Mathematical model identification of self-excited systems using experimental bifurcation analysis data

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ABSTRACT

Self-excited vibrations can be found in many engineering applications such as flutter of aerofoils, stick-slip vibrations in drill strings, and wheel shimmy. These self-excited vibrations are generally unwanted since they can cause serious damage to the system. To avoid such phenomena, an accurate mathematical model of the system is crucial. Self-excited systems are typically modelled as dynamical systems with Hopf bifurcations. The identification of such nonlinear dynamical system from data is much more challenging compared to linear systems.

In this research, we propose two different mathematical model identification methods for self-excited systems that use experimental bifurcation analysis data. The first method considers an empirical mathematical model whose coefficients are identified to fit the measured bifurcation diagram. The second approach considers a fundamental Hopf normal form model and learns a data-driven coordinate transformation mapping the normal form state space to physical coordinates. The approaches developed are applied to bifurcation data collected on a two-degree-of-freedom flutter rig and the two methods show promising results. The advantages and disadvantages of the methods are discussed.

Keywords: Self-excited systems, Hopf bifurcation, Hopf normal form, Parameter estimation, Data-driven model, control-based continuation

INTRODUCTION

Self-excited systems exhibit periodic responses without any oscillating input. Self-excited oscillations can have catastrophic consequences, such as a plane crash due to wing flutter. Therefore, it is often essential to accurately model and predict the response of self-excited systems. Self-excited systems are usually modelled using parameter-dependent differential equations that capture changes in the system’s states over a so-called bifurcation parameter. A typical scenario leading to oscillatory responses in a self-excited system is the Hopf bifurcation. A Hopf bifurcation is a critical point where the system's equilibrium changes stability, and limit cycle oscillations (LCOs) are generated from the critical point. This paper will discuss two different mathematical modelling approaches to capture the LCOs of self-excited systems based on experimental bifurcation analysis data. The latter were collected using control-based continuation (CBC) as CBC allows to measure both stable and unstable LCOs of the system [1]. The two modelling methods are demonstrated on the experimental data collected on a fluttering aerofoil (Figure 1).

The first approach is based on a mechanistic model, i.e., a model constructed from physical principles. To determine the parameters of this model, we use centre manifold reduction and normal form theory to predict the bifurcation diagram of the model. Model parameters are then optimised to minimise the difference between model-predicted and experimentally-measured bifurcation diagrams.

The second approach is based on a phenomenological model and uses machine learning (ML) to establish a transformation from this simple model to the coordinates of the real system. We define the ML model as a prediction of observables made from the centre manifold. The reduced dynamics on the centre manifold captures the bifurcation structure of the data, and the observables are trained using neural networks to predict the time series accurately.

MODELING USING A MECHANISTIC MODEL

The unsteady flutter model is the basis mechanistic model of our modelling approach. To estimate the parameters of this model, we use a two-stage identification approach where the parameters of the linearized model are identified first and then the parameters of the nonlinear part are identified. We identify the linearization using the small amplitude free-decay response by minimizing the prediction error of the state-space model [2]. In the second stage, the nonlinear part of the model is identified by parametrizing the amplitude of the LCOs using the centre manifold reduction and simplest normal form of the Hopf bifurcation [3] (see Figure 2). Results show a very good agreement between measured and predicted LCOs, especially in the unstable region where the assumption of the mechanistic model are valid. For larger oscillation amplitudes, model predictions deteriorate. It is thought to result from the simplistic aerodynamic model considered.

MODELING USING A PHENOMENOLOGICAL MODEL: THE HOPF NORMAL FORM

Topologically equivalent dynamical systems are transformable to each system using invertible coordinate transformation. We can use this mathematical framework while building the model. The idea is to use a modified Hopf normal form-- subcritical Hopf normal form added with the quintic nonlinear term-- to capture the bifurcation structure of the experiment. The LCOs in coordinates of normal form is a circle, and the 2-dimensional observation vector transforms this circle to a measured closed curve. The mapping between the centre manifold and the observable is trained using a neural network (see Figure 3). The oscillation speed is trained to minimize the prediction of time-series response using the differential equation solver equipped with a machine learning package [4]. The trained model can predict the bifurcation diagram and the time series of the LCOs.

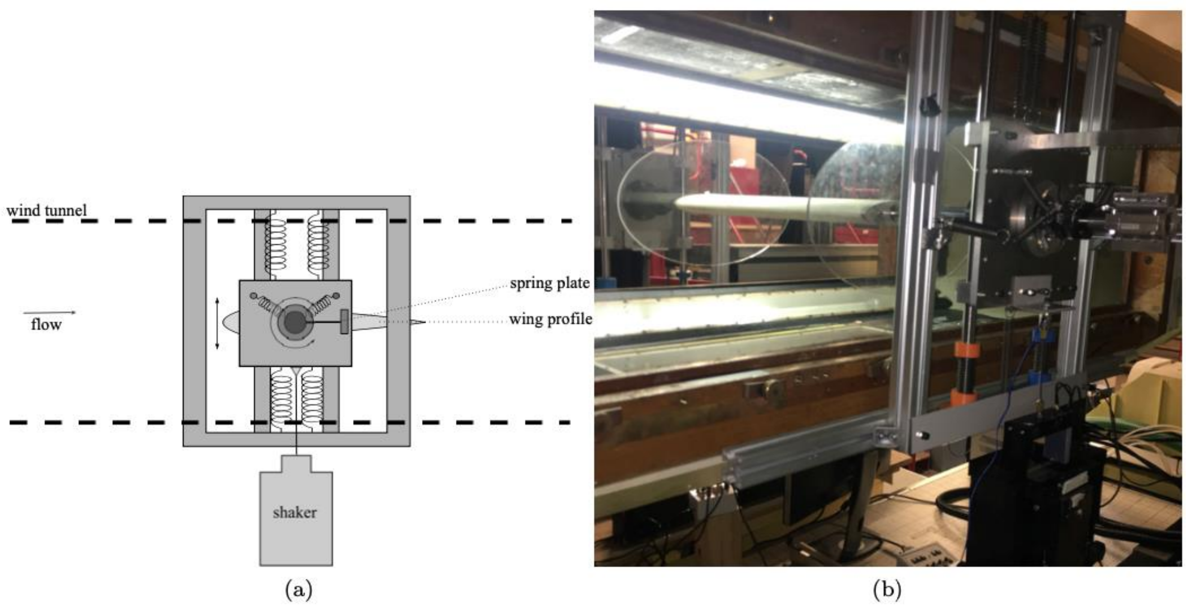

Figure 1: Flutter rig. (a) Schematic, (b) physical system in Bristol's wind tunnel facility.

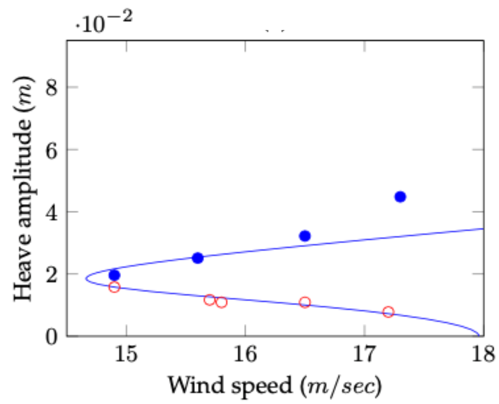

Figure 2: Comparison between measured and computed (mechanistic model) amplitudes of the LCOs. Red circles correspond to unstable LCOs measured using CBC, blue circles are stable LCOs (also measured using CBC) and the blue line is the numerical continuation of the model.

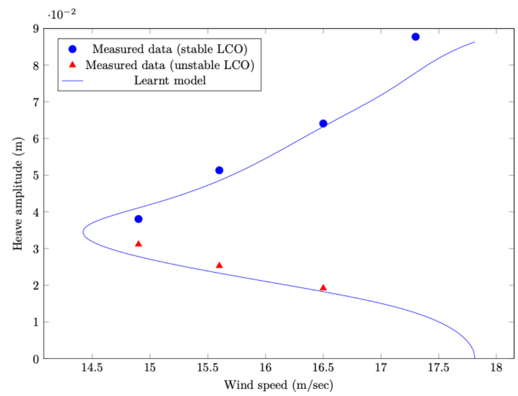

Figure 3: Comparison of the bifurcation diagram of the ML model and the measured data.

CONCLUSION

In this research, we show two different modelling approaches of a dynamical system with Hopf bifurcations. The first approach is using a mechanistic model and identifying the unknown parameters to match the bifurcation diagram. The second approach uses a Hopf normal form and ML techniques to train a mapping between the centre manifold and the observations. The first approach provides more physical insight than the second approach, while the second approach provides more modelling flexibility and hence accuracy since a mechanistic model is not required.

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