A new efficiency evaluation approach with rough data: An application to Indian fertilizer

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Abstract

In the world of chaos, nothing is certain. In such an unpredictable world, measuring the efficiency of any individual is inevitable. In a conventional data envelopment analysis (DEA) model, exact input and output quantity data are needed to measure the relative efficiencies of homogeneous decision-making units (DMUs). However, many real-world implementations show that this is not the case and exact knowledge of data might not be available. Rough DEA (RDEA) allows assessing the efficiencies of DMUs in an uncertain environment. This paper tries to construct a rough DEA model by combining conventional DEA and rough theory, all of which help provide a way to quantify uncertainty. DEA with rough data is developed in this study using positive and pessimistic β confidence values of rough variables. In the proposed method, the same set of constraints (production possibility sets) is employed to build a unified production frontier for all DMUs that can be used to properly assess each DMU's performance in the presence of rough input and output data. In addition, a ranking system is proposed based on the approaches that have been proposed. In the presence of uncertain conditions, this article investigates the efficiency of the Indian fertilizer supply chain for over a decade. The results of the proposed models are compared to the existing DEA models, demonstrating how decision-makers can increase the supply chain performance of Indian fertilizer industries.

Keywords: Rough data; Frontiers; Fertilizer industries; Data envelopment analysis; Ranking.

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1. Introduction

In today's business world, supply chain management (SCM) is playing an important role in increasing the sales and profitability of organizations. SCM first emerged in the 1990s, and it entailed overseeing and planning manufacturing/production, transportation, and distribution to meet customers [1].

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Although there is no universal concept of SCM ([2]), a common and well-known definition of a supply chain is a network of interested entities, via upstream and downstream linkages, in the various processes and activities that generate value in the form of goods and services in the hands of the ultimate consumer [3].

The chemical industry has an essential role in today's life. Chemical industries transformed raw materials into different products such as inorganic chemicals, polymers, petrochemicals, and fertilizers. A chemical factory that produces the fertilizer is known as the fertilizer industry since fertilizers are essential for plant growth, and fertilizer industries are of importance to the agriculture sector's success. Since India is hugely dependent on agriculture, fertilizer industries come to the fore in India. The fertilizer industry's objective is to identify the desirable quantities flow of elements for crop production.

The food industry has established a benchmark in India due to the technically competent fertilizer-producing companies in the country. India has many public and private fertilizer industries. Phosphate, nitrogen, and complex fertilizers are the products, which are manufactured in Indian fertilizer industries. The fertilizer industry is the first industry in which the Indian government's presence as a public sector in 1951 [4].

DEA is a frontier methodology to measure the relative efficiency of a set of firms- also called decision-making units (DMUs)- which is the most popular non-parametric methodology introduced by Charnes, Cooper, and Rhodes [5]. In DEA, the efficiencies of firms that produce multiple outputs by consuming multiple inputs are calculated based on the piecewise linear frontier's distance established by observations and some axioms.
Conventional DEA models mostly require the deterministic values of inputs and outputs. However, some observed data in many real-life cases is unclear, uncertain, or imprecise. In this paper, we took the rough set theory to cope with real-life problems in DEA. Rough set theory proposed by Pawlak [6] is a valuable tool to handle uncertain data in supply chain situations.

The rough DEA model was proposed by Xu et al. [1] to determine the furniture manufacturing industry's performance efficiencies. In their approach, the constraint set (production possibility sets) that is used to estimate the efficiency of each DMU is different. If the frontier is not unified, all DMUs' efficiencies may not be accurate and unbiased, making comparisons between efficiencies less meaningful. So, in the proposed method, we employ the same set of constraints to build a unified production frontier for all DMUs that can be used to objectively assess each DMU's performance in the presence of rough input-output data.

The results obtained from the proposed model are analyzed and then used to rank the fertilizer industries using the proposed ranking method. The contribution of this paper is fourfold: (1) we propose a rough DEA model when ambiguous, uncertain, and imprecise input and output data are available, (2) we consider the rough correlation coefficient between rough variables to ensure that the proposed rough approach is consistent, (3) we address the gap in the literature of rough sets theory by investigating a new ranking method, and (4) we study a real-life case study in the Indian fertilizer industries to demonstrate the applicability of the proposed framework. To the best of our knowledge, no research has been conducted on both supply chain and DEA with rough data to offer the foregoing contributions.

The paper is organized as follows: Section 2 offers the Literature review in Section 2. Preliminaries are present in Section 3. Section 4 offers the proposed DEA with rough data. Section 4 offers the proposed correlation coefficient between rough variables. Section 5 offers the proposed ranking approach and the advantages of the proposed approach. Section 6 provides an Indian fertilizer industry application to illustrate the proposed methods. The last section concludes the findings of this paper.

2. Issue

2.1. Problem statement

In India, 57 large fertilizer plants and some private sector companies are available. Large fertilizer plants produce Ammonium sulfate, DAP, Calcium ammonium nitrate, Urea, and Complex fertilizer. Three primary macronutrients are Phosphorus, Nitrogen, and Potassium. Phosphorus is used to grow in flowers, fruit, seeds, and roots, nitrogen is used to increase leaf growth, and Potassium is used to produce in plants' water movement and strongness in stem and promotion of fruiting and flowering. Fertilizer is a synthetic or natural origin material applied to plant tissues or soil to supply one or more plant nutrients essential to plants' growth. Fertilizer sources exist in both forms, either natural or industrial produced. The plants' growth goal is encountered in two ways: (i) some fertilizers modify the soils' aeration and water retention to enhance the soils' effectiveness (ii) being additives that provide nutrients. The primary goal of the Indian fertilizer industry is to supply the required amounts of primary and secondary nutrients. Due to India's technically capable fertilizer-producing industries, the food industry has created a benchmark. According to the environmental discussion context, the most energy-intensive sector in the Indian fertilizer industry. The fertilizer industry is crucial to increasing productivity.
The following methods can be used to assess fertilizer effectiveness: i) the use of best agronomic methods, (ii) the use of low-cost and effective fertilizer products, and (iii) integrated nutrient management, which involves the use of fertilizers, organic manures, biofertilizers, and other nutrient management techniques [7].
Researchers are not paying enough attention to the fertilizer industry's competitiveness. India's fertilizer use is steadily growing, and the country is now the world's largest producer and user of fertilizers. India, on the other hand, uses fertilizers at a far lower rate than many other countries [8]. Fertilizer use varies widely across the globe, from state to state, region to region, and district to district. Fertilizers play an important role in India's crop productivity because most soils are deficient in many macronutrients and micronutrients [8].

Figure 1 shows the estimated demand for food grains and fertilizers from 2020 to 2025, according to India's Department of Fertilizers' report for the 12th Plan. Until 2025, an additional 10 million tonnes of food grains and an additional 3 million tonnes of fertilizer will be needed.
Therefore, the nation's research and development policy should emphasize improving fertilizer company performance. Chemical fertilizers' significance in increasing agricultural production has already been noted and developed. Some claim that in the Green Revolution, fertilizer was just as essential as a crop [9]. It can account for as much as half of Asia's yield increase [10]. According to Bumb [11], fertilizer and related production factors account for one-third

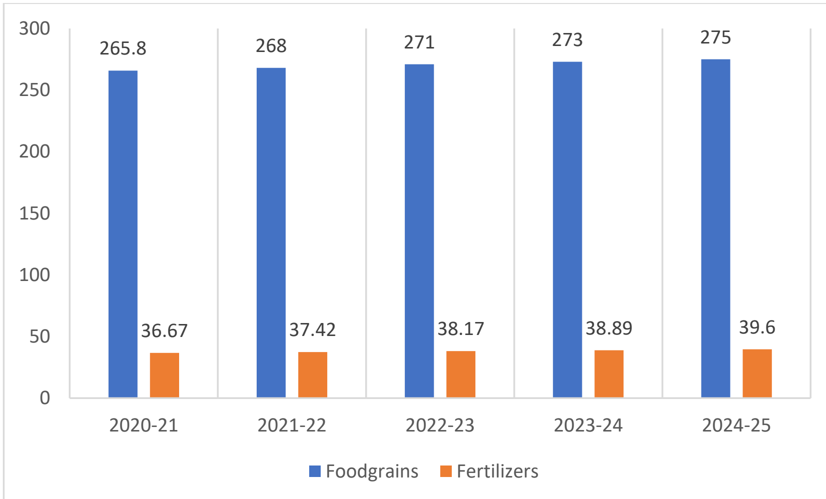

Figure 1. Foodgrains and fertilizers data

of global cereal production. There is a global shortage of major fertilizer constituents such as nitrogen (N), phosphorus (P), and potassium (K) [11]. As a result, fertilizer companies must be efficient in their activities to make the best use of these constituents. So, determining the fertilizer industry's performance is essential. However, some observed data in many real-life cases is unclear, uncertain, or imprecise. To deal with this situation we apply the rough set theory in DEA. This study aims to develop a new rough data envelopment analysis (DEA) model where the production possibility set is the same for all the DMUs to assess the efficiency of public and private fertilizer organizations in India.

2.2. Literature Review

Adewumin et al. [12] measured the availability, quality, and quantity of organic wastes in municipal solid wastes and farmyard manure in South-western Nigeria using effective means. Al et al. [13] collected the data with three types of plants based on their waste nitrogen fertilizers (WNFs) and determined Qatar's feasibility of biomass production with WNFs. Ahmad and Tariq [14] presented the literature review based on the effects of aquatic fern (known as bio-fertilizer) and environment detoxification.
Conventional DEA models mostly require the deterministic values of inputs and outputs [5]. However, some observed data in many real-life cases is unclear, uncertain, or imprecise. Considering uncertainty in inputs and outputs underlying any production process, five main approaches are available in the DEA literature to evaluate the efficiency of firms; imprecise DEA (see e.g., [15], [16], [17], [18]), fuzzy DEA (see e.g., [7], [19], [20]), stochastic DEA (see e.g., [21], [22], [23], [24]), robust DEA (see e.g., [25], [26], [27], [28]), and rough DEA (see e.g., [1], [29], [30]).
Due to the fact that many successful applications and case studies have been appeared in the DEA literature, it has been grown; examples include banking (see e.g., [31], [32], [33]), agriculture (see e.g., [34], [35], [36], [37], [38]), educations (see e.g., [39], [40]), public health (see e.g., [41], [42], [43]), sustainability ([44], [45]), and water and sanitation ([46], [47]). The

summary is provided in Table 1. China is the largest in consumption and rice production.

Table 1. **DEA in SCM**

Method	Data type	Application	Reference	Year
DEA	precise	Supply chain	[48]	1999
DEA network	precise	Swedish pharmacies	[49]	1999
DEA	precise	Supply chain	[50]	2001
DEA	precise	Petroleum industry	[51]	2002
DEA	precise	Taiwan hotels	[52]	2003
DEA	precise	Power plants	[53]	2004
DEA	precise	Automotive manufacturer	[54]	2008
Rough DEA	uncertain	Furniture industry	[1]	2009
DEA	precise	Enterprise Resource Planning	[55]	2009
Bilevel DEA	precise	Banking and manufacturing	[56]	2010
DEA	precise	Korean electronics industry	[57]	2012
DEA	precise	Green supply chain	[58]	2014
DEA	precise	Hyundai Steel Company and its suppliers	[59]	2015
Hybrid DEA	precise	Biodiesel supply chain	[60]	2017
Network DEA	precise	Tourism supply chain	[61]	2018
Bootstrap DEA	precise	Healthcare supply chain	[62]	2019
DEA	precise	Sustainability supply chain	[63]	2020
DEA	precise	Fairness in supply chain	[64]	2021

Sun and Li [65] determined the technical efficiencies of chemical fertilizer use in China's rice production provinces as the largest consumption and production of rice. The efficiency of irrigation fertilization schemes for wheat in China was calculated by Shang and Mao [66]. Abu [67] studied the effect of fertilizer on rice farms in Nigeria using the technical efficiency method. The rough set method has been used in DEA to capture the variance in the use of inputs and outputs for over a decade ([1], [29]).

Xu et al. [1] proposed a rough DEA model to determine the furniture manufacturing industry's performance efficiencies. Azadeh et al. [29] calculated banks' performance efficiencies using artificial neural network (ANN) and rough DEA. Shiraz et al. [30] proposed an integrated DEA and free disposal hull framework using rough data set and calculated Japanese banks' efficiencies.

2.3. Methodology

Conventional DEA was proposed by Charnes, Cooper, and Rhodes [5] to measure the relative efficiency of a set of firms. The majority of key components are ever-changing when assessing supply chain performance. Due to variations in data, it is difficult for conventional DEA to deal with the supply chain problem with uncertain information because the conventional DEA method often needs deterministic data. Considering uncertainty in inputs and outputs underlying any production process, five main approaches are available in the DEA literature to evaluate the efficiency of firms; imprecise DEA, fuzzy DEA, stochastic DEA, robust DEA, and rough DEA.

The uncertainty correlated with the available input-output data is incorporated by representing inputs and outputs as bounded intervals in the imprecise DEA approach. However, from a computational and practical point, solving imprecise DEA with bounded intervals (see e.g., [15], [16], [18]).
The stochastic DEA approach replaces the crisp data with interval data using probabilistic or statistical values. In this approach, to incorporate the uncertainty associated with the available input-output data, inputs and outputs are represented as a random variable with known probability density functions (pdf) and cumulative distribution functions (CDF), which are typically used in chance-constrained formulations. After all, solving stochastic DEA with various CDFs and pdfs is always very difficult from a computational and, therefore, practical standpoint (see e.g., [22], [23], [24]).
The uncertainty associated with the available input-output data is incorporated in the fuzzy DEA method by representing inputs and outputs as a membership function. However, solving fuzzy DEA with various membership functions is often quite tricky from a computational and practical point (see e.g., [7], [19], [20]).
The Robust DEA method incorporates the uncertainty associated with the available input-output data by treating inputs and outputs as perturbation vectors. However, from a computational and functional standpoint, solving Robust DEA with varying perturbation vectors is very difficult (see e.g., [25], [26], [28]).
The DEA approaches, as mentioned above, all concentrate on constructing interval values. In other words, imprecise, fuzzy, stochastic, and robust DEA are often interval methods. Alternatively, in the rough DEA method the uncertainty associated with the available input-output data is integrated into lower and upper approximations of crisp sets (see e.g., [1], [68], [29], [30], [69]). Since rough variables are used to describe lower and upper approximations of the interval values corresponding to the available inputs and outputs, the rough DEA method allows for a more general understanding of data uncertainty. As a result, the critical shortcoming of interval models, notably, can only be used in situations where inputs and outputs already have both a lower and upper bound, inherited directly from imprecise, fuzzy, stochastic, and robust DEA, although maintaining a pertinent problem in rough DEA.

In situations of uncertainty, the majority of key components are ever-changing when assessing supply chain performance. Shafiee [69] included rough variables in a two-stage SBM model to cope with the uncertainty involved in supply chains. Due to variations in data, this study uses the rough set theory in DEA to cope with inherent uncertainties and evaluate the supply chain performances of 17 private and public fertilizer industries in India over 10 years. In comparison with conventional DEA, some rough DEA virtues are (i) dealing with uncertain information in supply chain; (ii) dealing with incomplete information. So, a rough DEA is more appropriate to tackle some supply chain applications under uncertain situations.

Xu et al. [1] presented the rough DEA model to assess the performance efficiency of the furniture manufacturing industry. All DMUs in their approach have different production possibility sets. If the frontier is not unified, the efficiencies of all DMUs may differ, rendering efficiency comparisons redundant and meaningless. As a result, we proposed a rough DEA model with the same set of constraints to create a unified production frontier for all DMUs, which can be used to objectively assess each DMU's performance in the face of rough input-output data.

3. Preliminaries

In this section, the rough DEA model proposed by Xu et al. [1] is presented, which warrant the next sections. The basic key definitions of rough theory can be found in the appendix. More details on rough set theory can be found in [6], and [70].

Let $[x_{ij}^{\sup(\beta)}, x_{ij}^{\inf(\beta)}]$ be the i^{th} rough input and $[y_{rj}^{\sup(\beta)}, y_{rj}^{\inf(\beta)}]$ be the r^{th} rough output for DMU_j; $j = 1, 2, ..., n$. Using optimistic and pessimistic trust levels, Xu et al. [1] proposed a rough DEA model to determine the targeted DMU (DMU_k), as follows:

$$
\begin{aligned}
& X^{\sup(\beta)} = \min \theta \\
& s.t. \\
& \sum_{j=1, \neq k}^{n} x_{ij}^{\sup(\beta)} \lambda_j + x_{ik}^{\inf(\beta)} \lambda_k \leq x_{ik}^{\inf(\beta)} \theta, \quad \forall i, \\
& \sum_{j=1, \neq k}^{n} y_{rj}^{\inf(\beta)} \lambda_j + y_{rk}^{\sup(\beta)} \lambda_k \geq y_{rk}^{\sup(\beta)}, \quad \forall r, \\
& \lambda_j \geq 0, \forall j, \quad \theta \text{ unrestricted in sign.}
\end{aligned}
\tag{3.1}
$$

$$
\begin{aligned}
& X^{\inf(\beta)} = \min \theta \\
& \text{s.t.} \\
& \sum_{j=1, \neq k}^{n} x_{ij}^{\inf(\beta)} \lambda_j + x_{ik}^{\sup(\beta)} \lambda_k \leq x_{ik}^{\sup(\beta)} \theta, \quad \forall i \\
& \sum_{j=1, \neq k}^{n} y_{rj}^{\sup(\beta)} \lambda_j + y_{rk}^{\inf(\beta)} \lambda_k \geq y_{rk}^{\inf(\beta)}, \quad \forall r \\
& \lambda_j \geq 0, \ \forall j, \quad \theta \text{ unrestricted in sign.}
\end{aligned}
\tag{3.2}
$$

Where $X^{\sup(\beta)}$ and $X^{\inf(\beta)}$ are the best favorable and unfavorable situations' efficiencies of DMU_k; u_i and v_r are the assigned input and output weights. The above models (3.1) and (3.2) are referred to as optimistic (lower) and pessimistic (upper) models, respectively.

Models 3.3) and (3.4 are proposed models by Xu [1]. For easy understanding of the constraints sets in both models, we have written a fractional model to explain the efficient frontier. The fractional linear optimistic and pessimistic models (3.3) and (3.4) are equivalent to the envelopment form of models (3.1) and (3.2), respectively, as formulated below:

$$\max D^{\sup(\beta)} = \frac{\sum_{r=1}^{s} y_{rk}^{\sup(\beta)} v_r}{\sum_{i=1}^{m} x_{ik}^{\inf(\beta)} u_i}$$

s.t.

$$\frac{\sum_{r=1}^{s} y_{rj}^{\inf(\beta)} v_r}{\sum_{i=1}^{m} x_{ij}^{\sup(\beta)} u_i} \leq 1 \forall j, \neq k$$

$$\frac{\sum_{r=1}^{s} y_{rk}^{\sup(\beta)} v_r}{\sum_{i=1}^{m} x_{ik}^{\inf(\beta)} u_i} \leq 1,$$

$$u_i,\, v_r \geq 0, \forall i,\, r. \quad (3.3)$$

$$\max D^{\inf(\beta)} = \frac{\sum_{r=1}^{s} y_{rk}^{\inf(\beta)} v_r}{\sum_{i=1}^{m} x_{ik}^{\sup(\beta)} u_i}$$

s.t.

$$\frac{\sum_{r=1}^{s} y_{rj}^{\sup(\beta)} v_r}{\sum_{i=1}^{m} x_{ij}^{\inf(\beta)} u_i} \leq 1 \forall j, \neq k$$

$$\frac{\sum_{r=1}^{s} y_{rk}^{\inf(\beta)} v_r}{\sum_{i=1}^{m} x_{ik}^{\sup(\beta)} u_i} \leq 1,$$

$$u_i,\, v_r \geq 0, \forall i,\, r. \quad (3.4)$$

In the above models (3.3) and (3.4), k^{th} DMU is the targeted DMU under evaluation DMU_k; u_i and v_r are the assigned rough input and rough output weights. $E^{\sup(\beta)}$ and $E^{\inf(\beta)}$ are the best favorable and unfavorable situations' efficiencies of DMU_k. It can be referred to above models as optimistic (lower) and pessimistic (upper) models, respectively. Note that the constraint sets in (3.4) are different for each DMU to determine the DMUs' efficiencies. Also, the constraint sets are different in both models (3.3) and (3.4) to find the optimistic (lower) and pessimistic (upper) efficiency of a given DMU. In other words, the constraint set considers $\{(x_{ik}^{\inf(\beta)}, y_{rk}^{\sup(\beta)}), (x_{ij}^{\sup(\beta)}, y_{rj}^{\inf(\beta)})\}$ to determine the optimistic efficiency of DMU_k while the constraint set of model (3.4) is developed based on $\{(x_{ik}^{\sup(\beta)}, y_{rk}^{\inf(\beta)}), (x_{ij}^{\inf(\beta)}, y_{rj}^{\sup(\beta)})\}$ to identify the pessimistic efficiency of DMU_k.

As a simple example, let us take three DMUs (named DMU 1, DMU 2, and DMU 3) with rough input-output data. The optimistic and pessimistic approaches can be used to define the rough input-output data, i.e., $[\sup, \inf]$. Suppose that optimistic and pessimistic input and output data for 3 DMUs are $\{[1,3],[4,5],[6,7]\}$, $\{[1,3],[5,6],[7,8]\}$, respectively. In Figure 2, the pessimistic production frontier for DMU 1 is OD1, determined by the data set $\{(1,3),(5,5),(7,7)\}$, pessimistic production frontier for DMU 2 is OD2 and determined by the data set $\{(3,1),(4,6),(7,7)\}$ while OD3 is determined by the data set $\{(3,1),(5,5),(6,8)\}$. The relative efficiency of a DMU is determined based on its radial distance from the efficient frontier obtained from the input-output data. If the frontier is not unified, the efficiencies of all DMUs may be different and comparisons between efficiencies are not worthy and meaningful. The production frontier is by efficient DMUs, i.e., using the minimum inputs to produce maximum outputs. In Figure 2, the production frontier for DMU 1 is obtained based on three extreme points, i.e., $\{(1,3),(4,6),(6,8)\}$.

The above example is taken to check whether the efficient frontiers are the same or different and after getting the results we obtained the different efficient frontiers for both the

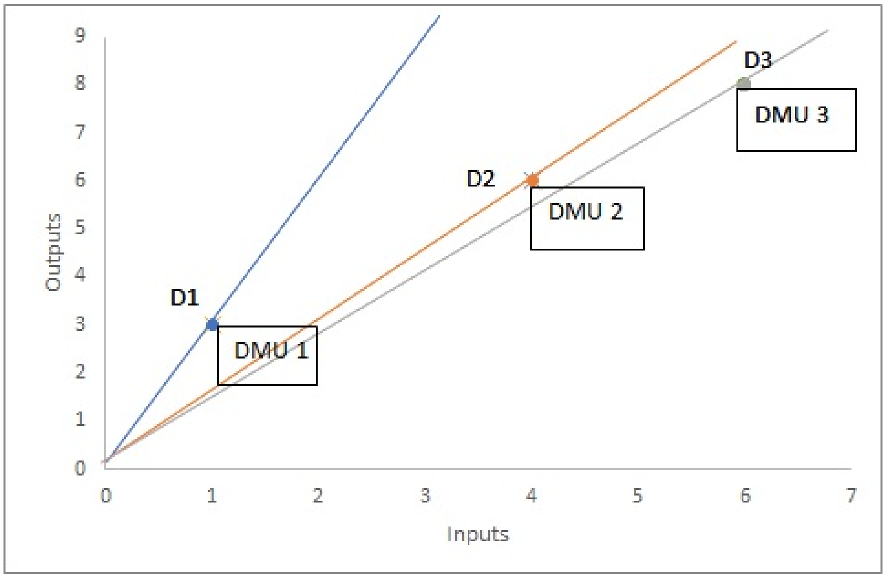

Figure 2. Production frontier

models as well as every DMU i.e. the frontier is not unified. Based on different frontiers we can not compare the DMUs and models. This is the disadvantage of Xus model [1]. Thus, a new optimistic and pessimistic interval DEA method will be proposed in the next section to neglect the different production frontiers to measure the performances of DMUs.

4. Proposed DEA with rough data

In this method, we introduce the same set of constraints (production possibility sets) to construct a unified production frontier for all DMUs that can be used to impartially assess every DMU's performance in the presence of rough input and output data. Assume $E_j = \frac{\sum_{r=1}^{s} y_{rj} v_r}{\sum_{i=1}^{m} x_{ij} u_i}$ is the j^{th} DMU efficiency. Given the optimistic and pessimistic data resulted from rough data, we develop the following pair of interval DEA models to cope with different production frontiers when measuring both the lower and upper bound efficiencies of DMU_k:

$$\max E_j^{\sup(\beta)} = \frac{\sum_{r=1}^{s} y_{rj}^{\sup(\beta)} v_r}{\sum_{i=1}^{m} x_{ij}^{\inf(\beta)} u_i}$$

$$\text{subject to} \quad E_j^{\inf(\beta)} = \frac{\sum_{r=1}^{s} y_{rj}^{\inf(\beta)} v_r}{\sum_{i=1}^{m} x_{ij}^{\sup(\beta)} u_i} \leq 1,$$

$$u_i, \;\; v_r \geq 0, \;\; \forall i, r. \qquad (4.1)$$

$$\max E_j^{\inf(\beta)} = \frac{\sum_{r=1}^{s} y_{rj}^{\inf(\beta)} v_r}{\sum_{i=1}^{m} x_{ij}^{\sup(\beta)} u_i}$$

$$\text{subject to} \quad E_j^{\inf(\beta)} = \frac{\sum_{r=1}^{s} y_{rj}^{\inf(\beta)} v_r}{\sum_{i=1}^{m} x_{ij}^{\sup(\beta)} u_i} \leq 1,$$

$$u_i, \;\; v_r \geq 0, \;\; \forall i, r. \qquad (4.2)$$

The Charnes-Cooper transformation [71] is first applied and then the dual of the above pair DEA models (4.1)-(4.2) is written:

$$E_k^{\sup(\beta)} = \min \theta$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij}^{\sup(\beta)} \lambda_j \leq x_{ik}^{\inf(\beta)} \theta, \;\; \forall i,$$

$$\sum_{j=1}^{n} y_{rj}^{\inf(\beta)} \lambda_j \geq y_{rk}^{\sup(\beta)}, \;\; \forall r,$$

$$\lambda_j \geq 0, \;\; \forall j, \;\; \theta \text{ is unrestricted in sign.} \qquad (4.3)$$

$$E_k^{\inf(\beta)} = \min \theta$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij}^{\sup(\beta)} \lambda_j \leq x_{ik}^{\sup(\beta)} \theta, \;\; \forall i,$$

$$\sum_{j=1}^{n} y_{rj}^{\inf(\beta)} \lambda_j \geq y_{rk}^{\inf(\beta)}, \;\; \forall r,$$

$$\lambda_j \geq 0, \;\; \forall j, \;\; \theta \text{ is unrestricted in sign.} \qquad (4.4)$$

$E_k^{\inf(\beta)}$ in model (4.4) intends to measure the most favorable relative efficiency of DMU_k when the other DMUs are in the position of the worst production activity and DMU_k is considered in the state of the best production activity. On the other hand, the objective of $E_k^{\sup(\beta)}$ in model (4.3) is to calculate the lower bound of the best possible relative efficiency of DMU_k when all the DMUs are in the state of worst production activity. The above optimistic model (4.3) and pessimistic model (4.4) are called the rough DEA (RDEA) models for a given $\beta \in [0.6, 1]$. Remarkably if $\beta \in (0, 0.5]$, the models (4.1) and (4.2) can be transformed into models (4.3) and (4.4), respectively using $E_k^{\inf(\beta)} = E_k^{\sup(1-\beta)}$ and $E_k^{\sup(\beta)} = E_k^{\inf(1-\beta)}$ (see Definition 6).

Let $E_k^{\sup(\beta)*}$ and $E_k^{\inf(\beta)*}$ be the optimal values of models (4.3) and (4.4), respectively. Efficient, weakly efficient and inefficient DMUs based on the trust value can be defined as follows:

- DMU_k is efficient if $E_k^{\sup(\beta)*} = 1$ for any $\beta \in [0.6, 1]$,
- DMU_k is weakly efficient if $E_k^{\inf(\beta)*} = 1$ and $E_k^{\sup(\beta)*} < 1$ for any $\beta \in [0.6, 1]$,
- DMU_k is inefficient if $E_k^{\inf(\beta)*} < 1$ for any $\beta \in [0.6, 1]$.

The constraint sets in (2.7) are different for each DMU to determine the DMUs' efficiencies. Also, the constraint sets are different in both models (2.7) and (2.8) to find the optimistic (lower) and pessimistic (upper) efficiency of a given DMU. In both models, the efficient frontier is not unified for all the DMUs and resultantly the efficiencies of all DMUs may be biased and comparisons between efficiencies are not worthy and meaningful. So, we propose models (3.1) and (3.2) in this paper where the constraint sets are not only similar for all DMUs but also similar in optimistic and pessimistic assessments of all DMUs.

Theorem 1. *Let* $E_k^{\sup(\beta)*}$ *and* $E_k^{\inf(\beta)*}$ *be the optimal values of models (4.3) and (4.4), respectively. Then,* $E_k^{\sup(\beta)*} \le E_k^{\inf(\beta)*}$.

Proof. The dual of models (4.3) and (4.4) are respectively expressed as follows:

$$\begin{aligned} & E_{1k}^{\sup(\beta)} = \sum_{r=1}^{s} y_{rj}^{\sup(\beta)} v_r \\ & \text{subject to } \sum_{i=1}^{m} x_{ij}^{\inf(\beta)} u_i = 1, \\ & \sum_{r=1}^{s} y_{rj}^{\inf(\beta)} v_r - \sum_{i=1}^{m} x_{ij}^{\sup(\beta)} u_i \le 0, \\ & u_i, \ v_r \ge 0, \ \forall i, r. \end{aligned} \tag{4.5}$$

$$\begin{aligned} & E_{1k}^{\inf(\beta)} = \sum_{r=1}^{s} y_{rj}^{\inf(\beta)} v_r \\ & \text{subject to } \sum_{i=1}^{m} x_{ij}^{\sup(\beta)} u_i = 1, \\ & \sum_{r=1}^{s} y_{rj}^{\inf(\beta)} v_r - \sum_{i=1}^{m} x_{ij}^{\sup(\beta)} u_i \le 0, \\ & u_i, \ v_r \ge 0, \ \forall i, r. \end{aligned} \tag{4.6}$$

Let u_i^*, $\forall i$ and v_r^*, $\forall r$ be the optimal values of model (4.5).

Let $\delta_k = \sum_{i=1}^{m} x_{ij}^{\inf(\beta)} u_i^*$, $U_i = \frac{u_i^*}{\delta_k}$, $V_r = \frac{v_r^*}{\delta_k}$.

$\sum_{i=1}^{m} x_{ij}^{\inf(\beta)} u_i^* = 1$ (from model 4.5) and $\sum_{i=1}^{m} x_{ij}^{\sup(\beta)} u_i^* \le \sum_{i=1}^{m} x_{ij}^{\inf(\beta)} u_i^*$

$$\delta_k = \sum_{i=1}^{m} x_{ij}^{\inf(\beta)} u_i^* = 1 \ge \sum_{i=1}^{m} x_{ij}^{\sup(\beta)} u_i^*, \tag{4.7}$$

$$\sum_{i=1}^{m} x_{ij}^{\sup(\beta)} U_i = \sum_{i=1}^{m} x_{ij}^{\sup(\beta)} \frac{u_i^*}{\delta_k} = \frac{1}{\delta_k} \sum_{i=1}^{m} x_{ij}^{\sup(\beta)} u_i^* \le 1$$

$$\sum_{r=1}^{s} y_{rj}^{\inf(\beta)} V_r - \sum_{i=1}^{m} x_{ij}^{\sup(\beta)} U_i = \frac{1}{\delta_k} (\sum_{r=1}^{s} y_{rj}^{\inf(\beta)} v_r^* - \sum_{i=1}^{m} x_{ij}^{\sup(\beta)} u_i^*) \le 0, \ \forall j, \tag{4.8}$$

$$u_i^* \ge 0, \forall i, \ \ v_r^* \ge 0, \forall r, \tag{4.9}$$

$$U_i = \frac{u_i^*}{\delta_k} \ge 0, \forall i, \quad V_r = \frac{v_r^*}{\delta_k} \ge 0, \ \forall r. \tag{4.10}$$

We can conclude from (3.7), (3.8) and (3.9) that (U_i, $\forall i$ and V_r, $\forall r$) is a feasible solution of model (4.6). Then, $\sum_{r=1}^{s} y_{rj}^{\inf(\beta)} V_r \leq E_{1k}^{\inf(\beta)*}$ and $E_{1k}^{\sup(\beta)*} = \sum_{r=1}^{s} y_{rj}^{\sup(\beta)} v_r \leq \sum_{r=1}^{s} y_{rj}^{\inf(\beta)} v_r = \delta_k \sum_{r=1}^{s} y_{rj}^{\inf(\beta)} V_r \leq \delta_k E_{1k}^{\inf(\beta)*} \leq E_{1k}^{\inf(\beta)*}$. So, $E_{1k}^{\sup(\beta)*} \leq E_{1k}^{\inf(\beta)*}$, which completes the proof.

□

Theorem 2. *$E_k^{\sup(\beta)}$ and $E_k^{\inf(\beta)}$ lie within* $(0, 1]$.

Proof. In models (4.1) and (4.2), $u_i \geq 0$, $v_r \geq 0$, $y_{rj}^{\sup(\beta)} > 0$, $x_{ij}^{\sup(\beta)} > 0$, $y_{rj}^{\inf(\beta)} > 0$ and $x_{ij}^{\inf(\beta)} > 0$. So, $\sum_{r=1}^{s} y_{rj}^{\sup(\beta)} v_r > 0$ and $\sum_{i=1}^{m} x_{ij}^{\sup(\beta)} u_i > 0$. Thus, $E_j^{\sup(\beta)} = \frac{\sum_{r=1}^{s} y_{rj}^{\sup(\beta)} v_r}{\sum_{i=1}^{m} x_{ij}^{\inf(\beta)} u_i} > 0$ and $E_j^{\sup(\beta)} \leq E_j^{\inf(\beta)} = \frac{\sum_{r=1}^{s} y_{rj}^{\inf(\beta)} v_r}{\sum_{i=1}^{m} x_{ij}^{\sup(\beta)} u_i} > 0$. From models (4.1) and (4.2), $E_j^{\inf(\beta)} = \frac{\sum_{r=1}^{s} y_{rj}^{\inf(\beta)} v_r}{\sum_{i=1}^{m} x_{ij}^{\sup(\beta)} u_i} \leq 1$ and $1 \geq E_j^{\inf(\beta)} \geq E_j^{\sup(\beta)} = \frac{\sum_{r=1}^{s} y_{rj}^{\sup(\beta)} v_r}{\sum_{i=1}^{m} x_{ij}^{\inf(\beta)} u_i}$.

So, $E_j^{\sup(\beta)}$ and $E_j^{\inf(\beta)} \in (0, 1]$, $\forall j$. Thus, $E_k^{\sup(\beta)}$ and $E_k^{\inf(\beta)} \in (0, 1]$. □

The efficient DMUs can not be fully ranked while in many situations the decision-maker seeks to discriminate the performance among efficient DMUs. To improve discriminatory the power of DEA, a super-efficiency DEA model was first developed by Andersen and Petersen [72]. Andersen-Petersen ranking method does not consider the data variations it handles only the conventional data so, we can not apply the conventional Andersen-Petersen ranking method due to (i) data variation in our method, (ii) efficiency values with trust values of rough variables in our proposed method. Here, we propose the following super-efficiency RDEA models for a given $\beta \in [0.6, 1]$ to rank the efficient DMUs in the presence of rough input and output data.

$$
\begin{aligned}
&SE_k^{\sup(\beta)} = \min \theta \\
&\text{subject to} \quad \sum_{j=1,\neq k}^{n} x_{ij}^{\sup(\beta)} \lambda_j \leq x_{ik}^{\inf(\beta)} \theta, \ \forall i, \\
&\sum_{j=1,\neq k}^{n} y_{rj}^{\inf(\beta)} \lambda_j \geq y_{rk}^{\sup(\beta)}, \ \forall r, \\
&\lambda_j \geq 0, \ \forall j, \ \theta \text{ is unrestricted in sign.}
\end{aligned} \tag{4.11}
$$

$$
\begin{aligned}
&SE_k^{\inf(\beta)} = \min \theta \\
&\text{subject to} \quad \sum_{j=1,\neq k}^{n} x_{ij}^{\sup(\beta)} \lambda_j \leq x_{ik}^{\sup(\beta)} \theta, \ \forall i, \\
&\sum_{j=1,\neq k}^{n} y_{rj}^{\inf(\beta)} \lambda_j \geq y_{rk}^{\inf(\beta)}, \ \forall r, \\
&\lambda_j \geq 0, \ \forall j, \ \theta \text{ is unrestricted in sign.}
\end{aligned} \tag{4.12}
$$

5. Correlation coefficient between rough variables

The isotonicity test is often used to validate the DEA model specification (see e.g., [73], [74]). Isotonicity means that if an input variable rises while keeping other input variables constant, this causes a rise in the value of at least one output variable and vice versa. The isotonicity test is passed and the inclusion of inputs and outputs is justified if positive intercorrelations are identified between all input and output data variables. Due to data complexity, certain real-world applications may not reveal positive correlations (see e.g., [75], [76]).

The standard isotonicity test considers only crisp data values (no variation and no uncertainty) and this test does not work in our proposed method due to the presence of data variations. To the best of our knowledge, no DEA research has been conducted on the relationship between rough inputs and rough outputs. Thus, the rough correlation coefficient is proposed in this section to examine the relationship between rough input-output variables. Let x_i and y_r be two rough variables. The rough correlation coefficient (RCC) between x_i and y_r is denoted as $\alpha(x_i, y_r)$ and can be defined as follows:

$$
\alpha(x_i, y_r) = \frac{n \sum_{j=1}^{n} x_{ij} y_{rj} - \sum_{j=1}^{n} x_{ij} \sum_{j=1}^{n} y_{rj}}{\sqrt{n \sum_{j=1}^{n} (x_{ij})^2 - (\sum_{j=1}^{n} x_{ij})^2} \sqrt{n \sum_{j=1}^{n} (y_{rj})^2 - (\sum_{j=1}^{n} y_{rj})^2}} \tag{5.1}
$$

where x_{ij} and y_{rj}, $j = 1, 2, ..., n$ are the x_i^{th} and y_r^{th} rough variables, respectively, for the j^{th} DMU. In situations of big observations, it is computationally expensive to apply (5.1) in our proposed models. We therefore propose a new approach using the expected value method to obtain the RCC between the rough variables based on trust values.

Let $[x_{ij}^{\sup(\beta)}, x_{ij}^{\inf(\beta)}]$ represents the optimistic and pessimistic bounds of x_i and $[y_{rj}^{\sup(\beta)}, y_{rj}^{\inf(\beta)}]$ represents the optimistic and pessimistic bounds of y_r, where $\beta \in [0.6, 1]$. Given trust values $\beta = [0.6, 1]$ of x_{ij} and y_{rj}, the optimistic and pessimistic RCC denoted by $\alpha^{\sup(\beta)}(x_i, y_r)$ and

$\alpha^{\inf(\beta)}(x_i, y_r)$, respectively, are calculated as follows:

$$\alpha^{\sup(\beta)}(x_i, y_r) = \frac{n\sum_{j=1}^{n} x_{ij}^{\sup(\beta)} y_{rj}^{\sup(\beta)} - \sum_{j=1}^{n} x_{ij}^{\sup(\beta)} \sum_{j=1}^{n} y_{rj}^{\sup(\beta)}}{\sqrt{n\sum_{j=1}^{n}(x_{ij}^{\sup(\beta)})^2 - (\sum_{j=1}^{n} x_{ij}^{\sup(\beta)})^2}\sqrt{n\sum_{j=1}^{n}(y_{rj}^{\sup(\beta)})^2 - (\sum_{j=1}^{n} y_{rj}^{\sup(\beta)})^2}} \tag{5.2}$$

$$\alpha^{\inf(\beta)}(x_i, y_r) = \frac{n\sum_{j=1}^{n} x_{ij}^{\inf(\beta)} y_{rj}^{\inf(\beta)} - \sum_{j=1}^{n} x_{ij}^{\inf(\beta)} \sum_{j=1}^{n} y_{rj}^{\inf(\beta)}}{\sqrt{n\sum_{j=1}^{n}(x_{ij}^{\inf(\beta)})^2 - (\sum_{j=1}^{n} x_{ij}^{\inf(\beta)})^2}\sqrt{n\sum_{j=1}^{n}(y_{rj}^{\inf(\beta)})^2 - (\sum_{j=1}^{n} y_{rj}^{\inf(\beta)})^2}} \tag{5.3}$$

Some key properties of $\alpha^{\sup(\beta)}(x_i, y_r)$ and $\alpha^{\inf(\beta)}(x_i, y_r)$ are listed as follows:

i) $\alpha^{\sup(\beta)}(x_i, y_r) \in [-1, 1]$ and $\alpha^{\inf(\beta)}(x_i, y_r) \in [-1, 1]$ for a given $\beta \in [0.6, 1]$.

ii) $\alpha^{\sup(\beta)}(x_i, y_r) = 1$ and $\alpha^{\inf(\beta)}(x_i, y_r) = 1$ if $x_i = y_r$ for a given $\beta \in [0.6, 1]$.

iii) $\alpha^{\sup(\beta)}(x_i, y_r) = \alpha^{\sup(\beta)}(y_r, x_i)$ and $\alpha^{\inf(\beta)}(x_i, y_r) = \alpha^{\inf(\beta)}(y_r, x_i)$ for a given $\beta \in [0.6, 1]$.

In interval RCC assessment, since the final measure for a particular beta is characterized by an interval, a simple and practical approach is to use the expected value as defined below:

$$\alpha^{\beta}(x_i, y_r) = \frac{1}{2}[\alpha^{\sup(\beta)}(x_i, y_r) + \alpha^{\inf(\beta)}(x_i, y_r)],\ \beta \in [0.6, 1], \tag{5.4}$$

where $\alpha^{\beta}(x_i, y_r)$ satisfies the following properties; i) $\alpha^{\beta}(x_i, y_r) \in [-1, 1]$, ii) $\alpha^{\beta}(x_i, y_r) = 1$ if $x_i = y_r$, and iii) $\alpha^{\beta}(x_i, y_r) = \alpha^{\beta}(y_r, x_i)$. The properties of the above correlation coefficient are similar to that of the crisp correlation coefficient (see e.g. [73], [74]).

6. Proposed ranking approach

The practice of ranking a group of complex units characterized by multiple variables is full of difficulties and almost always contentious. DEA can be used as a powerful tool for classifications and rankings. In this study, we first find the performances of DMUs using the proposed approach, then we rank the DMUs based on their indexes. To the best of our knowledge, there is no ranking method for rough variables in the literature. The proposed ranking algorithm is developed based on optimistic and pessimistic models. To this end, optimistic and pessimistic efficiency values with trust values of rough variables are used as outlined in Table 2.

Definition 1. Let $A = [f^{\sup(\beta)}, g^{\inf(\beta)}]$ and $B = [p^{\sup(\beta)}, q^{\inf(\beta)}]$ be two rough intervals. Then the difference between two rough variables A and B is $A - B = [f^{\sup(\beta)} - q^{\inf(\beta)}, g^{\inf(\beta)} - p^{\sup(\beta)}]$. For example, let $A = [10, 25]$ and $B = [11, 30]$ be two rough intervals; and $-B = [-30, -11]$. Then, $A - B = [10 - 30, 25 - 11] = [-20, 14]$.

Table 2. **Algorithm based on optimistic and pessimistic**

Algorithm based on optimistic	Algorithm based on pessimistic
Step O.1. Run model (4.3) for each DMU using rough input-output data and determine $E_k^{\sup(\beta)*}$, $\beta \in [0.6, 1]$.	**Step P.1.** Run model (4.4) for each DMU using rough input-output data and identify $E_k^{\inf(\beta)*}, \beta \in [0.6, 1]$.
Step O.2. Let $I = \{1, 2, ..., n\}$ be an index set. Let $S^\beta = \{DMU_j : E_k^{\sup(\beta)*} = 1, \beta \in [0.6, 1], j \in \bar{I}\}$ and $S_1^\beta = \{DMU_j : E_k^{\sup(\beta)*} < 1, \beta \in [0.6, 1], j \in I - \bar{I}\}$.	**Step P.2.** Let $I = \{1, 2, ..., n\}$ be an index set. Let $T^\beta = \{DMU_j : E_k^{\inf(\beta)*} = 1, \beta \in [0.6, 1], j \in \bar{I}\}$ and $T_1^\beta = \{DMU_j : E_k^{\inf(\beta)*} < 1, \beta \in [0.6, 1], j \in I - \bar{I}\}$.
Step O.3. Run model (4.11) for $DMU_j \in S^\beta$.	**Step P.3.** Run model (4.12) for $DMU_j \in T^\beta$.

Assume that $I = \{1, 2, ..., n\}$ represents an index set for DMUs. Let $f^{\sup(\beta)} = \min_{k \in I} SE_k^{\sup(\beta)*}$ and $g^{\inf(\beta)} = \max_{k \in I} SE_k^{\inf(\beta)*}$ for $\beta \in [0.6, 1]$, and β denotes the trust value. Obviously, $SE_k^{\inf(\beta)*} - f^{\sup(\beta)}$ and $SE_k^{\sup(\beta)*} - g^{\inf(\beta)}$ are positive and negative, respectively. So, the index of ranking for DMU_k is defined as follows:

$$R_k = \frac{\sum_\beta (SE_k^{\inf(\beta)*} - f^{\sup(\beta)})}{\sum_\beta (SE_k^{\inf(\beta)*} - f^{\sup(\beta)}) - \sum_\beta (SE_k^{\sup(\beta)*} - g^{\inf(\beta)})}, \tag{6.1}$$

where equation (6.1) applies only when Steps O.3 and P.3 are determined in Table 2. If Step O.3 does not apply, $SE_k^{\sup(\beta)*}$ is replaced with $E_k^{\sup(\beta)*}$ in equation (6.1) and if Step P.3 is not achievable, $SE_k^{\inf(\beta)*}$ is replaced with $E_k^{\inf(\beta)*}$ in equation (6.1).

According to the aforesaid discussion, the PR ranking algorithm can be described through a series of structured and successive steps as follows:

Step R.1. Determine the efficiency values of $E_k^{\sup(\beta)}$, $E_k^{\inf(\beta)}$, $SE_k^{\sup(\beta)}$ and $SE_k^{\inf(\beta)}$, $k = 1, 2, 3, ..., n$ using models 4.3, 4.4, 4.11, and 4.12, respectively.
Step R.2. Let $[E_k^{\sup(\beta)}, E_k^{\inf(\beta)}]$ and $[SE_k^{\sup(\beta)}, SE_k^{\inf(\beta)}]$ be the efficiency and super-efficiency intervals, respectively.
Step R.3. Find R_k, $k = 1, 2, 3, ..., n$ using equation (6.1).
Step R.4. DMUs are ranked based on the decreasing values of R_k.

This ranking method which is based on rough variables is suitable for determining the performance of DMUs because it considers the intervals of rough variables.
The proposed optimistic models (4.3) and (4.11), and pessimistic models (4.4) and (4.12) are applicable for rough input-output data. Xu et al. [1] proposed the rough DEA model for rough data, but did not develop the models for determining the super-efficiencies. The production frontier is different for every DMU and both optimistic and pessimistic models,

according to Xu et al. [1] approach (see Section 2.3). As a consequence, it's meaningless to compare efficiencies. The proposed approach uses the same production frontier to evaluate the efficiency of each DMU. While existing techniques often use distinct production frontiers for ranking DMUs, the proposed ranking method considers both performance efficiencies by integrating β optimistic and pessimistic trust values in the same production frontier.

7. Indian fertilizer's industries

The fertilizer supply chains have an essential role in improving food availability in India. Needless to say that India is an agriculture-based developing country and the performance of the fertilizer industry has a remarkable impact on people's nutrition. Hence, the 12th plan of the fertilizer department in India accentuates an increase in foodgrains and fertilizers till 2025 (see Figure 1). In this section, we address the question of how to measure the performance of the fertilizer supply chain in India. The selection of inputs and outputs is a focal step in DEA and there is no consensus in the literature and among practitioners. Without loss of generality, this case study includes the three inputs and three outputs ([7]) that are regarded to estimate the respective technology of the fertilizer supply chain (see Figure 3).

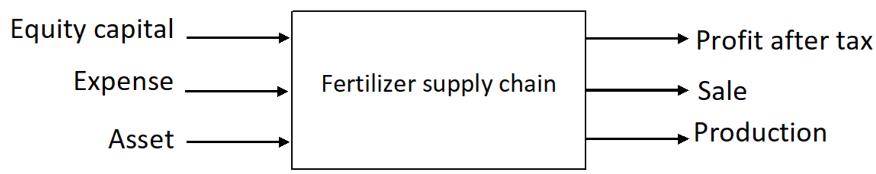

Figure 3. inputs-outputs of the fertilizer supply chain

In this application, we consider 17 Indian fertilizer supply chain industries for ten years and the input-output data are collected from the Centre for Monitoring Indian Economics (CMIE). According to the rough sets discussed in Section 2, the rough variables are written in interval form to make the optimal decision in an uncertain environment. This study utilizes a way to form rough data which helps the decision-makers cope with the inherent uncertainty associated with input and output factors. To this end, $([p, q], [s, t])$ presents the rough data of each observation including two intervals in which the former lies in minimum (s) and maximum (t) data values, and the latter lies in the first quartile (p) and third quartile (q) over ten years. Let's look at an example that represents 11 years of input data: 2,10,45,49,52,56,78,83,86,90,148. We must first establish the quartiles (Q1, Q2, Q3, interquartile range (IQR=Q3-Q1), lower fence (Q1-1.5*IQR), and higher fence (Q1+1.5*IQR) in order to find the outlier. The first quartile (Q1) is 47, the second quartile (Q2) is 56, the third quartile (Q3) is 84.5, the lower fence is -9.25, and the upper fence is 140.75, and we know that outliers are data values that are lower than the lower fence and greater than the upper fence. So, according to this, we have one outlier, 148, and if we do not enter this data, it will be detrimental to decision-makers. As a result, we used rough set theory to tackle this problem. We don't skip outliers in this theory, and the data is taken as is ([47,84.5],[2,148]). The resulting rough data for 17 Indian fertilizer supply chains are shown in Tables 3 and 4. A brief description of the responding fertilizer supply chains is given in the appendix.

Table 3. **Rough input data of 17 fertilizer supply chains**

Supply chains	Equity capital* (I1)	Expense* (I2)	Asset* (I3)
F1	([7509.2,7509.2],[7509.2,7509.2])	([240.5, 603.8],[211.5, 736.8])	([4373.5,6321.3],[3862.9,6382.1])
F2	([289.4,289.4],[289.4, 289.4])	([20206.1, 22745.8],[17372.7, 23111.5])	([16753.3, 23778.3],[14329.7, 23826.8])
F3	([1554.2, 1554.2],[1554.2, 1554.2])	([49408.6, 55172.2],[49380.9, 57958.6])	([79968.6, 87233.5],[76429.3, 106448.6])
F4	([797, 797],[797, 797])	([60184.5, 75291.4],[52649.7, 86734.3])	([94936, 107283.5],[86621.2, 109167.3])
F5	([6865.4, 6865.4],[6865.4, 6865.4])	([213.1, 3886],[55, 3887.4])	([2376.9, 2537.5],[1959.2, 2546.5])
F6	([0.5, 0.5],[0.5, 0.5])	([24.5, 52],[21.9, 75.5])	([103, 174.3],[94.4, 174.4])
F7	([1185.5, 1185.5],[1185.5, 1185.5])	([25125.8, 31324.5],[24489.5, 31515])	([26084.9, 29497.8],[23998.3,30706.6])
F8	([598.1, 598.1],[598.1, 598.1])	([24220.7, 39241.2],[22108.4, 41624])	([42063.1, 48246.3],[38859.1, 55072.3])
F9	([4905.8, 4905.8],[4905.8, 4905.8])	([82817.4, 129528],[78476.1, 132068.2])	([105554.7, 143484.9],[100050.6, 149129.1])
F10	([5516.9, 5516.9],[5516.9, 5516.9])	([74661.9, 90973.8],[70526.9, 95530.9])	([66844.3, 91464.8],[65371.9, 104968.3])
F11	([4162.1, 4162.1],[4162.1, 4162.1])	([74774.8, 104192.5],[71595, 111565.3])	([81230.6, 161534.8],[79681.5, 168028])
F12	([291.7, 292.5],[291.3, 293])	([106353.4, 118939.7],[94563, 133142.9])	([91745.1, 104428.4],[87303.5, 108638])
F13	([4208.5, 4249.8],[4205.5, 6275.7])	([224320.6, 289446.7],[221938.8, 295581.9])	([229687.6, 305328.3],[208104.5, 353605.4])
F14	([5754.5, 5754.5],[5754.5, 5754.5])	([36494.8, 48006.1],[36223.8, 48545.5])	([41315.1, 49726.4],[35857.2, 56277.5])
F15	([10, 170.5],[10, 170.5])	([927.3, 30459.4],[442.9, 39144.7])	([1204, 53449.8],[755.5, 60297.2])
F16	([2036.4, 2036.4],[2036.4, 2036.4])	([18625.4, 21720.7],[15415.4, 26060.8])	([14791.5, 20521],[12659.8, 21011.4])
F17	([2548.2, 2548.2],[2548.2, 2548.2])	([51823, 79787.4],[27860.5, 96902.6])	([132773.1, 143680],[129836.8, 153486])

* Million Indian rupees

Table 4. **Rough output data of 17 fertilizer supply chains**

Supply chains	Profit after tax* (O1)	Sale* (O2)	Production** (O3)
F1	([301, 1647.8],[86.5, 1909.8])	([3.1, 11.675],[1.3,25.1])	([7.78, 8.52],[6.08, 10.72])
F2	([96.4, 200.3],[18.4, 342])	([20148.8, 21829.4],[16663.3, 22826])	([5.16, 5.53],[4.02, 5.83])
F3	([4988.5, 7411.7],[1726.8, 7895.2])	([50137, 61345.1],[48489, 61863.7])	([8.66, 8.98],[8.42, 9.11])
F4	([4093.5, 4757.3],[987, 4936.8])	([61559.3, 76279.7],[54825.3, 85836.4])	([14.6, 16.47],[14.41, 16.9])
F5	([-3805, 221.7],[-3807.5, 93401.4])	([19.25, 21],[17,21.6])	([2.3, 3.03],[2.05, 3.24])
F6	([-9.5, -2.4],[-11.3, -2.2])	([18.2, 43.9],[12, 68.7])	([11.37, 11.84],[11.15, 12.08])
F7	([194.1, 605.8],[-2401.3, 645.5])	([26937.4, 29820.8],[24948.1, 30743.9])	([6.42, 6.73],[5.85, 6.85])
F8	([-4723.5, -927.4],[-4907.3, -215.2])	([8188.1, 16560.4],[4611.5, 39776.5])	([3.65, 7.11],[1.99, 7.92])
F9	([1986.2, 2127.7],[-1710.1, 2984.5])	([78146.3, 122427.9],[76619.8, 128111.3])	([37.97, 38.1],[37.27, 39.61])
F10	([1391.7, 1792.6],[788, 2081.5])	([72511.5, 88707.2],[36887.3, 94173.6])	([29.37, 30.17],[29.18, 31.87])
F11	([4251, 5452.7],[-111.4, 12243.1])	([74678.2, 100961.4],[74323.4, 122088.8])	([20.94, 25.03],[20.02, 32.66])
F12	([4767.8, 7139.1],[3578.6, 10591.7])	([101767.1, 131165.2],[82741, 131922.4])	([22.71, 27.27],[22.21, 27.55])
F13	([6847, 8415.8],[6368.7, 9371.7])	([158206, 222900.2],[124865, 279049.9])	([64.84, 69.25],[61.39, 72.02])
F14	([650.9, 1505.9],[433.3, 1590.5])	([30863.3, 41876.1],[24684.2, 43579.1])	([12.02,13.15],[10.63, 13.21])
F15	([22.8, 270.1],[-60.9, 545.9])	([950, 30707.2],[375.1, 37593.8])	([2.54, 5.04],[0.89, 5.65])
F16	([263.5, 533.4],[247.7, 569.4])	([18422.5, 21110.3]) ([15026.6, 26377.7])	([5.63, 6.52],[5.5, 6.59])
F17	([6927.1, 17669.6],[6662, 68402.2])	([31860.1, 38370.4],[30205.6, 84695])	([16.31,18.62],[15.71, 18.7])

* Million Indian rupees; ** Lakh metric tonnes

Let β trust level be 0.6. The β optimistic and β pessimistic values of the rough variables are first determined using Definition 6, then the resulting rough variables are in turn used in the proposed models (4.3) and (4.4). The upper and lower triangular matrix in Table 5 shows the correlation coefficients (CC) between the optimistic and pessimistic rough data that are determined by the proposed equations (5.2) and (5.3), respectively. The CC is calculated[1] by the proposed equation (5.4) and the results are shown in Table 6.

Note that the diagonal elements of the matrix in Table 5 are equal to 1. For instance, it demonstrates that the equity capital-equity capital connection is neutral. The CC between optimistic rough values of $I1$ (equity capital) and $O1$ (profit after tax) is 0.9767 which shows

[1]The calculations are done in MATLAB R2020b.

the positive and strong relationship between equity capital and profit after tax and indicates that as equity capital raises, profit after tax rises as well. The CC between optimistic rough values of $I1$ (equity capital) and $O2$ (sale) is 0.1153 which shows the positive and weak relationship between equity capital and sale, meaning that equity capital raises, and sales rise as well. The CC between optimistic rough values of $I3$ (asset) and $O1$ (profit after tax) is -0.2131 which shows the negative and weak relationship between asset and profit after tax, meaning that asset rises when profit after tax falls and vice versa. The CC between pessimistic rough values of $I1$ (equity capital) and $O1$ (profit after tax) is 0.9792 which shows the positive and strong relationship between equity capital and profit after tax and indicates that as equity capital raises, profit after tax rises as well. The CC between pessimistic rough values of $I1$ (equity capital) and $O2$ (sale) is 0.1156 which shows the positive and weak relationship between equity capital and sale, meaning that equity capital raises, and sales rise as well. The CC between pessimistic rough values of $I3$ (asset) and $O1$ (profit after tax) is -0.2373 which shows the negative and weak relationship between asset and profit after tax, meaning that asset rises when profit after tax falls and vice versa. The aforementioned CCs

Table 5. **CC between the optimistic rough data and CC between the optimistic rough data**

	I1	I2	I3	O1	O2	O3
I1	1	0.9495	-0.1234	0.9767	0.1153	0.9498
I2	0.9587	1	-0.0671	0.9133	0.0966	0.8906
I3	-0.1583	-0.1199	1	-0.2131	-0.3323	-0.0909
O1	0.9792	0.9246	-0.2373	1	0.0946	0.9253
O2	0.1156	0.1015	-0.2509	0.0943	1	0.2584
O3	0.9553	0.9005	-0.093	0.934	0.2531	1

are based on either optimistic or pessimistic rough values. In this paper, our motive is to find the CCs between rough variables by taking optimistic CCs and pessimistic CCs together. So, the CCs are determined between rough variables using aggregation (equation 5.4) of the optimistic CCs and pessimistic CCs and are shown in Table 6. For example, the CC between rough values of $I1$ (equity capital) and $O1$ (profit after tax) is 0.9781 showing that as equity capital raises, profit after tax rises as well.

Table 6. **PRCC: CC between rough data**

	I1	I2	I3	O1	O2	O3
I1	1					
I2	0.9546	1				
I3	-0.1457	-0.098	1			
O1	0.9781	0.9195	-0.2295	1		
O2	0.11547	0.0992	-0.2865	0.0945	1	
O3	0.9527	0.896	-0.0924	0.9299	0.2558	1

Since the data is in interval form, it is supposed to arrive at the interval efficiencies $[E_k^{\sup(\beta)}, E_k^{\inf(\beta)}]$ for each β trust level. The efficiencies are determined by using models (4.3) and (4.4) as reported in Table 7. Apart from $\beta = 1$, the interval efficiency of the fertilizer supply chains F1, F5, and F6 are $[1, 1]$ i.e. no improvement is required in their inputs and outputs, and the interval efficiency of the fertilizer supply chain F1 associated with $\beta = 1$ is $[0.3425, 1]$, implying that some improvement in inputs and outputs of the optimistic side is required to reach the frontier. As viewed in Table 7, the fertilizer supply chain F15 is only efficient from the optimistic viewpoint. We point out that the resulted interval efficiencies are non-increasing by the rise in β and it evidences that the lower β level would be often preferred by optimistic decision-makers who take high risks. It should be noted that if the

Table 7. **Optimistic-pessimistic efficiencies of 17 fertilizer supply chains**

Supply chains	$[E^{\sup}, E^{\inf}]$				
	$\beta = 0.6$	$\beta = 0.7$	$\beta = 0.8$	$\beta = 0.9$	$\beta = 1$
F1	[1,1]	[1,1]	[1,1]	[1,1]	[0.3425,1]
F2	[0.9872,1]	[0.8179,1]	[0.6183,1]	[0.3632,0.9967]	[0.0328,0.6352]
F3	[0.7056,0.7569]	[0.4495,0.5212]	[0.259,0.3301]	[0.1211,0.1728]	[0.0134,0.0238]
F4	[0.6851,0.7649]	[0.4494,0.5482]	[0.2721,0.3567]	[0.1283,0.303]	[0.0134,0.0548]
F5	[1,1]	[1,1]	[1,1]	[1,1]	[1,1]
F6	[1,1]	[1,1]	[1,1]	[1,1]	[1,1]
F7	[0.7648,0.8211]	[0.5905,0.6737]	[0.4265,0.5737]	[0.2035,0.4648]	[0.0282,0.1084]
F8	[0.2804,0.3701]	[0.1719,0.3134]	[0.0974,0.2717]	[0.0405,0.1758]	[0.005,0.0258]
F9	[0.662,0.7939]	[0.456,0.6442]	[0.2936,0.5055]	[0.138,0.3888]	[0.0234,0.0489]
F10	[0.6951,0.7816]	[0.5282,0.669]	[0.3626,0.5807]	[0.2322,0.5154]	[0.0268,0.0955]
F11	[0.6829,0.8046]	[0.4391,0.6369]	[0.2563,0.4842]	[0.1207,0.2977]	[0.0169,0.0492]
F12	[0.7725,0.8524]	[0.6113,0.7439]	[0.4437,0.5948]	[0.193,0.3024]	[0.0181,0.0333]
F13	[0.5416,0.6307]	[0.3856,0.5308]	[0.2498,0.4122]	[0.1279,0.2601]	[0.0132,0.0358]
F14	[0.6408,0.739]	[0.504,0.6875]	[0.3826,0.6373]	[0.2596,0.5622]	[0.0397,0.0955]
F15	[0.5755,1]	[0.3489,1]	[0.1943,1]	[0.0794,1]	[0.0046,1]
F16	[0.7656,0.8421]	[0.6133,0.7544]	[0.4707,0.6803]	[0.3119,0.6086]	[0.0445,0.0907]
F17	[0.3663,0.468]	[0.2219,0.4166]	[0.1242,0.3285]	[0.0567,0.2402]	[0.0101,0.0398]

efficiency measure of a fertilizer supply chain is unity, then it is known as an efficient supply chain. The ranking of efficient fertilizer supply chains is at the time required and the weak discriminatory power results in confusion when informed decisions should be made. To this end, the super-efficiency models (4.11) and (4.12) are proposed for various β values and the respective results are presented in Table 8. It is observable that the efficient supply chains take a value greater than 1 and it enables us to gain excellent discriminatory power. Remarkably, the inefficiency measures remained unchanged in Table 8 as viewed in the traditional super-efficiency models. Apart from the optimistic view of $\beta = 1$, the fertilizer supply chain F1 has super-efficiency measures greater than 1 as illustrated in Table 8. As shown in Table 8, F6 as the largest private fertilizer supply chain in Mumbai has the highest efficiency measure for all pre-determined βs.

To estimate the aggregated efficiency of the fertilizer supply chain using the proposed ranking approach outlined in Section 5 and presented in Table 9 on both the optimistic and pessimistic sides. Based on the proposed ranking approach, the efficiencies of all fertilizer supply chains fall within $(0, 1)$ as shown in Table 9. As envisaged, the fertilizer supply chain F6 has the best performance even after aggregating the pessimistic and optimistic viewpoints

Table 8. **Optimistic-pessimistic super-efficiencies of 17 fertilizer supply chains**

Supply chains	$[SE^{\text{sup}}, SE^{\text{inf}}]$				
	$\beta = 0.6$	$\beta = 0.7$	$\beta = 0.8$	$\beta = 0.9$	$\beta = 1$
F1	[7.3013,9.9515]	[4.9053,9.2883]	[3.0676,8.2182]	[1.5116,6.2238]	[0.3425,1.0961]
F2	[0.9872,1.2397]	[0.8179,1.2952]	[0.6183,1.0878]	[0.3632,0.9967]	[0.0328,0.6352]
F3	[0.7056,0.7569]	[0.4495,0.5212]	[0.259,0.3301]	[0.1211,0.1728]	[,0.0238]
F4	[0.6851,0.7649]	[0.4494,0.5482]	[0.2721,0.3567]	[0.1283,0.303]	[0.0134,0.0548]
F5	[1.8877,1.9]	[1.7865,1.9]	[1.6869,1.9]	[1.5879,1.9]	[1.386,4.7603]
F6	[10.2122,12.2894]	[9.0896,13.1988]	[7.8615,14.1662]	[6.124,14.8885]	[4.3193,15.2126]
F7	[0.7648,0.8211]	[0.5905,0.6737]	[0.4265,0.5737]	[0.2035,0.4648]	[0.0282,0.1084]
F8	[0.2804,0.3701]	[0.1719,0.3134]	[0.0974,0.2717]	[0.0405,0.1758]	[0.005,0.0258]
F9	[0.662,0.7939]	[0.456,0.6442]	[0.2936,0.5055]	[0.138,0.3888]	[0.0234,0.0489]
F10	[0.6951,0.7816]	[0.5282,0.669]	[0.3626,0.5807]	[0.2322,0.5154]	[0.0268,0.0955]
F11	[0.6829,0.8046]	[0.4391,0.6369]	[0.2563,0.4842]	[0.1207,0.2977]	[0.0169,0.0492]
F12	[0.7725,0.8524]	[0.6113,0.7439]	[0.4437,0.5948]	[0.193,0.3024]	[0.0181,0.0333]
F13	[0.5416,0.6307]	[0.3856,0.5308]	[0.2498,0.4122]	[0.1279,0.2601]	[0.0132,0.0358]
F14	[0.6408,0.739]	[0.504,0.6875]	[0.3826,0.6373]	[0.2596,0.5622]	[0.0397,0.0955]
F15	[0.5755,1.2517]	[0.3489,1.7588]	[0.1943,2.6778]	[0.0794,4.7892]	[0.0046,10.3892]
F16	[0.7656,0.8421]	[0.6133,0.7544]	[0.4707,0.6803]	[0.3119,0.6086]	[0.0445,0.0907]
F17	[0.3663,0.468]	[0.2219,0.4166]	[0.1242,0.3285]	[0.0567,0.2402]	[0.0101,0.0398]

(see Table 9). The fertilizer supply chain F15 is the second higher efficient supply chain, which is a private organization in Gujarat (the western coast of India). The F8 and F17 are the lowest supply chains belonging efficient among all the 17 supply chains, and it is the government and located in Andhra Pradesh and Mumbai, respectively (see Table 9). There would be a great perspective across public and private sectors to optimize the production process further to improve the fertilizer supply chain efficiency and this study can shed some light on possible solutions.

Let us compare the efficiency results of the supply chain of fertilizer industries under uncertain situations obtained by the developed rough DEA approach in this paper with an original DEA method. Supply chain performance analysis can not be normally carried out by the conventional DEA method when uncertainty exists. So, the proposed rough DEA results are obtained by optimistic and pessimistic β trust values for rough variables to cope with uncertainty. The rough input-output data (see Tables 3 and 4) is converted to crisp ones by aggregating the rough data values, and the crisp data is then used in the original DEA method to determine the efficiencies of fertilizer supply chains. The proposed and existing models are compared and the overall ranking is shown in Table 9.

The non-parametric Kruskal-Wallis test is widely used to test if there is a significant difference statistically between independent samples ([77], [78]). We employ this rank test, which is dominantly observed in the DEA setting since it does not require assumptions on the efficiency score distribution [79]. More details on the Kruskal-Wallis test can be found in [77], [78], and [80]. The Kruskal-Wallis test results based on the two groups' efficiencies are determined. The Kruskal-Wallis test has 1 degree of freedom and the asymptotic p-value is 0.000 which is less than the significance level (0.05). So, we conclude that there is no evidence to show that the results are similar in terms of efficiency measures.

In a nutshell, the fertilizer supply chain F6 is the most competitive and productive among

Table 9. **Comparision between the original DEA and proposed model**

Supply chains	Original DEA model ([5])	Rank	Proposed model	Rank
F1	24.4374	2	0.3025	3
F2	3.6104	4	0.0477	5
F3	1.0227	6	0.0127	15
F4	1.0157	7	0.0149	13
F5	1	8	0.1184	4
F6	158.8404	1	0.5425	1
F7	1	8	0.0213	8
F8	1	8	0.0059	17
F9	1	8	0.0186	11
F10	1	8	0.0213	9
F11	1	8	0.0175	12
F12	1.5248	5	0.0202	10
F13	1	8	0.0133	14
F14	1	8	0.0221	7
F15	4.5376	3	0.3512	2
F16	1	8	0.0248	6
F17	1	8	0.0094	16

all fertilizer industries and we would say that the proposed method's evaluation results are more appropriate, accurate, and true to reality. According to the above discussion, some rough DEA virtues are (i) dealing with uncertain information in the supply chain; (ii) dealing with outliers. So, a rough DEA is more appropriate to tackle some supply chain applications under uncertain situations.

8. Conclusion

In this study, we developed the rough DEA models in the presence of rough variables. The proposed rough DEA models assess the performance of the supply chain network. We explored the correlation coefficient between rough variables using optimistic and pessimistic β trust values of rough variables, we first transformed the rough data into interval form using optimistic and pessimistic β trust values of rough variables, then calculated the efficiency interval $[E_j^{\sup(\beta)}, E_j^{\inf(\beta)}]$ of each DMU for a given β trust level. Additionally, the ranking method has been proposed to rank the DMUs. The proposed rough DEA uses the same set of constraints (production possibility sets) to create a single production frontier for all DMUs, which can be used to accurately estimate each DMU's performance in the presence of rough input and output data.

Finally, we applied the proposed methods to the Indian fertilizer supply chains to illustrate the applicability and efficacy of the developed method in the presence of rough data. The findings suggest that the efficiency values of the Indian fertilizer supply chain range from 0.0059 to 0.5425 implying that, the fertilizer supply chain is generally very inefficient. The fertilizer supply chain F6 (private) is the most competitive and productive among all fertilizer

industries. F15, a private organization in Gujarat, is the second most efficient fertilizer supply chain. Among the 17 supply chains, the F8 (government) and F17 (government) are the least efficient. According to the Kruskal-Wallis test, there is no evidence that the efficiency measures from standard DEA and proposed rough DEA are comparable. There would be a great perspective to optimize the production process further to improve the fertilizer supply chain efficiency in India. The developed evaluation method can aid decision-makers substantially in improving the operational effectiveness and efficiency of the Indian fertilizer supply chain.
This research has some limitations: uncertain data is taken as rough variables, and the efficiencies are determined only for homogeneous units. These limitations are also a future scope for researchers.

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Declaration of interest

None

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Appendix

Rough set theory

The rough set theory aims to circumvent impreciseness and vagueness that is quite often observed in a myriad of real-world problems. Let us here provide an overview of the rough sets theory.

Definition 2 (Rough set). A rough set $(\underline{Z}, \bar{Z})$ is a subset of all sets that have the same lower and upper approximations. The lower approximation is a subset of the set that contains the objects that belong to it. The upper approximation, on the other hand, is a superset containing the objects that may belong to the set, and $\underline{Z} \subset Z \subset \bar{Z}$ [70].

Definition 3 (Rough space [70]). Let γ be a nonempty set, R as a $\sigma-$ algebra of subsets of γ, and χ an element in R, and ϕ a trust measure.

Definition 4 (Trust [70]). Let $(\gamma, \chi, \mathrm{R}, \phi)$ be a rough space. Then the upper trust of an event A is defined by $\bar{T}r\{A\} = \frac{\phi\{A\}}{\phi\{\gamma\}}$. The lower trust of the event A is defined by $\underline{Tr}\{A\} = \frac{\phi\{A \cap \chi\}}{\phi\{\chi\}}$ and the trust of the event A is defined by $Tr\{A\} = \frac{1}{2}(\bar{T}rA + \underline{Tr}A)$. If ϕ a trust measure, then $(\gamma, \chi, \mathrm{R}, \phi)$ is called a rough space.

Definition 5 (Rough variables [70]). A rough variable ξ is a measurable function from the rough space $(\gamma, \chi, \mathrm{R}, \phi)$ to the set of real numbers. That is, for every Borel set B of $\mathbb{R}$, $\{\lambda \in \gamma | \xi(\lambda) | \in B\} \in A$. The lower and the upper approximations of the rough variable are defined as $\bar{\xi} = \{\xi(\lambda) | \lambda \in \gamma\}$.

A rough variable $([p, q], [s, t])$ with $s \leq p < q \leq t$ represents the identity function $\xi(\lambda) = \lambda$ from the rough space $(\gamma, \chi, \mathrm{R}, \phi)$ to the set of real numbers, where $\gamma = \{\lambda | s \leq \lambda \leq t\},\ \chi = \{\lambda | p \leq \lambda \leq q\}$, A is the Borel algebra on λ, and ϕ is the Lebesgue measure.

Definition 6 (Expected value of a rough variable [1]). Let ξ be a rough variable. Then the expected value of ξ is defined by $E[\xi] = \int_0^{\infty} Tr\{\xi \geq r\}dr - \int_{-\infty}^{0} Tr\{\xi \leq r\}dr.$

Provided that at least one of the two above integrals is finite [1]. Example for how to find the expected value of a rough variable, let $\xi = ([p, q], [s, t])$ be a rough variable with $s \leq p < q \leq t$, we then have $E[\xi] = \frac{1}{4}(p + q + s + t)$.

Definition 7 (β-optimistic and pessimistic values [70]). Let ξ be a rough variable, and $\beta \in (0, 1]$. Then, $\xi_{sup}(\beta) = sup\{r | Tr\{\xi \geq r\} \geq \beta\}$ is called the β-optimistic value of ξ, and $\xi_{inf}(\beta) = inf\{r | Tr\{\xi \leq r\} \geq \beta\}$ is called the β-pessimistic value to ξ.

Let rough variable $([p, q], [s, t])$ with $s \leq p < q \leq t$ represents $\xi = ([p, q], [s, t])$. Then the β-optimistic value and the β-pessimistic value of ξ are calculated as follows:

$$\xi_{sup}(\beta) = \begin{cases} (1-2\beta)t + 2\beta s, & if \beta \leq \frac{t-q}{2(t-s)}, \\ 2(1-\beta)t + (2\beta - 1)s, & if \beta \geq \frac{2t-p-q}{2(t-s)}, \\ \frac{t(q-p)+q(t-s)-2\beta(q-p)(t-s)}{(q-p)+(t-s)}, & elsewhere. \end{cases} \quad \xi_{inf}(\beta) = \begin{cases} (1-2\beta)s + 2\beta t, & if \beta \leq \frac{p-s}{2(t-s)}, \\ 2(1-\beta)s + (2\beta - 1)t, & if \beta \geq \frac{q+t-2s}{2(t-s)}, \\ \frac{s(q-p)+p(t-s)+2\beta(q-p)(t-s)}{(q-p)+(t-s)}, & elsewhere. \end{cases}$$

Some basic relationships and statements on $\xi_{sup}(\beta)$ and $\xi_{inf}(\beta)$ are expressed as follows [1]:

i. $Tr\{\xi \geq \xi_{sup}(\beta)\} \geq \beta$ and $Tr\{\xi \leq \xi_{inf}(\beta)\} \geq \beta$,

ii. $\xi_{inf}(\beta)$ is a left-continuous and increasing function of β,

iii. $\xi_{inf}(\beta)$ is left-continuous and decreasing function of β,

iv. If $0 < \beta \leq 1$, then $\xi_{inf}(\beta) = \xi_{sup}(1-\beta)$, and $\xi_{sup}(\beta) = \xi_{inf}(1-\beta)$,

v. If $0 < \beta \leq 0.5$, then $\xi_{inf}(\beta) \leq \xi_{sup}(\beta)$,

vi. If $0.5 < \beta \leq 1$, then $\xi_{inf}(\beta) \geq \xi_{sup}(\beta)$.

Assume that ξ is a rough variable, and let $0.5 < \beta \leq 1$. Obviously, $\xi_{inf}(\beta) \geq \xi_{sup}(\beta)$, so the β-pessimistic and β-optimistic values of the ξ are in an interval, β-optimistic value is the lower bound and the β-pessimistic value is the upper bound, i.e. $[\xi_{\mathrm{sup}}(\beta), \xi_{\mathrm{inf}}(\beta)]$.

F1: The several state fertilizer industries combined into a single drive in 1961 by the Indian Government that time F1 industry started. Now, it is under the Department of Fertilizers, Ministry of Chemicals and Fertilizers. This industry was closed by the Indian Government in 2002 due to sick industry in 1992. The industry has been restarted operations in 2010 after the approval of the Indian Government. The industry has five units in different states of India: Odisha (Talcher Complex), Jharkhand (Sindri complex), Telangana (Ramagundam complex), Uttar Pradesh (Gorakhpur complex), Chhattisgarh (project). Greenstar Fertilizers Ltd.
F2: It is the Indian public sector fertilizer industry. It runs in Tamilnadu, India.
F3: It is under the fertilizers and Chemicals, India, and was founded in 1976. It is promoted by Gujarat and the fertilizers and chemicals of Gujarat state, and it is in Bharuch.
F4: It was founded in 1962. It is also under the fertilizers and Industrial Chemicals of the Indian Government. The headquarter of the F4 industry is in Vadodara, Gujarat.
F5: It is under the Indian Government. It is situated in Bihar and runs under the Indian Government.
F6: It is the largest private fertilizer industry which is based in Mumbai, Maharashtra, India, and the part of Aditya Birla Groups.
F7: It is the largest manufacturer of chemical fertilizers in Karnataka, India, and was founded in 1974. It is situated in Panambur, Mangalore. The operating offices of the F7 industry are located in Telangana, Kerala, Karnataka, Andhra Pradesh, Maharashtra, and Tamil Nadu.
F8: It is under the Indian Government and was founded in 1973. In August 2011, the industry name was changed to Kakinada Fertilizers limited. It is situated in Kakinada, Andhra Pradesh, India.
F9: It is under the Ministry of Chemicals and Fertilizers, India, and founded in 1979. As of 2018, it is the second-largest industry in the production of urea in India. It has two plants in Punjab, two plants in Madhya Pradesh, and one plant in Haryana.
F10: It is a Public sector industry under the Ministry of Chemicals and Fertilizers, India, and founded in 1978. It is operated in Mumbai, India.
F11: It is one of the largest Indian private sector fertilizer industries and was founded in 1985. It has three plants situated in Kota, Rajasthan.
F12: It is under the Ministry of Chemicals and Fertilizers, India, and founded in 1960. It is situated in Hyderabad, Telangana, India.
F13: It is under the Ministry of Chemicals and Fertilizers, India, and founded in 1967. It is the multi-state society of fertilizer's marketing and manufacturing and operating in New Delhi, India.
F14: It was founded in 1979 and is situated in Jagatsinghpur, Odisha, India. In 2002, it was converted as Municipality.
F15: It was founded in 1979 by Chimanlal Mehta, Gujarat, as private industry. It became a public industry in 1982 and ran under the Indian Government. F15 industry has marketed fertilizers under the brand name of "Mahadhan" since 1990. Its office is in Pune, Maharastra, India. It has four plants in India: Andhra Pradesh, Maharashtra, Gujarat, and Haryana.
F16: It is under the Indian Government and was founded in 1969. Its headquarter is in Chennai, India.
F17: It is under the Indian Government and was founded in 1938. Its headquarter is in

Mumbai, Maharashtra, India. It is one of India's most extensive chemical industry, with operations in India, Africa, North America, and Europe.