrishnan, and R. Rastogi. Holistic aggregates in a networked world: Distributed tracking of approximate quantiles. In Proceedings of the 2005 ACM SIGMOD international conference on Management of data, pages 25–36, 2005.
- [16] G. Cormode, Z. Karnin, E. Liberty, J. Thaler, and P. Vesely. Relative error streaming quantiles. In Proceedings of the 40th ACM SIGMOD-SIGACT-SIGAI Symposium on Principles of Database Systems, pages 96–108, 2021.
- [17] G. Cormode, F. Korn, S. Muthukrishnan, and D. Srivastava. Diamond in the rough: Finding hierarchical heavy hitters in multi-dimensional data. In Proceedings of the 2004 ACM SIGMOD international conference on Management of data, pages 155–166, 2004.
- [18] G. Cormode, F. Korn, S. Muthukrishnan, and D. Srivastava. Space-and timeefficient deterministic algorithms for biased quantiles over data streams. In Proceedings of the twenty-fifth ACM SIGMOD-SIGACT-SIGART symposium on Principles of database systems, pages 263–272, 2006.
- [19] G. Cormode and P. Vesely. A tight lower bound for comparison-based quantile summaries. In Proceedings of the 39th ACM SIGMOD-SIGACT-SIGAI Symposium on Principles of Database Systems, pages 81–93, 2020.
- [20] G. Dittmann and A. Herkersdorf. Network processor load balancing for highspeed links. In Proceedings of the 2002 International Symposium on Performance Evaluation of Computer and Telecommunication Systems, volume 735. Citeseer, 2002.
- [21] N. Duffield, C. Lund, and M. Thorup. Priority sampling for estimation of arbitrary subset sums. *Journal of the ACM (JACM)*, 54(6):32-es, 2007.
- [22] D. Felber and R. Ostrovsky. A randomized online quantile summary in  $o(\frac{1}{\epsilon} \log(\frac{1}{\epsilon}))$  words. arXiv preprint arXiv:1503.01156, 2015.
- [23] P. Garcia-Teodoro, J. Diaz-Verdejo, G. Maciá-Fernández, and E. Vázquez. Anomalybased network intrusion detection: Techniques, systems and challenges. *comput*ers & security. 28(1-2):18-28. 2009.
- [24] N. K. Govindaraju, N. Raghuvanshi, and D. Manocha. Fast and approximate stream mining of quantiles and frequencies using graphics processors. In *Proceedings of the 2005 ACM SIGMOD international conference on Management of data*, pages 611–622, 2005.
- [25] M. Greenwald and S. Khanna. Space-efficient online computation of quantile summaries. ACM SIGMOD Record, 30(2):58–66, 2001.
- [26] M. B. Greenwald and S. Khanna. Power-conserving computation of orderstatistics over sensor networks. In Proceedings of the twenty-third ACM SIGMOD-SIGACT-SIGART symposium on Principles of database systems, pages 275–285, 2004.

- [27] A. Gupta and F. Zane. Counting inversions in lists. In SODA, volume 3, pages 253–254, 2003.
- [28] Z. Huang, L. Wang, K. Yi, and Y. Liu. Sampling based algorithms for quantile computation in sensor networks. In *Proceedings of the 2011 ACM SIGMOD International Conference on Management of data*, pages 745–756, 2011.
- [29] A. Kabbani, M. Alizadeh, M. Yasuda, R. Pan, and B. Prabhakar. Af-qcn: Approximate fairness with quantized congestion notification for multi-tenanted data centers. In 2010 18th ieee symposium on high performance interconnects, pages 58–65. IEEE, 2010.
- [30] Z. Karnin, K. Lang, and E. Liberty. Optimal quantile approximation in streams. In 2016 ieee 57th annual symposium on foundations of computer science (focs), pages 71–78. IEEE, 2016.
- [31] K.-H. Li. Reservoir-sampling algorithms of time complexity o (n (1+ log (n/n))). ACM Transactions on Mathematical Software (TOMS), 20(4):481–493, 1994.
- [32] Z. Liu, R. Ben-Basat, G. Einziger, Y. Kassner, V. Braverman, R. Friedman, and V. Sekar. Nitrosketch: Robust and general sketch-based monitoring in software switches. In Proceedings of the ACM Special Interest Group on Data Communication, pages 334–350. 2019.
- [33] G. Luo, L. Wang, K. Yi, and G. Cormode. Quantiles over data streams: experimental comparisons, new analyses, and further improvements. *The VLDB Journal*, 25(4):449–472, 2016.
- [34] G. S. Manku, S. Rajagopalan, and B. G. Lindsay. Approximate medians and other quantiles in one pass and with limited memory. ACM SIGMOD Record, 27(2):426–435, 1998.
- [35] G. S. Manku, S. Rajagopalan, and B. G. Lindsay. Random sampling techniques for space efficient online computation of order statistics of large datasets. ACM SIGMOD Record, 28(2):251–262, 1999.
- [36] A. McGregor, A. Pavan, S. Tirthapura, and D. P. Woodruff. Space-efficient estimation of statistics over sub-sampled streams. *Algorithmica*, 74(2):787–811, 2016.
- [37] A. Metwally, D. Agrawal, and A. El Abbadi. Efficient computation of frequent and top-k elements in data streams. In *International Conference on Database Theory*, pages 398–412. Springer, 2005.
- [38] B. Mukherjee, L. T. Heberlein, and K. N. Levitt. Network intrusion detection. IEEE network, 8(3):26-41, 1994.
- [39] J. I. Munro and M. S. Paterson. Selection and sorting with limited storage. *Theoretical computer science*, 12(3):315–323, 1980.
- [40] S. Narayana, A. Sivaraman, V. Nathan, P. Goyal, V. Arun, M. Alizadeh, V. Jeyakumar, and C. Kim. Language-directed hardware design for network performance monitoring. In Proceedings of the Conference of the ACM Special Interest Group on Data Communication, pages 85–98, 2017.
- [41] A. Roy, H. Zeng, J. Bagga, G. Porter, and A. C. Snoeren. Inside the social network's (datacenter) network. In Proceedings of the 2015 ACM Conference on Special Interest Group on Data Communication, pages 123–137, 2015.
- [42] R. J. Serfling. Probability inequalities for the sum in sampling without replacement. *The Annals of Statistics*, pages 39–48, 1974.
- [43] N. Shrivastava, C. Buragohain, D. Agrawal, and S. Suri. Medians and beyond: new aggregation techniques for sensor networks. In Proceedings of the 2nd international conference on Embedded networked sensor systems, pages 239–249, 2004.
- [44] V. N. Vapnik and A. Y. Chervonenkis. On the uniform convergence of relative frequencies of events to their probabilities. In *Measures of complexity*, pages 11–30. Springer, 2015.
- [45] J. S. Vitter. Random sampling with a reservoir. ACM Transactions on Mathematical Software (TOMS), 11(1):37–57, 1985.
- [46] L. Wang, G. Luo, K. Yi, and G. Cormode. Quantiles over data streams: An experimental study. In Proceedings of the 2013 ACM SIGMOD International Conference on Management of Data, pages 737–748, 2013.
- [47] K. Yi and Q. Zhang. Optimal tracking of distributed heavy hitters and quantiles. Algorithmica, 65(1):206–223, 2013.
- [48] Q. Zhang and W. Wang. An efficient algorithm for approximate biased quantile computation in data streams. In Proceedings of the sixteenth ACM conference on Conference on information and knowledge management, pages 1023–1026, 2007.
- [49] Y. Zhang, X. Lin, J. Xu, F. Korn, and W. Wang. Space-efficient relative error order sketch over data streams. In 22nd International Conference on Data Engineering (ICDE'06), pages 51-51. IEEE, 2006.