
A numerical approach to investigating the mechanisms behind tonotopy in the bush-cricket inner-ear

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3 ABSTRACT

4 Bush-crickets (or katydid) have sophisticated and ultrasonic ears located in the tibia of their
5 forelegs, with a working mechanism analogous to the mammalian auditory system. Their inner-
6 ears are endowed with an easily accessible hearing organ, the *crista acustica* (CA), possessing
7 a spatial organisation that allows for different frequencies to be processed at specific graded
8 locations within the structure. Similar to the basilar membrane in the mammalian ear, the CA
9 contains mechanosensory receptors which are activated through the frequency dependent
10 displacement of the CA. While this tonotopical arrangement is generally attributed to the gradual
11 stiffness and mass changes along the hearing organ, the mechanisms behind it have not been
12 analysed in detail. In this study, we take a numerical approach to investigate this mechanism
13 in the *Copiphora gorgonensis* ear. In addition, we propose and test the effect of the different
14 vibration transmission mechanisms on the displacement of the CA. The investigation was carried
15 out by conducting finite-element analysis on a three-dimensional, idealised geometry of the
16 *C. gorgonensis* inner-ear, which was based on precise measurements. The numerical results
17 suggested that (i) even the mildest assumptions about stiffness and mass gradients allow for
18 tonotopy to emerge, and (ii) the loading area and location for the transmission of the acoustic
19 vibrations play a major role in the formation of tonotopy.

20 Keywords: bush-cricket hearing, *crista acustica*, finite-element analysis, numerical modelling, bioacoustics, acoustic vibration
21 transmission

1 INTRODUCTION

22 Tonotopic organisation, or frequency maps, arise in the auditory systems of many species (1). As is well
23 known, the main purpose of this phenomenon is to facilitate frequency analysis of the acoustic vibrations
24 entering the hearing chamber (2). First discovered in the mammalian ear, a tonotopic hearing organ has
25 recently also been observed in the ears of bush-crickets ((3), (4), (5)), which have been shown to have a
26 hearing system analogous to mammals (6). For bush-crickets, whose ears are located in their forelegs, a
27 non-invasive investigation of the biomechanism governing the inner-ear processes has been possible through
28 the transparent cuticle some species are endowed with (4). In contrast, the structure and location of the
29 mammalian inner-ear makes it experimentally challenging to investigate this mechanism non-invasively (7),
30 (8). Hence, the convergent evolution between the bush-cricket and the mammalian ears provides a unique
31 opportunity to enhance our understanding of these analogous hearing systems, making the investigation of
32 the mechanism behind the workings of the bush-cricket inner-ear a timely and worthy pursuit.

33 Through his Nobel prize winning work, Georg von Békésy showed that the fluid-immersed basilar
34 membrane in the mammalian inner-ear (the cochlea) acted as a biological Fourier transform, performing
35 a frequency analysis on the mechanical travelling wave containing the frequency information (2), (9).
36 As the travelling wave moved down the organ, it was observed that the cochlear hair cells (or auditory
37 sensory cells) lying along the membrane would receive mechanical input at specific frequencies, due to
38 an amplitude maxima response dependent on the stiffness and mass gradients of the basilar membrane
39 (10). Some properties of the basilar membrane leading to such gradients include a tapering in width, and
40 a gradual increase in thickness and elasticity (1). Since then, travelling waves have also been observed
41 directly in the basilar papilla of birds (11), and indirectly via the timing of responses of auditory-nerve
42 fibres in the auditory organs of some reptiles and frogs ((12), (13)). Analogous travelling waves have
43 also been measured invasively and non-invasively in the ears of bush-crickets ((3), (4), (14), (15)). The
44 underlying mechanism is likely more ancient since it has been observed in grigs, suggesting it was shared
45 by a common ancestor (16).

46 Bush-crickets have ultrasonic ears that are located in the prothoracic tibia of their forelegs, and each ear
47 is endowed with two tympanic membranes (TMs). The TMs are backed by an air-filled tube (the acoustic
48 trachea), and their outer-ear allows for sound to be received on both sides of the tympana: directly to the
49 external side, and through the acoustic trachea to the internal side (17). The acoustic trachea bifurcates
50 near the TM, with the anterior tracheal branch backing the anterior tympanic membrane, and similarly
51 the posterior tympanic branch lies behind the posterior tympanic membrane. After travelling through the
52 tracheal tube, the sound arrives at the TM with a phase difference and a pressure differential compared to
53 the external input (17), leading to the TM acting as pressure-time difference receivers (18). The acoustic
54 vibrations are then transmitted to the bush-cricket inner-ear, which is a separate, fluid-filled chamber above
55 the TM end of the tracheal tube (see Figure 1). The mechanosensory organ of the bush-cricket, the *crista*
56 *acustica* (CA), is located inside this chamber and lies on the dorsal wall of the anterior tracheal branch.
57 While in other insect hearing systems such as locusts and moths the mechanoreceptors are in direct contact
58 with the TMs (20), this is not the case for bush-crickets, demonstrating another likeness to the mammalian
59 ear (21), (22).

60 The CA has been observed to resemble an uncoiled basilar membrane (4), (19). Similar to mammals, in
61 the bush-cricket inner-ear the transmitted acoustic vibrations activated the hearing organ, leading to the
62 formation of travelling waves moving along this structure (6). These waves were observed to travel from the
63 narrowest part (distal part) of the sensory organ (the CA) containing the high frequency sensory cells, to the
64 broader (proximal) region containing the low frequency cells ((3), (9), (19)). As the travelling wave moved

65 along the hearing organ, the CA maximum displacement was seen to occur in a frequency-dependent
66 fashion, after which the wave dissipated, showing clear characteristics of a tonotopically ordered structure
67 (4), (6). Consequently, the frequency dependent movement of the CA activated the relevant mechanosensory
68 cells. Even though the tonotopy observed in the bush-cricket ear is generally attributed to the stiffness and
69 mass gradients of the CA (3), a more detailed analysis, combining experimental and numerical approaches,
70 is required for more conclusive evidence on the mechanism behind this phenomenon.

71 As is well known, for tonotopical vibrations to emerge along the hearing organ, first the acoustic vibrations
72 have to be transmitted into this chamber through a mechanical phenomenon. For the mammalian ear, the
73 chain of transmission by which sound is captured by the eardrum, via the ossicles to the inner ear is
74 well-studied ((23), (24)). However, in bush-cricket auditory mechanics, this process is more controversial.
75 To understand the workings of the bush-cricket auditory system, it becomes important to discern the process
76 of vibration transmission into the bush-cricket inner-ear. There are two main arguments in relation to
77 the air-to-liquid conversion of acoustic vibrations in the bush-cricket ear, which are based on a lever-like
78 system (25), where a higher output force is generated through mechanical advantage (26).

79 The first mechanism was proposed by Bangert et al. (27) by using the species *Polysarcus denticauda* and
80 *Tettigonia viridissima* as model systems, who likened the impedance conversion in the bush-cricket ear to
81 the TM acting as a type 2 lever. A type 2 lever is described as having a fulcrum located at one end with the
82 force applied at the other end. The resulting force is then sensed at the middle of the lever (26). Hence,
83 according to Bangert et al. (27), the force load was sensed at the area of intersection between the TM and
84 the dorsal wall, transferring the airborne acoustic energy to the fluid medium. It was also suggested that
85 there was a contribution to this mechanism from the sound pressure acting on the dorsal wall, emanating
86 from the acoustic trachea (27). A similar mechanism was also put forward by Nowotny et al. (28) for the
87 species *Mecopoda elongata*. Palghat Udayashankar et al. (19), however, proposed that the dorsal wall
88 played a more central role in activating the CA in the *M. elongata* ear, through the pressure of the sound
89 wave on the dorsal wall as it travelled in the acoustic trachea. Hence, it was suggested that the CA obtained
90 a displacement magnitude proportional to its local resonance while stimulated through a pressure parallel
91 to the hearing organ, rather than a travelling wave. A similar idea was also considered for the mammalian
92 inner-ear (29).

93 A second transmission model was proposed by Montealegre-Z et al. (6), using the species *Copiphora*
94 *gorgonensis* as a model system. For the neotropical bush-cricket *C. gorgonensis*, the CA is located in the
95 auditory vesicle (see Figure 1) which is believed to be derived from the hemolymph channel and is filled
96 with fluid, also bathing the CA ((6), (25)). The proposed transmission mechanism in (6) was centered
97 around the tympanal plate (TP), a cuticular patch attached to the TM in contact with the distal end of the
98 auditory vesicle (25). The TP was recorded to have an out-of-phase response to its hosting TM, hence
99 performing an air-to-liquid impedance conversion by transmitting the vibrations from the air backed TM to
100 the fluid-filled auditory vesicle. It is suggested that the TP governs this transmission process by acting as a
101 type 1 lever: a lever with the fulcrum in the middle, force applied at one end and the resulting force in the
102 other end.

103 Therefore, while the role of TPs are not considered by Bangert et al. (27), Montealegre-Z and colleagues
104 (6) suggested that TPs play a central role in the impedance conversion of acoustic vibrations to the
105 fluid medium. Hence, in (6) (see also (25)), the TPs were posited to have an analogous function to the
106 mammalian middle-ear. However, a middle-ear or the auditory vesicle were not observed in the ears of the
107 bush-cricket *M. elongata* (19). There is also a considerable difference in the load areas between the species
108 *M. elongata* and *C. gorgonensis*. For both the species a part of the TM ((19), (5)) or TP (6) is in contact

109 with the inner-ear fluid, however, the TM has contact along the length of the CA (28), whereas the TP is in
110 contact with only the distal end of the auditory vesicle (6). Thus, in addition to a different lever system, the
111 mechanisms suggested also propose differing sizes of load areas to the inner-ear. While these different
112 models were proposed, it has never been tested whether such mechanical configurations actually give rise
113 to the observed tonotopic behaviour.

114 In this study, using *C. gorgonensis* as our model system we investigated the underlying mechanism of the
115 bush-cricket inner-ear. By incorporating simple observable properties, such as mass gradient and tapering
116 (width and height) CA morphology in the geometry of the system, we tested mechanical features that are
117 crucial to the development of tonotopy. We also used the constructed models to numerically investigate
118 the effect of the transmission load of acoustic vibrations to the formation of tonotopy. Based on micro-
119 computed tomography (μ -CT) measurements of the *C. gorgonensis* inner-ear an idealised geometry was
120 constructed, on which numerical simulations were carried out by manipulating the “middle-ear” conditions.
121 Figure 2 demonstrates the idealised auditory vesicle and CA geometry. We used this 3D model to test
122 the hypothesis that for *C. gorgonensis*, the existence of a separate chamber, the auditory vesicle, makes
123 it necessary to have a load area with a smaller region as offered by the TP. Further, the role of the dorsal
124 wall in sound transmission was investigated. A comparison with the experimental results in the literature
125 suggested that our numerical results gave the closest match to the experimental data when the impedance
126 conversion took place with the influence of both the TP and the dorsal wall.

2 MATERIALS AND METHODS

127 2.1 Model Geometry

128 2.1.1 Morphological measurements

129 To produce an idealised geometry of the *C. gorgonensis* inner-ear, a female specimen was scanned using
130 (i) a SkyScan 1172 X-ray μ -CT scanner (Bruker Corporation, Billerica, MA, USA) with a resolution of
131 $1.6 \mu\text{m}$ (40 kV source voltage, $165 \mu\text{A}$ source current, 2200 ms exposure and 0.1° rotation steps) and (ii)
132 synchrotron X-ray CT imaging at the Diamond Manchester Imaging Branchline (I13-2, Diamond Light
133 Source, Oxford). We used monochromatic light and a $4\times$ objective with a pco.edge 5.5 detector, providing
134 a voxel size of $0.8125 \mu\text{m}$. The obtained images were then reconstructed with NRecon (v.1.6.9.18, Bruker
135 Corporation, Billerica, MA, USA) for a series of orthogonal slices.

136 The 3D segmentation of the inner-ear was performed with the software Amira-Aviso 6.7 (Thermo Fisher
137 Scientific, Waltham, Massachusetts, USA), and were used for obtaining the dorsal wall thickness and
138 the dimensions of the auditory vesicle, through the Center Line Tree module in AMIRA. For the 2D
139 measurement of cap cell surface area, scolopale cell radius and dendrite length, an Alicona InfiniteFocus
140 microscope (G5, Bruker Alicona Imaging, Graz, Austria) at $\times 5$ objective magnification was used, with a
141 resolution of about 100 nm.

142 2.1.2 Idealised geometry

143 The idealised geometry was constructed in the *Geometry* node of COMSOL Multiphysics, v. 5.6 (30),
144 with parameter dimensions based on the measurements obtained as described in Section 2.1.1. The actual
145 shape of the geometry can be seen in Figure 1 and Figure 3. The acquired geometry is given in Figure 2,
146 and the used dimensions are presented in Figure 3 and Table 1. The auditory vesicle itself was represented
147 as an oblong hexahedron. The geometry included 28 individual cap cells and corresponding dendrites of
148 varying dimensions (see Table 1), located inside the auditory vesicle. For cap cell geometry, we assumed

149 that they were shaped as cuboids (see Figure 2b). Conjoined to the cap cells from the bottom were the
150 scolopale cells which were modelled as spheres, and were also attached to the dendrite. On the other end,
151 it was assumed that dendrites were directly connected to the dorsal wall. The cap cells, scolopale cells
152 and dendrites made up the structure of the modelled CA. The CA was covered by a surface representing
153 the tectorial membrane. Near the distal end of the constructed CA, the TP was modelled to intersect with
154 the auditory vesicle wall. A second geometry with the TM intersecting the acoustic trachea along the
155 length of the CA was also constructed for comparison (see Supplementary Materials, Figure S1). A block,
156 intersecting at the dorsal wall, was added representing the acoustic trachea.

157 2.2 Mathematical Model

158 The mathematical models were developed using the Acoustics and Structural Mechanics modules of
159 COMSOL Multiphysics (v. 5.6) (30), and were set-up as an acoustic-structure interaction problem. All
160 the calculations were carried out in the frequency domain so that the system of equations was not time
161 dependent. The auditory vesicle wall and the tectorial membrane were represented with a shell formulation
162 (31), and were coupled with the fluid inside the auditory vesicle using the arbitrary Lagrangian-Eulerian
163 method (32). The fluid was assumed to be water, and the pressure in the fluid was represented by the
164 solution to the linearised Navier-Stokes equations (33). Hence, any possible viscous effects due to the fluid
165 were accounted for. The CA components (cap cells, scolopale cells and dendrites) were assumed to be
166 linear elastic and were represented by the elastic Helmholtz equation (32). The CA was in turn coupled
167 with the fluid inside the auditory vesicle, as well as the auditory vesicle boundary (the shell formulation).
168 In addition, we modelled the propagation of sound in the block representing the acoustic trachea. The block
169 was assumed to be filled with air, and the sound pressure was represented by the solution to the acoustic
170 Helmholtz equation (34). The block was also coupled with the dorsal wall to reflect the influence of the
171 sound pressure in the auditory vesicle due to the propagation of sound in the trachea.

172 The boundary of the auditory vesicle facing the proximal end of the CA was assumed to act as a pressure
173 release facilitator (6) (see Supplementary Materials, Figure S2), as a result of a *Free boundary condition*
174 (34) defined there. This condition ensures that the boundary moves based on its material properties and
175 applied forces, and is not constrained except at the edges. A free boundary condition was also defined on
176 the dorsal wall. The remainder of the auditory vesicle walls were fixed. At the proximal end face of the
177 idealised acoustic trachea, the sound wave travelling through the tracheal tube was modelled as a harmonic
178 wave of frequency f and amplitude 1 Pa (see Supplementary Materials, Figure S2). On this face, we also
179 defined a *radiation boundary condition* so that there were no reflections there (34). The transmission of
180 vibrations through the TP were represented with an *acceleration* condition with a magnitude of $\omega^2 \mu\text{m/s}^2$,
181 where $\omega = 2\pi f$ is the angular frequency (34). The frequency was considered in the interval 10-90 kHz
182 with a resolution of 10 kHz.

183 Table 2 summarises the material properties employed in the mathematical models. These values were
184 selected through parametric sweeps, as a result of giving the closest numerical results to the experimental
185 data (see Supplementary Materials, Section 1 for more details). The material properties were assumed to
186 be isotropic and homogeneous.

187 The model described above was also adapted to represent the condition of a TM transmission of acoustic
188 vibrations to the auditory vesicle rather than a TP transmission, where a larger area along the length of the
189 auditory vesicle received the force. This was achieved by a manipulation of the idealised geometry (see
190 Supplementary Materials, Figure S1). TP and TM entrances without the influence of the dorsal wall were
191 also considered by removing the idealised acoustic trachea from the geometry.

192 2.3 Numerical Simulations

The variational form of the developed mathematical models were solved using the finite-element method. Linear Lagrange elements were used for the solution so that a second order accuracy was obtained in the L^2 -norm (35). For the constructed finite-element mesh, the tetrahedral mesh radii were between 1-20 μm . This mesh-size ensured that there were at least 10 tetrahedral elements per wavelength for the considered frequency range of 10-90 kHz. The mesh size was also based on the thickness of the potential viscous boundary layers forming near the boundaries. From the linearized Navier-Stokes equations, the thickness of the viscous boundary layer can be obtained as

$$\delta = \sqrt{\frac{2\mu}{\omega\rho_0}},$$

193 where μ = dynamic viscosity, $\omega = 2\pi f$ is the angular frequency with f frequency, and ρ_0 the background
 194 density (36). It is clear that the thickness δ depends on the properties of the fluid considered, and that it
 195 becomes thinner with the increase of frequency. Since the auditory vesicle is assumed to be filled with
 196 water, at 20 °C and 90 kHz the viscous layer thickness $\delta = 1.8853 \mu\text{m}$. Hence, the adopted mesh size
 197 allows for capturing the solution even in the thinnest boundary layer.

198 A mesh stability analysis is presented in Supplementary Materials, Section 3. The COMSOL Multiphysics
 199 (30) inbuilt feature of mixed interpolation of tensorial components (MITC) shell elements were used for
 200 the meshing of the auditory vesicle boundary. MITC shell elements have been established as effectively
 201 capturing different shell behaviours with varied and complex stress conditions (37). The obtained mesh
 202 was a conforming finite-element mesh. The constructed mesh is demonstrated in Supplementary Materials,
 203 Figure S3.

3 RESULTS

204 In this study, we undertook the numerical investigation of the mechanism behind the workings of the *C.*
 205 *gorgonensis* inner-ear, the auditory vesicle. We found that a mass gradient generated by the changing size
 206 of the cap cells, and the geometry of the CA and dorsal wall were sufficient to generate tonotopic vibrations.
 207 In addition, the two main hypotheses of vibration transmission, (i) through the TP and dorsal wall, and (ii)
 208 through the TM and dorsal wall, were tested numerically to determine their influence on the formation of
 209 tonotopy. To investigate the individual contributions of the dorsal wall, TP and TM, their effects on the
 210 system were also simulated separately.

211 Throughout this section, distance measurements refer to the distance from the proximal end, as defined in
 212 Figure 2c.

213 3.1 Combined methods of sound transmission

214 The mathematical models were set-up to represent a combined method of sound transmission: through
 215 the (i) TP and dorsal wall, and the (ii) TM and dorsal wall. For an analysis of tonotopy along the CA, we
 216 checked for monotonicity in the relationship between frequency and distance, in the range of frequencies
 217 and positions sampled. Figure 4 demonstrates the results obtained for vertical displacement maxima
 218 location along the CA, for vibrations in the frequency range 10-90 kHz. As can be observed, the tonotopic
 219 arrangement, or a frequency dependent displacement maxima, was most pronounced when the TP and
 220 dorsal wall transmissions were considered together.

221 For the TP and dorsal wall input, the three-dimensional image of CA vertical displacement at various
222 frequencies is presented in Figure 5. For this input system, an increase in frequency in the interval 20-90
223 kHz lead to the movement of the displacement maxima location from 246 μm to 471 μm (see Figure 4a).
224 However, between 10 kHz and 20 kHz a discontinuity was observed in the tonotopic vibrations, and the
225 displacement maxima location was at 408 μm for both 50 kHz and 60 kHz. The displacement magnitude
226 with respect to distance is given in Figure 6a. The results demonstrate a non-decreasing displacement
227 with the increase of frequency. From Figure 5 and Figure 6a it can also be observed that the displacement
228 maxima are smooth peaks, dissipating a short distance from the maxima.

229 A tonotopical pattern of vibration was not observed when both the TM and dorsal wall served as
230 vibrational inputs to the CA (see Figure 4a and Supplementary Materials, Figure S4). In the frequency
231 range 30-90 kHz, the displacement maxima lied within the short interval 369 - 408 μm from the proximal
232 end, showing the spatial frequency gradient to be less differentiated. Further, displacement maxima at 30,
233 40 kHz were located at 360 μm , and similarly for 60-80 kHz they were located at 390 μm . Tonotopy was
234 also not present in the frequency range 10-30 kHz. The distance versus vertical displacement at the CA, for
235 the TM and dorsal wall input is given in Figure 6b. A correlation between frequency and displacement
236 magnitudes was not present.

237 3.2 Single method of sound transmission

238 To determine the individual effects of the dorsal wall, TP and TM input on the development of tonotopic
239 vibrations along the CA, the mathematical models were solved with these three parameters as the separate
240 excitation methods of the system. Figure 4b demonstrates the displacement maxima location along the CA
241 with respect to vibration frequency. As can be observed from Figure 4b, when these input methods are
242 considered separately, there is a lack of monotonicity between frequency and distance. More precisely,
243 some displacement maxima locations were obtained as:

- 244 (i) For TP input 390 μm at 50 kHz; 369 μm at 60 kHz; 443 μm at 70 kHz; 408 μm at 80 kHz;
245 (ii) For DW input 299 μm at 20 kHz; 272 μm at 40 kHz; 427 μm at 50 kHz; 408 μm at 60 kHz; 443 μm at
246 70 kHz;
247 (iii) For TM input 299 μm at 40 kHz; 408 μm at 50 kHz; 369 μm at 60 kHz; 427 μm at 70 kHz; 408 μm at
248 80 kHz;

249 While a TP input leads to displacement maxima closest to tonotopy, a fluctuation of the displacement
250 location is present throughout the considered frequency range. These fluctuations become smaller in
251 magnitude as the frequency increases. Similar fluctuations also resulted from TM and dorsal wall inputs in
252 the frequency range 40-90 kHz. For the dorsal wall and TM excitations a correlation between frequency
253 and displacement maxima location can not be discerned in the range 10-40 kHz.

254 The displacement magnitude along the CA obtained through the three inputs is given in Supplementary
255 Materials, Figure S5. The dorsal wall excitation led to displacement magnitudes that were orders of
256 magnitude smaller than those generated by the TP or TM excitation of the system.

4 DISCUSSION

257 In this study, we numerically investigated the mechanism behind the formation of tonotopical vibrations
258 along the CA. Further, we analysed how the transmission load area of acoustic vibrations influenced this
259 formation in the inner-ear.

260 4.1 Factors sufficient for developing tonotopy

261 The local resonance frequency (f) of an acoustic structure can be determined by the ratio of its stiffness
 262 (s) to its mass (m) in the form

$$f = \frac{1}{2\pi} \sqrt{\frac{s}{m}}, \quad (1)$$

263 where the stiffness is dependent on the elasticity (Young's modulus) and the dimensions of the structure,
 264 and mass is the product of the structure's density and volume. From equation (1) it is easy to see that a
 265 change in stiffness or mass leads to the change of the resonant frequency of the structure.

266 Due to a decrease in width and increase in thickness and elasticity from the base to the apex, stiffness and
 267 mass gradients are also present along the mammalian basilar membrane (9). As a result, these stiffness
 268 and mass gradients contribute to the formation of the observed tonotopical vibrations there (12). Using
 269 a combined experimental and numerical approach, Olson and Nowotny (14) demonstrated that the bush-
 270 cricket CA also had a similar stiffness magnitude to the basilar membrane, and that the stiffness decreased
 271 from the high-frequency region to the low-frequency region. Nevertheless, when such a structure is located
 272 inside a complex system, there are several factors that can modify the local behaviour. In particular, for
 273 the bush-cricket inner-ear such other factors can be the oscillators in the system being all coupled to each
 274 other, the standing wave cavity resonance of the auditory vesicle, and the standing wave resonances of
 275 the TM and the dorsal wall. Hence, while equation (1) is generally true for a single degree of freedom, it
 276 may not hold when the degrees of freedom increase. Applying a three-dimensional numerical analysis, we
 277 aimed to determine the extent to which the CA and dorsal wall geometry contributed to the emergence of
 278 the experimentally observed tonotopy there.

279 To incorporate the dimensions of the CA into the models, we used the precise μ -CT measurements
 280 of *C. gorgonensis* inner-ear parts (see Figure 3). While it was not possible to directly use the scanned
 281 geometry in our models due to the difficulties of forming a finite-element mesh on the detailed features of
 282 the CA, we constructed an idealised geometry based on the obtained measurements (Figure 2). As a result,
 283 the numerical models contained a CA tapering in width and height. A similar property was also present
 284 for the dorsal wall, due to the narrowing of the auditory vesicle towards the distal end of the chamber.
 285 Hence, the model included the geometrical factors considered to be necessary for the frequency dependent
 286 displacement of the CA. The elasticity of these parts, however, were assumed to be homogeneous and
 287 isotropic in the simulations. In addition, the geometry did not include the curvature present along the
 288 bush-cricket dorsal wall and CA, which could potentially add another gradient through its effect on the
 289 stiffness (38).

290 Since the geometry of the TP and dorsal wall input system matches the *C. gorgonensis* ear morphology,
 291 and TP input is the mechanism identified for vibration transmission to the *C. gorgonensis* inner-ear
 292 experimentally (6), in this section we refer to this set of data when talking about our results. This data
 293 set demonstrates a frequency based displacement along the CA (see Figure 4a). Further, the maximum
 294 displacement peak is a smooth peak dissipating a short distance after passing through the point of resonance
 295 (Figure 6), a tonotopical property also observed experimentally (6). Hence, we can see that tonotopy
 296 emerge as a result of changes in the CA and dorsal wall geometry that produce simplified stiffness and
 297 mass gradients.

298 Nevertheless, a discontinuity can be observed in the tonotopic organisation around 10 kHz (see Figure
 299 4), implying the requirement of additional features to the model for more definitive tonotopy. Such a
 300 discontinuity implies that certain properties have more significant effects at smaller frequencies, for instance

301 anisotropic elasticity, curvature or damping properties. Another discrepancy between the experimental
302 data ((3), (6)) and our results can also be seen at the tonotopy placement. While Montealegre-Z et al. (6)
303 recorded the displacement maxima for 30 kHz at a distance of about 200 μm from the proximal end, for
304 the numerical results it was located at a distance of 320 μm , showing a shift in tonotopy at the numerical
305 results. Based on formula (1), a shift in the maxima location points at a difference in stiffness and mass
306 between the actual and numerical geometry, which once again suggests the requirement of more realistic
307 material and geometric properties in the model. Hence, for the investigation of these discrepancies with the
308 experimental data, as well as for the further understanding of the inner-ear mechanism, an enhancement
309 of the model is certainly worth pursuing. Still, it is worth noting that the distances presented in (6) were
310 measured through the insect cuticle rather than directly on the CA, which was the case with the numerical
311 results.

312 We believe our numerical approach exhibits the utility of employing three-dimensional numerical models
313 for investigating a complex system alongside an experimental approach. For the bush-cricket hearing
314 system, while there are many studies successfully explaining the general workings of the inner-ear ((3), (6),
315 (14), (25), (27) and references therein), it is not experimentally possible to pick inner-ear parts apart to
316 determine their individual functions, without compromising the underlying mechanism of the system to a
317 certain extent. Our numerical technique provides an alternative approach to such an analysis, which can
318 be a powerful tool in obtaining reliable predictions on the inner-ear mechanism. Our approach is a first
319 attempt to represent the system with a simpler structure in order to understand the contribution of specific
320 properties, namely the basic morphology of the chamber and its components.

321 Another simplifying assumption we applied for setting up the mathematical models was that the TM
322 and TP were comprised of homogeneous and isotropic materials. Through experimental investigations
323 it has been observed that the tympana boost frequencies relevant for the communication of conspecific
324 bush-crickets (5), indicating a more complex material structure. In the mathematical models, properties
325 like mass, stiffness and damping of the TM and the TP, essential for capturing the impedance of the system
326 were also not based on measured values. Hence, the magnitude of the displacement maxima presented in
327 Figure 6 and Supplementary Materials, Figure S5 are not necessarily a true reflection of the displacement
328 magnitude. While the model predictions are not directly comparable to the observed data, the constructed
329 model provides a simplified adaptation of the biomechanics of the inner-ear, from which it is still possible
330 to obtain qualitative information related to tonotopy. Primarily the model suggests that it is possible to
331 develop a tonotopic vibrational map, based on the mass and spatial gradients that result purely from
332 geometrical changes.

333 4.2 Differentiation in CA morphology and the role of TP, TM and DW inputs

334 In the mammalian hearing system, the transmission of acoustic vibrations into the inner-ear is well-studied
335 (see (23), (24), (39) and references therein). In particular, the middle ear is comprised of three tiny bones
336 (the ossicles) which through a lever action pass the eardrum vibrations to the fluid filled cochlea, performing
337 an air-to-liquid impedance conversion (39). For bush-crickets, however, there are still multiple untested
338 ideas in the literature with regards to the sound transmission to the bush-cricket inner-ear. Nevertheless, the
339 two main propositions for transmission mechanisms are both based on lever systems. For instance, for the
340 species *M. elongata*, it is believed that the whole TM acts as the main input for sound transmission (3), (5)
341 through functioning as a type 2 lever. Hence, the *M. elongata* ear will receive the maximum load at the
342 intersection of the TM with the dorsal wall. It has been observed that a large section of the TM is in contact
343 with the air-filled acoustic trachea, and a smaller section is attached to the fluid-filled hemolymph channel,

344 thus allowing for impedance conversion. For *M. elongata*, The TM is in contact with the hemolymph
345 channel along the length of the CA (28). In contrast, for *C. gorgonensis*, vibration transmission was
346 identified to be facilitated by the type 1 lever action of the TP (6). Thus, the large deflections of the
347 airborne TM are transformed into smaller deflections of the fluid-bound TP, demonstrating clear impedance
348 conversion characteristics in the ear. Moreover, the TP is observed to be in contact with the fluid only
349 towards the distal end of the CA. Our numerical results suggest that the load area, or the dimensions and
350 location of the TM or TP in contact with the inner-ear, also play a significant role in the formation of
351 tonotopy inside the inner-ear.

352 As can be observed from Figure 5, the frequency dependent change in displacement maxima along the
353 CA is evident when the vibrations enter through the TP, with the maxima moving from the proximal end
354 to the distal end of the CA as frequency increases. This is less pronounced for a TM entrance to the *C.*
355 *gorgonensis* inner-ear (Supplementary Materials, Figure S4). Nonetheless, some differences between the
356 bush-cricket *C. gorgonensis* (established TP input, (6)), and *M. elongata* (established TM input, (5), (28))
357 are worth pointing out. The species *C. gorgonensis* have a separate inner-ear chamber, the auditory vesicle.
358 Whereas for *M. elongata*, the CA is located in the hemolymph channel, hence the mechanics driving the
359 formation of tonotopy may be different. In our numerical simulations, the geometry only reflected the
360 dimensions of the *C. gorgonensis* auditory vesicle. Interestingly, Bangert et al. noted that for the species
361 *Polysarcus denticauda* and *Tettigonia viridissima*, while the CA was activated through the TM, the tympana
362 was in contact with the hemolymph channel only where the high frequency receptor cells of the middle
363 and distal crista acustica were located (27). In this study, the outer surface of the TM for the considered
364 species were identified to have an area called the 'inner plate', which was a dark oval and stiff area that was
365 surrounded by a paler area, assumed to be more elastic. However, no out-of-phase response between the
366 inner-plate and the hosting tympana was recorded, and the tympana was identified as a uniformly vibrating
367 membrane. Hence, there may be greater variation in auditory mechanics even within bush-crickets than
368 previously assumed. Accordingly, we appreciate the differences in morphologies between bush-cricket
369 species and from this point on limit our conjectures to the species *C. gorgonensis*.

370 The model also elucidates the transmission properties of the TM, TP, and the role of the dorsal wall
371 which has also been a topic of interest (19), (27). While vibrational input from the wall was not reported
372 in (6), the influence of sound pressure from the dorsal wall was suggested in (5) and (27). In (19), it was
373 further proposed that the dorsal wall activated the CA before vibrations even reached the TM. To analyse
374 the dorsal wall influence numerically, we simulated the transmission of sound by removing the acoustic
375 trachea from underneath the dorsal wall in the geometry, and activated the system only through the TM
376 or the TP (see Figure 4b). In addition, we also considered a simulation where the transmission was only
377 through the dorsal wall. As can be observed in Figure 4, the removal of the dorsal wall pressure for a TP
378 and TM excitation did not lead to a tonotopical arrangement, highlighting the role of the wall displacement
379 itself on the movements of the CA. A solitary dorsal wall (Figure 4) excitation of the system did not lead
380 to the experimentally observed tonotopy at the CA either. Hence, we conjecture that in addition to the
381 effect of the stretching and squeezing of the dorsal wall through the TM or the TP entrance, the sound
382 pressure from the acoustic trachea below also plays a significant role in activating the mechanoreceptors.
383 However, the small displacement magnitude from the dorsal wall input (Supplementary Materials, Figure
384 S5c) leads us to believe that a stronger mechanism than a solitary dorsal wall stimulation is needed for
385 the excitation of the system. These results suggest that the resonance approach proposed in (19) is not a
386 probable mechanism for the stimulation of the *C. gorgonensis* CA.

387 Our attempt to simulate the biomechanical processes in the bush-cricket ear, using a 3-dimensional
388 idealised geometry, has resulted in a reasonable qualitative match to the experimental results in the
389 literature. This suggests that for the investigation of such processes, a numerical approach can provide a
390 cost efficient alternative and validation method to empirical studies. Reliable numerical models, validated
391 through experimental data, also provide a new platform for analysing the individual effects of the parameters
392 comprising the bush-cricket inner-ear, which is not possible through experimentation. Therefore, the further
393 numerical investigation of the bush-cricket inner-ear, through improving the incorporated quantitative
394 (material properties, fluid viscosity, potential non-Newtonian fluid properties) and geometrical (curvature
395 of the dorsal wall and CA) properties is certainly worth pursuing.

CONFLICT OF INTEREST STATEMENT

396 The authors declare that the research was conducted in the absence of any commercial or financial
397 relationships that could be construed as a potential conflict of interest.

AUTHOR CONTRIBUTIONS

398 EC, NM and FMZ contributed to conception and design of study. CW took morphological measurements.
399 EC developed numerical models, ran simulations and analysed obtained results. EC and CW developed
400 figures and images. EC wrote the first draft of the manuscript. All authors contributed to manuscript
401 revision, read, and approved the submitted version.

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DATA AVAILABILITY STATEMENT

406 The COMSOL models developed for this study can be found in the link
407 <https://drive.google.com/file/d/1xC7xfYp9896oEItITREDaAfGn5zLx-ZK/view?usp=sharing>

FIGURES

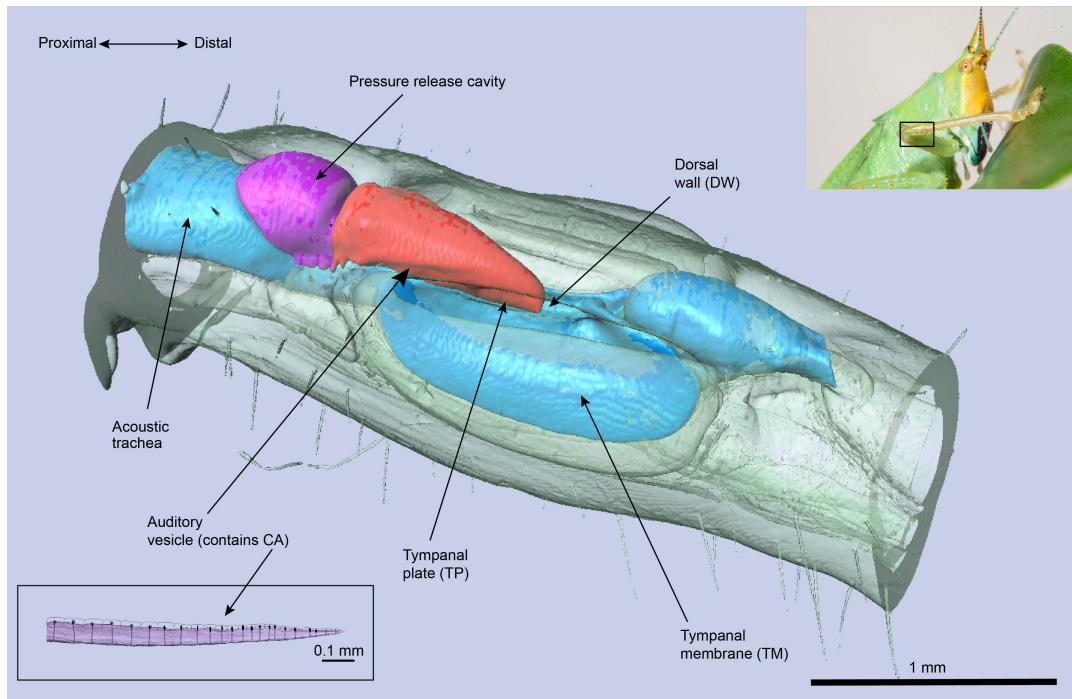


Figure 1. The μ -CT image of the *Copiphora gorgonensis* ear and its components.

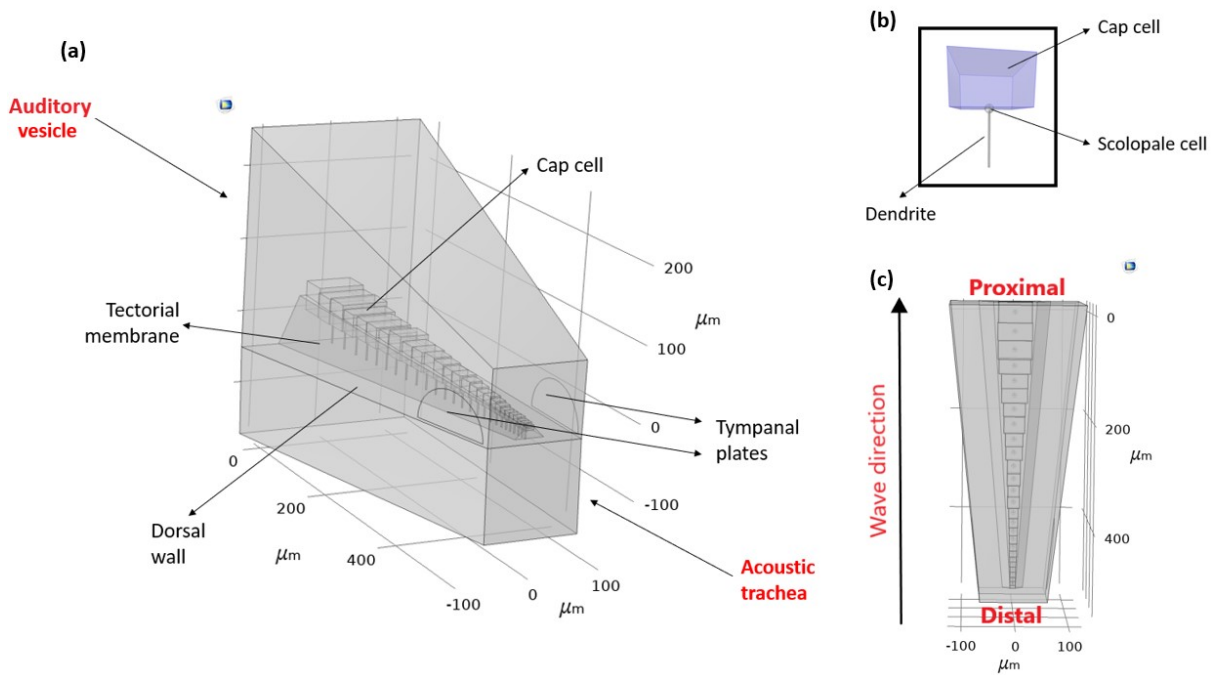


Figure 2. An idealised geometry of *C. gorgonensis* inner-ear. (a) The geometry parameters (b) A mechanosensory cell and its components (c) The travelling wave direction observed in the bush-cricket inner-ear.

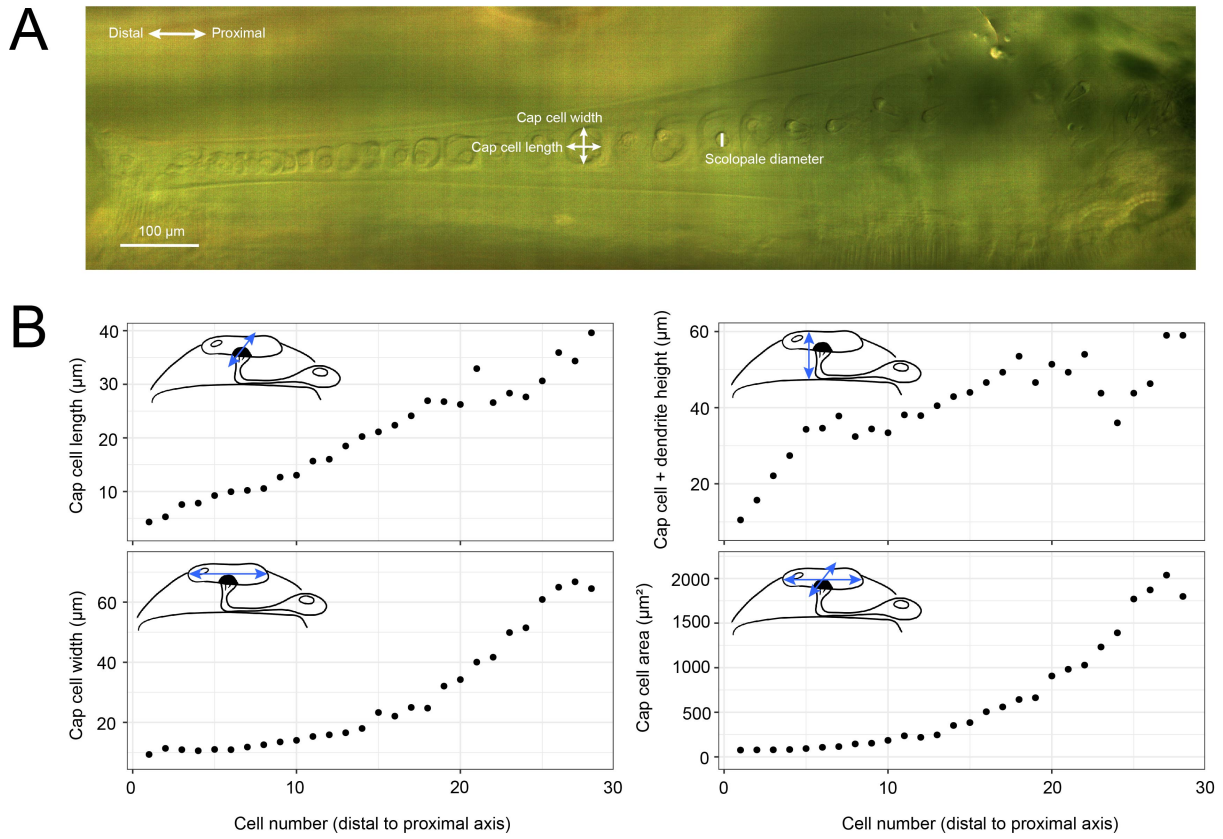


Figure 3. *Crista acustica* (CA) measurements. A) an Alicona InfiniteFocus microscope image of *Copiphora gorgonensis* CA. B) The measurements of *C. gorgonensis* CA components using the Alicona InfiniteFocus microscope. Cap cell area refers to the surface area of the cap cell top surface.

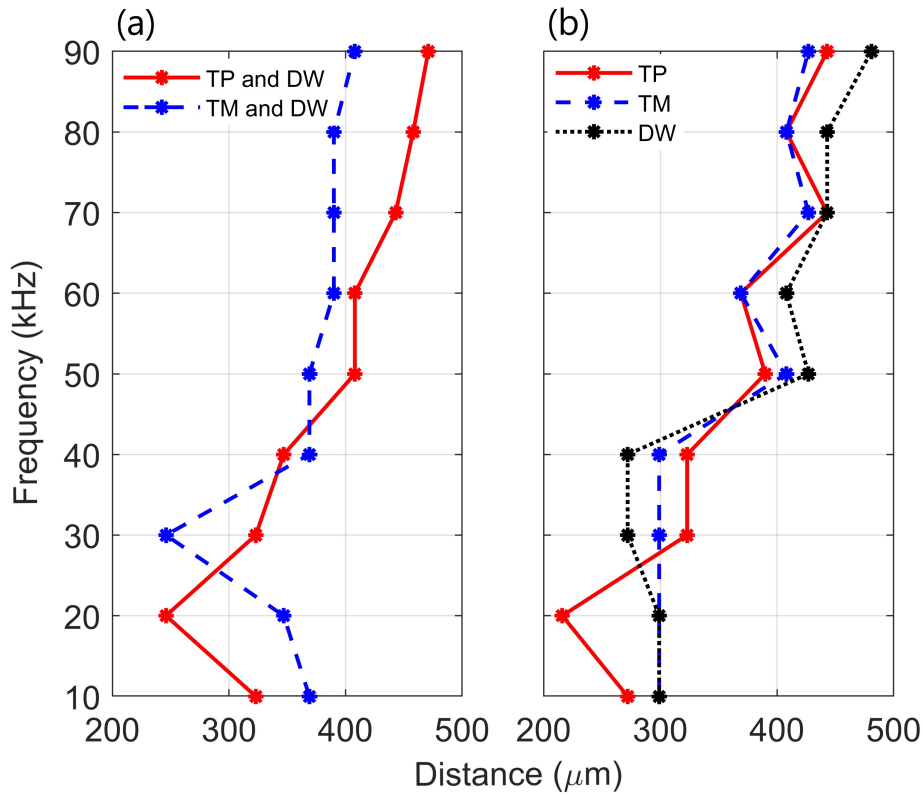


Figure 4. Location of displacement maxima. Distance, from the proximal end, of the vertical displacement maxima of *crista acustica* when vibrations are transmitted (a) through the tympanal plate and dorsal wall (TP and DW) or the tympanic membrane and dorsal wall (TM and DW), (b) through the tympanal plate (TP), tympanic membrane (TM) or the dorsal wall (DW).

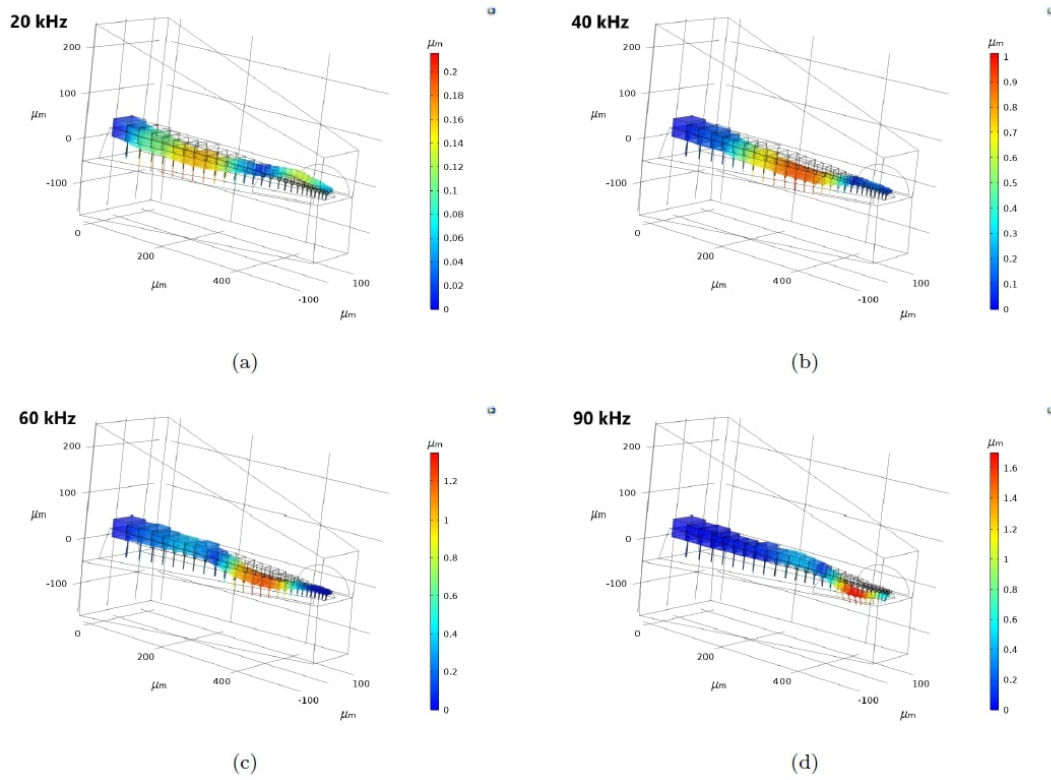


Figure 5. The three-dimensional *crista acustica* (CA) vertical displacement facilitated by the tympanal plate (TP) and dorsal wall (DW) transmission of acoustic vibrations at (a) 20 kHz, (b) 40 kHz, (c) 60 kHz, (d) 90 kHz.

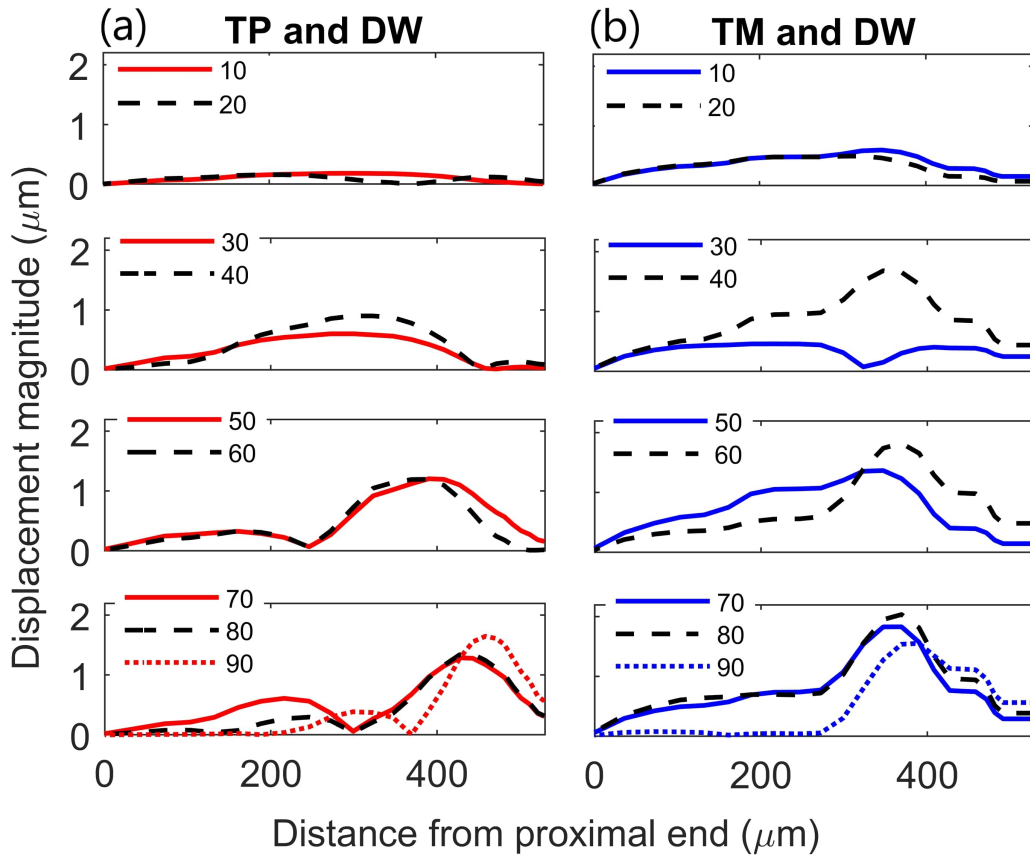


Figure 6. *Crista acustica* vertical displacement magnitude due to transmission of acoustic vibrations at various frequencies. Displacement magnitude due to transmission of vibrations facilitated by the (a) tympanal plate and dorsal wall (TP and DW) shown in the left column, (b) tympanic membrane and dorsal wall (TM and DW) shown in the right column.

TABLES

Table 1. The parameter dimensions applied in constructing the idealised geometry.

Parameter	Dimensions
Auditory vesicle volume	$2.22 \times 10^7 \mu\text{m}^3$
Dorsal wall thickness	Varying linearly in the interval $3.25\text{-}6.5 \mu\text{m}$ (proximal to distal)
Auditory vesicle wall thickness	$15 \mu\text{m}$
Cap cell dimensions	Largest $64 \times 40 \times 40 \mu\text{m}^3$ (proximal), Smallest $4 \times 9 \times 4 \mu\text{m}^3$ (distal)
Scolopale cell radius	$9 \mu\text{m}$
Tectorial membrane thickness	$2 \mu\text{m}$
Dendrite length	$59\text{-}11 \mu\text{m}$ (proximal to distal)
Tympanal plate area	$5493 \mu\text{m}^2$

Table 2. The parameter material properties applied in the mathematical models.

Parameter	Young's Modulus
Dorsal wall	0.5 GPa
Auditory vesicle wall	1 GPa
Cap cell, scolopale cell	50 MPa
Dendrite	1 GPa
Tectorial membrane	10 MPa
Tympanal plate	1 GPa
Tympanic membrane	1 GPa

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