

The LOFAR Two-metre Sky Survey Deep fields:

The Mass Dependence of the Far-Infrared Radio Correlation at 150MHz using Deblended *Herschel* fluxes

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ABSTRACT

The Far Infrared Radio Correlation (FIRC) is one of the strongest correlations in astronomy, yet a model that explains this comprehensively does not exist. The new LOFAR all Sky Survey (LoTSS) deep field, ELAIS-N1, allows exploration of this relation in previously unexplored regions of parameter space of radio frequency (150MHz), luminosity ($L_{150} < 10^{24.7}$), redshift ($z \sim 1$) and stellar mass $M_* < 10^{11.4}$. We present accurate, deblended far infrared (FIR) flux measurements with robust errors at 24, 100, 160, 250, 350 and 500 μ m from *Spitzer* and the *Herschel Space Observatory* using XID+. We found that the FIRC has a strong mass dependence, whose evolution takes the form $q_{TIR}(M_*) = (2.00 \pm 0.01) + (-0.22 \pm 0.02)(\log(M/M_*) - 10.05)$. This matches recent findings of the SFR-radio luminosity relation at 150MHz and results from radio observations in COSMOS at 1.4GHz with the Jansky Very Large Array (JVLA). Our results provide tighter constraints on the low redshift end of the FIRC and at lower frequency than the COSMOS observations. In addition we found a mild evolution with redshift, with a best fit relation $q_{TIR}(z) = (1.94 \pm 0.01)(1+z)^{-0.04 \pm 0.01}$. This evolution is shallower than previous results at 150MHz with the differences being previous studies not accounting for the mass dependence. Finally, we present deblended FIR fluxes for 79,609 galaxies across the LoTSS deep fields: Boötes, ELAIS-N1 and Lockman Hole.

Key words. Surveys – Catalogs – Galaxy: Evolution – Infrared: Galaxies – Radio Continuum: Galaxies

1. Introduction

It has long been known that radio emission from normal galaxies is mostly from synchrotron radiation produced by electrons in the interstellar medium (ISM) that are accelerated by supernova remnants (Condon 1992). Emission in the infrared (IR) is produced by dust heated by ultra-violet (UV) and optical radiation, which is then re-emitted in the IR (Casey et al. 2014). As both supernova and UV radiation are produced by massive stars, with short lifetimes, they are both tracers of recent star for-

mation (SF). This provides the theoretical underpinning of the well-known far infrared radio correlation (FIRC) which was first observed by van der Kruit (1971) and later by Condon (1992); Helou & Bicay (2002); Yun et al. (2002) who found a tight correlation between radio and total infrared (TIR 8 – 1000 μ m) luminosity in local galaxies. Since then the FIRC has been observed across a wide range of luminosities and redshifts (Murphy 2009; Sargent et al. 2010; Delhaize et al. 2017; Calistro Rivera et al. 2017). This relatively tight correlation has been utilised as a boundary to identify radio-loud active galactic nuclei (AGN)

(Del Moro et al. (2013); Calistro Rivera et al. (2017)) and as a tool to calibrate the star formation rate (SFR)-Radio luminosity relation (Davies et al. 2017; Gurkan et al. 2018). Radio surveys can play a unique role in studying SF in galaxies as they are not affected by dust extinction, and can cover very wide areas of the sky with high angular resolution. A well calibrated SFR-radio luminosity relation is therefore of vital importance for a full exploitation of the next-generation surveys with SKA (Square Kilometre Array) precursors (ASKAP and MeerKAT), as well as for the LOFAR all sky survey (LoTSS; Shimwell et al. (2019)), whose second data release will cover several thousand square degrees. Understanding whether the FIRC varies with a galaxies properties such as redshift or stellar mass can give insight into creating a SFR-radio luminosity relation.

The FIRC is thought to originate from SF and is broadly explained by the "calorimeter" theory developed by Voelk (1989). This proposes that cosmic rays (CR) accelerated by supernova shocks lose all, or most, of their energy as synchrotron emission while they spiral in the galactic magnetic field. The majority of UV emission from SF regions, is reprocessed by dust and emitted in the FIR - from where the supernova also originate. However, this model begins to break down when considering smaller galaxies with a lower mass, as you would expect more CR to escape the galaxy before they have radiated most of their energy. Similarly galaxies with a low dust mass would emit a higher amount of UV radiation as less is reprocessed, which could lead to a break of the FIRC. More complicated models have been developed to try and explain these discrepancies such as the conspiracy model by Bell (2003). The "conspiracy" is that the "missing" FIR and radio emission counterbalance each other. While this preserves the FIRC it affects its ability to calibrate the SFR-radio luminosity relation. This theory was expanded by Lacki et al. (2010) to examine the range of galactic parameters that would support a linear FIRC. They investigated how the radio spectra is modified by a number of different parameters including: gas density, the initial energy of CR injected into the ISM and the escape time of the CR. They also model the effect of CR protons produced by supernova and their decay products. They found that the majority of galaxies (with $\rho_{gas} > 0.01 \text{ g cm}^{-3}$) are UV and CR calorimeters and that below this the FIRC is maintained by the standard 'conspiracy'. However, starburst galaxies have a more complicated relationship. Their radio emission is reduced by CR losing the majority of their energy through inverse Compton scattering and bremsstrahlung radiation. This is counteracted by the emission from secondary CR electrons produced by the decay of CR protons. These findings are supported by those of Magnelli et al. (2015) who found no change in the FIRC with specific star formation rate (sSFR).

Whether the FIRC is redshift dependant is still a point of contention. Some theoretical works predict redshift evolution, caused by inverse Compton scattering of cosmic rays with the warmer cosmic microwave background (CMB), leading to higher flux ratios (q_{TIR} defined as the FIR luminosity divided by the radio luminosity, to calculate q_{TIR} see 4) (Murphy 2009). Others suggest the FIRC should remain unchanged up to $z \approx 1.5$ (Lacki et al. 2010). Observationally there is also tension, with some studies finding that the median q_{TIR} decreases with redshift, (Delhaize et al. 2017; Calistro Rivera et al. 2017) whilst other studies have found no significant evolution with redshift (Sargent et al. 2010; Smith et al. 2014; Delvecchio et al. 2021). This topic is further complicated by potential evolution in the FIRC with luminosity (either radio or IR). If the median FIRC is different for more luminous galaxies, then when observing at higher redshift, where there is a bias towards more lumi-

nous galaxies, an apparent redshift evolution would be observed where none exists (unless steps are taken to account for this in sample selection). Using infrared astronomical satellite (IRAS) data and DR1 of the LoTSS, Wang et al. (2019) found the FIRC to be sub linear at 150MHz for the most IR luminous galaxies (their sample had a median redshift of $z \sim 0.05$) lending evidence to this theory. Similarly, Heesen et al. (2019) also found a sub linear relation between radio and SFR (which is correlated with IR luminosity) for spatially resolved galaxies in the local universe. In addition, the degeneracy between a redshift and luminosity evolution could result in a degeneracy between redshift and other galactic parameters, such as stellar mass. Resolving these degeneracies requires a sample of galaxies whose selection effects, with respect to all these parameters, is understood. Then the evolution of the FIRC with respect to each parameter could be studied in isolation.

Recent studies have found evidence that the FIRC is mass dependent, i.e that for a fixed IR luminosity a galaxy with a higher stellar mass will have a greater radio luminosity. Using deep COSMOS data at 1.4GHz Delvecchio et al. (2021) found that q_{TIR} is linearly proportional to M_* over a wide range of redshifts and stellar masses ($0.1 < z < 4$ and $8.5 < \log(M_*) < 11.5$). Due to the small area of COSMOS ($< 2.0 \text{ deg}^2$) their low redshift measurements have a large uncertainty. In particular, the high mass, low redshift galaxies ($z < 0.4$) have a higher normalisation than the higher redshift ($z > 0.4$) galaxies. In addition, using the LoTSS deep field data in ELAIS-N1, Smith et al. (2021) found that the radio luminosity - SFR relation is mass dependant i.e for a given radio luminosity a higher mass galaxy will have a lower SFR than a galaxy with a lower stellar mass.

Smith et al. (2014) and Read et al. (2018) showed that as dust temperature increases the FIRC decreases for monochromatic fluxes (fluxes were measured at 100, 160, 250, 350 and 500 μm). In addition Molnár et al. (2018) showed that for a sample of star forming galaxies that the FIRC decreases more sharply with redshift for spheroid dominated galaxies compared to spiral galaxies. This discrepancy could be attributed to possible low level AGN activity that is boosting the radio emission but is undetected at other wavelengths. This is qualitatively supported by Sabater et al. (2020) who showed, using a mass selected sample, that all massive galaxies have low radio emission from AGN activity. In addition the recent findings of the FIRC's mass dependence would help explain these findings as for a given stellar mass a spheroidal galaxy will usually have a higher mass.

The majority of existing studies of the FIRC have been undertaken at 1.4GHz (Terzian (1972); Bell (2003); Sargent et al. (2010); Delhaize et al. (2017); Delvecchio et al. (2021) to name a few), whereas there are relatively few studies of the FIRC at lower frequencies. Low frequency (150MHz) observations have the advantage that they are not affected by contamination from thermal emission from gas ionised by nearby massive stars. This emission is more relevant at higher frequencies, and while both synchrotron and thermal emission mechanisms are linked to massive stars, the interplay between these two mechanisms is not known. Using wide area LOFAR data combined with SDSS, Gurkan et al. (2018) found the radio-SFR relation was best fitted by a broken power law at 150MHz as well as finding that for a given SFR there is a mass dependence on the radio luminosity. They also found that galaxies not classified as SF had a greater radio luminosity than would be predicted from their SFR, implying low level AGN activity. This work was expanded on by Read et al. (2018); Smith et al. (2021) which found the FIRC to explicitly vary with stellar mass and redshift. A redshift evolution was

also found by Calistro Rivera et al. (2017) using a radio-selected sample from LOFAR data at 150MHz in Boötes.

This work expands on previous studies at 150MHz by using new deep LOFAR observation in ELAIS-N1 (with an rms noise of $20\mu\text{Jy}$) that overlap with deep optical data, that covers 7.15deg^2 in total, (Sabater et al. 2021) and new deblended *Herschel* fluxes provided through the *Herschel* Extragalactic Legacy Project (HELP; Hurley et al. (2017); Vaccari & Consortium (2015)). This covers 3 times the area of the Jansky Very Large Array (JVLA) survey in COSMOS (Smolčić et al. 2017) while reaching a similar depth (assuming a spectral index of -0.7) allowing us to probe brighter radio and FIR luminosities, which are rarer (Grupponi et al. 2013; Sabater et al. 2020). In addition, making use of the XID+ tool (Hurley et al. 2017) to obtain more accurate deblended fluxes for the FIR allows us to push to fainter galaxies in the FIR. Finally new aperture matched optical and photometric redshifts, produced as part of the LoTSS deep fields data release (Kondapally et al. 2021; Duncan et al. 2021), allow us to create multiple mass-complete samples out to a redshift of one and investigate whether the presence of low level AGN activity biases the FIRC by observing the relation with stellar mass. Section 2 will discuss the optical, radio and FIR datasets used and the physical parameters that are estimated from them. Section 3 will discuss how these datasets are crossmatched to form a joined catalogue and give details on how FIR fluxes were measured for each radio galaxy in the three deep fields (Boötes, ELAIS-N1 and Lockman). Section 4 details how our mass complete sample was constructed in ELAIS-N1 and how AGN were removed from this sample. In Section 5 we present our measurement of the FIRC and validate our methods by comparing with the simulated infrared dusty extragalactic sky (SIDES) simulation (Béthermin et al. 2017). In Section 6 we compare our results with previous measurements of the FIRC and discuss the implications of the FIRC dependence on stellar mass. In addition we examine the non-linearity of the FIRC at 150MHz and investigate the dispersion around the FIRC. Throughout this work we assume a ΛCDM with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $H_0 = 70\text{km s}^{-1}\text{Mpc}^{-1}$.

2. Data

Here we present an outline of the datasets and data products used for this work.

2.1. 150 MHz Radio Data

As part of the LOFAR all Sky Survey, (Shimwell et al. 2017) deep radio observations have been taken in selected fields (Boötes, ELAIS-N1 and Lockman Hole). A full description of the method used to produce the radio image and source catalogue can be found in Tasse et al. (2021) for Boötes and Lockman-Hole and Sabater et al. (2021) for ELAIS-N1. The radio coverage extends well beyond the optical and FIR datasets so only a subset of the radio sources can be used. The overlapping area and the properties of the radio catalogues are summarised in Table 1. All radio observations have an angular resolution of $6''$.

2.2. Optical and Near Infrared

To create a homogeneous catalogue from the UV to near infrared (NIR), Kondapally et al. (2021) produced new photometric catalogues in the ELAIS-N1 and Lockman-Hole (Brown et al. (2008) have already created a homogenous catalogue in Boötes).

In ELAIS-N1 and Lockman Hole, this was done by first generating pixel-matched mosaics from a range of deep, wide-area optical, NIR and MIR surveys. Sources were then detected using deep χ^2 detection images (Szalay et al. 1999) which incorporated multi-band information, with photometry extracted using SExtractor (Bertin 1996) in UV to mid infrared (MIR) bands at various apertures. The full details of the catalogue generation process are described in Kondapally et al. (2021). In Boötes, the existing I-band and $4.5\text{-}\mu\text{m}$ catalogues of Brown et al. (2007, 2008) were cross-matched to generate the multi-wavelength catalogue.

2.3. Far Infrared

The FIR data used in this project are from three different instruments: Spectral and Photometric Imaging Receiver (SPIRE, Griffin et al. (2010)) and Photodetector Array Camera and Spectrometer (PACS, Poglitsch et al. (2010)) taken from the *Herschel* Multi-Tiered Extragalactic Survey (HerMes) project (Oliver et al. 2012) as well as the Multi-band Imaging Photometer for Spitzer (MIPS Rieke et al. (2004)), taken from the *Spitzer* mission. These images have been collected and homogenised as part of the HELP (*Herschel* Extragalactic Legacy Project) project. All FIR fluxes were measured using XID+ (Hurley et al. 2017). XID+ is a Bayesian probabilistic deblending tool that measures the contribution of galaxies detected at shorter wavelengths (with higher resolutions) to the FIR images. This is done by modelling the observed maps as a combination of flux from every source as well as contribution from instrumental and confusion noise (Σ_{int} and Σ_{conf}). Using Bayes Theorem (Equ 1) XID+ explores the flux posterior for every galaxy in the prior list (\mathbf{s}_i) with Hamiltonian Monte Carlo (HMC, Safarzadeh et al. (2014)),

$$P(\mathbf{s}_i, \Sigma_{\text{int}}, \Sigma_{\text{conf}}|d) \propto P(\mathbf{s}_i, \Sigma_{\text{int}}, \Sigma_{\text{conf}})P(d|\mathbf{s}_i, \Sigma_{\text{int}}, \Sigma_{\text{conf}}) \quad (1)$$

where d is the map, \mathbf{s}_i is the flux of the sources in the prior list, Σ_{int} is the instrumental noise and Σ_{conf} is the confusion noise. There are several advantages of using XID+ to measure the flux posterior for sources from a prior list instead of crossmatching blind FIR catalogues to optical catalogues. Firstly it allows for detecting multiple galaxies within a single beam. As well as sources whose pixel SNR is below 3σ . In addition, the full posterior allows us to accurately measure flux errors.

The prior list from the HELP XID+ run is constructed from galaxies detected at optical wavelengths, which are less affected by confusion. Fluxes for each galaxy are taken as the 50th percentile of the flux posterior and the 84th and 16th percentiles are taken as the upper and lower errors respectively. To ensure that the measured fluxes are not dominated by the prior we can look at the faintest sources. If a galaxy included in the prior list doesn't contribute to the FIR maps then its flux posterior will be dominated by a convolution of the prior and the flux distribution of the map (a Gaussian skewed towards zero). These faint sources would all have a high skew (defined here as the 84th-50th percentile divided by the 50th-16th percentile as opposed to measuring the skew of Gaussian fit to the posterior). By looking at the skew as a function of flux we can see at what flux the majority of galaxies become skewed (see Figure 1) by eye. The flux posterior of galaxies whose flux is below these cutoffs are dominated by the uniform prior, as the map is providing little to no information. As the posterior flux is dominated by the uniform flux prior and noise level of the map, the posterior of

Table 1. Description of the radio data available in each of the LoTSS deep fields

Field	Overlap area	150MHz rms depth	No. 150MHz Sources	SPIRE limit 250, 350, 500 μ m	PACS limit 100, 160 μ m	MIPS limit 24 μ m
	[deg ²]	[μ Jy]		[mJy]	[mJy]	[μ Jy]
Boötes	9.50	30	18766	5, 5, 10	12.5, 17.5	20
ELAIS N-1	7.15	20	31059	4, 4, 6	12.5, 17.5	20
Lockman	10.73	23	29784	4, 4, 6	12.5, 17.5	20

Overlap area is the area of the radio observations that overlap with the ancillary multi-band (Kondapally et al. 2021) data. Depth is the average rms noise in the overlap region (Mandal et al. 2021). The SPIRE, PACS and MIPS limit is the faintest a source can be and still have a reliable flux measurement from XID+.

these sources has not been informed by the data and so XID+ flags these sources. These flags are included in the FIR flux catalogues.

The FIR fluxes that are available from HELP are at 24 (from Spitzer MIPS observations), 100, 160, 250, 350 and 500 μ m (from *Herschel* PACS and SPIRE respectively, Pilbratt et al. 2010) which required two different prior lists for XID+, one for 24 μ m and one for the longer wavelengths (due to the differences in resolution between MIPS and PACS + SPIRE). The prior list for 24 μ m was constructed from a subset of the HELP masterlist. This was created by merging optical/NIR/MIR catalogues that overlap with the *Herschel* observations, the details of which can be found in Shirley et al. (2019). This is necessary because if the 24 μ m prior list was comprised of the entire HELP masterlist then XID+ would find a large degeneracy in the galaxy fluxes due to the high source density relative to the MIPS resolution. To remove this degeneracy we exclude sources from the prior list that are expected to have a negligible 24 μ m flux density. To this end a number of cuts were applied to the merged catalogue to select sources that have significant emission at 24 μ m. The prior list used for deblending the MIPS observations was constructed by only including objects which were detected by *Spitzer* (3.6-8.0 μ m) and at shorter wavelengths. These cuts select objects most likely to be bright at 24 μ m while removing artifacts from *Spitzer*. The PACS and SPIRE prior list was constructed by only considering objects from the 24 μ m prior list whose 24 μ m flux density was greater than the cutoff flux (Figure 1, this is the same 24/*mum* flux density cut used to define the PACS and SPIRE prior list) as there is a correlation between flux at 24 μ m and flux measured from PACS and SPIRE. For both MIPS, PACS and SPIRE a uniform flux prior was used.

For each galaxy we can also examine the goodness of fit of the flux posterior. By looking at the flux distribution of a pixel, we can evaluate how likely we are to draw the measured pixel flux. If on average the pixels to which a given source is contributing to are more than 2σ away from the true value then we flag that source as having an unreliable flux. This can be caused by missing a source from our prior list that is contributing to the map or because the included source does not contribute to the map.

Additional FIR bands were available from MIPS (70 and 160 μ m) and PACS (70 μ m) as well as 850 μ m from SCUBA-2. However the additional data from PACS and MIPS is of a very low sensitivity so would add very little value. The SCUBA-2 data typically probes higher redshift galaxies while LOFAR predominantly observes lower redshift star forming galaxies. In addition there are only several thousand galaxies detected in the SCUBA-2 Cosmology Legacy Survey (S2CLS) (Geach et al. 2017) survey in Lockman-Hole and these can be cross-matched to the LOFAR catalogue by a simple nearest neighbour crossmatch. This

was done in Ramasawmy et al. (2021) in prep and found only tens of matches, and so it has not been included in the data release.

2.4. Physical Parameters and SED Fitting

Photometric redshifts were measured for the full multi-wavelength catalogues using the method outlined in Duncan et al. (2021) which is briefly summarised here. By combining Gaussian processes and template fitting using a hierarchical Bayesian framework, photometric redshifts were estimated from multi-band photometry spanning from the near ultraviolet (NUV) to NIR. The Gaussian processes were trained on galaxies with a spectroscopically confirmed redshift.

SEDs were computed for each radio source with a multi-wavelength crossmatch using five different SED fitting codes: BAGPIPES (Carnall et al. 2019), MAGPHYS (Da Cunha et al. 2008), AGNFITTER (Calistro Rivera et al. 2016) and CIGALE using two different AGN models: Fritz et al. (2006), Stalevski et al. (2016). A full comparison of the results of each of the SED fitting codes is presented in Best et al. (2021) in prep. Here we give a brief description. Good agreement was found between all the codes for the stellar mass, SFR and dust luminosity (with some small scatter, but no systematic offsets). Where the codes differ is in their treatment of AGN as both BAGPIPES and MAGPHYS do not contain any AGN components in their models whereas AGNFITTER and CIGALE do. As a general rule when there is no significant AGN activity in a galaxy MAGPHYS and BAGPIPES do obtain better fits than either AGNFITTER or CIGALE as with fewer parameters they are able to better explore their parameter space. On the other hand they do not provide good fits for the majority of AGN that are better fit by CIGALE and AGNFITTER. A comparison of the results of the different codes therefore allows identification of sources with AGN features; a full description of the AGN identification method is presented in section 4.2.

3. Cross-Matching

3.1. Radio to Multi-Wavelength

The radio sources were crossmatched to the multi-wavelength catalogue using a combination of likelihood ratio (LR) and manual crossmatching as described in Kondapally et al. (2021). LRs are calculated using the formula from (Sutherland & Saunders 2014):

$$LR = \frac{q(m, c)f(r)}{n(m, c)}, \quad (2)$$

where $q(m, c)$ is the combined magnitude colour distribution of the true radio counterparts, $n(m, c)$ is the normalised magnitude

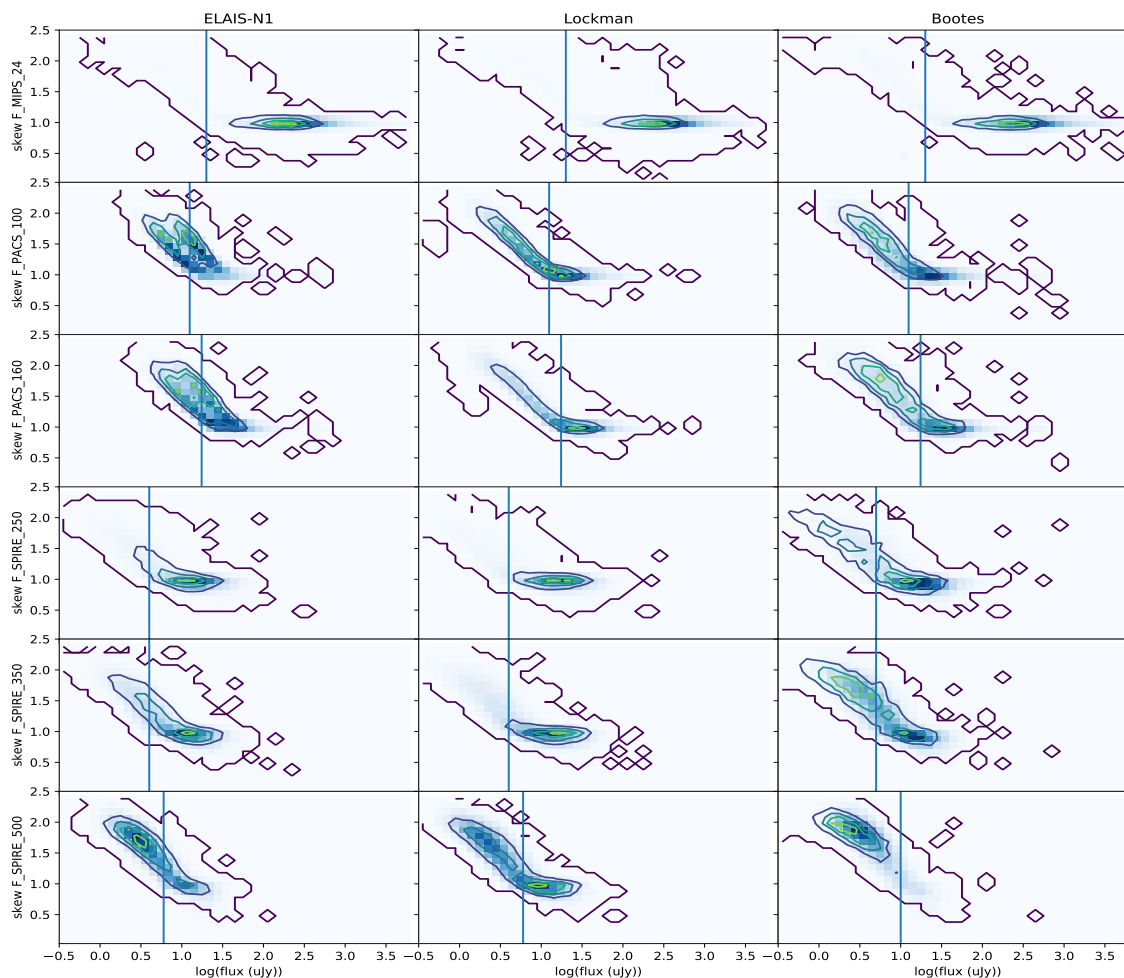


Fig. 1. Skew of the XID+ fluxes for all MIPS, PACS and SPIRE bands in the three LOFAR deep fields. The contours show the density of the LOFAR sources whose multi-wavelength counterpart was originally undetected in the FIR. The histogram beneath the contours shows the distribution of all FIR sources in the LOFAR catalogues. The vertical lines show the flux below which a source’s posterior is dominated by the prior.

colour distribution of all multi-wavelength sources and $f(r)$ is the probability of the separation between the radio and multi-wavelength source.

Manual crossmatching was carried out for sources that were deemed unsuitable for LR crossmatching and were then visually inspected using the LOFAR galaxy zoo platform. Unsuitable sources included extended sources, sources with multiple Gaussian components or clustered sources. Members of the LOFAR consortium inspected the optical/NIR postage stamps around the radio source overlaid with the radio contours and made their best interpretation about the true multi-wavelength counterpart. Each radio galaxy was inspected by five different members and then if a consensus had been reached on the galaxies counterpart(s) then this was taken. If no clear consensus had been reached, then either the galaxy would not be assigned a counterpart, or it would be sent to an expert workflow where it was inspected in more

detail. This choice depended on why no consensus was reached and is discussed in full in Kondapally et al. (2021).

3.2. Far infrared crossmatching

To measure the FIR flux from the radio sources we used the following procedure. First we crossmatched the LOFAR multi-wavelength counterparts with the HELP prior list using a search radius of $0.5''$, which corresponds to the positional uncertainty of the *Spitzer* IRAC sources. If there is a source from the HELP prior list within $0.5''$ then the FIR fluxes measured for that source were assigned to the radio counterpart. If the radio source has no multi-wavelength counterpart or there is no HELP prior list source within $0.5''$ then XID+ is rerun around each of these sources using the same prior list as HELP, but with the addition of the radio source (using the position of the multi-wavelength

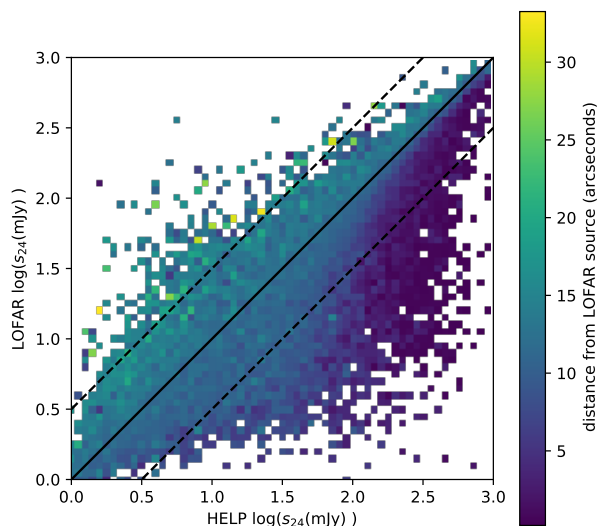


Fig. 2. Comparison between the flux measured with XID+ using the HELP and LOFAR prior lists. The colour shows the average distance from the LOFAR source for all sources in that bin (and hence the centre of the rerun region). Note the asymmetry about the $x=y$ line on the right hand side of the plot where sources are brighter with the HELP prior compared to the LOFAR prior. This discrepancy is caused by the inclusion of the radio source in the LOFAR prior that causes sources in the HELP prior to have less flux assigned to them. The flux measurement from the LOFAR prior list is consistent with the measurement using the HELP prior list.

counterpart if there is one, otherwise the radio position is used) and a uniform flux prior is used. For the HELP XID+ run the FIR maps were divided into equal area tiles, based on the HEALPIX tiling system, that were individually run through XID+. As the XID+ run for LOFAR uses circular regions (with a radius of $60''$), because we did not need to rerun XID+ across the entire deep fields, we compared the fluxes we measured for the non radio galaxy counterparts to their fluxes measured in HELP. The results can be seen in Figure 2 and show a good agreement between the two measurements, within their measured uncertainty (i.e. $\approx 60\%$ of sources agree within one uncertainty). The results are symmetric about the $x = y$ line apart from a subset of sources that are brighter in HELP than in the LOFAR run. These sources are all close to the new radio galaxy that was added to the prior list. It would be expected that they would be fainter in the LOFAR run as the flux in the map is being assigned to the radio source.

In total we have FIR fluxes for 79,609 galaxies with detections at 150MHz across the three deep fields.

4. Sample Creation

4.1. Mass Selection

To investigate how the FIRC depends on redshift and stellar mass we need to construct mass complete samples. We use the MAGPHYS SED fits from Smith et al. (2021) in ELAIS-N1 to construct a mass complete sample. Smith et al. (2021) used MAGPHYS to measure the SED for galaxies in the LOFAR catalogues with

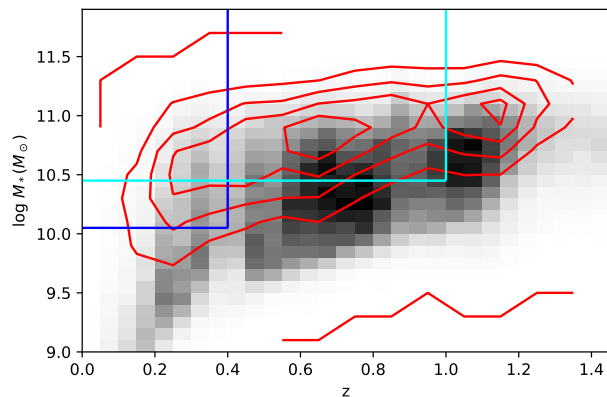


Fig. 3. Stellar mass - redshift distribution of the Smith et al. (2021) SED fits. The red contours show the distribution for galaxies with a detection at 150MHz and the grey histogram shows the distribution for the galaxies without a detection at 150MHz. The dark blue lines show the region selected for our mass sample and the light blue lines show the region selected for our redshift sample.

a $3.6\mu\text{m}$ flux density $> 10\mu\text{Jy}$ and estimated the completeness in stellar mass as a function of redshift using the method from Pozzetti et al. (2010). We created two samples using this information, one with $z < 1.0$ and $M_* > 10^{10.45}$, and the other with $z < 0.4$ and $M_* > 10^{10.05}$ to investigate the FIRC's dependence on redshift and stellar mass respectively (Figure 3). We shall refer to these samples as the redshift sample and the mass sample.

MAGPHYS uses energy balance to match UV/optical and FIR templates so that the energy attenuated by dust in the UV/optical is equal to the energy radiated in the FIR. This energy balance allows an estimate of the TIR luminosity even when there are no individual FIR flux measurements, this ensures that all galaxies in our sample have a TIR luminosity measurement (Appendix A.1 shows the distribution of galaxies with less than one detection in SPIRE and PACS). The effect of using the energy balance approach when a galaxy has no individual detections from PACS or SPIRE is validated in Appendix A.1 to ensure that no systematic bias is introduced from the SED fitting. In addition Malek et al. (2018) tested the use of energy balance in measuring total infrared luminosities by measuring the TIR luminosity for galaxy when their FIR fluxes were included and excluded and found the results to be in good agreement. This analysis was carried out using CIGALE rather than *magphys* but the energy balance principle used in both SED fitters is the same and gives us confidence that there is no systematic bias in the TIR luminosity measurements for galaxies with no PACS or SPIRE fluxes.

Not all galaxies have a radio detection and therefore an upper limit on the radio flux is calculated (detailed in next paragraph). We then impose a cut on specific star formation rate (sSFR, SFR/M_*) to select star forming galaxies. To be considered a star forming galaxy a galaxy must be no more than 0.6 dex below the main sequence. We calculate the MS for each galaxy using Equ A1 from Sargent et al. (2014) depending on its stellar mass and redshift. We use 0.6 dex as this corresponds to two standard deviations away from the MS (the effects of changing the offset from 0.6 dex is discussed in section 6.1). This cut removes passive galaxies from our sample while not excluding starburst galaxies. Lacki et al. (2010) showed that starburst galaxies should theoretically lie along the FIRC and Magnelli et al. (2015) found this to be true in COSMOS, therefore we do

not apply a cut above the SF main sequence. Table 2 shows how the different cuts reduce the number of galaxies in our sample (we also remove AGN from our sample and this is discussed in section 4.2).

Galaxies with a radio detection had their rest-frame luminosity at 150MHz calculated using:

$$L_{150} = \frac{4\pi D_l^2}{(1+z)^{\alpha+1}} S_{150}, \quad (3)$$

where L_{150} is the rest frame luminosity at 150MHz, α is the radio spectral index and S_{150} is the measured flux at 150MHz. We take the spectral index to be -0.60 which is the spectral index measured between 150MHz and 325MHz in Calistro Rivera et al. (2017) which is appropriate to use for our sample of $z < 1$ galaxies as the highest rest frame frequency in our sample is 300Hz. For the sources with no radio detection we calculate a limit on their radio luminosity by taking the flux at 150MHz to be 5σ , where σ is the rms noise in the pixel that contains the galaxy. This assumes that the undetected radio sources are spatially unresolved, which is valid due to the large angular resolution ($6''$) and is consistent with the threshold used to detect radio sources with PYBDSF (Tasse et al. 2021; Sabater et al. 2021).

4.2. AGN identification

The FIRC is believed to originate from radio and FIR emission linked to SF. The removal of AGN is essential for measuring this relation and its scatter accurately. AGN can contribute to a galaxy's SED in the IR and radio and thus could contaminate the FIRC.

Galaxies in our sample were identified as AGN if they were flagged as AGN in Duncan et al. (2021), which include AGN identified using their NIR colours by the criteria in Donley et al. (2012), had bright x-ray counterparts, were spectroscopically identified as AGN or are part of the Million Quasar Catalog (Flesch 2019). In addition, Best et al. (2021) in prep identified AGN in radio detected sources (at 150MHz) by using the four different SED fits described in 2.4 as well as looking for evidence of excess radio emission based on the SFR-radio relation or extended radio emission. Finally as part of selecting galaxies with a reliable MAGPHYS SED we excluded fits that had a poor χ^2 (see Smith et al. (2012) for more details). It was found in Smith et al. (2021) that 94% of galaxies flagged as AGN in Best et al. (2021) in prep are also flagged by this method, so we can be confident that AGN have been removed even if they are undetected in the radio.

5. Far Infrared Radio Correlation

We parameterize the FIRC using q_{TIR} (the log ratio of the TIR to radio luminosity), defined as,

$$q_{TIR} = \log\left(\frac{L_{TIR}}{3.75 \times 10^{12} Hz}\right) - \log\left(\frac{L_{150MHz}}{WHz^{-1}}\right) \quad (4)$$

noting that L_{TIR} is divided by the central frequency of 3.75×10^{12} to make it dimensionless, and calculate q_{TIR} for every galaxy in our sample. If a galaxy has an upper limit on its radio luminosity then we calculate a lower limit of q_{TIR} for that galaxy. We can then measure the median q_{TIR} of our sample of galaxies, using survival analysis to account for the lower limits in q_{TIR} . Survival analysis works by redistributing the lower limits assuming they follow the same underlying distribution, without making

any assumptions as to the form of this distribution. This is implemented using the LIFELINES PYTHON package (Davidson-Pilon et al. 2021) which uses the Kaplan Meier estimator (Kaplan E. L. & Meier Poul 1958) which has had multiple applications within astronomy (Schmitt 1985). The median q_{TIR} of our sample was measured by bootstrapping the survival analysis. This was done by repeating the measurement of the median, using survival analysis 1000 times on a random 90% of our sample and the median measured for each of these samples. The median q_{TIR} was taken as the mean of the distribution of medians from our bootstrapped samples and the error was taken as the standard deviation of the samples.

In the following sections we detail how we validated our methodology by adding radio emission to the simulated infrared dusty extragalactic sky (SIDES) (B  thermin et al. 2017) simulation using a simple model. We then present detail how the FIRC varies with different physical parameters.

5.1. SIDES FIRC simulation

To validate our methodology we created a simulation of our samples using the SIDES simulation (B  thermin et al. 2017). SIDES simulates galaxies by generating their stellar mass from stellar mass functions. These galaxies are then matched to dark matter halos by abundance matching. Galaxies are then assigned to be either star-forming or quiescent with the probability of a galaxy being star-forming depending on its stellar mass and redshift. The galaxies' FIR properties are then calculated based on its SFR, if a galaxy is quiescent then it has no FIR emission. It should be noted that AGN are not included in the SIDES simulation.

To ensure that our sample selection does not introduce any significant bias we selected a mass complete sample from the SIDES simulation using the same selection criteria used to create our real mass sample (i.e $M_* > 10^{10.05}$ and $z < 0.4$) and removed galaxies based on their sSFR using the method described in section 4.1. As the SIDES simulation does not include radio emission we created a simple model to add emission at 150MHz to the SIDES galaxies that have $\log(M_*) > 10.05$ and $z < 0.4$. This was done by first giving each galaxy a q_{TIR} value drawn from a Gaussian distribution with mean 1.8 and standard deviation 0.35. These parameters were chosen to provide a reasonable match to the q_{TIR} distribution of the mass complete sample. It should be noted that the distributions do not completely match and it is clear that the real q_{TIR} distribution is not Gaussian. However, for the purposes of validating our method we do not need to perfectly reproduce our data. As we are measuring the median q we are unaffected by the small number of galaxies with low q values. Since every galaxy has a q_{TIR} value assigned to it we can compute its radio luminosity and flux at 150MHz and then determine whether it would be included in our radio catalogue (i.e if $S_{150MHz} > 0.1mJy$). If a galaxy would not have been detected then we computed the upper limit of its radio luminosity using equation 3 and taking the upper limit on its flux to be 0.1mJy (0.1mJy is the average 5σ rms noise) and then recalculated its q_{TIR} using equation 4. We then performed survival analysis on the simulated galaxies to measure the median of the 'observed' q_{TIR} distribution, taking lower limits of q_{TIR} into account, and found it to agree to the true median within the measured uncertainty (Figure 4).

We then modified our simple model to incorporate an evolution of q_{TIR} with stellar mass. To do this, we set the mean of the Gaussian distribution used to draw q_{TIR} to a function of stellar mass, $\mu = N(1 + M_*)^\gamma$.

Table 2. Number of galaxies in our sample after each cut is made

	z, M_*	sSFR	AGN
Mass sample detections ($z < 0.4, M_* > 10^{10.05}$)	3464	2417	2353
Mass sample limits ($z < 0.4, M_* > 10^{10.05}$)	10447	1798	
Redshift sample detections ($z < 1.0, M_* > 10^{10.45}$)	10046	7190	6856
Redshift sample limits ($z < 1.0, M_* > 10^{10.45}$)	41763	9784	

Each column shows the number of galaxies in our sample after each successive cut is made. The cuts are made in z, M_* plane, sSFR and to remove AGN. The two rows for each sample show the detections at 150MHz and limits at 150MHz. There is no number in the AGN column for the limits because AGN are excluded automatically based on their `MAGPHYS` chi2 flag which identifies bad fits making impossible to identify which of those galaxies are AGN.

To investigate whether q_{TIR} evolves with stellar mass we divided our simulated into 10 stellar mass bins such that the bins had an equal number of galaxies in them. The median q_{TIR} in each bin was then measured using survival analysis. We can then fit a power law of the form,

$$q_{TIR} = N(1 + M_*)^\gamma \quad (5)$$

with N and γ as free parameters. To measure the value of the free parameters we bootstrapped our data by taking a random 90% of our data and fitting the power law to it using the method outlined above. We then took the value of the free parameters to be the median of the 1000 individual values measured, with an uncertainty equal to the standard deviation of the individual values. Using this method we were able to recover the intrinsic (measured by using all the radio luminosities, rather than a combination of radio luminosities and upper limits for those with $S_{150MHz} > 0.1mJy$) parameters of the power law within 1σ (figures 5 and 6). So if q_{TIR} evolves with stellar mass in our LOFAR sample we are confident that we could recover the power law parameters for that evolution.

Additionally we are able to check whether a dependence of the FIRC on stellar mass would cause an observed evolution of the FIRC with redshift. We measured the evolution of q_{TIR} with redshift for a simulated sample of galaxies whose q_{TIR} varies with stellar mass. We found no observed evolution of q_{TIR} with redshift in this scenario. This gives us confidence that if we observe an evolution of q_{TIR} with redshift in our redshift sample that the evolution is genuine and not an artifact of any observed variation with stellar mass.

Finally, we investigated the case where we have more galaxies with upper limits on their radio luminosity than measurements. To change this we artificially raised the the flux limit (0.5mJy for the mass sample and 0.25 for the redshift sample, this gives a ratio of ≈ 0.5 of detections to upper limits), above which a galaxy is detectable at 150MHz, and remeasured the evolution of q_{TIR} with stellar mass and redshift. We found that having significantly more lower limits on q than direct detections does not cause a significant bias in our results. Even in the lowest mass bin, or highest redshift bin respectively, where the ratio of detections to upper limits is less than 0.5. These simulations give us confidence that we can correctly measure the underlying parameters that describe the evolution of q_{TIR} with both stellar mass and redshift.

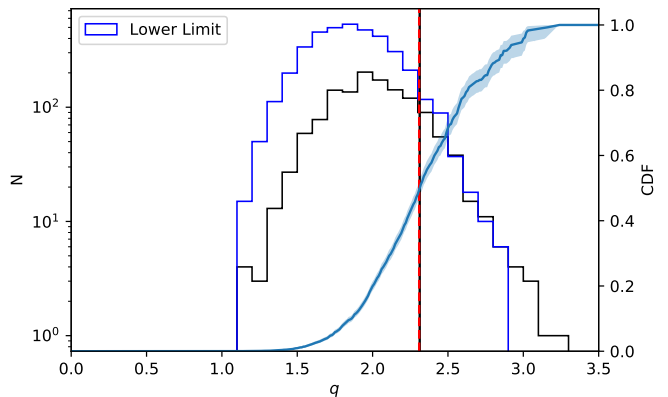


Fig. 4. The distribution of q_{TIR} values from the SIDES simulation for galaxies with $M_* > 10^{10.45}$ and $z < 1$. The blue histogram is for lower limits of q_{TIR} and the black histogram is for direct detections. The black vertical line (behind the dashed red line) marks the median of the sample, as measured using survival analysis and the red dashed vertical line shows the true median. The slight overestimation of the median is caused by the greater number of lower limits compared to direct detections. The blue line is the CDF of the entire sample, measured using survival analysis.

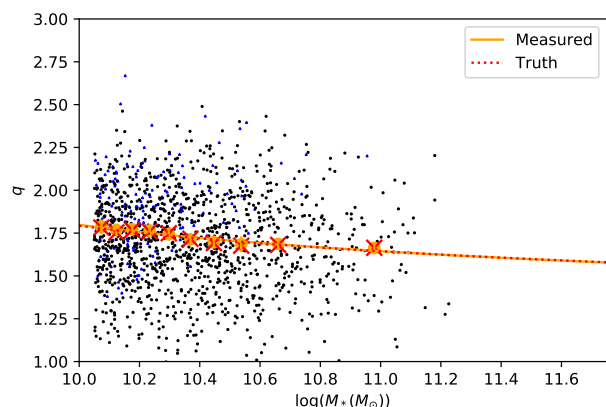


Fig. 5. Evolution of q_{TIR} with M_* for the SIDES simulation ($M_* > 10^{10.05}$ and $z < 0.4$) when q_{TIR} is drawn from a Gaussian distribution whose mean is M_* dependent. The orange points are the measured medians in their bins and the orange line is the fit to the medians. The red crosses and line are the true medians and fit, note that the red line is underneath the orange line.

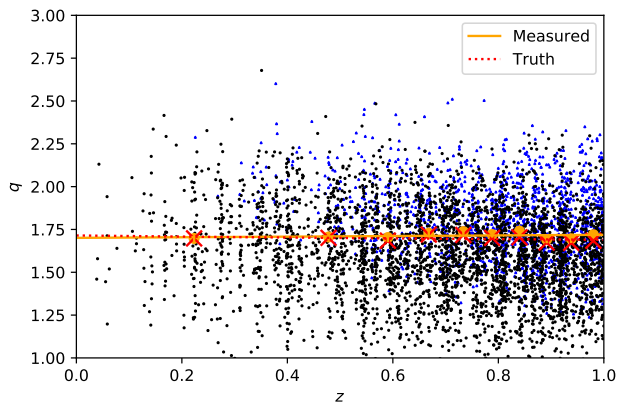


Fig. 6. Evolution of q_{TIR} with redshift for the SIDES simulation ($M_* > 10^{10.45}$ and $z < 1.0$) when q_{TIR} is drawn from a Gaussian distribution whose mean is M_* dependent. The orange points are the measured medians in their bins and the orange line is the fit to the medians. The red crosses and line are the true medians and fit.

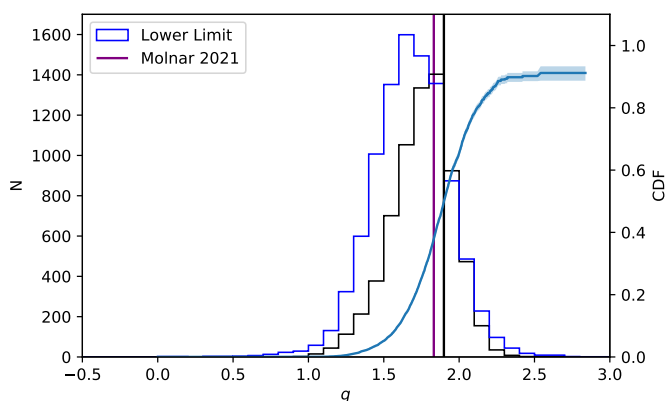


Fig. 7. Distribution of q_{TIR} values for the redshift sample with $M_* > 10^{10.2}$ and $z < 1$. The black histogram is the q_{TIR} distribution for galaxies that are detected at 150MHz. The blue histogram is the distribution of lower limits of q_{TIR} for galaxies that have an upper limit at 150MHz and therefore their q_{TIR} is a lower limit. The vertical line marks the median of the sample, as measured using survival analysis and the purple line shows the median q_{TIR} from Molnár et al. (2021), measured at 1.4GHz and converted to 150MHz using $\alpha = -0.78$.

5.2. FIRC with stellar mass and redshift

For our redshift sample we measured the median $q = 1.879 \pm 0.002$ (Figure 7) with typically uncertainties on individual measurements of q of 0.1 – 0.2dex.

We investigated the evolution of q_{TIR} with redshift, with our redshift sample, using the method outlined in the above section. By fitting a power law to measure the redshift evolution we can compare with the results of Delhaize et al. (2017) and Calistro Rivera et al. (2017) (we also fit a linear function whose parameters can be seen in table 3). The fit parameters can be seen in Table 3. The results can be seen in Figure 8 where we show how the median q_{TIR} of our sample changes with redshift. The orange line show our measured relation which shows a much shallower evolution compared to Calistro Rivera et al. (2017) and Delhaize et al. (2017) (red and green lines respectively). We also investigated the evolution of q_{TIR} with stellar mass. This was done by

repeating the above methodology with our mass sample to investigate how q_{TIR} depends on stellar mass and the results can be seen in Figure 9.

Finally we investigated the joint evolution of the FIRC with redshift and stellar mass using a sample of galaxies with $z < 0.4$ and $M_* > 10^{10.45}$ (giving 1171 galaxies detected at 150MHz and 281 galaxies with upper limits on their flux at 150MHz). The sample was split into 4 redshift bins with an equal number of galaxies in each bin and then each redshift bin is split into 4 stellar mass bins that all have an equal number of galaxies in them. We then fit the combined evolution using the method described above and find an evolution of the form $q_{TIR}(z, M_*) = (1.98 \pm 0.02)(1 + z)^{0.02 \pm 0.04} + (-0.22 \pm 0.03)\log(M_*) - 10.45$.

To further validate our results we remeasured the median q_{TIR} in our stellar mass bins using two alternative methods. Firstly we employed the method from Smith et al. (2021), which we will briefly describe. In this method the PDF of the stellar mass vs q_{TIR} plane is constructed by sampling from an individual galaxy’s radio flux posterior and TIR luminosity posterior. These posteriors, for galaxies detected at 150MHz, are taken to be Gaussian distributions with mean equal to the measured value (for the TIR luminosity we used to 50th percentile output from *magphys*) and standard deviation equal to the error on that mean. For each posterior sample we can calculate its radio luminosity and therefore its q_{TIR} . The 150MHz flux posterior for galaxies undetected at 150MHz is taken to be a Gaussian with mean equal to the flux in the pixel that the galaxy is located, and standard deviation equal to the error of the flux in that pixel. Samples with a negative 150MHz luminosity are set to the arbitrary limit of, 10^{17}WHz^{-1} and similarly negative TIR luminosities are set to $10^6 L_\odot$. The samples are then summed in the stellar mass vs q_{TIR} plane to measure the combined PDF. Within each of our stellar mass bins we can then measure the median q_{TIR} from the combined PDF and compare it to the median q_{TIR} measured using survival analysis (Figure 9). The second method employed is stacking. Instead of measuring the upper limit of each undetected radio galaxies flux at 150MHz we measured the average flux at 150MHz for all galaxies within the same stellar mass bin. This was done for all galaxies regardless of whether they were detected or not as the vast majority of our sample are unresolved at 150MHz so the pixel flux is equal to the total flux. This was done by taking the mean of the fluxes in the pixels that the undetected galaxies (undetected at 150MHz) were located in (we tested using the median flux instead of the mean and found the answers to be within one standard deviation of each other). This mean flux was converted into a radio luminosity using the mean redshift of those galaxies. The mean q_{TIR} was similarly calculated using the mean TIR luminosity and the results can be seen in Figure 9. We note that our crude stacking method does not account for any bias caused by galaxy clustering. Therefore our stacked radio luminosities are likely higher than the truth, which would cause the median q_{TIR} to be lower than the truth and could explain why stacking produces a systematically lower q_{TIR} . The good agreement between all three methods give us further confidence that our method (survival analysis) is able to accurately measure the true evolution of our samples.

5.3. FIRC non linearity

There has been little research done into whether the FIRC is non linear at 150MHz. Wang et al. (2019) found a sub linear relation between radio luminosity for a sample of low redshift galaxies with high IR luminosities. To test whether the slope of the FIRC is 1, we binned our galaxies, from our redshift sample (as this has

Table 3. Fit parameters for different samples

Sample	Function	Assumptions	Power Law Params		Linear Fit Params	
			γ	N	β_1	β_0
$z < 0.4, M_* > 10^{10.05}$	$q(M_* - 10^{10.05})$	$\alpha_{150}^{325} = -0.60$	-0.16 ± 0.01	1.99 ± 0.01	-0.22 ± 0.01	2.00 ± 0.01
$z < 1.0, M_* > 10^{10.45}$	$q(z)$	$\alpha_{150}^{325} = -0.60$	-0.04 ± 0.01	1.94 ± 0.01	-0.06 ± 0.01	1.93 ± 0.01
Delvecchio 2020	$q(M_* - 10^{10})$	$\nu = 1.4GHz$			-0.12 ± 0.02	2.59 ± 0.01
Rivera 2017	$q(z)$		-0.22 ± 0.05	1.72 ± 0.04		

The sample column shows the sample considered and where it originates from. The Function column the variables which q_{TIR} is a function of. The Assumptions column lists any extra details the reader should know about the fit, such as the frequency of the observations and the spectral index used. The slope and normalisation of the power law fit are shown in the γ and the N columns respectively while the slope and intercept of the linear fit are shown in β_1 and β_0 .

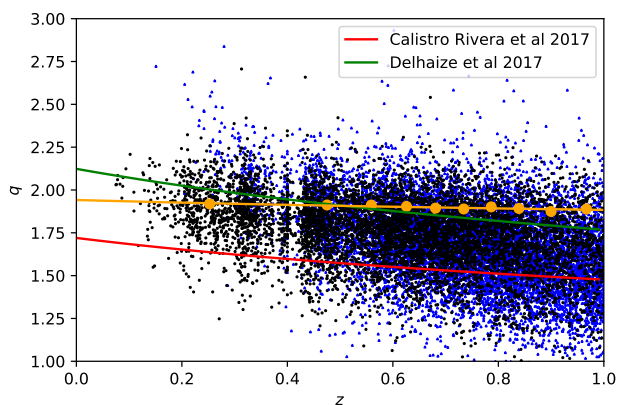


Fig. 8. The variation of the FIRC at 150MHz with redshift for the redshift sample. The black points are direct detections in the radio and the blue points are undetected in the radio and are therefore lower limits in q . The orange points are the median q_{TIR} , measured using survival analysis, in each redshift bin and the orange line is the power law fit to those points. The red line is the fit from Calistro Rivera et al. (2017) and the green line is the fit from Delhaize et al. (2017) converted to 150MHz with a spectral index of -0.78.

more galaxies in it) into ten dust luminosity bins, spaced so that there is an equal number of galaxies in each bin. In each bin we found the median q_{TIR} using the method detailed in 5.2 and converted that median q_{TIR} to a radio luminosity using equation 4 and taking L_{TIR} to be at the center of the bin. We then fit a linear relation to these data points of the form $L_{TIR} = \beta_1 L_{150MHz} + \beta_0$ and found $\beta_1 = 0.90 \pm 0.01$, $\beta_0 = -9.8 \pm 0.3$ (Figure 11). This shows a statistically significant deviation from a linear FIRC. However, due to the variation of q_{TIR} with mass it is difficult to disentangle the two relations and say whether the variation with mass causes the non linearity or vice versa, as our sample is not complete in L_{TIR} (we consider this in more detail later). Smith et al. (2021) found a non linear relation between SFR and L_{150} (equation 1 in their paper) with slope 0.945 ± 0.07 (when substituting SFR with L_{TIR} and converted to the same form as our relation). While the slopes are $> 5\sigma$ apart the difference can be attributed to the assumption that $L_{TIR} \propto SFR$. Any slight deviations from this assumption would cause a difference in the slopes between our work and that of Smith et al. (2021). In addition, we find a shallower slope than that observed by Wang et al. (2019) who found a slope of 0.77. However, their sample was constructed from IRAS detected galaxies across the hetdex region ($\sim 400\text{deg}^2$) that, by construction will be bright in the IR

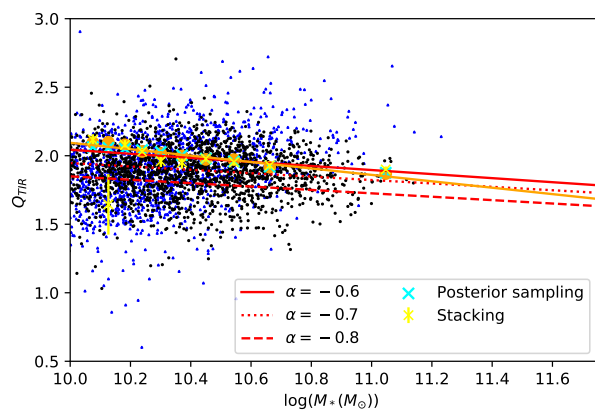


Fig. 9. The variation of the FIRC at 150MHz with stellar mass for our mass sample. The blue points are upper limits of radio luminosity (lower limit on q) and the black points are for direct detections. The orange points are the median q_{TIR} , measured using survival analysis, in each stellar mass bin and the orange line is the linear fit to those points. The light blue crosses are the median q_{TIR} within each bin measured using the method described in Smith et al. (2021) while the yellow points are the median q_{TIR} s measured with stacking. The red lines are the linear fit from equation 6 of Delvecchio et al. (2021), at the median redshift of our sample, converted to 150MHz with three spectral indices (see legend).

and have a median redshift of ~ 0.05 . It is hard to match their sample selection as we have few galaxies with that low a redshift due to the smaller area covered by the LoTSS observations in ELAIS-N1.

6. Discussion

Here we present our results in the context of past studies (section 6.1), how our sample selection and assumptions affect the observed variation of q_{TIR} with stellar mass (section 6.2), whether the FIRC is non linear at 150MHz is consistent with the main sequence (section 6.3) and causes for the dispersion around the FIRC (section 6.4).

6.1. Comparison with Past Measurements

Our results have a similar normalisation to those of Delhaize et al. (2017) (study done in COSMOS at 1.4GHz) but have a mild evolution compared to theirs. Our shallower evolution is likely caused by the observed evolution in stellar mass that re-

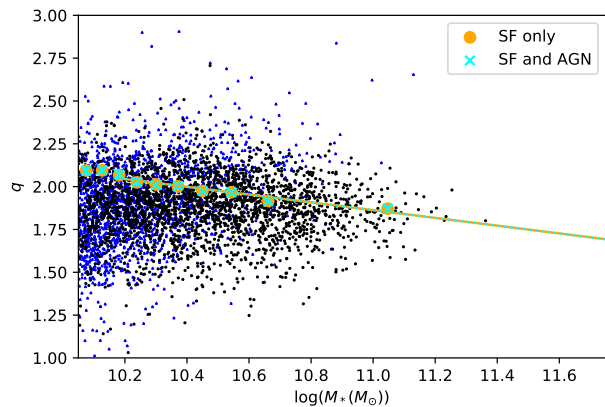


Fig. 10. The variation of the FIRC at 150MHz with stellar mass for our mass sample. The blue points are upper limits of radio luminosity (lower limit on q) and the black points are for direct detections. The orange points are the median q_{TIR} within each stellar mass bin, measured using survival analysis on only SF galaxies, and the orange line is linear fit to these medians. The light blue crosses and line are the same but when AGN are included in the sample. Note how the blue crosses overlap with the orange points

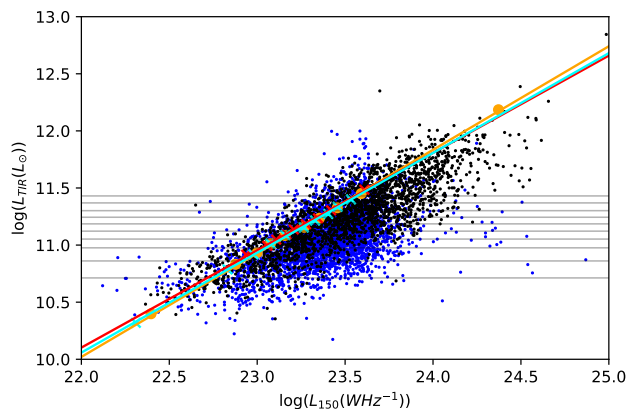


Fig. 11. Total infrared luminosity ($8 - 1000\mu\text{m}$), measured using MAGPHYS, vs radio luminosity at 150MHz for all galaxies. The horizontal lines show the bins that the galaxies were divided into such that each bin has an equal number of galaxies in it. The orange points are the median q_{TIR} , measured using survival analysis, in each L_{TIR} bin and the orange line is the linear fit to those points (note that the orange points are often behind the cyan and red crosses). Blue data points are upper limits on radio luminosity and black points are direct detections at 150MHz. The red line is the linear relation we would expect if our galaxies followed the main sequence measured by Schreiber et al. (2015) and the light blue line if the followed the main sequence measured by Leslie et al. (2020).

sults in galaxies with lower stellar masses having higher q_{TIR} . The sample used by Delhaize et al. (2017) didn't have a cut in stellar mass so they would expect to see higher q_{TIR} values at low redshift, where low mass galaxies are observable and conversely lower q_{TIR} at high redshift. This would explain their apparently stronger evolution with redshift. Similarly Calistro Rivera et al. (2017) did not use a mass complete sample. This similarly biased their results towards lower q_{TIR} values and explains the majority of the differences between our results and theirs. We can directly confirm this by modifying our sample to match the selections

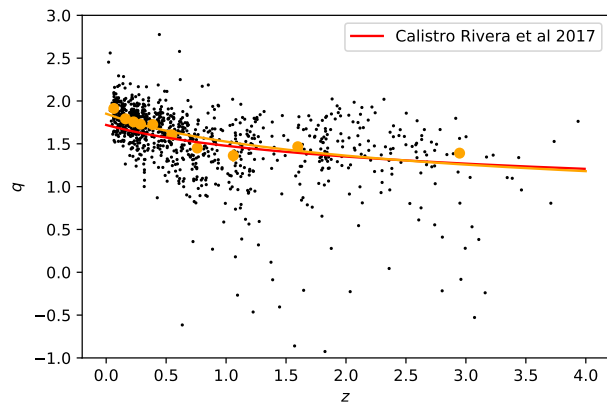


Fig. 12. Evolution of q_{TIR} with redshift only using radio sources with peak flux $> 0.6\text{mJy}$ to mirror the sample selection of Calistro Rivera et al. (2017). Note the improved agreement between our fit (orange line) and the fit from Calistro Rivera et al. (2017) (red line) compared to using a mass complete sample (Figure 8)

used in Calistro Rivera et al. (2017). To do so we only included galaxies with a peak flux at 150MHz $> 0.6\text{mJy}$ and then reran the analysis described in section 5.2, with $\alpha_{150}^{1400} = -0.63$, the results are shown in Figure 12. Using a radio selected sample brings us into much better agreement with Calistro Rivera et al. (2017). In conclusion the evolution in redshift observed in most other studies appears to be a result of their incomplete sample selection with respect to stellar mass.

This contrasts with the results of Delvecchio et al. (2021) (Figure 9), which used a mass complete sample measured at 3GHz and then corrected to 1.4GHz with $\alpha_{1.4}^3 = -0.75$, showing relatively little evolution with redshift but a clear dependency on stellar mass. They found good agreement between their sample and the results of Delhaize et al. (2017) when considering high redshift galaxies. These are dominated by high mass galaxies so the sample used in Delhaize et al. (2017) is close to the mass complete one used in Delvecchio et al. (2021). Meanwhile we found a steeper relation between q_{TIR} and stellar mass and a higher normalisation than Delvecchio et al. (2021). Our tests with the SIDES simulation showed that our use of survival analysis is able to accurately recover the true relation of our sample. So unless some of the underlying assumptions used in our work are incorrect (such as choice of spectral index, which are examined in the next section) then we are confident that the differences are caused by the different frequencies used in our works. We expect to have a different ratio of thermal to synchrotron emission at our respective frequencies and these differences could result in a different normalisation and slope.

Finally Figure 7 shows a reasonable agreement between the median q_{TIR} of our redshift sample and that of Molnár et al. (2021). Their sample was constructed from low redshift SDSS galaxies a 1.4GHz so some differences are expected. We converted their median to 150MHz using a spectral index of -0.73 and a higher spectral index would also explain the difference (see the next section for a more in depth discussion on the choice of spectral index).

6.2. Mass Evolution

Our results are based on several choices and assumptions made during our analysis and sample selection. Here we examine

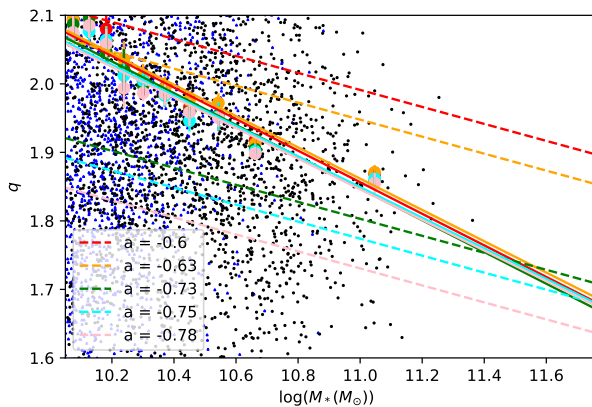


Fig. 13. The effect of different spectral indices on the linear fit of the FIR with stellar mass and how our fits compare to those of Delvecchio et al. (2021). The solid lines show our relation depending on the choice of spectral index (see legend) and the dashed line show the fit from Delvecchio et al. (2021) when converted to 150MHz with the same spectral index. The black points show galaxies that are detected at 150MHz and the blue points show galaxies that have upper limits on their flux at 150MHz.

how changing those assumptions affects our results focusing on: choice of spectral index, different FIR vs radio depths, removal of AGN and selection of SF galaxies.

We used the spectral index measured in Calistro Rivera et al. (2017) of $\alpha_{150}^{325} = -0.60 \pm 0.05$, the mean of a Gaussian fitted to the observed distribution. Alternatively we could have taken the mean of the distribution itself (-0.63), however Figure 13 shows that the choice of spectral index make little difference to the normalisation of our measured relation and no difference to the slope (solid lines). This is due to the small redshift range that our stellar mass sample was constructed in ($z < 0.4$) so any correction to the rest frame frequency will be minor even at $z = 0.4$. However, when comparing to Delvecchio et al. (2021) the choice of α_{150}^{1400} has a big impact on the difference in normalisation between our results. We used $\alpha_{150}^{1400} = -0.78$ as it is the mean of the α_{150}^{1400} distribution measured in Calistro Rivera et al. (2017). They also fit a Gaussian to the spectral index distribution with a mean of -0.73. Regardless, even if we use the higher spectral index we still find that their $q_{1.4} = 2.53$ becomes $q_{150} = 1.82$ which is still 0.2 dex below our results and can only explain 0.07 dex of the difference. Alternatively Calistro Rivera et al. (2017) show α_{150}^{1400} as a function of redshift and find a large variation at low redshift, with results varying between -0.85 and -0.59 for $z < 0.5$. Taking the highest spectral index would put our results and those of Delvecchio et al. (2021) in good agreement (shown in Figure 9). In addition, we investigated whether the different choices of spectral index used to convert to rest frame frequencies of 150MHz and 1.4GHz respectively could account for the difference. We converted their relation back to the observed frequency of 3GHz using their assumed spectral index of -0.75 and then re-converted to 1.4GHz. The subsequent conversion from 1.4GHz to 150MHz and converting our results from the observed frame to 150MHz were all done with the same spectral index and the results can be seen in Figure 13. This shows that the choice of spectral index only has a minor effect on the normalisation of our fit, which is to be expected due to the low redshift of our stellar mass sample. However, if the spectral index is closer to -0.6 then our normalisation would be in better agreement with Delvecchio

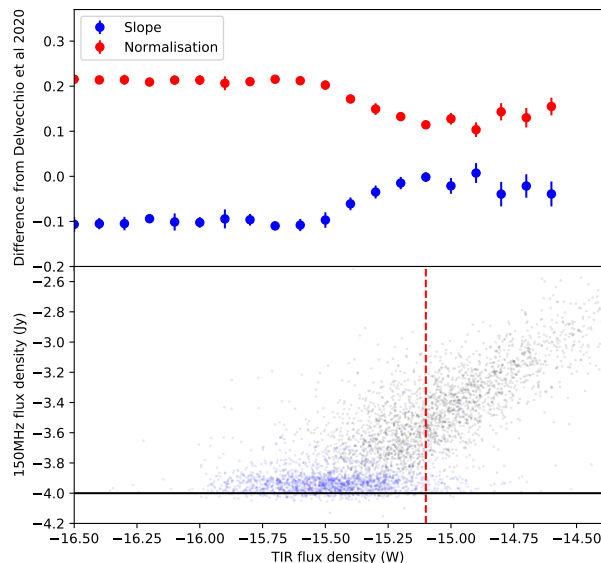


Fig. 14. Top: Difference of linear fit parameters compared to Delvecchio et al. (2021) depending on the limit applied to the TIR flux. The red and blue points show the normalisation and slope of the linear fit respectively for all galaxies with a TIR flux density to the right of the point. It can be seen that imposing increasingly strict cuts on the TIR flux density does not cause any significant bias in our results and does not explain the difference between our fit and that of Delvecchio et al. (2021) until the TIR flux cut starts to significantly cut into our radio detected galaxies. Bottom: TIR flux density, calculated by dividing the TIR luminosity by $4\pi D_L^2$, against flux density at 150MHz. The black points are galaxies that are detected at 150MHz and the blue points are galaxies with an upper limit at 150MHz. It can be seen that the limit at 150MHz ($\sim 100\mu\text{Jy}$) has a greater effect on our sample than the limits of the TIR flux.

et al. (2021). Overall we are confident that varying α_{150}^{325} , within reasonable values, has little to no impact on our measured relations but the choice of α_{150}^{1400} does greatly affect comparisons with work done at higher frequencies.

Next we consider the difference in FIR vs radio depths. Our sample is created from an IRAC selected sample and then limited to a mass complete sample of star-forming galaxies. Since the measurements of IR luminosity are done using MAGPHYS, using energy balance, there are many galaxies with TIR luminosity measurement that are not detected in any individual FIR bands. This makes our sample deeper in the IR than the radio, in contrast to the sample used by Delvecchio et al. (2021) whose radio data was deeper than their IR data. We investigated whether this difference in depths could explain the measured discrepancy between our results. This was done by imposing a cut on the IR flux (calculated dividing the IR luminosity by $4\pi d_L^2$) and then increasing this cut over a range of 2.0 dex. The greatest change in normalisation was ~ 0.1 dex obtained by imposing a harsh cut in our sample (Figure 14) that has no physical motivation. In addition such a harsh cut in TIR flux would likely be introducing significant bias towards lower q_{TIR} values. Finally it is worth emphasising that the maximum effect this can have on the normalisation of our linear fit is to decrease it by ~ 0.1 dex which still leaves us with a significant disagreement with Delvecchio et al. (2021), when taking the nominal value of $\alpha_{150}^{1400} = 0.73$.

We consider the impact that removing AGN has on our results. We remeasured the evolution of q_{TIR} with stellar mass with AGN host galaxies included in the sample (all AGN were re-added to the sample) to see how they affect our results (Figure 10). We can see that the inclusion of AGN does not significantly affect our results. This is due to the fact that we are fitting to the median q_{TIR} within each redshift or stellar mass bin so we aren't as affected by outliers as well as the relatively low fraction of AGN within our sample ($< 10\%$). The galaxies with the lowest q_{TIR} values are potentially unidentified radio loud AGN or galaxies with a low dust mass (and therefore the TIR luminosity does not map the full SFR).

Finally we investigated the impact of how we remove passive galaxies from our sample. Both our mass sample and redshift sample define a passive galaxy as one that is more than 0.6 dex below the main sequence. This offset below the main sequence was chosen to match $\approx 2\sigma$ where σ is the dispersion in the main sequence. However, choosing to set the offset between 0.3 to 0.9 dex we be equally justified. We investigated the effect that changing this offset has on our results and found that the slope and normalisation of the linear fit to $q_{TIR}(M_*)$ varies by 0.02 and 0.01 respectively. The change in normalisation is the same as our uncertainty in the normalisation and the variation in slope is only marginally greater than our uncertainty. While varying our SF galaxy criteria does have a minor impact on the measured relation, the impact is similar to the random error of our measured distributions.

6.3. Main Sequence Prediction of FIRC

The star formation rate of a galaxy is known to be correlated with its mass and this main sequence has been observed many times (Schreiber et al. 2015; Leslie et al. 2020). For a given galaxy in our redshift sample we know its radio luminosity (or upper limit), mass and redshift, so we are able to calculate its predicted SFR, using the MS relations measured in Schreiber et al. (2015) and Leslie et al. (2020) and therefore its TIR luminosity, using the relation from Kennicutt, Jr. (1998). We can then bin our galaxies on their predicted TIR luminosity so that there is an equal number of galaxies in each bin. In each bin we can compute the median q_{TIR} using survival analysis and see how we would expect the slope of the FIRC to vary based purely on the main sequences prediction. We can see from Figure 11 that the predicted non-linearity is in agreement with our observed non-linearity. This could imply that the physics causing the observed non-linearity is the same that causes the main sequence. This in turn could help calibrate the SFR - radio luminosity relation. However, if the relation between SFR and stellar mass, which in turn causes a change in TIR luminosity, results in a change in the relation between radio and TIR luminosity why would the radio luminosity not be similarly affected. Resolving such issues is beyond the scope of this work. However, we hope to address such topics in the near future.

6.4. FIRC dispersion

A little-explored property of the FIRC is the cause for the dispersion. While the FIRC is surprisingly linear, with the standard deviation being 0.2, q_{TIR} values still span a range of 1.5 dex, when just considering the detected galaxies. Understanding what affects a galaxy's deviation from the median q_{TIR} can provide some insight into the underlying physics behind the FIRC. For high mass galaxies stellar mass does affect the median q_{TIR} , and

it could also affect the dispersion. These galaxies are most likely to have low level AGN emission (Sabater et al. 2020) and large low mass stellar population. Either of these could boost the radio or FIR emission of the galaxy through mechanisms that are unrelated to star formation. Other parameters that could affect where a galaxy lies on the FIRC are SFR and sSFR, as the mechanisms governing the FIRC may begin breaking down for starburst galaxies (though this was not observed by Magnelli et al. (2015) or predicted by Lacki et al. (2010)). Finally, the steepness of the radio spectra is indicative of how recent the SF in a galaxy has occurred and may cause a galaxy to lie off the FIRC as the FIR and radio may be less sensitive to SF that has occurred more or less recently.

In order to investigate for this effect, we also included proxies for the angular and physical size of the galaxy. These were taken as the ratio of the total radio flux divided by the peak radio flux and as the difference in magnitudes in the r band between 1-2, 1-3 and 1-6". These proxies for angular size were converted to a measure of physical size by multiplying them by the distance to the galaxy. Finally a proxy for gas surface density was created by dividing SFR by physical size as this is one of the key parameters used in Lacki et al. (2010). All of these parameters were fed into a neural network to predict the difference between a galaxy's q_{TIR} value and the median q_{TIR} (Δq). We found that the neural network had no significant ability to predict Δq when compared to the intrinsic scatter of the Δq distribution.

The lack of predictive power of the neural network could be caused by several factors. Firstly the parameters that govern the radio and FIR emission, such as magnetic field strength, are difficult to directly measure. Without a way to proxy these measurements it is difficult to constrain exactly the q_{TIR} value that a given galaxy should have. In addition, the measurements of SFR from MAGPHYS measure the combined SFR detected in the UV and optical and from the FIR. If the measured SFR is imperfect, due to the dust geometry of the galaxy, then the galaxy may lie off the FIRC.

Further work is required to explain the scatter in the FIRC. We are unable to accurately assess the error in L_{TIR} caused by unusual dust geometry. In addition, we have no measure of the radio spectral index, which may explain a portion of the scatter.

If some of the SFR is unobserved, either due to lower than expected dust mass or the geometry of the galaxy, then the galaxy may not lie on the FIRC and we would be unable to measure this from the features we provided to the neural network.

7. Conclusions

In this work we measured FIR fluxes for all radio sources in the LoTSS deep fields. These fluxes are then used to derive the FIRC at 150MHz using data from LoTSS (Shimwell et al. 2019) and *Herschel* (Oliver et al. 2012) used in combination with a mass complete sample (Kondapally et al. 2021; Duncan et al. 2021) of star-forming galaxies to measure the FIRC. The total infrared luminosity is measured using MAGPHYS (Da Cunha et al. 2008), which uses energy balance to infer the IR luminosity when FIR fluxes aren't measured, enabling IR luminosity measurements for all galaxies in our sample. To account for non-detections in the radio and FIR we use survival analysis and upper limits on the radio flux for galaxies that are undetected at 150MHz. We investigate the evolution of q_{TIR} with redshift and stellar mass and compare to the recent results from Delvecchio et al. (2021) using the JVLA. Our main results are listed below.

1. We measure fluxes at 24, 100, 160, 250, 350 and 500 μ m for 79609 galaxies across the LoTSS deep fields. In addition, we provide flags to identify sources whose flux is unreliable. The flags are based on the goodness of fit of the flux posterior to the map and whether it is dominated by the prior. The fluxes are available in the LOFAR deep fields data release (column descriptions can be found at https://github.com/H-E-L-P/dmu_products/tree/master/dmu26).
2. We validate our methods by creating a simple model based on galaxies created as part of the SIDES simulation (B ethermin et al. 2017). Radio luminosities were added to star-forming galaxies by generating q_{TIR} values from a Gaussian distribution, whose mean evolved as a power law, that mimicked the observed distribution from our mass complete sample. We determined whether a galaxy would be detected in the radio or not. We found that we were able to recover the true distribution parameters (the true mean and power law parameters) from the simple model when we had more detected galaxies than undetected galaxies (at 150MHz).
3. The FIRC was measured at 150MHz with a mass complete sample of star-forming galaxies. We also examined the evolution of q_{TIR} with redshift and stellar mass, finding a power law relation for both of the form $q(z) = 1.94 \pm 0.01(1+z)^{-0.04 \pm 0.01}$. Similarly the evolution with stellar mass was found to have a linear relation of the form $q(M_*) = (2.00 \pm 0.01) + (-0.22 \pm 0.02)(\log(M_*) - 10.05)$.
4. Comparing with previous results we found a significant difference between our evolution with stellar mass and that of Delvecchio et al. (2021). Neither our different treatment of AGN nor the different radio vs FIR depths can explain the difference. However, our choice of spectral index from Calistro Rivera et al. (2017) could affect our result as we used the median $\alpha_{150}^{1400} = -0.78$ instead of $\alpha_{150}^{1400} = -0.59$ which was observed at low redshift. Using a higher spectral index would explain the discrepancy between our results, but more work is needed to better understand the radio spectra before we can say this conclusively. We also observe significant difference between our redshift evolution and that of Calistro Rivera et al. (2017). However, this can be explained by our different sample selections.
5. We investigate the non-linearity of the FIRC at 150MHz and found that $L_{TIR}(M_*) = 0.90 \pm 0.01 L_{150} - 9.8 \pm 0.3$. This relation is in good agreement with that found by Smith et al. (2021) in the SFR - radio luminosity plane and Moln ar et al. (2021) at 1.4GHz. In addition, the non linearity observed agrees with predictions from the main sequence (Schreiber et al. 2015; Leslie et al. 2020) future work is needed to explore whether both relations originate from the same physical processes.

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Appendix A: Energy Balance Assumption

Using the SED fits from Smith et al. (2021) we are relying on the energy balance approach used by *magphys* when there are no PACS or SPIRE fluxes to help constrain the FIR. Firstly, Figure A.1 shows that in no part of parameter space are we dominated by galaxies that are dependant on the energy balance, so the affect of any systematic bias, if any exist, would be minimal. As a second check we can compare the νL_ν at $24\mu\text{m}$ with the SFR measured from *magphys*. νL_ν at $24\mu\text{m}$ is a tracer of SF Rieke et al. (2004) so by comparing their ratio for different sub samples we can see whether there are any significant deviations in this ratio between the subsamples. To calculate νL_ν at $24\mu\text{m}$ we k corrected the measured L_ν using the red galaxy template from Berta et al. (2013). Figure A.2 shows the comparison between galaxies with one or less detection from PACS or SPIRE compared to galaxies with more than one detection. We can see that the peak of both distributions are within $\approx 0.2dex$ of each other, lending further evidence that relying on the energy balance from *magphys* does not cause a bias in measurements of SFR (and hence TIR luminosity as the two are tightly correlated). The small offset is likely causes by faulty choice of SED template for the k correction. The subsample with few FIR detections are more likely to be fainter galaxies or at higher redshift and a different template may be needed to accurately measure their νL_ν at $24\mu\text{m}$.

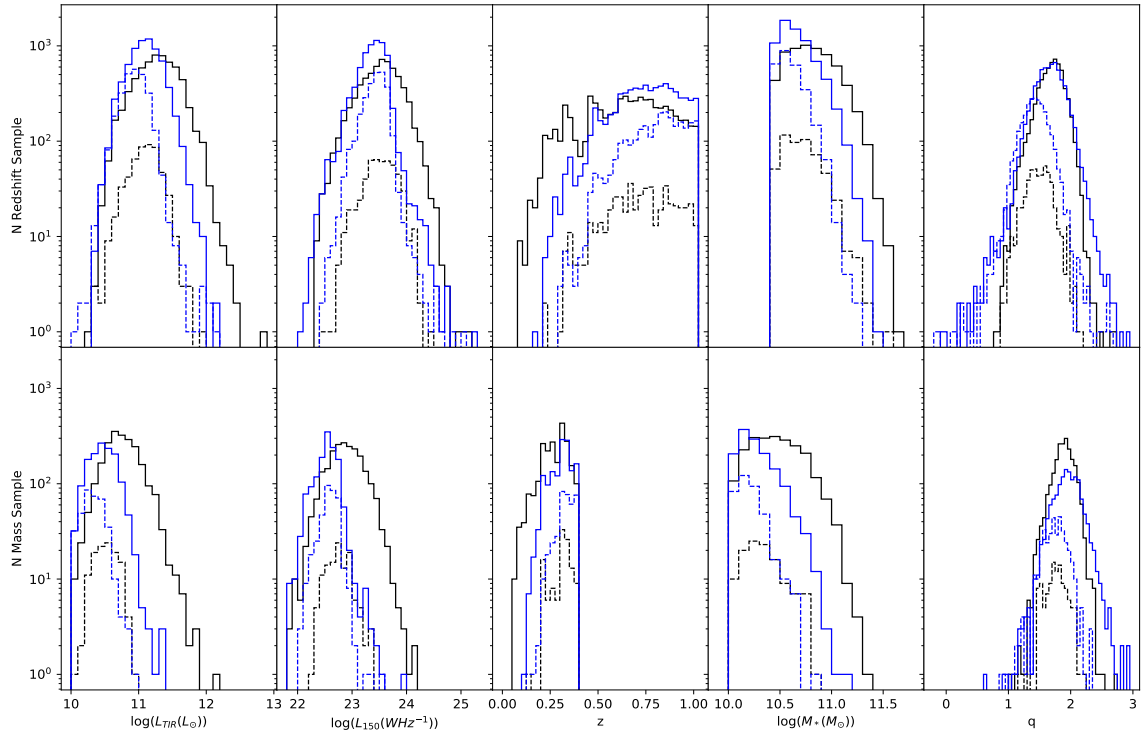


Fig. A.1. How galaxies are distributed, split by their observed fluxes and which sample they are in. The black line shows how the galaxies that are detected at 150MHz are distributed and the blue line is for galaxies undetected at 150MHz. If the line is dotted then it shows galaxies that have one unflagged (see section 2.3) flux from PACS or SPIRE and if solid then it has more than one unflagged flux from PACS or SPIRE. We can see that in no part of parameter space are we dominated by galaxies that have one or less unflagged flux in PACS or SPIRE.

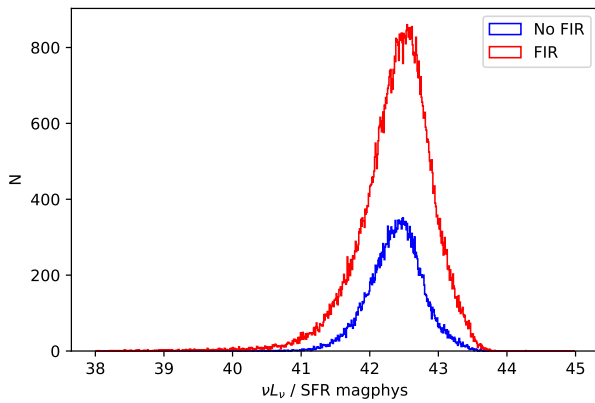


Fig. A.2. Comparison of $\nu L_\nu / \text{SFR}$ measured from *magphys* for the $3.6\mu\text{m}$ selected sample in ELAIS-N1. The blue histogram shows the results for galaxies with one or less unflagged flux from PACS or SPIRE while the red histogram shows the results for galaxies with more than one unflagged flux from PACS or SPIRE.