Modelling patient flow and outcomes in community healthcare – a fluid approximation of a stochastic queueing system

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Context: Community healthcare and healthcare policy

In recent decades, an ambition of healthcare policy has been to deliver more care in the community sector [6].

- Diverse range of services, operating in different physical locations
- Common for patients to use a range of services which they may re-use
- Considered to be crucial in meeting the current and future challenges that face modern health care services [3]

Challenge: how to organise and deliver these services given: physical distribution, patients using multiple services, increased referrals, case mix, and long term care requirements [7].

Quality measurement in community healthcare

- Operational capability - waiting time, queue length, resource utilisation, capacity
- Outcome measures - aspects of a patient's health or experience, influenced by a care interaction
  - i.e. measurable behaviours, opinions, medical characteristics or health status
  - Used to monitor and evaluate the progression of patients and the quality of care received

Increasingly used by managers, clinicians and commissioners to inform quality improvement [2].

Outcomes in practice and modelling

Uses in healthcare:

- Regularly evaluated as periodically calculated proportions
  - Misses time dependent variability in output of service and outcomes achieved
- Often considered in isolation from other services
- Misleading when considering dynamic patient flow of stochastic healthcare systems

Common modelling assumptions:

- Operational improvements positively affect outcomes
- Uniform patients

Rationale and aims

Understand dynamics of patient flow and use of patient outcomes in evaluating community healthcare

1. Unify two perspectives of quality in a single modelling framework
   - Account for flow of patients in different health states
   - Transition in health throughout queueing process important to understanding demand for and effect of system
2. Establish a concept of the flow of outcomes - how individual services contribute to a system's performance
   - Flow: bottlenecks, required capacity, waiting times
   - Outcomes: how they accrue over time through a combination of services

Dynamics of patient flow in community healthcare

- Multiple services of varying configuration
- Patients reusing the same services - either sequentially or after care within another service
- Possibility of patients abandoning queue - impatient; seek healthcare in a non-community setting
- Patients arrive in different health states
  - Different capacity to benefit/resource requirement
  - Health may improve or decline throughout
- Time dependent demand

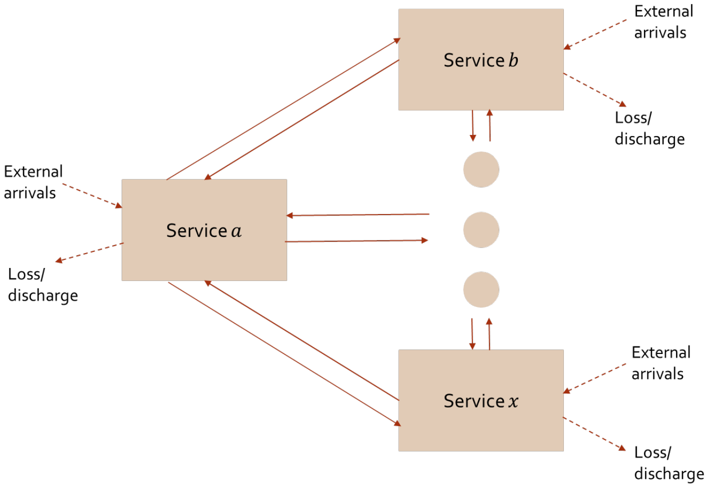
Traditional methods do not cope well with these dynamics - computationally expensive

Extension and application of a fluid approximation

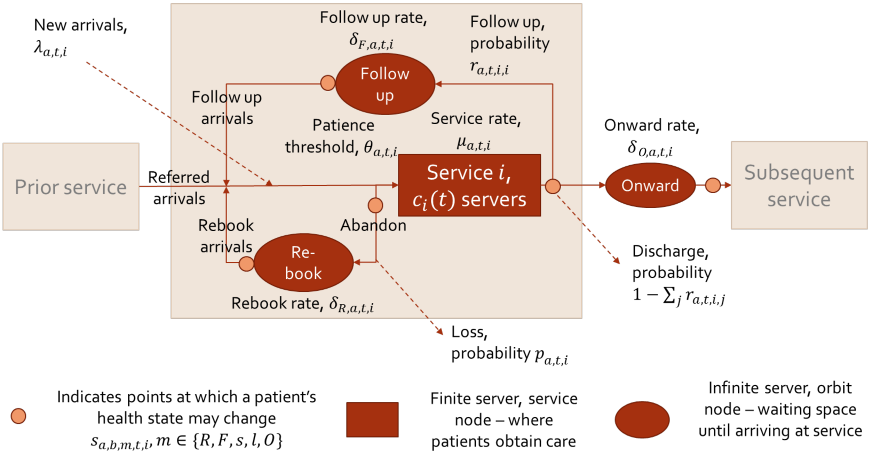
We present the application of a deterministic fluid and diffusion approximation for a stochastic queueing system. A novel application and extension of work by both S Ding et al. (2015)[1] and A Mandelbaum et al. (1998, 2002) [4, 5].

- Network of multiple services
- Health states - different parameters
- Application of diffusion equation

The system - a series of stochastic processes



The system - a series of stochastic processes



Number of servers split across A parallel queues

Fluid approximation of patient flow

For analytical tractability, patients served first come first serve in parallel queues according to health state, $a \in A$.

Capacity $C_{a,i}(t)$ allocated to each queue at time t, with $\sum_{a \in A} C_{a,i}(t) = c_i(t)$.

$$C_{a,i}(u, \mathbf{Z}(u)) = \begin{cases} \frac{c_i Z_{a,Q,i}(u)}{\sum_{b=1}^{A} Z_{b,Q,i}(u)}, & \text{if } c_i < \sum_{b=1}^{A} Z_{b,Q,i}(u) > 0 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$C_{a,i}(u, \mathbf{Z}(u)) = \begin{cases} \frac{c_i \mu_{a,i} Z_{a,Q,i}(u)}{\sum_{b=1}^{A} \mu_b Z_{b,Q,i}(u)}, & \text{if } c_i < \sum_{b=1}^{A} Z_{b,Q,i}(u) > 0 \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

$$C_{a,i}(u, \mathbf{Z}(u)) = \frac{Z_{a,Q,i}(0) + \lambda_{a,i}}{\sum_{b=1}^{A} Z_{b,Q,i}(0) + \lambda_{b,i}} \times c_i \quad (3)$$

Fluid approximation of patient flow

- Work within Skorokhod space with J1 metric: intuitively provides wiggle space within both space and time.
- A natural and convenient formalism for describing trajectories of stochastic processes that may admit discontinuities, i.e. trajectories of Poisson processes.

Consider a sequence of models where the n-th model denoted by the superscript (n) has an arrival rate of $\lambda_{a,i} n$ for new patients in health state a and the total number of servers is nc_i. The scaled fluid process is defined as:

$$\bar{Z}_{a,m,i}(t) := \frac{Z^{(n)}_{a,m,i}(t)}{n},$$

where $a \in \{1, ..., A\}$, $i = 1, ..., J$ and $m \in \{Q, R, F, O, D, L\}$

Within this space and scaling the system for $n \to \infty$, we can define the fluid limit:

$$
\begin{aligned}
z_{a,Q,i}(t) = z_{a,Q,i}(0) + \lambda_{a,i} t &+ \sum_{b=1}^{A} s_{b,a,R,i}\ \delta_{b,R,i} \int_0^t z_{b,R,i}(u)\ du \\
&+ \sum_{b=1}^{A} s_{b,a,F,i}\ \delta_{b,F,i} \int_0^t z_{b,F,i}(u)\ du \\
&+ \sum_{b=1}^{A} s_{b,a,O,i}\ \delta_{b,O,i} \int_0^t z_{b,O,i}(u)\ du \\
&- \mu_{a,i} \int_0^t \min\left(z_{a,Q,i}(u), C_{a,i}(u, \mathbf{z}(u))\right)\ du \\
&- \theta_{a,i} \int_0^t \left(z_{a,Q,i}(u) - C_{a,i}(u, \mathbf{z}(u))\right)^+\ du
\end{aligned}
$$

Fluid approximation of patient flow

$$
\begin{aligned}
z_{a,R,i}(t) &= z_{a,R,i}(0) - \delta_{a,R,i} \int_0^t z_{a,R,i}(u)\ du \\
&\quad + p_{a,i} \sum_{b=1}^{A} s_{b,a,l,i}\ \theta_{b,i} \int_0^t \left(z_{b,Q,i}(u) - C_{b,i}\left(u, \mathbf{z}(u)\right)\right)^+\ du \\
z_{a,F,i}(t) &= z_{a,F,i}(0) - \delta_{a,F,i} \int_0^t z_{a,F,i}(u)\ du \\
&\quad + r_{a,i,i} \sum_{b=1}^{A} s_{b,a,s,i} \int_0^t \mu_{b,i} \min\left(z_{b,Q,i}(u), C_{b,i}(u, \mathbf{z}(u))\right)\ du
\end{aligned}
$$

Fluid approximation of patient flow

$$z_{a,O,i}(t) = z_{a,O,i}(0) - \delta_{a,O,i} \int_0^t z_{a,O,i}(u)\ du$$
$$+ \sum_{j=1;\ j\neq i}^{J} \sum_{b=1}^{A} s_{b,a,s,j} r_{a,j,i} \int_0^t \mu_{b,j} \min\left(z_{b,Q,j}(u), C_{b,j}(u, \mathbf{z}(u))\right)\ du$$

$$z_{a,L,i}(t) = z_{a,L,i}(0)$$
$$+ (1 - p_{a,i}) \sum_{b=1}^{A} s_{b,a,l,i}\ \theta_{b,i} \int_0^t \left(z_{b,Q,i}(u) - C_{b,i}\left(u, \mathbf{z}(u)\right)\right)^+\ du$$

$$z_{a,D,i}(t) = z_{a,D,i}(0) + r_{a,i,D} \sum_{b=1}^{A} s_{b,a,s,i}\ \mu_{b,i} \int_0^t C_{b,i}(u, \mathbf{z}(u))\ du$$

Fluid approximation of patient flow

Analytical expressions cannot be found for the above, however can be solved iteratively using common numerical schemes.

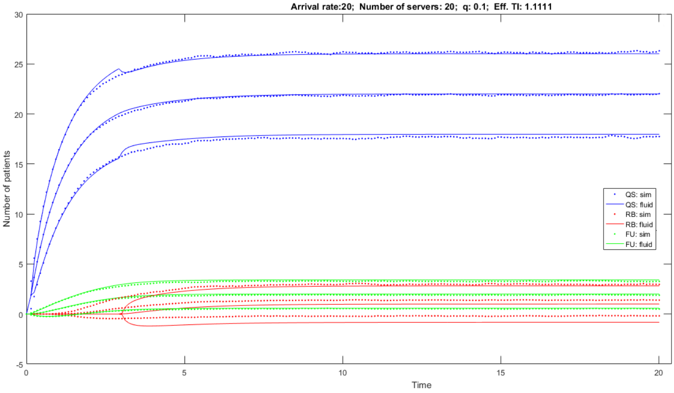
- Rewrite $z_{a,Q,i}(t), z_{a,R,i}(t), z_{a,F,i}(t), z_{a,O,i}(t),\ \ i = 1, ..., J,\ \ a = 1, ..., A$ as $z(t) = \phi(z(t))$
- Let $z^{(0)}(0) = 0$
- Calculate $z^{(k+1)} = \phi(z^{(k)}), k = 0, 1, ...$ using a common numerical scheme
- Stop when difference between $z^{(k+1)}$ and $z^{(k)}$ is deemed sufficiently small

Computed in MATLAB, produced a Discrete Event Simulation for the basic stochastic system and extension with health states

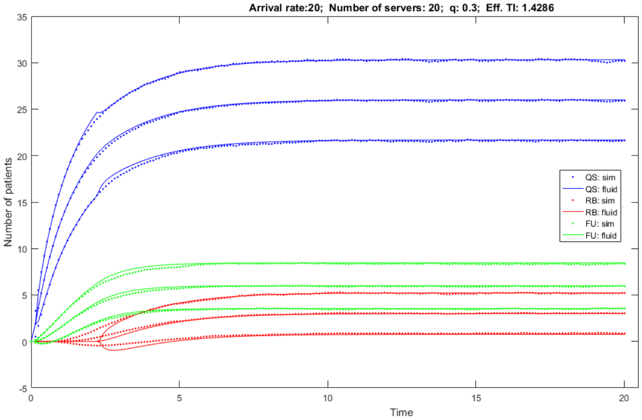
- Compare fluid model to simulation
- Basic model - rebook and follow up [1]
  - Explore parameter space for community healthcare
  - Triangulation of models
- Extended to include health states - single service
  - Assess accuracy of extension

Simulations - Basic case
Small system with small probability of re-use

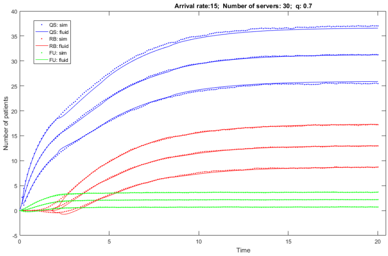
Effective traffic intensity: $\hat{\rho} = \frac{\lambda}{c\mu(1-q)}$

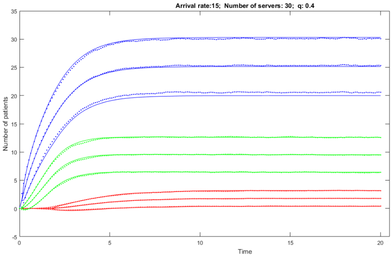


Simulations - Basic case
Larger probability of re-use

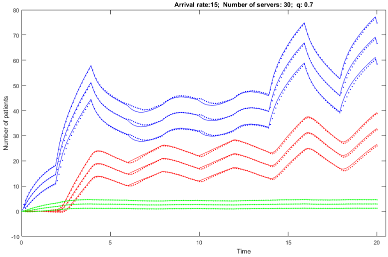


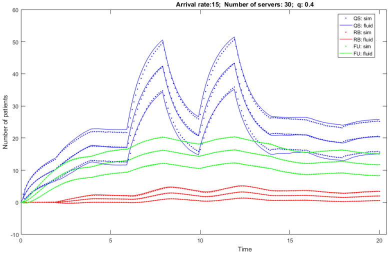
Simulations - Introduction of outcomes





Simulations - System with outcomes
Time dependent behaviour - arrival spikes





Measures from fluid approximation of patient flow

- Number of patients, over time, in:
  - Services (or orbits)
  - Health states
- Waiting times, number in the system, waiting time distribution - using Erlang A or R, Virtual Waiting Time [5]
  - Per service
- Production of outcomes: number of patients discharge from a service/system in a given health state over time
- Loss over time

Types of analyses

- Capacity allocation - i.e. optimisation:
  1. Balance across queues for equitable wait times
  2. Reduce net loss
  3. Prevent resource intensive re-joins
- Production of outcomes:
  1. Begin to understand how a network of services work together to "produce" patients with good health
  2. Seek balance across multiple services
     - Identify bottlenecks
     - Holistic view of operational measures

Limitations and future work

Limitations:

- A deterministic analogue of a stochastic system
- Errors for none heavily loaded systems
- Less accurate for smaller systems

Future directions:

1. Joint use of services - could an extension capture this?
2. Can the patient flow and outcomes of patients with multiple morbidities be informatively modelled?
3. Combining with optimisation methods - can useful information be gained for service planning?

Conclusions

Extending a fluid approximation of stochastic systems to a network of services, including patient health, is beneficial since:

- Model key dynamics of community healthcare
- Overcomes computational burden and time expense of other methods
- Provides time dependent analysis of system outputs

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Thank you for your attention

Are there any questions?