Procurement Mechanisms with Post-Auction Pre-Award Cost-reduction Investigations

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A buyer seeking to outsource production may be able to find ways to reduce a potential supplier's cost, e.g., by suggesting improvements to the supplier's proposed production methods. We study how a buyer could use such "cost-reduction investigations" by proposing a three-step supplier selection mechanism: First, each of several potential suppliers submits a price bid for a contract. Second, for each potential supplier, the buyer can exert an effort to see if she can identify how the supplier could reduce his cost to perform the contract; the understanding is that if savings are found, they are passed on to the buyer if the supplier is awarded the contract. Third, the buyer awards the contract to whichever supplier has the lowest updated bid (the supplier's initial bid price minus any cost-reduction the buyer was able to identify for that supplier). For this proposed process, we characterize how the buyer's decision on which suppliers to investigate cost reductions for in step 2 is affected by the aggressiveness of the suppliers' bids in step 1. We show that even if the buyer does not share the cost savings she identifies in step 2, *ex ante* symmetric suppliers are actually better off (*ex ante*) in our proposed mechanism than in a setting without such cost-reduction investigations, resulting in a win-win for the buyer and suppliers. When suppliers' cost and cost-reduction distributions become very heterogeneous, the win-win situation may no longer hold, but every supplier still has an incentive to allow the buyer to investigate him in step 2 because it increases his chance of winning the contract. Using an optimal mechanism analysis, our numerical studies show that our proposed Bid-Investigate-Award mechanism helps the buyer achieve near-optimal performance, despite its simplicity.

Key words: procurement, cost-reduction investigation, win-win, mechanism design, first-price sealed-bid auction, optimal mechanism

History: .

1. Introduction

In 2019, U.S. manufacturers spent about 55% of their revenues on procuring components and parts from their suppliers (U.S. Department of Commerce 2021). To drive down procurement costs, manufacturers (buyers) have increasingly relied on *electronic reverse auctions* in their supplier selection process (CAPS 2009). The intuition is simple: price competition among suppliers can lower the buyer's procurement cost. In practice, however, buyers employ many other approaches to further reduce the procurement costs. One common approach during the life of the contract is to conduct *cost-reduction investigations* on the suppliers (e.g., inspecting the suppliers' facilities, investigating the suppliers' production processes, etc.) to find cost-saving opportunities. The idea is that by investigating the suppliers' facilities and production processes, the buyer can help the suppliers identify cost-saving opportunities to reduce production cost; the buyer also benefits from this because she can then expect a reduction in her payment to the supplier.

Cost-reduction investigations are common in manufacturing industries. For example, with its superior experience in manufacturing, Toyota has been able to identify "waste" in its suppliers' production processes and help improve their efficiencies by regularly visiting suppliers' plants and investigating suppliers' production processes. The goal of Toyota's partnering with its suppliers is not to squeeze their profits but to make them more efficient and keep them as profitable or more profitable than before (Kalkoffen et al. 2007). Ford also has similar programs where supplier development engineers work with the suppliers to improve their efficiency. The important point is that the buyer is providing its expertise to the supplier, rather than vice-versa.[1] We have recently worked with a Fortune 500 Tier-1 auto supplier who designs and manufactures subsystems for an automaker and bids out long-term contracts to its Tier-2 suppliers for components. This company has cost-reduction programs where its *cost reduction manager* leads a team of technicians that works closely with one or more of the company's suppliers. Typically the cost reduction manager has a background in engineering and is able to identify significant cost-reduction opportunities for suppliers by examining the suppliers' production proposals and by visiting the suppliers' plants and collaborating with the suppliers' engineers. For example, for one of the subassemblies it manufactures, the company we worked with outsourced to a Tier-2 supplier the production and the assembly of two parts that needed to be securely fastened. The supplier initially used a crimping process, but this required exerting tremendous force on the parts and could cause the parts to crack, so some very expensive material was blended in to increase the strength of the parts. However, this approach was much more expensive than the state-of-art practice in industry: By dropping the temperature during assembly using liquid nitrogen, the parts will shrink, making it much easier to assemble via press-fit, and will be tightly fastened after the temperature goes back to normal. This change in production process identified millions of dollars of savings for the company by removing the whole crimping process and the need for the expensive material over the life of a multi-year contract.

[1] Indeed, protection against a buyer learning and misappropriating a supplier's production practices that are not generally known outside the supplier is provided by the Uniform Trade Secrets Act of 1979 and the Defend Trade Secrets Act of 2016 in Title 18 PART I Chapter 90 of United States Code (http://uscode.house.gov/). Vice versa, such protection also applies to the buyer's trade secrets.

We now explain how cost-reduction investigations and supplier selection can interface with each other. The simplest approach can be described as Bid-Award-Investigate (BAI). That is, the suppliers bid, the winning bid wins the contract, and then throughout the life of the contract the buyer conducts cost-reduction investigations. Intuitively, some cost reductions (e.g., optimizing maintenance schedules based on historical performance for the machines producing particular part) can only be done after production has been running awhile. But other types of savings could be identified prior to awarding the contract. For instance, in the example above, if the buyer investigated the supplier's proposed production process, the buyer would have discovered that the supplier planned to use a crimp fitting instead of shrink-to-fit approach, and significant savings could be found at the outset of the contract. This paper explores, for the first time in the auctions/OM literature, the possibility that the buyer weaves such cost reductions into its supplier selection process. More specifically, we model and analyze inserting cost-reduction investigations *after* bidding and *before* awarding the contract[2]. More concretely, we consider the following three-step Bid-Investigate-Award (BIA) supplier selection process: In the first step Bid, each supplier simultaneously submits a bid to the buyer; we call these *initial* bids. In the second step Investigate, the buyer determines which suppliers to investigate for cost-reduction opportunities via an *investigation policy* (see §3 for more details of the investigation policy), and the supplier-specific cost savings identified in these investigations will be used to adjust respective suppliers' initial bids in the following way: An investigated supplier's *updated bid* equals his initial bid minus the cost reduction identified by the buyer for him; an un-investigated supplier's *updated bid* equals his initial bid. In the last step Award, the supplier with the lowest updated bid wins the contract and gets paid his updated bid.

The BIA supplier selection process has attractive features. By conducting investigations *before* awarding the contract, the buyer is selecting suppliers not just on their initial pricing, but also identified cost savings which further reduce the price the buyer would pay. Furthermore, conducting investigations *after* bidding is a practical way to foster supplier participation. The buyer is not attempting to understand a supplier's cost prior to the supplier bidding, which presumably would make the supplier very nervous since he would be worried that the buyer would utilize this cost information to squeeze the supplier's profits, e.g., by setting the maximum bid she is willing to accept just above the supplier's cost. In our setting, the buyer is not trying to reduce the supplier's profits, but rather to take waste out of the system without reducing the supplier's profits. In fact, finding cost reductions with a supplier makes that supplier's updated bid more competitive which

[2] Note that it is actually quite common for buyers to investigate suppliers' plants *before* the contract is awarded. For example, the common procedure of supplier qualification screening (Wan and Beil 2009) in the auto industry usually involves investigating suppliers' plants before awarding the contract in order to make sure that the supplier has the manufacturing capabilities and capacity to perform the contract. The level of engagement from suppliers in cost-reduction investigations we study in this paper is not too different from supplier qualification screenings.

increases the chance that supplier wins the contract, and the contract payment specified in the Award step ensures that the buyer preserves the supplier's profit margin. Thus a supplier stands to benefit from letting the buyer into its facility to identify cost savings in the Investigate step.

While the potential benefit of cost-reduction investigations is clear, it does not come for free; in fact, to help a supplier identify how to reduce production cost, the buyer has to spend effort and resources to, for example, visit the supplier's facilities, figure out potential production process reconfigurations, and validate their feasibility. These activities result in the buyer's travel costs, personnel costs, and the opportunity cost of time as cost-reduction investigations may delay the start of production. Moreover, due to limited supply of certain resources that are critical to the execution of cost-reduction investigations (e.g., the buyer's specialized engineers), the buyer may not even be able to investigate all of the suppliers, even if she wanted to. Finally, the buyer is not *guaranteed* that these investigations will *unequivocally* bring about cost-reductions. The cost reduction manager might find that they simply cannot identify any feasible methods to reduce the supplier's overall production cost.

The above operational frictions of weaving cost-reduction investigations in supplier selection (i.e. BIA) naturally lead to our first research question: how should the buyer manage BIA's Investigate step – i.e., which supplier(s) should be investigated and in what sequence? Meanwhile, it is also important to realize that in BIA the suppliers will bid during the Bid step anticipating the buyer's use of cost-reduction investigations during the Investigate step. This leads to our second research question: in BIA, how would suppliers' strategic bidding behaviors affect the buyer's procurement cost? To address these questions, we analytically characterize the buyer's investigation policy and the suppliers' equilibrium bidding strategies in BIA. Intuitively, having BIA in addition to the even simpler BAI provides the buyer another option, and can only help the buyer improve her profits. Yet, one may wonder if these two simple mechanisms[3] are sufficient for the buyer, or if the buyer could do better by combining cost-reduction investigations in more elaborate ways into the award step. This motivates us to ask how much additional cost savings a buyer could achieve by employing a more sophisticated mechanism. For example, what if the buyer allowed for two rounds of bidding, one *before* and one *after* the Investigate step? We conduct an optimal mechanism analysis to evaluate the effectiveness of our simple approach, which is to simply use BIA or BAI – whichever is cheaper, and in our numerical study we see that our simple approach performs strikingly well despite its simplicity.

[3] Note that BIA is very simple and straightforward to implement in practice as it combines, in a novel way, common cost-reduction investigation practices with an auction format (i.e., first-price sealed-bid) that is very familiar to practitioners.

While the buyer determines the format of the procurement and supplier selection process, it is helpful for the buyer to consider the broader implication of switching from BAI to our proposed simple approach. Specifically, it is helpful to understand the impact on suppliers' expected profits, because this affects the extent to which such mechanisms could be successful in practice. We find that when suppliers are ex ante symmetric, they make more expected profits under BIA than BAI. Thus, including BIA as an option in the buyer's sourcing toolkit results in a win-win situation for the suppliers and the buyer. As the *ex ante* asymmetry among the suppliers becomes very large, we show that some suppliers may become worse off under BIA than BAI; intuitively, it is helpful for the buyer to be aware of such challenges when implementing BIA so that she can manage potential supplier concerns accordingly. However, we also show that, although win-win may break down, the supplier still has incentive to let the buyer investigate him, since it increases his chance of winning the contract. This suggests that the buyer can still roll out BIA even in situations where win-win breaks down.

Before moving on, we would like to point out that, in order to highlight the idea that a careful choice of the timing of cost-reduction investigations can be beneficial to the buyer, we have chosen to focus on a model where cost-reduction efforts can be *fully* conducted before awarding the contract. However, in practice, not *all* cost-reduction activities can be conducted before contract award. Thus, it is worth mentioning that the cost-reduction activities that we focused on are not the improvement of the execution of the production plan, but more on the improvement of the plan itself such as our motivating example whereby the buyer suggested the supplier to replace a crimping process with press-fit with liquid nitrogen cooling, and other workflow control adjustment (e.g., replacing make-to-stock with make-to-order to reduce inventory cost and production lead time between stages). In contrast, for long-term contracts, many buyers actually have long-term continuous cost-reduction programs with their suppliers to improve efficiency, because certain process improvements may be hard to identify unless the supplier has run the production for a while. For example, suppose there is a labor-intensive step of the proposed production process from the supplier. When the production starts, the supplier may find out that the particular work task causes significant fatigue on factory workers and results in a lot of absenteeism and capacity loss, which cannot be foreseen at the time when the supplier maps out the production plan. However, once such inefficiency is identified after the supplier has started executing the production plan, it can potentially be alleviated by improving industrial ergonomics through workstation equipment improvement and re-design of the work process itself. Thus, such improvement of the execution of the production plan is not the type of cost-reduction activities which can be conducted before contract award. Hence, we would like to clarify that, in those settings in practice, our simple approach should also be integrated with buyer's continuing long-term cost-reduction efforts with the supplier who wins the contract.

2. Literature Review

Auctions have been thoroughly studied in the economics literature for decades (Krishna (2009) summarizes the classic results in this vast literature). More recently, a growing body of literature in operations management studies the applications of auctions in sourcing. Motivated by operational considerations faced by supply chain practitioners (Rothkopf and Whinston 2007, Elmaghraby 2007), this stream of work features a variety of strategic issues when running procurement auctions that go beyond simply collecting bids and transacting. For example, prior work has investigated how multi-attribute supplier selection criteria should be incorporated to optimize supplier selection via auctions (Che 1993, Chen et al. 2008, Kostamis et al. 2009, Santamaria 2015). More broadly, there is a growing line of work focusing on structuring and optimizing the procurement process itself. For example, in an assembly context where a buyer needs to *contract with multiple suppliers*, Jiang (2015), Hu and Qi (2018), and Davis et al. (2021) investigate the procurement *process design* of the timing and sequence the buyer should follow to reach out to suppliers for contracting. In a dual sourcing setting, Chu et al. (2020) investigates how the strategic addition of a *request for proposal/quotation* before contract negotiations can help reduce the buyer's purchasing cost even in situations where there is no information asymmetry about suppliers' costs. Several papers (Wan and Beil 2009, Wan et al. 2012, Chen et al. 2018, Zhang et al. 2021) study how to incorporate *supplier qualification screenings*, a key activity in procurement in the manufacturing industries, into the procurement process to reduce the buyer's overall cost. Beil et al. (2018) reveals how *new supplier recruitment* activities could be effectively incorporated into the procurement process via a test auction approach. Gal-Or et al. (2007) investigates the buyer's effort in searching for new suppliers which fit with the buyer's procurement requirement and compares sequential and parallel search processes. Building on this line of work, our paper is the first to study procurement process optimization for cost-reduction investigation activities, and its welfare implications on the buyer and suppliers.

Our paper is related to the literature on cost-reduction investments/activities in supply chains. Most work in this literature focuses on cost-reduction activities initiated by the suppliers, rather than by the buyer. A handful of papers study cost-reduction investments made by suppliers *before* competing for contracts. For example, Tan (1992) and Piccione and Tan (1996) consider a R&D setting and investigate how the nature of the suppliers' R&D investment affects suppliers' investment incentives and the contract competition that follows. Bag (1997) and Arozamena and Cantillon (2004) focus on how the suppliers' cost-reduction investments affect the asymmetry of the suppliers and the impact on the competitive landscape of the contract competition. Li and Wan (2017) investigate how various information structures affect the interplay between competition and suppliers' cost improvement efforts and the implications on supply base design. Li (2020) studies, in a supplier selection process that is subject to the hold-up problem, the buyer's procurement auction and supply base design in the face of the endogenous information asymmetry generated by suppliers' randomized equilibrium cost-reduction strategies. In contrast, other papers have studied the cost-reduction efforts made by the suppliers after being awarded the contract. McAfee and McMillan (1986) use the principal-agent framework to study the issues related to contracting with moral hazard risks from the suppliers. Bernstein and Kok (2009) study, in an assembly network, the dynamics of suppliers' cost-reduction efforts over the life cycle of a product under different contractual arrangements. Krahmer and Strausz (2011) study how a principal should contract out a project to an agent, who not only has a more accurate estimate of project cost a priori due to his professional expertise but can also exert unobservable costly pre-project investigations to further uncover the cost of the project. In all these papers, the buyer does not directly engage in cost-reduction investments, whereas in our paper, the cost-reduction investigation decision can be viewed as an endogenous investment by the buyer.

To the best of our knowledge, there is very limited work that studies the cost-reduction efforts exerted by the buyer. Iyer et al. (2005) study joint process improvement in a buyer-supplier partnership where the buyer and the supplier both decide how much effort to put into the development of a production process. However, in their paper, the cost-reduction efforts are made *after* supplier selection (the supplier selection step is assumed to have already occurred) whereas our paper focuses on cost-reduction investigations *before* supplier selection. Cantillon (2008) develops a framework to analyze the impact of *ex ante* asymmetry of bidders on the revenue the auctioneer receives in a first-price sealed-bid auction and, as an application, investigates how a buyer should allocate a fixed budget to sponsor suppliers' cost-reduction investments *before* a reverse auction. In contrast, we study a very different setting where the buyer gets suppliers' initial price bids first and then decides which suppliers to investigate for cost improvement. Jin et al. (2019) study the interplay between manufacturers' decisions on supplier development and supplier integration in the face of competition for the end consumer market. In contrast to our model, they simplify the process of contract allocation and contract price discovery by assuming that suppliers' production costs are public information, and focus on the implications of buyer's supplier development effort on more strategic supply chain integration decisions.

We would like to also point out that the buyer's investigation problem, a sub-problem in our paper that we tackle to analyze the BIA approach, is closely related to the classical optimal sequential search problem studied in Weitzman (1979) where a decision maker (DM) is faced with a collection of n alternatives each of which can be explored at most once at a cost and yields a random reward; the DM needs to find the optimal exploration policy to maximize her terminal surplus defined as the maximum reward of the alternatives she explored minus the cumulative cost of exploration.[4] Weitzman (1979) establishes an elegant result that, in spite of the complex dynamics in this problem, a simple index rule (i.e., "Pandora's rule" in his terminology) is optimal if there is no limit on how many alternatives the DM can explore. In practice, however, the DM may have other constraints which disallow her to explore more than $m < n$ alternatives (this happens in our setting when critical resources limit the maximum number of suppliers the buyer can investigate for cost reductions); in this situation, the optimal policy may no longer possess the simple index structure and can be computationally extremely challenging, as implied by the following quote from Weitzman (1979) Page 650: *"In the general case $n > m \geq 2$, an involved permutational exercise would be required to determine which m [alternatives] should be potentially [explored]"*.[5] In our paper, using a novel conditional sample path based induction argument, we show that under the rather mild assumption that the distributions of the alternatives' rewards have the same shape, even though the decision maker is not allowed to explore all of the alternatives, Weitzman's elegant result can nonetheless be established: A simple index rule with an intuitive stopping criterion is optimal.

Finally, the optimal mechanism analysis we conduct in our paper is related to the broad literature on mechanism design. In our setting, as the buyer proceeds (dynamically) with each investigation in the Investigate step, the cost-reduction information of the investigated supplier gets revealed to both the investigated supplier and the buyer. After the buyer commits to a mechanism, (some of) the suppliers as well as the buyer receive additional payoff-relevant information (e.g., cost-reductions of the investigated suppliers), and the *optimal mechanism* could *hypothetically* be one in which the buyer discloses information to the suppliers and the suppliers update their bids over time. We utilize a dynamic version of the Revelation Principle which has been developed in Myerson (1986) and later refined and formalized in Sugaya and Wolitzky (2021) to analyze our mechanism design problem.

[4] Recent papers have made notable contributions by generalizing Weitzman (1979)'s result to different settings. For example, Doval (2018) assumes that an alternative can be chosen even without exploration; however, in Doval (2018)'s setting, the DM receives the (random) benefit by choosing that alternative even if it was not explored whereas in our setting, the engineers have to conduct their exploration in order to achieve cost savings, thus we allow an unexplored bid to be chosen as the winning bid but do not receive the cost reduction benefit if we do so which makes our setting different than Doval (2018)'s. Balseiro and Brown (2019) studied another extension where the DM can receive the reward from the top m alternatives, which is different from our sourcing setting where we can only pick one supplier.

[5] Note that in the simpler case where the decision maker has to a priori select m boxes to be potentially opened and then conduct optimal search among the m boxes, the simple index rule is optimal for any a priori selected m boxes. However, if the decision maker does not need to a priori select m boxes to be potentially opened but is only restricted to open no more than m boxes (which is the setting in our model), the simple index rule does not hold in general as the optimal sequence to search is path dependent.

3. The Model

Consider a cost-minimizing risk-neutral buyer who wishes to award a contract to a single supplier from a set of risk-neutral profit-maximizing suppliers. The supplier selection process commonly employed by practitioners, which we call Bid-Award-Investigate Mechanism (BAI) is outlined below.

Bid-Award-Investigate (BAI)

Step 1 (Bid)
All potential suppliers submit a sealed price bid for the contract.

Step 2 (Award)
The buyer awards the contract to one of the suppliers.

Step 3 (Investigate)
The buyer decides whether or not to investigate the supplier who is awarded the contract, and pays the supplier his updated bid (i.e., the supplier's initial bid price minus any cost-reduction the buyer was able to identify for that supplier).

In BAI, only the contract winner is ever investigated for cost-savings. However, there may be other alternative supplier selection processes in which the buyer could potentially investigate more suppliers for cost-savings and leverage the findings to inform supplier selection. In fact, anticipating the opportunity to reduce suppliers' production costs and the uncertainties about the cost-savings that can be identified, we contemplate, for the buyer, a different three-step supplier selection process called Bid-Investigate-Award Mechanism (BIA) outlined below.

Bid-Investigate-Award (BIA)

Step 1 (Bid)
All potential suppliers submit a sealed price bid for the contract.

Step 2 (Investigate)
For each potential supplier, the buyer chooses whether or not to conduct a cost-reduction investigation at an expense; if she chooses to do so, the cost-reduction (if any) is revealed to the buyer and that supplier, and the supplier's production cost to execute the contract gets reduced accordingly.

Step 3 (Award)
The buyer awards the contract to the supplier with the lowest updated bid (i.e., the supplier's initial bid price minus any cost-reduction the buyer was able to identify for that supplier) and pays this supplier his updated bid.

Note that while the buyer may get additional information about suppliers' private production cost during the investigations, in both BIA and BAI, her final payment to the contract winner equals the winner's *initial bid price* minus any cost-reduction the buyer was able to identify for that supplier; in other words, we make the implicit assumption that the buyer does not leverage the additional information to squeeze the supplier's profit (e.g., by lowering supplier's initial bid). The rationale behind this assumption is two-fold. First and foremost, the additional information the buyer may learn about a supplier's cost from investigation is limited in many cases. For instance, in our motivating example from the Introduction, the cost-savings opportunity that the buyer identified for one of the suppliers was to replace the crimping process by a new approach based on adjusting temperature using liquid nitrogen. Although it is true that to quantify the amount of cost-savings, the buyer needs to work with the supplier to work out the supplier's cost of running the original crimping process, the buyer cannot simply use this information to figure out the production cost of the whole manufacturing process because the crimping process is only a small part of it. Thus, it is practically difficult, if not infeasible, to fully infer the supplier's production cost and then use it to replace the supplier's initial bid. Secondly, even if the buyer could update her posterior of a supplier's cost significantly based on the new information she obtains during investigation, we assume that the buyer would not use this information to her advantage by adjusting the supplier's initial bid. Recall that the goal of such investigations is not to squeeze the suppliers' profits but to make them more efficient and keep them as profitable or more profitable than before; these cost reductions of course benefit the buyer, too, by lowering the payment to the supplier, but not by lowering the supplier's margin. Intuitively, if the buyer starts to leverage information about suppliers' costs that she learns from investigations to shrink supplier margins (e.g., by lowering suppliers' initial bids and hence squeezing their profit margin), such behavior may create tension in her relationship with the suppliers. If buyers in practice carried out such behavior, suppliers may start resisting cost-reduction investigations.

Before explaining both BAI and BIA in more detail in the remainder of this section, we first explain the model preliminaries: how we model supplier production costs, cost-reduction opportunities, and costs of cost-reduction investigations.

3.1. Model Preliminaries

We assume that the buyer has already identified a set $\mathcal{N} = \{1, \dots, N\}$ of qualified suppliers. For supplier $i \in \mathcal{N}$, his (nonnegative) *cost* (of production) is $c_i = \Delta_i^c + \epsilon_i^c$, where $\Delta_i^c := \mathbf{E}[c_i] \in \mathbb{R}_+$ is public information that reflects the *ex ante* inefficiency of supplier i's production process (i.e., a larger Δ_i^c corresponds to a less efficient supplier i) and ϵ_i^c is supplier i's private information, a zero mean continuous random variable with a cumulative distribution function (c.d.f.) (resp.

probability density function (p.d.f.)) $F(\epsilon_i^c)$ (resp. $f(\epsilon_i^c)$) on the support $[\underline{c}, \bar{c}]$. Therefore, c_i follows a distribution with c.d.f. $F_i(c_i) := F(c_i - \Delta_i^c)$, p.d.f. $f_i(c_i) := f(c_i - \Delta_i^c)$ on the support $C_i = [\underline{c}_i, \bar{c}_i] := [\underline{c} + \Delta_i^c, \bar{c} + \Delta_i^c]$. We assume that the cost types $\{c_i\}_{i=1}^N$ are independent across suppliers. Denote by $\mathbf{f}(\mathbf{c}) = \prod_{i=1}^N f_i(c_i)$ (resp. $\mathbf{F}(\mathbf{c}) = \prod_{i=1}^N F_i(c_i)$) the joint p.d.f. (resp. c.d.f.) for the cost vector $\mathbf{c} = (c_1, \cdots, c_N)$. By convention, we use $-i$ to denote the indices of suppliers other than supplier i. Let $C := \otimes_{i \in \mathcal{N}} C_i$ denote the set of all possible cost realizations for all suppliers. We make the following assumption on the distribution of cost.

Assumption 1. *$F_i(c_i)/f_i(c_i)$ is nondecreasing in c_i for all i.*

Note that this assumption is satisfied by many distributions (e.g., the family of log-concave distribution functions that includes, among others, uniform, normal, exponential, Gamma, Beta distributions, and their truncations). For expositional simplicity, we assume that the buyer must transact with one of the suppliers. (Our analysis easily generalizes when this is not the case.)

To incorporate the opportunity to reduce suppliers' production costs into our model, we assume that after all suppliers submit their initial price bids in Bid, the buyer can choose to exert an effort to perform a one-time cost-reduction investigation on any supplier at a cost d per supplier investigated. (Our analysis generalizes to the case when the investigation costs are heterogeneous across suppliers.) To capture the practical considerations that cost-reduction investigations may require certain critical but limited resources such as specialized engineers, lean experts who can visit the supplier, etc., we assume that there is a cap $\hat{N} \in \{1, 2, \dots, N\}$ on the maximum number of suppliers the buyer can investigate cost reduction for. For example, to determine $\hat{N}$ in practice, the buyer would assess both the time needed for each investigation and the deadline to award the contract to figure out the maximum number of investigations that can be feasibly carried out.

After an investigation of, say supplier i, a nonnegative random cost-reduction opportunity for supplier i is discovered, denoted by $t_i = \Delta_i^t + \epsilon_i^t$, where $\Delta_i^t := \mathbf{E}[t_i] \in \mathbb{R}_+$ is a publicly known constant that reflects the *ex ante* cost-reduction opportunity by investigating supplier i (i.e., a larger Δ_i^t means that supplier i's production process is expected to have more cost-reduction opportunities) and ϵ_i^t is a random variable of mean zero, and with c.d.f. (resp. p.d.f.) $G(\epsilon_i^t)$ (resp. $g(\epsilon_i^t)$) on the support $[\underline{t}, \bar{t}]$.[6] Note that if Δ_i^c and Δ_i^t are the same across all suppliers, then our model reduces to the *ex ante* symmetric suppliers setting; otherwise, it also captures other realistic settings with *ex ante* asymmetric suppliers, e.g., if a large Δ_i^c is coupled with a large Δ_i^t, then it captures the setting where a higher cost production process may have more inefficiencies and is thus associated with

[6] Note that supplier i's cost distribution support is $[\Delta_i^c + \underline{c}, \Delta_i^c + \bar{c}]$ and his cost-reduction distribution support is $[\Delta_i^t + \underline{t}, \Delta_i^t + \bar{t}]$. Thus, the model parameter should satisfy the condition $\Delta_i^c + \underline{c} > \Delta_i^t + \bar{t}$ to ensure that cost is always larger than cost-reduction.

more cost-reduction opportunities. We assume that for each supplier i, ϵ_i^c and ϵ_i^t are independent. (In §9.2, we consider an extension where the cost and cost-reduction distributions of the same supplier are *stochastically* correlated.) We assume that $\{\epsilon_i^t\}_{i\in\mathcal{N}}$ are independent across suppliers. This is to reflect that different suppliers may have different opportunities of reducing cost: Simply because the buyer can reduce supplier 1's cost by reconfiguring his production work flow does not necessarily mean that the same configurations would work for supplier 2 who may have a very different factory shopfloor layout. We also assume that for any $i \in \mathcal{N}$, ϵ_i^t is unknown to everyone before the investigation, but is observed *only to* the buyer and supplier i after the buyer completes her investigation on supplier i; this captures the fact that the cost saving opportunity cannot be had until the buyer completes her investigation and uncovers some cost-reduction ideas that the supplier was not aware of. Note that t_i follows a distribution with c.d.f. $G_i(t_i) := G(t_i - \Delta_i^t)$, p.d.f. $g_i(t_i) := g(t_i - \Delta_i^t)$ on the support $T_i = [\underline{t}_i, \bar{t}_i] := [\underline{t} + \Delta_i^t, \bar{t} + \Delta_i^t]$. Denote by $\mathbf{g}(\mathbf{t}) = \prod_{i=1}^N g_i(t_i)$ (resp. $\mathbf{G}(\mathbf{t}) = \prod_{i=1}^N G_i(t_i)$) the joint p.d.f. (resp. c.d.f.) for the cost-reduction vector $\mathbf{t} = (t_1, \ldots, t_N)$. Let $T := \otimes_{i\in\mathcal{N}} T_i$ denote the set of all possible cost-reduction realizations for all suppliers.

In both BAI and BIA, if, say supplier i, is investigated *and* is awarded the contract, then the *updated cost* for supplier i to perform the contract becomes $c_i - t_i$. The assumption here is that *all of the cost savings* t_i identified by the buyer will be passed on to the buyer in the form of a reduction of final contract payment; in other words, the buyer's payment to supplier i will be his *updated bid* which equals his initial price bid minus t_i. (In §9.1, we consider an extension of the setting where the identified cost savings are shared between the buyer and the contract winning supplier.) Otherwise, i.e., the contract winning supplier is not investigated, the buyer's payment will be the supplier's initial price bid.

Also, note that a buyer would not investigate a supplier j if the average cost-savings benefit is smaller than the investigation cost; thus, we make the following assumption to avoid the trivial cases where the cost of investigation is prohibitively high so that it is never optimal for our risk-neutral buyer to investigate any supplier.

Assumption 2. $d < \max_{i\in\mathcal{N}} \Delta_i^t$.

3.2. The Analysis of BAI Mechanism

Note that BAI is a natural enhancement of the classic first-price sealed bid auction: After awarding the contract, the buyer could choose to investigate the contract winner for potential reduction in production cost. We analyze BAI in this subsection via backward induction. In the Investigate step, suppose supplier i is the contract winner, then the risk-neutral buyer would investigate supplier i if the average cost-reduction is larger than the investigation cost, i.e., $\mathbf{E}[t_i] = \Delta_i^t > d$. This implies that if the buyer awards the contract to supplier i, then the expected procurement cost equals supplier

i's initial bid minus supplier i's expected payment reduction which equals $r_i = \max\{0, \Delta_i^t - d\}$. Hence, after each supplier i submits a sealed initial bid b_i to the buyer, the buyer should award the contract to supplier $j = \arg\min_{i\in\mathcal{N}} \lambda_i$ where $\lambda_i := b_i - r_i$ is the buyer's expected procurement cost if supplier i is the contract winner. (In case of a tie, the buyer randomly selects a supplier in $\arg\min_{i\in\mathcal{N}} \lambda_i$. Note that it can be formally shown that a tie occurs with probability zero in equilibrium). Finally, in the Bid step, each supplier i submits a sealed bid $b_i \in B_i := [0, \bar{c}_i]$ to the buyer, where $\bar{c}_i$ is the highest cost-type of supplier i (note that if supplier i bids higher than $\bar{c}_i$, the buyer could give a counter-offer $\bar{c}_i$ and would know the supplier would accept the offer; we capture this by simply assuming that supplier i cannot bid higher than $\bar{c}_i$). Then, due to the buyer's optimal contract award decision, supplier i wins the contract if $\lambda_i < \min_{j\neq i} \lambda_j$; if he wins, he earns a profit of $b_i - c_i$ regardless of whether he is investigated or not (i.e., even when the buyer investigates him and identifies some cost savings, his profit margin still equals $b_i - c_i$ because the reduction in buyer's payment is cancelled out with the reduction of his cost of executing the contract). Suppose there exists a Bayesian Nash equilibrium where supplier i bids according to $\tilde{\beta}_i : C_i \to B_i$, and let $u_i(b_i; c_i)$ denote supplier i's expected profit when his cost is c_i, he bids b_i and each supplier $j \neq i$ bids according to $\tilde{\beta}_j$. Then the following must hold:

$$\tilde{\beta}_i(c_i) \in \arg\max_{b_i\in B_i} u_i(b_i; c_i)$$
$$\text{where } u_i(b_i; c_i) = (b_i - c_i)\, \mathbf{P}_{\mathbf{c}_{-i}} \left(b_i - r_i < \tilde{\beta}_j(c_j) - r_j, \forall j \neq i\right).$$

It turns out that BAI is, in essence, strategically equivalent to an N-supplier first-price sealed bid auction where supplier i's private cost equals $\check{c}_i := c_i - r_i$. Specifically, let $\{\check{\beta}_i\}_{i\in\mathcal{N}}$ denote the equilibrium bidding functions in this first-price sealed bid auction (note that the existence of the equilibrium has been established in Athey (2001)); the following result characterizes the connection between this auction and BAI.

Proposition 1. *There exists a Bayesian Nash equilibrium in BAI in which the equilibrium bidding functions are* $\tilde{\beta}_i(c_i) := \check{\beta}_i(c_i - r_i) + r_i$ *for all* $i \in \mathcal{N}$ *and* $c_i \in C_i$.

3.3. The BIA Mechanism in Detail

For any set $\mathcal{A} \subseteq \mathcal{N}$, let $\overline{\mathcal{A}} := \mathcal{N} - \mathcal{A}$, and denote by $\mathbf{t}^{\mathcal{A}}$ a vector in $\mathbb{R}_+^{|\mathcal{A}|}$ that consists of elements t_i for all $i \in \mathcal{A}$. For any event $\mathcal{E}$, denote by $\mathbf{1}_{[\mathcal{E}]}$ the indicator function of whether $\mathcal{E}$ occurs. We now explain each step of BIA in more detail.

Bid. This step proceeds in a standard sealed-bid fashion: Each supplier i submits a sealed initial bid $b_i \in \mathbb{R}_+$ to the buyer. We denote by $\mathbf{b} = (b_1, \ldots, b_N) \in \mathbb{R}_+^N$ the vector of bids of all suppliers.

Investigate. In this second step, the buyer follows an *investigation policy* to structure her investigation process (e.g., which supplier(s) to investigate). The class of feasible investigation policies we focus on is the so-called *sequential investigations* where the cost-reduction investigations are carried out one by one. In practice, it may be too time-consuming to carry out many rounds of investigations when the buyer needs to ensure timely product launch. Therefore, $\hat{N} \leq N$ captures the maximum number of investigations that can be carried out. Essentially, upon knowing the cost-reduction of an investigated supplier, the buyer has two options: (a) continue another round of investigation of one of the remaining uninvestigated suppliers, or (b) stop further rounds of investigations and award the contract to one of the suppliers. This captures a wide range of dynamics in the investigation process. To illustrate, consider the case where $\mathcal{N} = \{1,2,3\}$, the buyer can investigate no more than $\hat{N} = 2$ suppliers and the initial bids are $\mathbf{b} = (b_1, b_2, b_3)$. In the first investigation round, the buyer decides, based on $\mathbf{b}$, whether to investigate any supplier, and if so, which supplier to investigate. After she chooses whom to investigate in the first round, say supplier 1, she incurs an investigation cost of d and learns t_1. Then, based on $\mathbf{b}$ and t_1, she needs to decide in the second round whether to proceed with further investigations and if so whether to investigate supplier 2 or supplier 3 next. If she chooses to investigate supplier 3, then an investigation cost of d is incurred and t_3 is revealed. At this point, she has exhausted the maximum number of investigations that she can carry out and cannot investigate supplier 2 even if it is worthwhile to do so based on $\mathbf{b}, t_1$ and t_3. As this example illustrates, throughout all the investigation rounds, whether and whom to investigate depend on not only the initial bids $\mathbf{b}$ but also the revealed cost-reductions of those already-investigated suppliers. Naturally, we assume that in BIA, the buyer is sequentially rational: *based on the suppliers' initial bids* $\mathbf{b}$, the buyer chooses her investigation policy (which can be adaptive to the revealed cost-reduction information of the investigated suppliers) to minimize her expected total cost (i.e., contract payment plus investigation cost).

We now formalize the above ideas mathematically. Since the buyer can conduct at most $\hat{N}$ investigations, the maximum number of possible investigation rounds is $\hat{N}$. We index those rounds by *k forward*, i.e., $k = 1$ corresponds to the first investigation round and $k = \hat{N}$ corresponds to the last investigation round. An *investigation policy* is defined as $\pi = (\pi_1, \ldots, \pi_{\hat{N}})$ where

$$\pi_k : (\otimes_{i \in \mathcal{N}} B_i) \times \left(\otimes_{i \in \mathcal{I}_k} T_i\right) \to \{\emptyset\} \cup (\mathcal{N} - \mathcal{I}_k),$$

where $\mathcal{I}_k$ is the set of suppliers that have been investigated before round k. In other words, π_k uses all the information the buyer has at the beginning of round k (i.e., the initial bids from all suppliers and the cost-reductions of already investigated suppliers) to decide if no investigation will be conducted (denoted by $\emptyset$) or to investigate one of the not-yet investigated suppliers. Under any such investigation policy π, we denote by $\mathcal{I}_k^\pi$ the resulting set of suppliers that have been investigated before round k, i.e., $\mathcal{I}_1^\pi = \emptyset, \mathcal{I}_{k+1}^\pi = \mathcal{I}_k^\pi \cup \pi_k(\mathbf{b}; \mathbf{t}^{\mathcal{I}_k^\pi})$ for all $k = 1, \ldots, \hat{N}$. Let Π denote the set of all such sequencing strategies. Then, for any $\mathbf{b}$ and any investigation policy $\pi \in \Pi$, $\mathcal{I}_{\hat{N}+1}^\pi(\mathbf{b})$ is a random set. Define $S^\pi(\mathbf{b}) := \mathcal{I}_{\hat{N}+1}^\pi(\mathbf{b})$ for all $\mathbf{b} \in B$. Note that $S^\pi(\mathbf{b})$ is a random set induced by the investigation policy π; moreover, at the conclusion of the investigations, $S^\pi(\mathbf{b})$ is revealed, and we will denote by $\mathbf{t}^{S^\pi(\mathbf{b})}$ the vector of the cost-reduction realizations of all the investigated suppliers. (We will sometimes suppress the dependency of $S^\pi(\mathbf{b})$ on $\mathbf{b}$ and π for notational simplicity whenever there is no confusion.) As mentioned previously, at the conclusion of a cost-reduction investigation of some supplier i that the buyer chooses to investigate, the cost-reduction t_i is revealed to both the buyer and supplier i, and supplier i's *updated bid* becomes $b_i - t_i$ and his *updated cost* of production becomes $c_i - t_i$.

Award. In this last step, the supplier with the lowest updated bid (if a supplier is not investigated, his updated bid equals his initial bid) wins the contract and gets paid his updated bid. One nice feature of BIA is that even though the winning supplier's contract payment may be reduced from his initial bid due to the identified cost savings, his profit margin still equals $b_i - c_i$ because the identified cost savings also reduce his cost of executing the contract.

4. Analysis of Bid-Investigate-Award Mechanism

In this section, we characterize the buyer's investigation policy and the suppliers' equilibrium bidding strategies in BIA using backward induction.

4.1. Buyer's Investigation policy

In this subsection, we characterize the buyer's investigation policy π^*. Suppose that in the Bid step, each supplier i places an initial bid of $b_i \in B_i$. Then, a risk-neutral buyer's investigation policy $\pi^* \in \Pi$ should minimize her expected cost given $\mathbf{b} = (b_1, \ldots, b_N)$:

$$\pi^* = \arg\min_{\pi \in \Pi} \mathbf{E}\left[\min_{i \in \mathcal{N}} \left\{b_i - \mathbf{1}_{[i \in S^\pi(\mathbf{b})]} t_i\right\} + |S^\pi(\mathbf{b})| d\right], \tag{1}$$

and we denote $S^* := S^{\pi^*}$. Note that solving (1) is equivalent to solving a dynamic program which is characterized by a set of Bellman equations defined below. Before formulating the Bellman equations, define the *lowest updated bid before round k* as follows: For all k,

$$u_k := \min_{i \in \mathcal{N}} \left\{b_i - \mathbf{1}_{[i \in \mathcal{I}_k]} t_i\right\}.$$

Note that the lowest expected cost in round k onwards is only a function of the current lowest updated bid and the set of suppliers that have been investigated. We can then define the value function $V(u,\mathcal{I})$ and arrive at the following Bellman equations: For all $u\in\mathbb{R}_+,\mathcal{I}\subseteq\mathcal{N}$ and $|\mathcal{I}|<\hat{N}$,

$$V(u,\mathcal{I})=\min\left\{u,\min_{i\notin\mathcal{I}}\left\{d+\underbrace{V(u,\mathcal{I}+\{i\})\int_{-\infty}^{b_i-u}dG_i(t_i)}_{\text{lowest updated bid higher than } u}+\underbrace{\int_{b_i-u}^{\bar{t}_i}V(b_i-t_i,\mathcal{I}+\{i\})dG_i(t_i)}_{\text{lowest updated bid lower than } u}\right\}\right\} \quad (2)$$

with the boundary conditions $V(u,\mathcal{I})=u$, for all u and for all $\mathcal{I}\subseteq\mathcal{N}$ such that $|\mathcal{I}|=\hat{N}$. The following theorem states that the buyer's optimal investigation policy is an index rule, and provides comparative statics results of the investigation decisions.

THEOREM 1. *Let $\tau_i:=\max\{0,\tilde{\tau}_i\}$ where $\tilde{\tau}_i=\tau+\Delta_i^t$ and τ solves the equation $d=\int_\tau^\infty(\epsilon^t-\tau)dG(\epsilon^t)$. Define, for any $\mathbf{b}$, supplier i's index as $\mu_i(b_i):=b_i-\tau_i$. Rank suppliers by their indices from low to high and let $\kappa(i)$ denote supplier i's ranking and let $\iota(k)$ denote the supplier who is ranked k. Define $\tilde{N}:=\min\{n:\tau_{\iota(n+1)}=0\}$ and $\bar{N}:=\min\{\hat{N},\tilde{N}\}$.*

(a) The optimal investigation policy is: $\pi_k^*=\emptyset$ for all $k=\bar{N}+1,\ldots,\hat{N}$, and

$$\pi_k^*=\begin{cases}\emptyset, & \text{if } u_k<\min_{i\notin\mathcal{I}_k^{\pi^*}}\{\mu_i(b_i)\}\\ \arg\min_{i\notin\mathcal{I}_k^{\pi^*}}\{\mu_i(b_i)\}, & \text{otherwise}\end{cases}, \quad \text{for all } k=1,\ldots,\bar{N}. \quad (3)$$

(b) τ_i is nonnegative, nonincreasing in d, and weakly increases as G_i stochastically increases;

(c) Supplier i's probability of being investigated is nonincreasing in t_j for all j such that $\kappa(j)<\kappa(i)$, Δ_j^t for all $j\neq i$, b_i and d, and nondecreasing in Δ_i^t and b_j for all $j\neq i$.

Theorem 1 states that the optimal investigation policy is an index rule where each supplier i is assigned with an index $\mu_i(b_i)=b_i-\tau_i$. To understand the index, it helps to understand what τ_i represents. When $\tau_i=0$, it means that $\mathbf{E}[t_i]\leq d$ and obviously it is suboptimal to investigate supplier i. In contrast, when $\tau_i>0$, i.e., $\mathbf{E}[t_i]>d$, whether it is worthwhile to investigate supplier i depends on how his bid b_i compares to the current best bid. It is worthwhile only when the realized t_i exceeds d plus the gap between the current best bid and supplier i's bid b_i. Thus, we define τ_i so that if the current best bid were exactly τ_i below b_i, the buyer would be indifferent between investigating supplier i or not; algebraically, the equation that determines τ_i when $\tau_i>0$ in Theorem 1 is obtained from the condition $\mathbf{E}_{t_i}[\max\{0,t_i-\tau_i\}]=d$, where the expected benefit of investigating supplier i (i.e., the current best bid will improve only if $t_i>\tau_i$) and the cost of investigation are equated.

It is attractive that the optimal investigation policy has a simple index structure. Moreover, Theorem 1 part (c) suggests that the sensitivity of the policy with respect to suppliers' characteristics aligns with intuition, which also makes it attractive from a practical perspective: Supplier i is more likely to be investigated in the optimal investigation policy if he becomes more competitive (i.e., lower initial bid or larger cost-reduction expectation) and his opponents become less competitive (i.e., higher initial bid or smaller cost-reduction expectation).

Finally, we would like to mention that in contrast to BAI in which at most one supplier is investigated (i.e., the contract winner), in BIA it is possible that the buyer would investigate multiple suppliers. Thus, conditioning on the same realization of suppliers' bids, BIA helps the buyer reduce the *conditional expected procurement cost* since it essentially allows the buyer the extra option to explore more cost-saving opportunities and choosing, after investigations, the supplier with the lowest updated cost as the contract winner. However, we cannot assume that the suppliers' bids will be the same under BIA and BAI. In fact, the investigation policy and contract award decision in BIA will affect the suppliers' bidding strategy in a non-trivial way, which we investigate in the next subsection. We then study in §5 and §6 how the suppliers' strategic bidding in BIA affects supplier and buyer payoffs (respectively) in BIA compared to BAI.

4.2. Suppliers' Equilibrium Bidding Strategy

Having worked backwards from Award to Investigate to characterize the buyer's investigation policy, we now solve the suppliers' equilibrium bidding strategy in Bid. Let $W_i(b_i, \mathbf{b}_{-i})$ denote supplier i's winning probability when he bids b_i and his competitors bid $\mathbf{b}_{-i}$ in BIA. Then, we can show that

$$W_i(b_i, \mathbf{b}_{-i}) = \begin{cases} \bar{\omega}_i(\mathbf{b}), & \forall i : \kappa(i) > \bar{N} \\ \int_{T_i} \underline{\omega}_i(\mathbf{b}, t_i) dG_i(t_i), & \forall i : \kappa(i) \le \bar{N} \end{cases}, \tag{4}$$

where $\bar{N}$ is defined in Theorem 1, and $\bar{\omega}_i$ and $\underline{\omega}_i$ are defined as:

$$\bar{\omega}_i(\mathbf{b}) = \left[\prod_{j:\kappa(j)\le\bar{N}} G_j(b_j - b_i)\right]\left[\prod_{j:\kappa(j)>\bar{N}, j\ne i} \mathbf{1}_{[b_i<b_j]}\right],$$

$$\underline{\omega}_i(\mathbf{b}, t_i) = \left[\prod_{j:\kappa(j)\le\bar{N}, j\ne i} G_j(b_j - b_i + \tau_i \wedge t_i)^{\mathbf{1}_{[\mu_j<\mu_i]}} G_j(b_j - b_i + t_i)^{\mathbf{1}_{[\mu_i\le\mu_j<b_i-t_i]}}\right]\left[\prod_{j:\kappa(j)>\bar{N}} \mathbf{1}_{[b_i-t_i<b_j]}\right].$$

For expositional clarity, the derivation of the expressions above are deferred to Appendix A.2. One can easily verify that $W_i(b_i, \mathbf{b}_{-i})$ is nonincreasing in b_i.

Suppose that each supplier i, anticipating that the buyer will use the optimal investigation policy characterized in Theorem 1, follows a bidding strategy $\beta_i : C_i \to B_i$. Denote by $\boldsymbol{\beta} = (\beta_1, \dots, \beta_N)$ the bidding strategy profile of all suppliers. Recall that supplier i's margin upon winning always equals $b_i - c_i$ regardless of whether he is investigated. Hence, supplier i's expected profit when his cost is c_i and he bids b_i and all other suppliers bid according to $\boldsymbol{\beta}_{-i}$ is

$$u_i^{\boldsymbol{\beta}_{-i}}(b_i; c_i) = (b_i - c_i)\mathbf{E}_{\mathbf{c}_{-i}}\left[W_i(b_i, \boldsymbol{\beta}_{-i}(\mathbf{c}_{-i}))\right]. \tag{5}$$

A strategy profile $\tilde{\boldsymbol{\beta}}$ forms an equilibrium if $\tilde{\boldsymbol{\beta}}_i(c_i) \in \arg\max_{b_i\in B_i} u_i^{\tilde{\boldsymbol{\beta}}_{-i}}(b_i; c_i)$ for all $i \in \mathcal{N}$ and $c_i \in C_i$. The following result characterizes a pure strategy Bayesian Nash equilibrium where supplier i's equilibrium bidding strategy is nondecreasing in its cost c_i.

LEMMA 1. *There exists a pure strategy Bayesian Nash equilibrium* $\tilde{\boldsymbol{\beta}} = (\tilde{\beta}_1, \ldots, \tilde{\beta}_N)$ *where* $\tilde{\beta}_i$ *is nondecreasing for all* $i \in \mathcal{N}$. *In this equilibrium, for any supplier* i, *there exists* $\tilde{c}_i \leq \bar{c}_i$ *such that, the equilibrium bidding strategies are characterized by the following system of equations:*

$$\tilde{\beta}_i(c_i) = c_i + \frac{\int_{c_i}^{\bar{c}_i} \mathbf{E}_{\mathbf{c}_{-i}}\left[W_i(\tilde{\beta}_i(z_i), \tilde{\boldsymbol{\beta}}_{-i}(\mathbf{c}_{-i}))\right] dz_i}{\mathbf{E}_{\mathbf{c}_{-i}}\left[W_i(\tilde{\boldsymbol{\beta}}(\mathbf{c}))\right]}, \forall i \in \mathcal{N}, \forall c_i < \tilde{c}_i, \tag{6}$$

and the boundary conditions $\tilde{\beta}_i(c_i) = c_i$ *for all* i *and* $c_i \geq \tilde{c}_i$.

Unfortunately, the equilibrium does not permit a closed form solution. This is not surprising because the analysis of equilibrium bidding strategies for *ex ante* asymmetric bidders in the first-price sealed-bid auction is already analytically intractable in general (see Maskin and Riley (2000) for more details), except in some special cases (e.g., Lebrun (1999) and Mares and Swinkels (2014)); incorporating the cost-reduction investigations only makes the analysis more complex. Even if we focus on the symmetric equilibrium for the case where suppliers are *ex ante* symmetric (i.e., $\Delta_i^c = \Delta_1^c$ and $\Delta_i^t = \Delta_1^t$ for all $i \in \mathcal{N}$), it is still, in general, very difficult to characterize the equilibrium bidding function in closed-form: Unlike the classic first-price sealed-bid auction where suppliers' winning probability does *not* depend on the *functional form* of the monotone symmetric equilibrium bidding function but only depends on the ranking of the bids, in our setting, the winning probability *does* depend on the functional form of the symmetric equilibrium bidding function. (For example, if the symmetric equilibrium bidding function is less spread-out, the difference of two equilibrium bids under two different cost types is smaller; in this case, given the distribution of cost reductions, the lowest cost supplier needs a larger cost advantage to ensure his updated bid is also the lowest to secure the contract.)

Although analytically deriving the equilibrium strategy is difficult (if not impossible), it can be efficiently computed numerically. This allows the buyer to calculate the expected cost of BIA before the procurement process starts, compare it with the expected cost of the traditional approach BAI, and choose to use whichever is cheaper. *Therefore, the approach we are proposing for the buyer is to use BIA or BAI, whichever is ex ante more beneficial.* Obviously, including BIA as an additional option in sourcing would help the buyer reduce procurement cost. But how would it affect suppliers' profits? We explore this question in the next section.

5. Supplier Profit Implications

Suppose the buyer finds that BIA provides lower expected cost than BAI, and informs the suppliers that she will use BIA to source the contract before the formal procurement process starts; would this benefit the suppliers? In this section, we investigate the suppliers' *ex ante* profits comparison between BIA and BAI to shed light on the profit implications of BIA on suppliers.

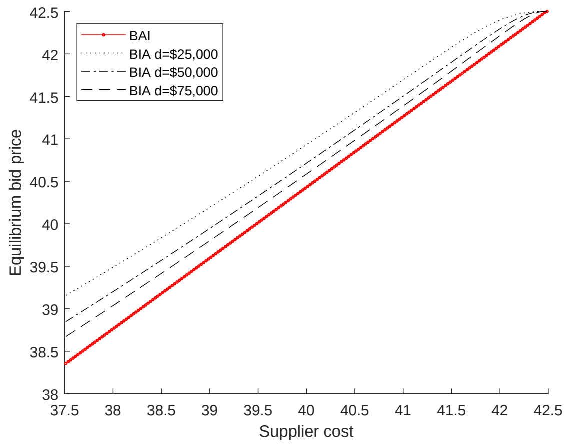
5.1. Ex ante Symmetric Suppliers: A Win-win Situation

At first glance, suppliers do not seem to benefit from BIA. Indeed, if the buyer finds some cost savings after investigating supplier i and supplier i ends up winning the contract, the contract payment to supplier i will be reduced by the identified cost savings; in other words, the buyer keeps all of the identified savings. Moreover, the buyer may also investigate other suppliers, making their bids more competitive and, in turn, decrease supplier i's chance of winning. Interestingly, we show that if suppliers are *ex ante* symmetric, all of them would prefer BIA to BAI.

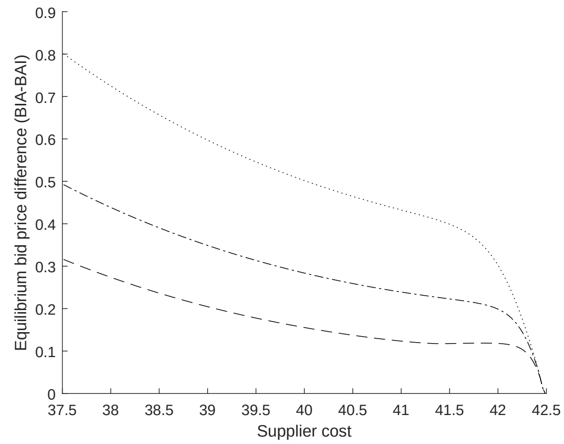
THEOREM 2. *Suppose that the suppliers are* ex ante *symmetric, i.e.,* $\Delta_i^c = \Delta^c$ *and* $\Delta_i^t = \Delta^t$ *for all* $i \in \mathcal{N}$*. Then all suppliers earn higher* ex ante *expected profits from BIA than BAI.*

The most intriguing implication of this result is that our proposed simple approach, i.e., the cheaper option between BAI and BIA, provides a win-win situation to *both* the buyer *and* all the suppliers compared to the conventional approach of only using BAI! As mentioned previously, the buyer always weakly benefits from the option of choosing between BAI and BIA; while the suppliers' ex ante profits remain the same when BAI is optimal to the buyer, Theorem 2 shows that their profits would increase when BIA is optimal to the buyer. Recall that in BIA, the buyer keeps all the *direct benefit* of the identified cost savings. Thus, the fact that the suppliers prefer cost-reduction investigations is not because they can benefit *directly* from the cost-savings identified by investigations. How do suppliers benefit from investigations? It turns out that conducting cost-reduction investigations *before* awarding contract has strategic implications on how suppliers bid in the Bid step. In particular, a supplier does not have to submit the lowest price bid in order to win the contract because the ranking of the updated bids could be different from the initial bids due to the cost-reductions revealed by investigations; this softens competition in the Bid step. This is illustrated in Figure 1, where, anticipating the buyer's investigation policy in BIA, suppliers would place higher bids than they would in BAI. The extent to which the suppliers inflate their bids in BIA depends on the extent to which the investigations affect the difference between the rankings of the initial bids and updated bids. Specifically, the difference between the bidding curves of BIA and BAI is smaller when the cost of investigation becomes higher; this is because the buyer would investigate fewer suppliers per Theorem 1 (i.e., Figures 1(a) and 1(b)); similarly, the difference is smaller when the cost-reduction distribution becomes less variable, because the ranking of the updated bids are less likely to be different from the initial bids (i.e., Figures 1(c) and 1(d)).

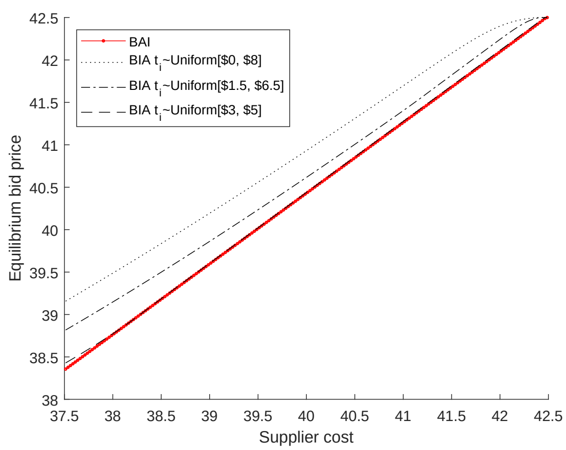
The win-win result may not hold when suppliers are *sufficiently* asymmetric: Intuitively, if one particular supplier is very likely to get a considerably smaller cost reduction than his competitors, then he has a disadvantage in BIA; similarly, if he is very likely to have a considerably lower production cost than his competitors, then he may lose his production cost advantage in BIA due to the uncertain cost reductions. We investigate the *ex ante* asymmetric suppliers case below.



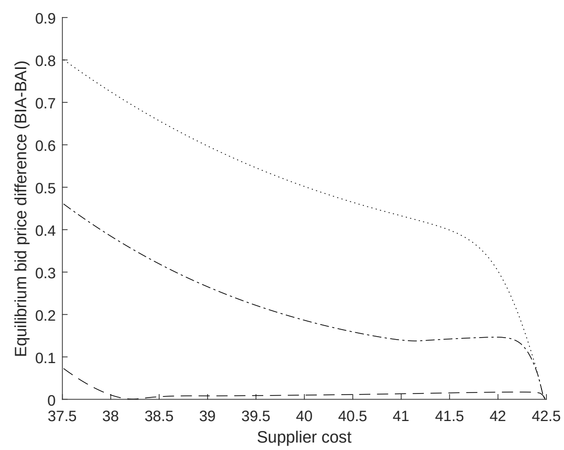
(a) Equilibrium bidding functions for BAI and BIA varying investigation costs



(b) Difference between equilibrium bidding functions for BAI and BIA varying investigation costs



(c) Equilibrium bidding functions for BAI and BIA varying cost-reduction distributions



(d) Difference between equilibrium bidding functions for BAI and BIA varying cost-reduction distributions

Figure 1 Comparison of suppliers' equilibrium bidding functions in BIA and BAI.

Note: In both Figures 1(a) and 1(c), the dotted line is supplier's equilibrium bidding function in BIA in an example where the number of units in the contract is 50,000, the investigation cost is $25,000, the number of suppliers and the maximum number of investigations are both 3, the per-unit cost distribution is uniformly distributed between $37.5 and $42.5, and the per-unit cost-reduction distribution is uniformly distributed between $0 and $8. In Figure 1(a), we illustrate how the bidding function in BIA varies as the investigation costs becomes higher, i.e., the dash-dot line (resp. the dashed line) corresponds to the same example but with investigation cost equals $50,000 (resp. $75,000); similarly, in Figure 1(c) we reduce the variability of the cost-reduction distribution, i.e., the dash-dot line (resp. the dashed line) corresponds to the same example but with cost-reduction uniformly distributed between $1.5 and $6.5 (resp. $3 and $5). The solid line in both Figures 1(a) and 1(c) corresponds to BAI which remains the same across all the examples. Finally, the dotted line (resp., dash-dot line, dashed line) in Figure 1(b) depicts the difference between the dotted line (resp., dash-dot line, dashed line) and the solid line in Figure 1(a); same for Figures 1(d) and 1(c).

5.2. Ex ante Asymmetric Suppliers

To investigate how the *ex ante* asymmetry affects suppliers' *ex ante* profit comparisons between BIA and BAI, we vary the level of asymmetry among the suppliers by changing either the cost-reduction distribution or the cost distribution of supplier 1, and compare the *ex ante* profits of all suppliers under BIA and BAI.

Heterogeneous Cost-Reduction Distributions. Consider N *ex ante* symmetric suppliers and stochastically increase (resp. decrease) the cost-reduction distribution of supplier 1 by increasing (resp. decreasing) Δ_1^t. The following result compares the *ex ante* profit changes of the suppliers under BIA and BAI.

PROPOSITION 2. *Suppose that all suppliers have the same production cost distribution, i.e., $\Delta_i^c = \Delta^c$ for all $i \in \mathcal{N}$, and all suppliers except supplier 1 have the same cost-reduction distribution, i.e., $\Delta_i^t = \Delta^t > d$ for all $i \neq 1$ whereas $\Delta_1^t = \Delta^t - \nu$. Then, there exists $\nu^{**} = \bar{c} - \underline{c} + \bar{t} - \underline{t} > 0$ such that, compared to BAI,*

(a) Supplier 1 (resp. other suppliers) earns higher (resp. lower) ex ante profit in BIA if $\nu < -\nu^{**}$,

(b) Supplier 1 (resp. other suppliers) earns lower (resp. higher) ex ante profit in BIA if $\nu > \nu^{**}$.

The result means that the win-win situation may break down as the suppliers become sufficiently asymmetric in their cost-reduction distributions. There are two elements driving this result. First, the buyer takes into account the cost-reduction distributions of all suppliers when choosing whom to investigate; as supplier 1's cost-reduction distribution becomes stochastically smaller, investigating him is less appealing. Second, even if supplier 1 *is* investigated, his winning probability (i.e., the probability of having the lowest updated bid) decreases as his cost-reduction distribution becomes stochastically smaller. In sum, our result suggests that suppliers with sufficiently stochastically smaller cost-reduction distributions than other suppliers may *not* benefit from pre-award investigations. Next, we analyze the effect of heterogeneity in suppliers' cost distributions.

Heterogeneous Cost Distributions. Consider N *ex ante* symmetric suppliers and stochastically increase (resp. decrease) supplier 1's cost distribution by increasing (resp. decreasing) Δ_1^t. Note that cost distributions affect suppliers' *ex ante* profits in *both* BIA and BAI. The result below shows that, interestingly, the difference of the *ex ante* profits between BIA and BAI for supplier 1 (resp. other suppliers) may *not* be monotonic.

PROPOSITION 3. *Suppose that all suppliers have the same cost-reduction distribution, i.e., $\Delta_i^t = \Delta^t$ for all $i \in \mathcal{N}$, and all suppliers except supplier 1 have the same cost distribution, i.e., $\Delta_i^c = \Delta^c$ for all $i \neq 1$ whereas $\Delta_1^c = \Delta^c - \nu$. Suppose that d is sufficiently small. Then, there exist constants $\nu^* = \bar{c} - \underline{c}$ and $\nu^{**} = \bar{c} - \underline{c} + \bar{t} - \underline{t}$ such that, compared to BAI,*

(a) When $\nu = 0$, every supplier earns higher ex ante profit in BIA;

(b) When $\nu = \nu^*$, supplier 1 earns lower ex ante profit in BIA, whereas other suppliers earn higher ex ante profits in BIA;

(c) When $\nu > \nu^{**}$, every supplier earns equal ex ante profits in both BIA *and* BAI*;*

(d) When $\nu = -\nu^{*}$, supplier 1 *earns higher ex ante profit in* BIA*;*

(e) When $\nu < -\nu^{**}$, supplier 1 *earns equal profits in both* BIA *and* BAI*, whereas other suppliers earn higher (resp. the same) profits in* BIA *than (resp. as)* BAI *when $N \geq 3$ (resp. $N = 2$).*

The result above shows that, similar to the effect of supplier asymmetry in cost-reduction distributions, when suppliers are sufficiently asymmetric in their *cost* distributions, win-win also breaks down. Interestingly, as suppliers become more asymmetric in their cost distributions, the profit difference between BIA and BAI is *non-monotonic*. The intuition is as follows. Consider supplier 1 first. When his cost distribution is *moderately* low (i.e., Proposition 3 part (b)), his cost advantage ensures that he will always win in BAI but does not guarantee him winning in BIA because one of his competitors may undercut him by having a sufficiently large cost reduction. However, once supplier 1's cost distribution becomes *extremely* low (i.e., Proposition 3 part (c)), he will win in both BIA and BAI with probability one and earn equal profits. Similarly, if supplier 1's cost distribution is *moderately* high (i.e., Proposition 3 part (d)), he will always lose in BAI due to the cost disadvantage but may win under BIA by having a sufficiently large cost reduction. Yet this cost-reduction investigation only helps him to a certain degree beyond which the cost disadvantage is too large to overcome and he will always lose in both BIA and BAI (i.e. Proposition 3 part (e)). The intuition for the other suppliers' profits follows a similar logic when supplier 1's cost distribution is stochastically smaller than other suppliers (Proposition 3 parts (b)-(c)). However, when supplier 1's cost distribution is *moderately* high (i.e., Proposition 3 part (d)), other suppliers' profit comparison between BIA and BAI can go either direction: (i) On the one hand, other suppliers can better capitalize their cost advantage over supplier 1 in BAI than BIA; (ii) on the other hand, they also benefit from BIA since the competition in the Bid step in BIA is less intense than in BAI (see the comments after Theorem 2 for more detail). However, in the case where supplier 1's cost disadvantage is too large to overcome (i.e., Proposition 3 part (e)), the effect in (i) vanishes (since in both BIA and BAI, supplier 1 is effectively out of the race), so other suppliers earn weakly higher profits in BIA.

We have shown so far that some suppliers may earn less *ex ante* profit in BIA than BAI when suppliers' cost distributions and cost-reduction distributions are sufficiently heterogeneous. However, we also would like to point out that in BIA, when the buyer sets out to investigate suppliers for cost-saving opportunities in Step 2, suppliers have an incentive to cooperate and let the buyer conduct cost-reduction investigations: Blocking buyer's investigation would weakly reduce supplier's chance of winning the contract and keep his expected profit conditioning on him winning unchanged. Formally, the following holds.

Proposition 4. *Under* BIA, ex post*, any supplier would weakly prefer engaging in the buyer's investigation regardless of the preferences of the others.*

Hence, we believe BIA is a practical option: the buyer can make a more informed supplier selection decision and enjoy cost savings by conducting cost-reduction investigations on the suppliers she chooses, knowing that it is also in the suppliers' interest to engage in such investigations.

6. Buyer's Choice Between BIA and BAI

The main trade-off the buyer faces in choosing between BIA and BAI is the tension between being able to identify a larger cost-saving and experiencing a higher level of bidding competition: On the one hand, BIA helps the buyer to identify, on average, a larger cost-saving since it allows the buyer to investigate *multiple* suppliers for cost-reduction and pick the best one; on the other hand, as discussed in §5, the possibility of multiple investigations gives rise to scenarios where a supplier with the lowest initial bid loses the contract to another supplier with higher initial bid but a large cost-reduction, which in turn reduces suppliers' incentives to bid aggressively in the first place and results in higher initial bids.

While this trade-off for the buyer's choice between BIA and BAI is quite natural, the factors of the buyer's procurement environment (i.e., different model parameters) *jointly* influence the cost comparison between these two mechanisms in a non-trivial way. To untangle these effects, it is instructive to start with the impact of the investigation cost. Note that when investigation is quite expensive, the buyer would only investigate one supplier in BIA, the same as in BAI; as a result, BIA has neither the advantage of identifying a larger cost-reduction nor the disadvantage of reducing the bidding competition, and will have the same expected cost as BAI. However, BAI and BIA will be quite different when the cost of investigation is not too expensive. In particular, in the extreme case when the cost of investigation is zero, the buyer would investigate all suppliers in BIA. In this case, the relative strengths of the advantage and disadvantage of BIA depend on other model parameters. For example, if the range of the cost distribution, which we denote by $\Xi^c := \bar{c} - \underline{c}$ (i.e., the length of the cost distribution support) is small, the disadvantage of BIA will be less pronounced since the extent to which the bidding could inflate in BIA is reduced. Similarly, if the number of suppliers or the variability of cost-reduction distribution is large, then the maximum of the multiple cost-reduction draws in BIA will take a more extreme value, making the advantage of BIA more pronounced. These insights are formally stated in the analytical results below (note that for expositional clarity, we focus on the setting where suppliers are ex ante symmetric and the maximum number of investigations equals the number of suppliers (i.e., $N = \hat{N}$), but similar insights extend naturally for the more general setting when suppliers are ex ante asymmetric and $\hat{N} \leq N$, which we omit for brevity).

THEOREM 3. *Suppose that $\Delta_i^c = \Delta^c$, $\Delta_i^t = \Delta^t$ for all $i = 1, \ldots N$, and $\hat{N} = N$. Then:*

(a) If $\Delta^t + \underline{t} > 0$, then there exists a threshold $d^\star < \Delta^t$ such that BIA and BAI result in the same expected cost for the buyer when $d > d^\star$.

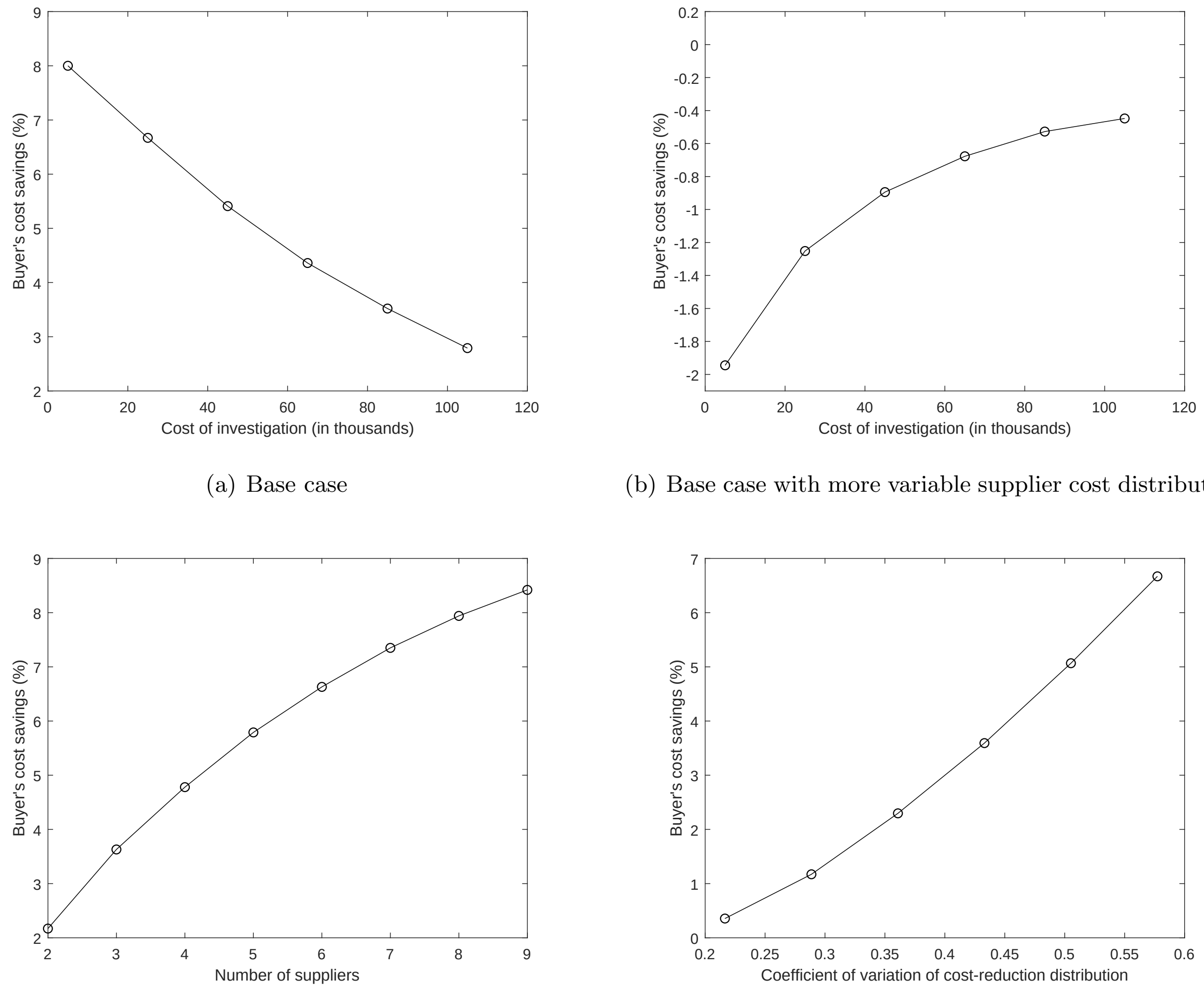
(b) If $d = 0$, and, in addition, $\Xi^c < \max\{\Delta^c + \bar{c} - \Delta^t - \bar{t}, \bar{t}\}$, then there exists a threshold $N^\star$ and a decreasing function $\sigma^\star : [N^\star, \ldots, \infty) \to [1, \infty)$ such that the buyer's expected cost is lower under BIA than BAI if $N \geq N^\star$, and $\{\epsilon_i^t\}_{i=1}^N$ are all multiplied by a scaling parameter σ that is greater or equal to $\sigma^\star(N)$.

While it is challenging to derive analytical results in scenarios when d is neither very high nor very low, our numerical examples in Figure 2 indicate that the insights we have discussed hold for these scenarios as well. For example, Figure 2(a) validates our analytical results on the two extreme scenarios of d (i.e., when d becomes very large, the difference between BIA and BAI converges to zero, which is consistent with Theorem 3 part (a); however, when d is zero and the cost distribution range is narrow, BIA is cheaper than BAI as Theorem 3 part (b) predicts). The figure also suggests that, as one might expect, when d is between the extreme scenarios, the percentage cost-savings of BIA over BAI is positive and changes monotonically. A similar monotone pattern shows up in Figure 2(b), except that because the cost distribution range is much wider than the example in Figure 2(a), BIA is more expensive than BAI. Moreover, Figures 2(c) and 2(d) further illustrate that when the cost distribution range is narrow, the results in Theorem 3 part (b) also hold when d is neither very high nor very low, i.e., the percentage cost-savings of BIA over BAI increases as the number of suppliers increases (see Figure 2(c)) and as the variability of cost-reduction distributions increases (see Figure 2(d)).

Our discussion in this section highlights, qualitatively, the main trade-off at play in the buyer's choice between BIA and BAI. In §8 we will turn to the task of more thoroughly quantifying the magnitude of the cost savings that BIA can provide when it is preferred by the buyer. Before doing so, in the next section we introduce an optimal mechanism analysis that serves as an additional benchmark.

7. Optimal Mechanism Analysis

Our simple approach, using the cheaper option between BIA and BAI, is just one feasible approach among many that a buyer can use to integrate investigations into supplier selection. It is possible that the buyer could achieve an even lower expected cost by a more complicated approach with multiple rounds of bidding as more cost-reduction information is revealed during the procurement process: For example, one possibility is that after identifying a large cost-reduction for a particular supplier, the buyer could disclose this cost-reduction information to other suppliers so as to lure



(a) Base case

(b) Base case with more variable supplier cost distribution

(c) Base case with varying number of suppliers

(d) Base case with more variable cost-reduction distribution

Figure 2 Percentage cost savings of BIA over BAI.

Note: In Figure 2(a), we take the same example in Figure 1 with a stretched cost-reduction distribution (i.e., uniform distribution between \$0 and \$16), and vary the cost of investigation. Figure 2(b) shows how the graph in Figure 2(a) changes when the support of cost distribution becomes much wider (i.e., uniform distribution between \$27.5 and \$52.5 instead of uniform distribution between \$37.5 and \$42.5). Figure 2(c) takes the example in Figure 2(a), and then varies the number of suppliers N while keeping $\hat{N} = N$. Figure 2(d) takes the example in Figure 2(a), and then varies the coefficient of variation of cost-reduction distributions by considering uniform distributions with the same mean of \$8 but different distribution range.

them into submitting a more competitive pricing bid for the contract. While it is not clear whether such a more complicated approach would be feasible for the buyer to use in practice, a buyer may still want to know, compared to our simple approach, how much additional savings a more complicated approach can provide. Understanding this provides a way to assess the effectiveness of our simple approach. Since it is impossible to exhaust all possible approaches and analyze them one by one, we resort to an optimal mechanism analysis to find the approach which minimizes the buyer's expected total cost. The idea is that while the optimal mechanism turns out to be very difficult (if not impossible) to implement in practice as will be evident shortly, analysis of the optimal mechanism provides a theoretical lower bound for the optimal cost that a buyer can achieve, providing a benchmark which we will use in our numerical study (see §8) to gauge the effectiveness of our simple approach.

Note that any approach (i.e., mechanism) the buyer uses induces a (dynamic) game with asymmetric information and strategic communications. Note that in our setting, the additional exogenous information which could be revealed in the game is the realization of the cost-reductions for each of the N suppliers. Thus, to introduce the optimal mechanism design problem that encompasses approaches that allow multiple rounds of communication (e.g., suppliers bidding) as more cost-reduction information is revealed, we consider a multistage dynamic game with $N+1$ players, i.e., one buyer and N suppliers, who interact in N stages. Below we describe, at a high level, the sequence of four events that occur in each stage of the game:

E1. A signal vector is drawn and each player observes a component of this signal vector (i.e., in the first stage, each supplier observes his production cost; in later stages, if an investigation occurs in the previous stage, say supplier i is investigated, then both the buyer and supplier i observes the investigated supplier i's cost-reduction; otherwise, no signal is observed);

E2. Each supplier chooses a message to send to the buyer (e.g., their bids);

E3. The buyer chooses a message to send to each supplier (i.e., the buyer, either fully or partially, discloses to the suppliers information that relates to suppliers' previous messages and previously revealed cost-reduction information; of course, the buyer can also choose to disclose no information);

E4. Each player takes an action (i.e., the buyer decides whether to investigate one more supplier and, if the answer is yes, she decides which supplier to investigate; otherwise, the buyer ends the process by awarding the contract and making payments; the suppliers decide, in each stage, whether or not to keep participating in the mechanism).

Following the literature on optimal mechanism design, we assume that the buyer has full commitment power[7]; thus, the buyer can induce different equilibrium outcomes by committing to different combinations of information disclosure rule (in **E3**), investigation rule (in **E4**), awarding of contract (in **E4**) and payment rule (in **E4**), which the buyer will strictly follow once the mechanism

[7] Note that this is different from the line of work on mechanism design with imperfect commitment (e.g., Bester and Strausz (2001), Bester and Strausz (2000)). However, we would like to note that, if a buyer does not have full commitment power, the equilibrium outcome in such a case can be replicated by a fully-committed buyer who commits to the equilibrium outcome of the former case; in other words, assuming full commitment power permits a wider class of feasible mechanisms which means that the optimal cost we derived is a lower bound of the optimal cost a buyer with limited commitment power can achieve. Thus, for our purpose of identifying a lower bound of the optimal cost, focusing on a buyer with full commitment power would only help us identify a stronger benchmark to compare our proposed approach with in §8, and provide a more conservative assessment of how effective our proposed simple approach is.

starts. Thus, the buyer's optimal mechanism design problem is to choose a mechanism (i.e., the combination of information disclosure, investigation, allocation and payment rules) to minimize her expected procurement cost. We would like to highlight that the class of mechanisms we discussed above is very general in that it not only encompasses BIA, BAI, and our simple approach (using the cheaper of the two) as special cases, but also allows a wide variety of possible dynamics with respect to communications between the buyer and the suppliers.

Note that since the suppliers may receive additional payoff relevant information (e.g., their cost-reductions) after the buyer commits to the mechanism, the mechanism design problem in our setting is akin to a dynamic mechanism design problem. To solve the optimal mechanism design problem, we find it convenient to invoke a dynamic version of the Revelation Principle which has been developed in Myerson (1986) and later refined and formalized in Sugaya and Wolitzky (2021). By using this Revelation Principle, an optimal mechanism, which we denote by OPT, can be characterized below:

THEOREM 4. *Define, for any* $\mathbf{c}$*, supplier* i*'s index as* $\hat{\mu}_i(c_i) := \psi_i(c_i) - \tau_i$*, where* $\psi_i(c_i) := c_i + \frac{F_i(c_i)}{f_i(c_i)}$ *is supplier* i*'s* virtual cost *and* τ_i *is as defined in Theorem 1. Rank suppliers by their indices from low to high and let* $\hat{\kappa}(i)$ *denote supplier* i*'s ranking and let* $\hat{\imath}(k)$ *denote the supplier who is ranked* k*. Define* $\tilde{N} := \min\{n : \tau_{\hat{\imath}(n+1)} = 0\}$ *and* $\bar{N} := \min\{\hat{N}, \tilde{N}\}$*. Then, an optimal mechanism OPT is characterized as follows:*

- *First, the buyer asks the suppliers to report their true cost.*
- *Then, based on suppliers' reported costs* $\mathbf{c}$*, the buyer investigates the suppliers according to the investigation policy* π^* *defined in Theorem 1 with the minor modifications that* μ_i *and* u_k *are replaced by* $\hat{\mu}_i$ *and* $\hat{u}_k := \min_{i\in\mathcal{N}}\{\psi_i(c_i) - \mathbf{1}_{[i\in\mathcal{I}_k^{\pi^*}]}t_i\}$ *respectively.*
- *Finally, after the investigation policy concludes, let* i *be the supplier with the lowest updated virtual cost* $\psi_i(c_i) - \mathbf{1}_{[i\in S^*(\mathbf{c})]}t_i$ *where* $S^*(\mathbf{c})$ *is the set of investigated suppliers. The buyer awards the contract to supplier* i *and pays him*

$$P_i^*(\mathbf{c}, \mathbf{t}^{S^*(\mathbf{c})}) = \underbrace{c_i - \mathbf{1}_{[i\in S^*(\mathbf{c})]}t_i}_{\textit{updated cost}} + \underbrace{\int_{c_i}^{\bar{c}_i} \mathbf{E}_{\mathbf{t}^{\overline{S^*(\mathbf{c})}}}[A_i^*(z_i, \mathbf{c}_{-i}, \mathbf{t}^{S^*(z_i,\mathbf{c}_{-i})})|\mathbf{t}^{S^*(\mathbf{c})}]\, dz_i}_{\textit{supplier } i\textit{'s markup } (\mathsf{Markup}_i)}, \tag{7}$$

where $\overline{S^*(\mathbf{c})} = \mathcal{N} - S^*(\mathbf{c})$*, and* A_i^* *is the probability of allocating the contract to supplier* i *given suppliers' reported costs and the cost-reductions of the investigated suppliers who are determined according to the investigation policy detailed in the previous bullet point; other suppliers are paid zero.*

Theorem 4 states that the optimal investigation policy of the optimal mechanism is the same as the investigation policy in BIA except that in computing the index for supplier i, the initial bid b_i is replaced by supplier i's virtual cost $\psi_i(c_i)$. The supplier with the lowest updated virtual cost wins the contract and gets paid his updated cost plus a mark-up. For expositional clarity, we provide a detailed derivation of $\mathsf{Markup}_i(\mathbf{c}, \mathbf{t}^{S^*})$ in (7) and a detailed discussion of what it represents in Appendix A.3. The main take-away from Appendix A.3 is that, inflating one's true cost not only would directly make his virtual cost (and hence his updated virtual cost) less competitive but may also de-prioritize his investigation which could decrease his chance of winning the contract; thus, we can interpret Markup_i as the extent to which the winning supplier i could inflate his bid but still win the contract. We want to point out that while OPT results in a lower expected total cost for the buyer compared to our simple approach, it may be very difficult to implement in practice because the payment rule characterized in (7) is very complex (see (A.20) in Appendix A.3 for the explicit expression in terms of model primitives) and the buyer may find it very difficult to even explain the mechanism to the suppliers. Furthermore, a priori, the buyer needs to commit to OPT which need not be sequentially rational for the buyer; this adds another layer of difficulty for implementation. By contrast, in BIA and BAI, the buyer's investigations are always sequentially rational. While OPT is hard to implement in practice, it does provide a lower bound of the best achievable cost for the buyer in practice. We use this lower bound to numerically evaluate the performance of our simple approach in §8.

8. Effectiveness of the Proposed Approach

In this section, we use an extensive numerical study to investigate the effectiveness of our proposed simple approach, i.e., using the cheaper of BAI and BIA, from two perspectives. First, we want to quantify the magnitude of the additional cost savings the buyer can achieve by adding BIA as another sourcing option alongside BAI. Second, we want to gauge the optimality gap between our simple approach and the theoretically optimal yet difficult to implement mechanism OPT from §7.

In order to help disentangle the effects of cost-reduction investigations from the competitive supplier selection, we will introduce Bid-Award (only a competitive supplier selection in the form of a first-price sealed-bid reverse auction), or BA for short, as a base benchmark. To that end, we calculated the buyer's expected total cost and the suppliers' *ex ante* expected profit in BA, BAI, BIA and OPT, respectively, in a wide range of situations by generating 162 problem scenarios with different model parameters. We let the per-unit production cost be uniformly distributed with an average of \$40, and let the range of suppliers' cost distribution support to be at most 5%, 12.5%, and 20% of the average. To capture a wide range of possible variability of the per-unit cost-reduction distribution, we vary its coefficient of variation (CV) by incorporating uniform distributions ($CV < 1$), exponential distributions ($CV = 1$), and hyperexponential distributions ($CV > 1$). (For exponential and hyperexponential distributions, we need to truncate them in order to ensure that the updated cost is nonnegative. To keep our numerical study tractable, we focus on the *ex ante* symmetric supplier case with $\hat{N} = N$.) Table 1 below summarizes the factorial design of our numerical study in more detail.

Table 1 Factorial design of the numerical study. $CU1, CU2, CU3, TU1, TU2$ **are all uniform distributions, and their respective supports are** $[\$39, \$41]$, $[\$37.5, \$42.5]$, $[\$36, \$44]$, $[\$0, \$8]$, $[\$0, \$16]$. $TE1, TE2$ **are exponential distributions with mean** 4 **and** 8 **respectively which are then truncated by a support of [\$0, \$34] to ensure that the updated cost is always non-negative.** $TH1, TH2$ **are both hyperexponential distributions which are truncated by the same support as before. Specifically, before truncation,** $TH1$ **is an equal chance mixture of two exponential distributions with mean 2 and 6, while** $TH2$ **is an equal chance mixture of two exponential distributions with mean 4 and 12.**

Parameters	Values
Number of units in the contract	50,000
Per-unit production cost distributions	CU1, CU2, CU3
Per-unit cost-reduction distributions	TU1, TU2, TE1, TE2, TH1, TH2
Investigation cost	\$5000, \$25,000, \$50,000
Number of suppliers	3, 6, 9

We report key summary statistics of the performance of different mechanisms for all 162 scenarios in Table 2. Compared to BA which does not conduct any cost-reduction investigations at all, investigating the winner of the contract (i.e., BAI) leads to a cost-saving of 13.3% on average. If the buyer also considers the BIA option, and chooses whichever of the two options (BIA and BAI) has lower cost, then the average cost savings over BA increases to 19.6%, a 47.7% improvement of the cost-savings achieved by only using BAI. More strikingly, the optimal mechanism OPT does not provide much additional savings: It achieves an average cost saving of 20.9% over BA, which corresponds to a 56.9% improvement of the cost savings achieved by only using BAI. In summary, our simple approach not only provides sizable improvement in cost savings compared to the more conventional approach BAI, but also achieves nearly the performance of the optimal mechanism which is, per our discussion in the previous section, extremely difficult to implement in practice. We would also like to reiterate that our simple approach is straightforward: It is a combination of a sealed-bid first-price auction and cost-reduction investigations, both of which are common in practice. Our numerical study highlights that a simple sourcing mechanism can achieve near-optimal results.

Not only does the buyer achieve significant cost-saving improvement using our simple approach, the whole supply chain also achieves a cost improvement of 11.9% compared to BAI. This indicates that, while both BAI and BIA have the feature of investigation which helps improve supply chain welfare compared to BA, the fact that BIA leverages the knowledge of the investigation outcomes to make a more informed contract allocation allows it to achieve more supply chain welfare gains over

Table 2 **Summary statistics for the buyer's absolute cost-savings (in thousand dollars) and percentage cost-savings of different mechanisms over BA.**

	BAI		min{BAI, BIA}		OPT	
	Absolute	Percentage	Absolute	Percentage	Absolute	Percentage
Mean	259.8	13.3%	383.7	19.6%	407.5	20.8%
Standard Deviation	91.1	4.7%	182.4	9.3%	181.1	9.3%
1^{st}-percentile	147.1	7.4%	147.1	7.4%	162.6	8.2%
25^{th}-percentile	174.7	8.9%	209.5	10.9%	235.0	12.1%
50^{th}-percentile	245.9	12.6%	389.1	19.6%	417.6	20.9%
75^{th}-percentile	350.0	17.7%	514.4	26.4%	540.7	27.3%
99^{th}-percentile	395.0	20.6%	818.2	41.8%	826.5	42.5%

BA than BAI does. Moreover, in contrast to BAI where the suppliers capture 0% of the added supply chain welfare, the suppliers capture 3.52% of the added supply chain welfare in BIA. While it may seem that suppliers only capture a small portion of the added supply chain welfare in BIA, due to the thin margins caused by contract competition, this small improvement in revenue actually has a very large effect on their profits: the suppliers achieve on average 165.9% improvement in profit in our proposed simple approach compared to BAI. Our numerical results not only validate our theoretical prediction but also illustrate that the "win-win" can be quite economically significant.

9. Extensions

In this section, we consider three extensions of the base model and show that our main analytical results generalize to these settings: In the first extension, we allow the buyer to share a fraction of the direct cost-saving benefit of investigation with the contract winner; in the second extension, we allow the suppliers' cost and cost-reduction distributions to be stochastically correlated; in the third extension, the buyer is allowed to conduct one single round of *parallel* investigations of multiple suppliers in BIA rather than sequential investigations.

9.1. Cost Savings Shared with the Contract Winner

In the base model, we assume that only the buyer enjoys all the direct benefit of the buyer's investigation, i.e., the buyer's final contract payment to the contract winner equals his initial bid minus *all* of the cost savings identified by the buyer. In practice, however, it is not uncommon for the buyer and the contract winner to split the benefit of the identified cost savings. To capture this, we consider an extension in which the buyer's final payment to the contract winner is different from the base model when the contract winner is investigated, i.e., the payment to the contract winner, say supplier i, equals $b_i - t_i(1-\eta)$ where $\eta \in [0,1)$ captures the fraction of the identified cost-savings that the buyer shares with the supplier. To avoid trivial cases where it is never worthwhile to do any investigations, we make the following assumption throughout this subsection in lieu of Assumption 2 of the base model.

Assumption 3. $d < \max_{i\in\mathcal{N}}(1-\eta)\Delta_i^t$.

Our next theorem shows that all the main results for the base model generalize to this extension.

Theorem 5. *Under Assumptions 1 and 3, the following statements hold:*

(a) Theorem 1 holds by replacing Δ_i^t, $G(\epsilon^t)$ *and* G_j *with* $(1-\eta)\Delta_i^t$, $G_\eta(\epsilon^t)$ *and* $G_{j,\eta}$ *respectively, where* $G_\eta(\epsilon^t) := G((1-\eta)^{-1}\epsilon^t)$ *and* $G_{j,\eta}(\epsilon) := G_j((1-\eta)^{-1}\epsilon)$.

(b) Theorem 2 holds.

(c) Theorem 3 holds by replacing $d^\star < \Delta^t$ *and* $\Xi^c < \max\{\Delta^c + \bar{c} - \Delta^t - \bar{t}, \bar{t}\}$ *by* $d^\star < (1-\eta)\Delta^t$ *and* $\Xi^c < \max\{\Delta^c + \bar{c} - \Delta^t - \bar{t}, (1-\eta)\bar{t}\}$, *respectively.*

9.2. Correlation between Cost and Cost-reduction

In our base model, we assume that supplier's cost and cost-reduction are stochastically independent. However, it is also possible that the supplier's production cost and cost saving opportunities are stochastically correlated. For example, positive correlation could arise because a supplier with higher production cost may have more room for cost improvement and thus a stochastically larger cost-reduction. Negative correlation could also arise because cost-reduction programs such as lean operations are more likely to be successful if the supplier has already been familiar with lean management and has developed the necessary culture in its workforce, and such amenability with lean operations is negatively correlated with supplier's cost.

To explore the impact of the correlation on our base model, we consider an extension where the cost and cost-reduction distributions have the following correlation structure:

$$t_i = \alpha(c_i - \mathbf{E}[c_i]) + \Delta_i^t + \epsilon_i^t, \tag{8}$$

where α parameterizes the level of correlation between cost and cost-reduction distributions. The correlation coefficient of c_i, t_i, which we denote by $\rho(c_i, t_i)$, equals

$$\rho(c_i, t_i) = \operatorname{sgn}(\alpha)\left(1 + \frac{\mathbf{Var}(\epsilon_i)}{\alpha^2\mathbf{Var}(c_i)}\right)^{-1/2},$$

where $\operatorname{sgn}(x)$ is the sign function. Thus, when $\alpha > 0$, cost and cost-reduction are positively correlated; when $\alpha < 0$, cost and cost-reduction are negatively correlated. Moreover, as $|\alpha|$ increases, the level of correlation strengthens. Note that the special case where $\alpha = 0$ reduces to the base model. For simplicity, throughout this subsection, we make the following assumption which can be viewed as adapting Assumption 2 to our correlation setting: Similar to Assumption 2 for the base model case, this assumption helps us to focus on the setting where the cost of investigation is not so large that it is never worthwhile to do any investigation for certain supplier cost types.

Assumption 4. $d < \max_{i\in\mathcal{N}}\{\min_{c_i\in C_i}\alpha(c_i - \mathbf{E}[c_i]) + \Delta_i^t\}$.

In the analysis to follow, we focus on symmetric increasing Bayesian Nash equilibria, i.e., higher cost-type suppliers bid higher prices than lower cost-type suppliers.

Note that correlation between cost and cost-reduction distributions introduces an additional layer of complexity to the buyer's investigation and contract allocation decisions in both BAI and BIA: Since the buyer's payment is affected by the winning supplier's cost-reduction, which is correlated with the winner's true cost, the buyer could, based on the observed bids, form posterior beliefs about suppliers' cost-reduction, and then make the investigation decisions and contract allocation accordingly. In equilibrium, such inference should be consistent with the equilibrium bidding functions. That is, suppose the equilibrium bidding functions are $\{\beta_i : C_i \to B_i\}_{i\in\mathcal{N}}$; then, for any i, let $\mathcal{B}_i := \{b : \beta_i(c_i), c_i \in C_i\} \subseteq B_i$ denote the range of β_i, and let $\beta_i^{-1} : \mathcal{B}_i \to C_i$ denote its inverse function. Thus if the buyer observes that supplier i bids $b_i \in \mathcal{B}_i$, then the buyer's posterior of supplier i's cost is $\hat{c}_i = \beta_i^{-1}(b_i)$ with probability one, and her posterior of supplier i's cost-reduction equals $\hat{t}_i = \alpha(\hat{c}_i - \mathbf{E}[c_i]) + \Delta_i^t + \epsilon_i^t$. Obviously, this updated belief will affect the buyer's investigation and contract allocation, which in turn will also affect the suppliers' equilibrium bidding strategy. In order to specify the buyer's belief on off-equilibrium paths (i.e., when supplier i bids $b_i \notin \mathcal{B}_i$), we simply set $\hat{c}_i = \underline{c} + \Delta_i^c$ when $\alpha < 0$, and $\hat{c}_i = \bar{c} + \Delta_i^c$ when $\alpha > 0$. This means that when a supplier's bid does not correspond to the equilibrium bid of any supplier cost type, the buyer would believe that this supplier has the stochastically lowest possible cost-reduction distribution; such beliefs provide an incentive for suppliers to follow the equilibrium.

THEOREM 6. *Suppose an equilibrium exists for BIA and BAI. Under Assumptions 1 and 4,*

(a) Theorem 1 holds by replacing $\tilde{\tau}_i = \tau + \Delta_i^t$ *with* $\tilde{\tau}_i = \alpha(\beta_i^{-1}(b_i) - \mathbf{E}[c_i]) + \tau + \Delta_i^t$.

(b) There exists some positive constant $\tilde{\alpha} > 0$ *such that for all* $\alpha < \tilde{\alpha}$, *Theorem 2 holds.*

(c) Theorem 3 holds by replacing Δ^t *and* $\Xi^c < \max\{\Delta^c + \bar{c} - \Delta^t - \bar{t}, \bar{t}\}$ *with* $\min_{c_1\in C_1}\{\alpha(c_1 - \mathbf{E}[c_1])\} + \Delta^t$ *and* $\Xi^c < \max\{\Delta^c + \bar{c} - \max_{c_1\in C_1}\{\alpha(c_1 - \mathbf{E}[c_1])\} - \Delta^t - \bar{t}, \frac{\bar{t}}{1+|\alpha|}\}$, *respectively.*

While the correlation of cost and cost-reduction distributions does not significantly change the result on the buyer's optimal investigation rule in BIA (i.e., Theorem 6 part (a)), nor the buyer's optimal choice between BIA and BAI (i.e., Theorem 6 part (c)), it does meaningfully change the result on suppliers' profit comparison, and thus the condition under which the inclusion of BIA results in win-win for the buyer and the suppliers. Specifically, Theorem 6 part (b) shows that the presence of win-win when suppliers are ex ante symmetric also depends on the correlation structure between cost and cost-reduction distributions: Win-win holds if cost and cost-reduction are negatively correlated, or independent, or are not strongly positively correlated. Why is this the case? Recall that we have explained in the base model (i.e., when $\alpha = 0$) that there is less competition in the Bid step in BIA which results in a higher supplier equilibrium bidding function than in BAI, and hence suppliers benefit from higher ex ante profit in BIA. The presence of correlation adds another layer of complexity to suppliers' bidding incentives. It can be shown that when an equilibrium exists in both BIA and BAI, the equilibrium bidding function in BAI remains the same as in the base model with no correlations; in contrast, the equilibrium bidding function in BIA changes as the correlation level varies. To see how it changes, note that in BIA, a supplier's winning probability not only depends on his bid (e.g., it affects whether, and if so, when the buyer investigates him) but also his true cost when it is correlated with cost-reduction (i.e., his assessment of his cost-reduction is correlated with his true cost, but not his bid). This means that, when correlation is positive, supplier i with cost c_i's chance of winning in BIA when he pretends to be a higher cost-type $c_i' > c_i$ is smaller than the equilibrium winning probability of cost-type c_i', because supplier i's cost-reduction distribution does not become stochastically larger if he bids as cost-type c_i'; in contrast, when there is no correlation, imitating a higher cost-type c_i' will result in the same winning probability a cost-type c_i' gets in equilibrium. As a result, there is less incentive to imitate higher cost-type suppliers in BIA when correlation is positive, so as correlation increases, the equilibrium bidding function stretches downward, reducing the supplier's ex ante profit in BIA. By a similar argument, when correlation is negative, then suppliers' equilibrium bidding curve is weakly higher than the no correlation case, so suppliers enjoy higher ex ante profit.

With these results, we now adapt the example in Figure 1 to illustrate the new insight that supplier's relative ex ante profit gains in BIA over BAI *decreases* as the correlation coefficient between cost and cost-distribution increases. As this example illustrates, suppliers earn less ex ante profit in BIA only when the correlation coefficient is higher than 0.6; this suggests that while in the ex ante symmetric case win-win may break down if cost and cost-reduction are correlated, such breakdown occurs only when the correlation is sufficiently high, i.e., when suppliers with high costs are highly likely to have cost-reductions that are large enough to put their updated costs on relatively even footing with suppliers whose initial production costs are much lower.

9.3. Parallel Investigations

When there is a very stringent deadline for awarding the contract (e.g., long production lead time, suppliers unwilling to wait due to other business opportunities in the market), sequential investigations may be too time-consuming. Instead, the buyer may investigate a subset of suppliers simultaneously. Note that the analysis of BAI remains the same as in the base model; in the remainder of this subsection, we formally introduce the BIA mechanism with parallel investigations and then show that the main managerial insights of the base model hold in this extension.

The BIA mechanism with parallel investigations, which we denote by BIA_p, works in the same way as BIA except that in the second step Investigate, the buyer may choose to investigate some

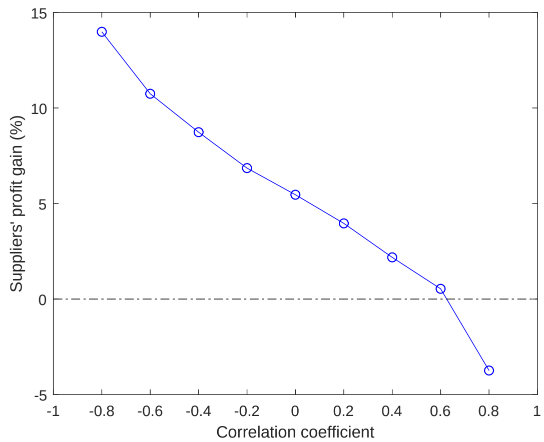


Figure 3 Percentage suppliers' ex ante profit gains of BIA over BAI.

Note: We use the same model parameters as in Figure 1 except that, in order to capture the impact of correlation between cost and cost-reduction distributions, we use the model in (8) for the per-unit cost-reduction distribution and vary α to obtain a range of different correlation coefficients between cost and cost-reduction distribution: In this model, $\Delta^t = 4$, and ϵ_i^t is uniform between $[-1.2, 1.2]$ (we chose a smaller range of ϵ_i^t than in Figure 1 to account for the extra variation of cost-reduction caused by correlation).

of the suppliers *simultaneously* for cost-reduction opportunities. Same as before, we use backward induction to conduct equilibrium analysis. In the Investigate step, since the investigation decisions must be made *simultaneously before* any cost reduction is revealed, the investigation policy only depends on the bids, i.e., $\mathbf{b}$. Denote by $S_p^*(.) : B \to \{\mathcal{I} \subseteq \mathcal{N} : |\mathcal{I}| \leq \hat{N}\}$ the buyer's parallel investigation policy that minimizes her expected total cost given the initial price $\mathbf{b}$, where $\hat{N}$ is the maximum number of suppliers that can be investigated. Then, anticipating that the buyer will conduct investigations according to S_p^*, each supplier i chooses a bidding strategy $\beta_i : C_i \to B_i$ that forms an equilibrium.

Theorem 7. *Under Assumptions 1 and 2, the following hold:*

(a) For any suppliers' bids $\mathbf{b} = (b_1, \ldots, b_N)$, define $\tilde{N}$, $\bar{N}$, $\mu_i(b_i)$, $\kappa(i)$ and $\iota(i)$ the same way as in Theorem 1. Define $\mathcal{I}_0 = \emptyset, \mathcal{I}_j = \{\iota(1), \ldots, \iota(j)\}$ for all $j = 1, \ldots, \bar{N}$. Then: (1) For any rankings of suppliers based on their indices, there exists a set of thresholds $\{\tau_{\iota(j)}(\mathbf{b}_{-\iota(j)})\}_{j=1}^{\bar{N}}$ such that $S_p^*(\mathbf{b}) = \mathcal{I}_{j^*(\mathbf{b})}$, where $j^*(\mathbf{b}) = \max\{j : b_{\iota(j)} \leq \tau_{\iota(j)}(\mathbf{b}_{-\iota(j)})\}$, is the optimal parallel investigation policy; (2)Supplier j's chance of being investigated, i.e., $\mathbf{1}_{[j \in S_p^*(\mathbf{b})]}$, is nonincreasing in d, b_j and Δ_i^t for all $i \neq j$, and nondecreasing in Δ_j^t and b_i for all $i \neq j$.

(b) Theorem 2 holds by replacing BIA with BIA$_p$.

(c) Theorem 3 holds by replacing BIA with BIA$_p$.

Theorem 7 part (a1) states that the buyer's parallel investigation policy has a nested structure: For any bid vector $\mathbf{b}$, the optimal investigation set can only be one of the $\bar{N} + 1$ candidate sets that are nested in the sense that the n^{th} set contains the first $n - 1$ suppliers that have the lowest indices. The optimal one among those candidate sets is jointly determined by $\bar{N}$ thresholds respectively designated for each supplier (for any given $\mathbf{b}$). Specifically, the optimal investigation set is the largest candidate set where the bids of the suppliers in that set are all smaller than their respective thresholds. Theorem 7 part (a2) states how the investigation decisions depend on the model parameters, which is in line with the insights in the sequential investigation case.

10. Closing Remarks

Observing that it is a common practice for buyers to investigate suppliers for cost-saving opportunities, we explore the idea that the buyer may want to incorporate cost-reduction investigations into supplier selection by inserting an investigation step in the middle of the conventional bid-award process, i.e., the BIA mechanism studied in this paper. We show that the buyer's optimal investigation policy in BIA takes the form of a simple index rule, and establish conditions under which front-loading investigations pre-award results in a win-win for the buyer and the suppliers compared to the more conventional BAI where only the contract winner can be investigated for cost-saving opportunity. We also investigate the tradeoff in the buyer's optimal choice between BIA and BAI and highlight how it is affected by the underlying business conditions (model parameters). Our numerical study reveals that BIA, which would be a straightforward concept for practitioners, can significantly reduce buyer's cost compared to BAI; moreover, our simple approach of choosing BIA or BAI (whichever is cheaper) captures most of the benefit offered by the theoretically optimal mechanism which is extremely difficult to implement in practice. We also consider several important model extensions to capture a broader range of realistic problem contexts, which enrich the managerial insights and underscore the robustness of our findings. We believe our simple approach can be a powerful tool for the buyers to reduce their procurement cost.

References

Arozamena, L., E. Cantillon. 2004. Investment incentives in procurement auctions. *Review of Economic Studies* **71** 1–18.

Athey, S. 2001. Single crossing properties and the existence of pure strategy equlibria in games of incomplete information. *Econometrica* **69** 861–890.

Bag, P. K. 1997. Optimal auction design and r & d. *European Economic Review* **41** 1655–1674.

Balseiro, S., D. Brown. 2019. Approximations to stochastic dynamic programs via information relaxation duality. *Oper. Res.* **67** 577–597.

Beil, Damian R, Qi Chen, Izak Duenyas, Brendan D See. 2018. When to deploy test auctions in sourcing. *Manufacturing & Service Operations Management* **20**(2) 232–248.

Bernstein, F., A. G. Kok. 2009. Dynamic cost reduction through process improvement in assembly networks. *Management Sci.* **55** 552–567.

Bester, Helmut, Roland Strausz. 2000. Imperfect commitment and the revelation principle: the multi-agent case. *Economics Letters* 165–171.

Bester, Helmut, Roland Strausz. 2001. Contracting with imperfect commitment and the revelation principle: The single agent case. *Econometrica* 1077–1098.

Cantillon, E. 2008. The effect of bidders' asymmetries on expected revenue in auctions. *Games and Economic Behavior* **62** 1–25.

CAPS, Research. 2009. Caps research snapshots: Use of reverse auctions. *Institute for Supply Management and W.P. Carey School of Business at ASU* .

Che, Y. K. 1993. Design competition through multidimensional auctions. *RAND Journal of Economics* **24** 668–680.

Chen, W., M. Dawande, G. Janakiraman. 2018. Optimal procurement auction under multi-stage supplier qualification. *Manufacturing Service Oper. Management* **20** 566–582.

Chen, Ying-Ju, Sridhar Seshadri, Eitan Zemel. 2008. Sourcing through auctions and audits. *Production and Operations Management* 121–138.

Chu, Leon Yang, Ying Rong, Huan Zheng. 2020. The strategic benefit of request for proposal/quotation. *Operations Research* .

Davis, Andrew M, Bin Hu, Kyle Hyndman, Anyan Qi. 2021. Procurement for assembly under asymmetric information: Theory and evidence. *Management Science* .

Doval, L. 2018. Whether or not to open pandora's box. *Journal of Economic Theory* **175** 127–158.

Elmaghraby, W. 2007. Auctions with e-sourcing events. *Production and Operations Management* **16** 409–422.

Gal-Or, Esther, Mordechai Gal-Or, Anthony Dukes. 2007. Optimal information revelation in procurement schemes. *The RAND Journal of Economics* **38**(2) 400–418.

Hu, Bin, Anyan Qi. 2018. Optimal procurement mechanisms for assembly. *Manufacturing & Service Operations Management* **20**(4) 655–666.

Iyer, A. V., L. B. Schwarz, S. A. Zenios. 2005. A principal-agent model for product specification and production. *Management Sci.* **51** 106–119.

Jiang, Li. 2015. Managing supplier competition and sourcing sequence for component manufacturing. *Production and Operations Management* **24**(2) 287–310.

Jin, Yannan, Qiying Hu, Sang Won Kim, Sean X. Zhou. 2019. Supplier development and integration in competitive supply chains. *Production and Oper. Management* **28** 1256–1271.

Kalkoffen, M., Z. Momin, X. Mosquet, J. Singh, G. Sticher. 2007. Getting to win-win: How toyota creates and sustains best-practice supplier relationships. *The Boston Consulting Group: Focus* .

Kostamis, D., D. R. Beil, I. Duenyas. 2009. Total-cost procurement auctions: Impact of suppliers' cost adjustments on aution format choice. *Management Sci.* **55** 1985–1999.

Krahmer, D., R. Strausz. 2011. Optimal procurement contracts with pre-project planning. *Review of Economic Studies* **78** 1015–1041.

Krishna, V. 2009. *Auction Theory*. Elsevier.

Lebrun, B. 1999. First price auctions in the asymmetric n bidder case. *International Economic Review* **40** 125–142.

Li, C., Z. Wan. 2017. Supplier competition and cost improvement. *Management Sci.* **63** 2460–2477.

Li, Cuihong. 2020. Supplier competition and cost reduction with endogenous information asymmetry. *Manufacturing & Service Operations Management* **22**(5) 996–1010.

Mares, V., J. M. Swinkels. 2014. On the analysis of asymmetric first price auctions. *Journal of Economic Theory* **152** 1–40.

Maskin, E. S., J. G. Riley. 2000. Equilibrium in sealed high bid auctions. *Review of Economic Studies* **67** 439–454.

McAfee, R. P., J. McMillan. 1986. Bidding for contracts: A principal-agent analysis. *The RAND Jounral of Economics* **17** 326–338.

Milgrom, P., I. Segal. 2002. Envelope theorems for arbitraty choice sets. *Econometrica* **70** 583–601.

Myerson, R. 1986. Multistage games with communication. *Econometrica* **54** 323–358.

Myerson, R. B. 1981. Optimal auction design. *Math. Oper. Res.* **6** 58–73.

Piccione, M., G. Tan. 1996. Cost-reducing investment, optimal procurement and implementation by auctions. *International Economic Review* **37** 663–685.

Rothkopf, M. H., A. B. Whinston. 2007. On e-auctions for procurement operations. *Production and Operations Management* **16** 404–408.

Santamaria, N. 2015. An analysis of scoring and buyer-determined procurement auctions. *Production and Operations Management* **24** 147–158.

Sugaya, Takuo, Alexander Wolitzky. 2021. The revelation principle in multistage games. *The Review of Economic Studies* **88**(3) 1503–1540.

Tan, G. 1992. Entry and r & d in procurement contracting. *Journal of Economic Theory* **58** 41–60.

U.S. Department of Commerce, Bureau of the Census. 2021. 2018-2019 annual survey of manufactures. available via *https://www.census.gov/programs-surveys/asm/data.html* retrieved august 22, 2021. .

Wan, Z., D. R. Beil. 2009. Rfq auctions with supplier qualification screening. *Oper. Res.* **57** 934–949.

Wan, Z., D. R. Beil, E. Katok. 2012. When does it pay to delay supplier qualification? theory and experiments. *Management Sci.* **58** 2057–2075.

Weitzman, M. L. 1979. Optimal search for the best alternatives. *Econometrica* **47** 641–654.

Zhang, Wen, Qi (George) Chen, Elena Katok. 2021. Now or later? when to deploy qualification screening in open-bid auction for re-sourcing. *Forthcoming in Operations Research* .

Online Appendix

Appendix A.1: Proof of Results

Proof of Proposition 1. Note that the equilibrium bidding functions $\{\check{\beta}_i\}_{i\in\mathcal{N}}$ of a first-price sealed bid auction in which suppliers' costs are $\check{c}_i = c_i - r_i$ for all i must satisfy the following:

$$\begin{aligned}\check{\beta}_i(\check{c}_i) &\in \arg\max_{\check{b}_i\in[\underline{c}_i-r_i,\bar{c}_i-r_i]}(\check{b}_i-\check{c}_i)\mathbf{P}(\check{b}_i\le\check{\beta}_j(\check{c}_j),\forall j\neq i)\\ &=\arg\max_{\check{b}_i\in[\underline{c}_i-r_i,\bar{c}_i-r_i]}(\check{b}_i+r_i-\check{c}_i-r_i)\mathbf{P}(\check{b}_i\le\check{\beta}_j(\check{c}_j),\forall j\neq i).\end{aligned}$$

Let $\tilde{\beta}_i(c_i) := \check{\beta}_i(c_i - r_i) + r_i$ for all i, then the equation above implies that

$$\begin{aligned}\tilde{\beta}_i(c_i) &= \check{\beta}_i(c_i-r_i)+r_i = \check{\beta}_i(\check{c}_i)+r_i\\ &\in \arg\max_{b_i\in[\underline{c}_i,\bar{c}_i]}(b_i-c_i)\mathbf{P}(b_i-r_i\le\tilde{\beta}_j(c_j)-r_j,\forall j\neq i)\\ &=\arg\max_{b_i\in B_i}(b_i-c_i)\mathbf{P}(b_i-r_i\le\tilde{\beta}_j(c_j)-r_j,\forall j\neq i),\end{aligned}$$

where the inclusion follows by a change of variable $b_i := \check{b}_i + r_i$, and the last equality follows since any $b_i\in[0,\underline{c}_i)$ is suboptimal. Thus, $\{\tilde{\beta}_i\}_{i\in\mathcal{N}}$ forms an equilibrium. □

Proof of Theorem 1. Part (a). Define $\delta(x) := \int_x^\infty(\epsilon^t - x)dG(\epsilon^t) - d$. Then, $\delta(x) = \mathbf{E}_{\epsilon^t}[\max\{0,\epsilon^t - x\}] - d$ where the c.d.f. of the random variable ϵ^t is G. Obviously, $\delta(x)$ is continuous in x. Note also that $\delta(x)$ is strictly decreasing in x on $(-\infty,\bar{t}]$, $\lim_{x\to-\infty}\delta(x)=\infty$, and $\delta(x) = -d < 0$ for all $x\ge\bar{t}$. This implies that τ is well-defined and $\delta(\tau)=0$. We now prove part (a) in two steps.

<u>Step 1: $\pi_k^* = \emptyset$ for $k = \bar{N}+1,\ldots,\hat{N}$.</u>

Note that we only need to prove the case when $\bar{N} = \tilde{N} < \hat{N}$. For any supplier j, the following identity holds

$$\delta(\tau-\tilde{\tau}_j) = \delta(-\Delta_j^t) = \int_{-\Delta_j^t}^\infty(\epsilon^t+\Delta_j^t)dG(\epsilon^t) - d = \int_0^\infty t_j dG_j(t_j) - d = \mathbf{E}[t_i] - d.$$

Hence, if $\tau_j = 0$, then $\tilde{\tau}_j \le 0$, which means $\mathbf{E}[t_j]\le d$; thus, it is obvious that investigating supplier j is always suboptimal. Next, take any supplier j such that $\kappa(j)\ge\tilde{N}+1$ and $\tau_j > 0$. Note that it is never optimal to investigate supplier j since the expected net cost savings by doing so is bounded from above by:

$$\begin{aligned}&b_{\iota(\tilde{N}+1)} - \mathbf{E}\left[\min\left\{b_{\iota(\tilde{N}+1)}, b_j - t_j\right\}\right] - d = \mathbf{E}\left[\max\left\{0, b_{\iota(\tilde{N}+1)} - b_j + t_j\right\}\right] - d\\ &\quad= \delta(b_j - \Delta_j^t - b_{\iota(\tilde{N}+1)}) = \delta(\tau + \mu_j(b_j) - b_{\iota(\tilde{N}+1)}) = \delta(\tau + \mu_j(b_j) - \mu_{\iota(\tilde{N}+1)}(b_{\iota(\tilde{N}+1)})) \le 0,\end{aligned}$$

where the last inequality holds since $\mu_j(b_j) - \mu_{\iota(\tilde{N}+1)}(b_{\iota(\tilde{N}+1)}) \ge 0$ (recall that $\kappa(j)\ge\tilde{N}+1$). Hence, it is never optimal to investigate any supplier who ranked $\tilde{N}+1$ or higher. This completes the proof of Step 1.

<u>Step 2: For $k\le\bar{N}$, the optimal investigation policy is (3).</u>

We prove the optimality of the index rule using backward induction. At the beginning of round $k=\bar{N}$ (this can be viewed as the "last round" since it is never optimal to investigate more suppliers afterwards by Step 1), suppose there are m uninvestigated suppliers, and the best cost so far is u. Without loss of generality, assume these suppliers are suppliers $1,2,\ldots,m$, and their indices satisfy $\mu_1(b_1)\le\mu_2(b_2)\le\cdots\le\mu_m(b_m)$. Note that since $\bar{N} \leq \tilde{N}$, $\tau_j = \tilde{\tau}_j > 0$ for all $j = 1, \ldots, m$. If the buyer investigates supplier j in round $\bar{N}$, then the expected additional net cost savings is $u - \mathbf{E}_{t_j}[\min\{u, b_j - t_j\}] - d = \mathbf{E}_{t_j}[\max\{0, u - b_j + t_j\}] - d = \delta(\tau + \mu_j(b_j) - u)$. Hence, the index rule is optimal in the last round. This is the basis of the backward induction.

Suppose that the index policy is optimal for rounds $k+1, k+2, \ldots, \bar{N}$; we now show it is also optimal in round k. Without loss of generality, assume that at the beginning of round k, the lowest cost so far is u, and there are m suppliers, i.e., suppliers $1, 2, \ldots, m$, that have not been investigated; moreover, their indices satisfy $\mu_1(b_1) \leq \mu_2(b_2) \leq \cdots \leq \mu_m(b_m)$. Note that the following lemma holds:

Lemma A.1. *Suppose that there are $m \geq \bar{N} - k + 1$ uninvestigated suppliers whose rankings are no larger than $\bar{N}$ (without loss of generality, assume these are suppliers 1, 2, ..., m, and $\mu_1(b_1) < \cdots < \mu_m(b_m)$) at the beginning of round k, and the lowest cost so far is u. Then for any $j = 1, \ldots, m-1$, investigating supplier j and following the optimal investigation policy in future rounds yields lower cost than investigating supplier $j+1$ and following the optimal investigation policy in future rounds.*

The proof of this lemma is a bit technical, and we defer it to the end of the proof of Theorem 1. Note also that if $\mu_j(b_j) > u$ for *all* supplier $j \leq m$, it is suboptimal to investigate anyone in round k and in later rounds. Indeed, if the buyer investigates some supplier j, then the immediate net cost savings in round k is on average $\delta(\tau + \mu_j(b_j) - u) < 0$, and the future net cost savings will be zero (this is because the lowest cost after this round's investigation is $u' = \min\{u, b_j - t_j\} \leq u \leq \min_{1 \leq j \leq m} \mu_j(b_j)$; so according to the induction hypothesis and the definition of index policy, there will not be any investigations in the future.) This completes the induction step. Hence, the index rule is optimal.

Part (b). By definition of $\tilde{\tau}_i$ and τ, we have $d = \int_{\tilde{\tau}_i}^{\infty}(t_i - \tilde{\tau}_i)dG_i(t_i)$. By integration by parts, $\int_{\tilde{\tau}_i}^{\infty}(t_i - \tilde{\tau}_i)dG_i(t_i) = \int_{\tilde{\tau}_i}^{\infty}(1 - G_i(t_i))dt_i$. Define $K(x, d, G_i) := \int_{x}^{\infty}(1 - G_i(t_i))dt_i - d$. Then, for given d and G_i, $\tilde{\tau}_i$ satisfies $K(\tilde{\tau}_i, d, G_i) = 0$. Obviously, $K(.,.,.)$ is decreasing in the first two arguments. Hence, as d increases, $\tilde{\tau}_i$ decreases. Moreover, fix x and d, as G_i becomes stochastically smaller, K weakly decreases, and hence $\tilde{\tau}_i$ weakly decreases. Since $\tau_i = \max\{0, \tilde{\tau}_i\}$ and is thus nonnegative and nondecreasing in $\tilde{\tau}_i$, the result follows.

Part (c). As μ_i increases, $\kappa(i)$ weakly increases, and hence $u_{\kappa(i)}$ stochastically decreases. As a result, i is less likely to be investigated. Since μ_i is increasing in b_i and nonincreasing in Δ_i^t, supplier i's probability of being investigated is nonincreasing in b_i and nondecreasing in Δ_i^t. Conversely, as μ_j increases for some $j \neq i$, $\kappa(i)$ weakly decreases, and hence $u_{\kappa(i)}$ stochastically increases. As a result, i is more likely to be investigated. Since μ_j is increasing in b_j and nonincreasing in Δ_j^t, supplier i's probability of being investigated is nondecreasing in b_j and nonincreasing in Δ_j^t. As for the comparative statics for cost reduction realizations, supplier i's probability of being investigated weakly decreases in t_j for all j such that $\kappa(j) < \kappa(i)$ since the ranking is independent of $\mathbf{t}$ but $u_{\kappa(i)}$ weakly decreases; whereas it is independent of t_i and t_j for all j such that $\kappa(j) > \kappa(i)$ because whether or not to investigate supplier i is decided before the realization of the cost reductions of supplier i and all other suppliers that have larger rankings. Finally, as d increases, τ decreases. Therefore, $\tilde{N}$ decreases but suppliers' relative rankings do not change. Thus, supplier i is less likely to be investigated due to the increase in d. $\square$

Proof of Lemma A.1 For any $j=1,\dots,m$, let π^j denote the policy that investigates supplier j in round k, and then follows the optimal investigation policy (i.e., the index rule by induction hypothesis) afterwards. We denote by $V(\pi)$ the expected *period-k-cost-to-go* (i.e., sum of payment cost and the investigation costs for all of the rounds starting from k) for any policy π, and by $V(\pi|\mathcal{E})$ the expected *period-k-cost-to-go* conditioning on event $\mathcal{E}$. Then, the sequence of suppliers to be investigated (if they are ever investigated) under π^j (resp. π^{j+1}) is (†) (resp. (††)) defined below:

$$\begin{array}{rl}(\dagger): & j \;\to 1\to 2\to 3\to \dots \to j-1\to j+1\to j+2\to \dots \to m\\ (\dagger\dagger): & j+1\to 1\to 2\to 3\to \dots \to j-1\to \;\; j \;\;\to j+2\to \dots \to m\end{array}$$

For any $i=0,\dots,m-1$, let $\iota(i;\dagger)$ (resp. $\iota(i;\dagger\dagger)$) denote the $(i+1)^{st}$ supplier in sequence (†) (resp. (††)). For any supplier j, let $a_j:=b_j-t_j$. Define $\mathcal{E}_n:=\{\min_{0\le i\le n}\{a_{\iota(i;\dagger)}\}<\mu_{\iota(n+1;\dagger)}\}$ and $\tilde{\mathcal{E}}_n:=\{\min_{0\le i\le n}\{a_{\iota(i;\dagger\dagger)}\}<\mu_{\iota(n+1;\dagger\dagger)}\}$, where we write μ_i in lien of $\mu_i(b_i)$ for notational simplicity. We need to show that $V(\pi^j)\le V(\pi^{j+1})$, which we do below.

For policy π^j (resp. π^{j+1}), consider a collection of straw policies $\{\bar{\pi}^j(n)\}_{n=0}^{\min\{\bar{N}-k,j\}}$ (resp. $\{\bar{\pi}^{j+1}(n)\}_{n=0}^{\min\{\bar{N}-k,j\}}$) defined as follows: The buyer always first investigates the first $n+1$ suppliers according to (†) (resp. (††)), and then follows the index rule. Note that $\pi^j=\bar{\pi}^j(0)$ (resp. $\pi^{j+1}=\bar{\pi}^{j+1}(0)$), and $V(\bar{\pi}^j(n+1)|\mathcal{E}_n)\ge V(\bar{\pi}^j(n)|\mathcal{E}_n)$ (resp. $V(\bar{\pi}^{j+1}(n+1)|\tilde{\mathcal{E}}_n)\ge V(\bar{\pi}^{j+1}(n)|\tilde{\mathcal{E}}_n)$) for all $n=0,\dots,\min\{\bar{N}-k,j\}-1$ by the induction hypothesis. Note that the following holds: For any $n=0,\dots,\min\{\bar{N}-k,j\}-1$

$$\begin{aligned}
&V(\bar{\pi}^j(n+1))-V(\bar{\pi}^j(n))\\
&\quad= \left(V(\bar{\pi}^j(n+1)|\mathcal{E}_n)-V(\bar{\pi}^j(n)|\mathcal{E}_n)\right)\mathbf{P}(\mathcal{E}_n)\\
&\quad= \left\{d+\mathbf{E}\left[\hat{\delta}(u,n,\dagger)|\mathcal{E}_n\right]\right\}\mathbf{P}\left(\left\{\min_{0\le i\le n}\{a_{\iota(i;\dagger)}\}<\mu_{\iota(n+1;\dagger)}\right\}\right)\\
&\quad\ge \left\{d+\mathbf{E}\left[\hat{\delta}(u,n,\dagger\dagger)|\tilde{\mathcal{E}}_n\right]\right\}\mathbf{P}\left(\left\{\min_{0\le i\le n}\{a_{\iota(i;\dagger\dagger)}\}<\mu_{\iota(n+1;\dagger\dagger)}\right\}\right)\\
&\quad= \left(V(\bar{\pi}^{j+1}(n+1)|\tilde{\mathcal{E}}_n)-V(\bar{\pi}^{j+1}(n)|\tilde{\mathcal{E}}_n)\right)\mathbf{P}(\tilde{\mathcal{E}}_n)\\
&\quad= V(\bar{\pi}^{j+1}(n+1))-V(\bar{\pi}^{j+1}(n)),
\end{aligned}\tag{A.1}$$

$$\text{where } \hat{\delta}(u,n,\cdot):=\min\left\{u,\min_{0\le i\le n+1}\{a_{\iota(i;\cdot)}\}\right\}-\min\left\{u,\min_{0\le i\le n}\{a_{\iota(i;\cdot)}\}\right\}.$$

The first equality follows by law of total expectation, and the fact that supplier $\iota(n+1;\dagger)$ is always investigated under $\bar{\pi}^j(n)$ on events $\mathcal{E}_n^c$, so it coincides with $\bar{\pi}^j(n+1)$. The second equality follows since on $\mathcal{E}_n$, $\bar{\pi}^j(n)$ investigates the first $n+1$ suppliers in (†) while $\bar{\pi}^j(n+1)$ investigates the first $n+2$ suppliers in (†). The first inequality follows due to the following: First, for any $i=0,\dots,\min\{\bar{N}-k,j\}-1$, $a_{\iota(i;\dagger)}$ is stochastically smaller than $a_{\iota(i;\dagger\dagger)}$, and $\mu_{\iota(i+1;\dagger)}\ge\mu_{\iota(i+1;\dagger\dagger)}$, so $\mathbf{P}(\mathcal{E}_n)\ge\mathbf{P}(\tilde{\mathcal{E}}_n)$; second, note that the random variable $\omega:=\min\{u,\min_{0\le i\le n}\{a_{\iota(i;\dagger)}\}\}$ conditioning on $\mathcal{E}_n$ is stochastically smaller than the random variable $\tilde{\omega}=\min\{u,\min_{0\le i\le n}\{a_{\iota(i;\dagger\dagger)}\}\}$ conditioning on $\tilde{\mathcal{E}}_n$, and the random variable $a_{\iota(n+1,\dagger)}$ conditioning on $\mathcal{E}_n$ has the same distribution as $a_{\iota(n+1,\dagger\dagger)}$ conditioning on $\tilde{\mathcal{E}}_n$, so $\mathbf{E}[\tilde{\omega}|\tilde{\mathcal{E}}_n]-\mathbf{E}[\omega|\mathcal{E}_n]\ge\mathbf{E}[\tilde{\omega}\wedge a_{\iota(n+1,\dagger\dagger)}|\tilde{\mathcal{E}}_n]-\mathbf{E}[\omega\wedge a_{\iota(n+1,\dagger)}|\mathcal{E}_n]$ which, after rearranging the terms, leads to $d+\mathbf{E}[\hat{\delta}(u,n,\dagger)|\mathcal{E}_n]\ge d+\mathbf{E}[\hat{\delta}(u,n,\dagger\dagger)|\tilde{\mathcal{E}}_n]=$

$V(\bar{\pi}^{j+1}(n+1)|\tilde{\mathcal{E}}_n) - V(\bar{\pi}^{j+1}(n)|\tilde{\mathcal{E}}_n) \geq 0$. The last two equalities follow the same argument as in the first two equalities. The desired result then follows since

$$\begin{aligned} V(\pi^j) = V(\bar{\pi}^j(0)) &= \textstyle\sum_{n=0}^{\min\{\bar{N}-k,j\}-1} \left(V(\bar{\pi}^j(n)) - V(\bar{\pi}^j(n+1))\right) + V(\bar{\pi}^j(\min\{\bar{N}-k,j\})) \\ &\leq \textstyle\sum_{n=0}^{\min\{\bar{N}-k,j\}-1} \left(V(\bar{\pi}^{j+1}(n)) - V(\bar{\pi}^{j+1}(n+1))\right) + V(\bar{\pi}^{j+1}(\min\{\bar{N}-k,j\})) \\ &= V(\bar{\pi}^{j+1}(0)) = V(\pi^{j+1}), \end{aligned}$$

where the inequality follows by (A.1) and the observation that $V(\bar{\pi}^j(\min\{\bar{N}-k,j\})) \leq V(\bar{\pi}^{j+1}(\min\{\bar{N}-k,j\}))$ (Indeed, if $\bar{N}-k > j$, then both policies are the same so the equality holds; otherwise, $\bar{\pi}^j(\bar{N}-k)$ investigates the same number of suppliers as $\bar{\pi}^{j+1}(\bar{N}-k)$, but the updated costs of suppliers investigated by $\bar{\pi}^j(\bar{N}-k)$ are stochastically smaller than $\bar{\pi}^{j+1}(\bar{N}-k)$, so the inequality holds). $\square$

Proof of Lemma 1. Note that the distribution of suppliers' cost has a joint density with respect to the Lebesgue measure, $\mathbf{f}(.)$, and is bounded and atomless; moreover, for any supplier i, $b_i \in B_i$, and for any nondecreasing bidding strategy profiles of other suppliers $\boldsymbol{\beta}_{-i}$, $\mathbf{E}[u_i^{\boldsymbol{\beta}_{-i}}(b_i;c_i)|c_i \in \mathcal{E}] < \infty$ for all convex set $\mathcal{E}$. Hence, Assumption 1 in Athey (2001) holds. We now show that the Single Crossing Condition for games of incomplete information (SCC), defined in Definition 3 in Athey (2001) holds. This is equivalent to show that for any nondecreasing bidding strategy profiles of other suppliers $\boldsymbol{\beta}_{-i}$, $u_i^{\boldsymbol{\beta}_{-i}}(b_i;c_i)$ satisfies single crossing of incremental returns (SCP-IP) in (b_i,c_i) as defined in Definition 1 in Athey (2001). Indeed, SCP-IP holds since for any $b_i' > b_i$ and $c_i' > c_i$, if $u_i^{\boldsymbol{\beta}_{-i}}(b_i';c_i) - u_i^{\boldsymbol{\beta}_{-i}}(b_i;c_i) \geq 0$, then

$$\begin{aligned} &u_i^{\boldsymbol{\beta}_{-i}}(b_i';c_i') - u_i^{\boldsymbol{\beta}_{-i}}(b_i;c_i') \\ &\quad = (b_i'-c_i')\mathbf{E}_{\mathbf{c}_{-i}}[W_i(b_i',\boldsymbol{\beta}_{-i}(\mathbf{c}_{-i}))] - (b_i-c_i')\mathbf{E}_{\mathbf{c}_{-i}}[W_i(b_i,\boldsymbol{\beta}_{-i}(\mathbf{c}_{-i}))] \\ &\quad = c_i'\left(\mathbf{E}_{\mathbf{c}_{-i}}[W_i(b_i,\boldsymbol{\beta}_{-i}(\mathbf{c}_{-i})) - W_i(b_i',\boldsymbol{\beta}_{-i}(\mathbf{c}_{-i}))]\right) + b_i'\mathbf{E}_{\mathbf{c}_{-i}}[W_i(b_i',\boldsymbol{\beta}_{-i}(\mathbf{c}_{-i}))] - b_i\mathbf{E}_{\mathbf{c}_{-i}}[W_i(b_i,\boldsymbol{\beta}_{-i}(\mathbf{c}_{-i}))] \\ &\quad \geq c_i\left(\mathbf{E}_{\mathbf{c}_{-i}}[W_i(b_i,\boldsymbol{\beta}_{-i}(\mathbf{c}_{-i})) - W_i(b_i',\boldsymbol{\beta}_{-i}(\mathbf{c}_{-i}))]\right) + b_i'\mathbf{E}_{\mathbf{c}_{-i}}[W_i(b_i',\boldsymbol{\beta}_{-i}(\mathbf{c}_{-i}))] - b_i\mathbf{E}_{\mathbf{c}_{-i}}[W_i(b_i,\boldsymbol{\beta}_{-i}(\mathbf{c}_{-i}))] \\ &\quad = (b_i'-c_i)\mathbf{E}_{\mathbf{c}_{-i}}[W_i(b_i',\boldsymbol{\beta}_{-i}(\mathbf{c}_{-i}))] - (b_i-c_i)\mathbf{E}_{\mathbf{c}_{-i}}[W_i(b_i,\boldsymbol{\beta}_{-i}(\mathbf{c}_{-i}))] \\ &\quad = u_i^{\boldsymbol{\beta}_{-i}}(b_i';c_i) - u_i^{\boldsymbol{\beta}_{-i}}(b_i;c_i) \geq 0, \end{aligned}$$

where the inequality follows since $c_i' > c_i$ and $W_i(b_i,b_{-i})$ is nonincreasing in b_i. The stated existence result follows by invoking Theorems 1 and 2 in Athey (2001).

Having established the existence of such equilibrium, which we denote by $\tilde{\boldsymbol{\beta}}$, we now provide a characterization of it. Note that since β_i is nondecreasing in c_i and $W_i(b_i,\mathbf{b}_{-i})$ is nonincreasing in b_i, a supplier's equilibrium winning probability is nonincreasing in c_i. Let $\tilde{c}_i := \min\{\bar{c}_i, \min\{c_i : \mathbf{E}_{\mathbf{c}_{-i}}W_i(\tilde{\beta}_i(c_i),\tilde{\boldsymbol{\beta}}_{-i}(\mathbf{c}_{-i})) = 0\}\}$ (note that by convention, $\min\emptyset := \infty$). Consider two cases. If $\tilde{c}_i = \bar{c}_i$, then due to the maximum bid limit, $\beta_i(\bar{c}_i) = \bar{c}_i$; moreover, for all $c_i < \bar{c}_i$, $\mathbf{E}_{\mathbf{c}_{-i}}W_i(\beta_i(c_i),\boldsymbol{\beta}_{-i}(\mathbf{c}_{-i})) > 0$, so (6) holds by the envelope theorem. If $\tilde{c}_i < \bar{c}_i$, then for all $c_i \geq \tilde{c}_i$, $\mathbf{E}_{\mathbf{c}_{-i}}W_i(\tilde{\beta}_i(c_i),\tilde{\boldsymbol{\beta}}_{-i}(\mathbf{c}_{-i})) = 0$, while for all $c_i < \tilde{c}_i$, $\mathbf{E}_{\mathbf{c}_{-i}}W_i(\tilde{\beta}_i(c_i),\tilde{\boldsymbol{\beta}}_{-i}(\mathbf{c}_{-i})) > 0$. We claim that $\tilde{\beta}_i(\tilde{c}_i) = \tilde{c}_i$: if $\tilde{\beta}_i(\tilde{c}_i) < \tilde{c}_i$, supplier i can never make money; if $\tilde{\beta}_i(\tilde{c}_i) > \tilde{c}_i$, then since $\tilde{\beta}_i$ is continuous on $[\underline{c}_i, \tilde{c}_i]$ (due to the envelope theorem and (6)), supplier i with type $\tilde{c}_i$ is better off by reducing his bid a little bit so that he has a positive probability of winning and making a profit. Then, by the envelope theorem, (6) holds. Finally, for all cost-types $c_i > \tilde{c}_i$, as long as they bid higher than $\tilde{\beta}(c_i) \geq \tilde{c}_i$, then $\tilde{\boldsymbol{\beta}}$ forms an equilibrium (i.e., the equilibrium is not unique when $\tilde{c}_i < \bar{c}_i$). $\square$

Proof of Theorem 2: For any mechanism $\mathcal{M}$, let $A_i^{\mathcal{M}}(z_i;\mathbf{c}_{-i},\mathbf{t})$ denote supplier i's probability of winning when he behaves as if his cost-type was z_i, but all other suppliers behave according to their true cost-type. Then supplier i's probability of winning is $\phi^{\mathcal{M}}(z_i)=\mathbf{E}_{\mathbf{c}_{-i},\mathbf{t}}[A_i^{\mathcal{M}}(z_i;\mathbf{c}_{-i},\mathbf{t})]$. Let $m_i^{\mathcal{M}}(z_i)$ denote the expected payment to supplier i if he behaves as if his cost-type was z_i. Then, by incentive compatibility and the Envelope Theorem, supplier i's expected profit is

$$U_i^{\mathcal{M}}(c_i)=U_i^{\mathcal{M}}(\bar{c}_i)+\int_{c_i}^{\bar{c}_i}\phi_i^{\mathcal{M}}(y)dy. \tag{A.2}$$

By a standard derivation (e.g., Myerson (1981)), supplier i's *ex ante* expected profit is $\mathbf{E}_{c_i}[U_i^{\mathcal{M}}(c_i)]=\mathbf{E}_{c_i}[U_i^{\mathcal{M}}(\bar{c}_i)]+\mathbf{E}_{\mathbf{c},\mathbf{t}}[A_i^{\mathcal{M}}(\mathbf{c},\mathbf{t})\frac{F_i(c_i)}{f_i(c_i)}]$. So by symmetry, a supplier's *ex-ante* expected profit under BIA is

$$\frac{1}{N}\sum_{i=1}^{N}\mathbf{E}_{c_i}[U_i^{\mathsf{BIA}}(c_i)]=\frac{1}{N}\left(\sum_{i=1}^{N}U_i^{\mathsf{BIA}}(\bar{c}_i)+\mathbf{E}_{\mathbf{c},\mathbf{t}}\left[\sum_{i=1}^{N}A_i^{\mathsf{BIA}}(\mathbf{c},\mathbf{t})\frac{F_i(c_i)}{f_i(c_i)}\right]\right)=\frac{1}{N}\mathbf{E}_{\mathbf{c},\mathbf{t}}\left[\sum_{i=1}^{N}A_i^{\mathsf{BIA}}(\mathbf{c},\mathbf{t})\frac{F_1(c_i)}{f_1(c_i)}\right],$$

where the last inequality holds since $U_i^{\mathsf{BIA}}(\bar{c}_i)=0$ (i.e., for ex ante symmetric suppliers, the highest cost-type supplier would bid his true cost in equilibrium since he cannot bid higher due to maximum bid limit, and he will lose money if he bids lower and wins) and $F_i=F_1, f_i=f_1$ for all i. Similarly, under BAI, each supplier's ex ante expected profit equals the following:

$$\frac{1}{N}\sum_{i=1}^{N}\mathbf{E}_{c_i}[U_i^{\mathsf{BAI}}(c_i)]=\frac{1}{N}\sum_{i=1}^{N}\left(U_i^{\mathsf{BAI}}(\bar{c}_i)+\int_{c_i}^{\bar{c}_i}\phi_i^{\mathsf{BAI}}(y)dy\right)=\frac{1}{N}\mathbf{E}_{\mathbf{c}}\left[\sum_{i=1}^{N}A_i^{\mathsf{BAI}}(\mathbf{c})\frac{F_1(c_i)}{f_1(c_i)}\right],$$

where the second equality follows due to the observation that $U_i^{\mathsf{BAI}}(\bar{c}_i)=0$ for all i (see Krishna (2009)). (Note that under BAI, the allocation does not depend on $\mathbf{t}$, so A_i^{BAI} is only a function of $\mathbf{c}$.) Note that in BAI, $A_i^{\mathsf{BAI}}(\mathbf{c})=1$ if and only if (ignoring events of ties which occur with zero probability) $c_i<\min_{j\neq i}\{c_j\}$ which implies that $\frac{F_1(c_i)}{f_1(c_i)}\leq\min_{j\neq i}\left\{\frac{F_1(c_j)}{f_1(c_j)}\right\}$; moreover, $\sum_{i=1}^{n}A_i^{\mathcal{M}}=1$ for $\mathcal{M}\in\{\mathsf{BAI},\mathsf{BIA}\}$. Thus

$$\sum_{i=1}^{N}A_i^{\mathsf{BAI}}(\mathbf{c})\frac{F_1(c_i)}{f_1(c_i)}=\min_{1\leq i\leq N}\left\{\frac{F_1(c_i)}{f_1(c_i)}\right\}\leq\sum_{i=1}^{N}A_i^{\mathsf{BIA}}(\mathbf{c},\mathbf{t})\frac{F_1(c_i)}{f_1(c_i)},$$

almost everywhere (this is because a tie happens with zero probability due to the assumption that the joint distribution of $(\mathbf{c},\mathbf{t})$ is atomless). The desired result follows by taking expectation on both sides. □

Proof of Proposition 2. **Part (a).** Note that a supplier would never bid below his true cost (otherwise he loses money if he wins). Note also that when $\nu<-\nu^{**}$, for supplier 1, even if he bids the highest possible bid $\bar{c}+\Delta_1^c$, his cost index will always be the lowest since $\min_{i\neq 1}\mu_i(b_i)\geq\underline{c}+\Delta^c-\tau-\Delta^t>\bar{c}+\Delta_1^c-\tau-\Delta_1^t$, so he will be investigated first. Moreover, after investigation, his updated bid is lower than other supplier's index since, for any $i\neq 1$, $\bar{c}+\Delta_1^c-\Delta_1^t-\underline{t}<\bar{c}+\Delta^c-\Delta^t-\underline{t}-\nu^{**}=\underline{c}+\Delta^c-\bar{t}-\Delta^t<\mu_i(b_i)$. In other words, in BIA, supplier 1 is guaranteed to win even if he bids the highest possible bid (i.e., achieving the highest possible mark-up when he wins), and other suppliers will lose the contract for sure. In contrast, in BAI, supplier's ex ante profit is the same as in the first-price auction with ex ante symmetric suppliers. Thus, supplier 1 earns higher ex ante profit in BIA whereas other suppliers earn lower ex ante profit in BIA.

Part (b). Following a similar argument, one can show that when $\nu>\nu^{**}$, supplier 1 will lose the contract for sure in BIA; so supplier 1 earns lower ex ante profit in BIA. Given this observation, the analysis of other suppliers' ex ante profits is equivalent to analyzing the ex ante profit in BIA with $N-1$ ex ante symmetric suppliers. By Theorem 2, when there are $N-1$ suppliers, all $N-1$ suppliers earns a higher ex ante profit in BIA than in BAI; moreover, in BAI, each individual supplier earns higher ex ante profit when he is competing against fewer other suppliers. Thus, all other suppliers earn higher ex ante profit in BIA. □

Proof of Proposition 3. **Part (a).** It follows directly from Theorem 2.

Part (b). In this case, supplier 1's cost is always lower than other suppliers' cost. Under BAI, even when supplier 1 bids the highest feasible bid $\Delta_1^c + \bar{c}$, he still wins with probability one; in other words, this is the highest ex ante profit he can get in any mechanism where a supplier cannot bid higher than his cost distribution upper bound. In contrast, under BIA, there is a positive probability that supplier i's cost and at least one of the other suppliers' cost are sufficiently close to $\bar{c} + \Delta_1^c = \underline{c} + \Delta^c$. Given that, for sufficiently small investigation cost d, supplier 1 will be investigated first *but* there is a positive probability that his cost-reduction is close to $\underline{t} + \Delta^t$, and therefore the buyer will investigate another supplier. If the other supplier's cost-reduction is large enough, then supplier 1 will lose the contract. In other words, in BIA, there is a positive probability that supplier 1 may earn zero profit. Hence, supplier 1 expects lower ex ante profit in BIA. Conversely, other suppliers will always lose in BAI, but they have a positive chance of winning and earning a positive profit in BIA. Hence, they earn higher ex ante profit in BIA.

Part (c). Same as in the previous case, supplier 1 always wins in BAI. In BIA, even if supplier 1 bids the highest possible bid $\bar{c} + \Delta_1^c$, his index is still the lowest and will be investigated first. After investigation, no other suppliers will be investigated because supplier 1's updated cost is at most $\bar{c} + \Delta_1^c - \underline{t} - \Delta^t \leq \underline{c} + \underline{t} - \bar{t} + \Delta^c - \underline{t} - \Delta^t = \underline{c} + \Delta^c - \Delta^t - \bar{t} < \mu_i(b_i)$ for all i. Hence, in both mechanisms, supplier 1 always wins the contract. Thus, every supplier earns same ex ante profits in both mechanism.

Part (d). Following a similar argument as in Part (b), in this case, supplier 1 never wins in BAI and earns zero ex ante profit, but there is positive probability that he can win in BIA. The profit comparison for other suppliers can go either direction.

Part (e). In this case, supplier 1 always loses in both mechanisms and earns zero profit. If there are $N=2$ suppliers, then the other supplier always wins the contract and earns the same ex ante profit in both mechanisms. If there are more than 3 suppliers, then in both mechanisms there are, effectively, at least 2 ex ante symmetric suppliers in the competition. By Theorem 2, suppliers earns higher ex ante profit in BIA. □

Proof of Proposition 4. Take any supplier j. Note that supplier j blocking investigation affects supplier j's ex post profit only on sample paths where the buyer chooses to investigate supplier j. On any of such sample path $(\mathbf{c}, \mathbf{t})$, supplier j's margin (if he wins) does not depend on whether he engages in buyer's investigation. However, compared to engaging in investigation, if he blocks his investigation, then the following occur: For all $k > \kappa(j)$, u_k becomes stochastically larger, so $\mathbf{1}_{[i \in S^*(\mathbf{b})]}$ stochastically increases for i such that $\kappa(i) > \kappa(j)$. It means that the lowest updated bid of all other suppliers stochastically decreases, but supplier 1's updated bid increases. Thus, supplier i's chance of winning decreases. However, supplier 1's margin remains the same. Thus, he expects lower profit if he chooses to block investigation ex post. □

Proof of Theorem 3: Let $\mathrm{TC}(\mathcal{M})$ denote the buyer's expected cost under mechanism $\mathcal{M}$.

Part (a). By Assumption 2, $d < \Delta^t$. Let $d^\star = -\underline{t}$. Note that $d^\star < \Delta^t$ by $\Delta^t + \underline{t} > 0$, so the interval $(d^\star, \Delta^t)$ is not an empty set, and it is optimal to at least investigate the supplier with the lowest initial bid under BIA when $d \in (d^\star, \Delta^t)$. Suppose the lowest initial bid comes from supplier 1. Then it is suboptimal to investigate more suppliers under BIA because, for any supplier $i \neq 1$, $b_1 - t_1 \leq \mu_1 + \tau - \underline{t} \leq \mu_i + \tau - \underline{t} \leq b_i - \Delta^t - \underline{t} < b_i - \Delta^t + d$, where the first inequality follows by the definition of supplier 1's index and $\epsilon_1^t \geq \underline{t}$. Thus the lowest (initial) bid wins in both BIA and BAI. Moreover, the supplier's profit upon winning equals his bid minus his cost in both BIA and BAI. Therefore, suppliers's equilibrium bidding strategies are the same, so $\mathrm{TC}(\mathsf{BIA}) = \mathrm{TC}(\mathsf{BAI})$.

Part (b). When $d = 0$, it is optimal to investigate all suppliers under BIA. Consider the buyer's expected cost when there are N suppliers and the random variables $\{\epsilon_i^t\}_{i=1}^N$ are multiplied by a parameter $\sigma > 1$. Let $\tilde{\beta}_i^{\mathcal{M}}$ denote the equilibrium bidding function under mechanism $\mathcal{M}$ (we suppress its dependency on N and σ for notational simplicity). Then, since $d = 0$

$$\begin{aligned}
\mathrm{TC}(\mathsf{BIA}) &= \mathbf{E}\left[\min_{i\in\mathcal{N}}\{\tilde{\beta}_i^{\mathsf{BIA}}(c_i) - (\Delta^t + \sigma\epsilon_i^t)\}\right] \leq \mathbf{E}\left[\min_{i\in\mathcal{N}}\{\bar{c} + \Delta^c - \Delta^t - \sigma\epsilon_i^t\}\right] = \bar{c} + \Delta^c - \Delta^t - \sigma\mathbf{E}\left[\max_{i\in\mathcal{N}}\epsilon_i^t\right];\\
\mathrm{TC}(\mathsf{BAI}) &= \mathbf{E}\left[\min_{i\in\mathcal{N}}\{\tilde{\beta}_i^{\mathsf{BAI}}(c_i)\}\right] - \Delta^t \geq \underline{c} + \Delta^c - \Delta^t,
\end{aligned}$$

where the inequalities hold since the suppliers' bids lie between the maximum and minimum cost types. Taking the difference of the costs of both mechanisms yields:

$$\mathrm{TC}(\mathsf{BIA}) - \mathrm{TC}(\mathsf{BAI}) \leq \Xi^c - \sigma\mathbf{E}\left[\max_{i=1,\ldots,N}\epsilon_i^t\right]. \tag{A.3}$$

Note that the first term of (A.3) is constant and the second term of (A.3) is increasing in σ and N. Note also that when $N \to \infty$, the second term converges to $\sigma\bar{t}$. These observations combined with the assumption that $\Xi^c < \bar{t}$ implies that there exists $N^\star$ such that for all $N > N^\star$, there exists $\sigma^\star(N) \geq 1$ such that for all $\sigma \geq \sigma^\star(N)$, $\sigma\mathbf{E}[\max_{i\in\mathcal{N}}\epsilon_i^t] > \Xi^c$. The monotonicity result of $\sigma^\star(.)$ follows since the second term of (A.3) is also supermodular in σ and N. (The assumption $\Xi^c < \Delta^c + \bar{c} - \Delta^t - \bar{t}$ is equivalent to $\Delta^c + \underline{c} - \Delta^t - \bar{t} > 0$, which ensures the existence of $N^\star$ and $\sigma^\star(.)$ so that for all $N \geq N^\star$, there exist σ such that it is large enough to satisfy the condition $\sigma \geq \sigma^\star(N)$ but not so large that results in negative updated cost or negative cost-reductions.) $\square$

Proof of Theorem 4. To analyze the optimal mechanism design problem, we invoke the Revelation Principle established in Proposition 8 in Sugaya and Wolitzky (2021). This Revelation Principle basically says that any *conditional probability perfect Bayesian equilibrium* (please refer to §4 of Sugaya and Wolitzky (2021) for a formal definition of this solution concept; intuitively, it can be viewed as a type of perfect Bayesian Equilibrium that not only requires consistency of on-equilibrium beliefs and sequential rationality of player's actions, but also requires that both on-equilibrium and off-equilibrium beliefs are derived from a common conditional probability system on the set of complete histories of play) of the mechanism we described in §7 can be implemented by a canonical mechanism where communication between players and the mediator takes the following form: non-mediator players (suppliers in our setting) communicate only their private information to the mediator (buyer in our setting), and the mediator communicates only recommended actions to the players. Note also that, after the first stage, the suppliers in our setting do not observe *private* signal after the first stage (i.e., the cost-reduction of supplier i is not supplier i's private information, since it is also observed by the buyer). This observation combined with the Revelation Principle implies that the choice of information disclosure rule is irrelevant in finding the optimal *canonical mechanism*; hence, it is without loss of generality to optimize within the class of three-step direct mechanisms defined below.

DEFINITION 1. *A three-step direct mechanism* is determined by an investigation policy π (by abuse of notation π used in §3), an allocation rule A and a payment rule P. In the first step, all suppliers report their costs to the buyer (they can choose to be not truthful); in the second step, the buyer investigates the suppliers according to the investigation policy π; in the third step, the buyer allocates the contract and makes payment according to A and P. Specifically:

- **Investigation policy $\boldsymbol{\pi}$.** Similar to BIA's description in §3, the investigation policy determines which supplier to investigate and when to stop, i.e., $\pi := (\pi_1, \ldots, \pi_{\hat{N}})$ where for any investigation round $k = 1, 2, \ldots, \hat{N}$, $\pi_k : C \times (\otimes_{i \in \mathcal{I}_k} T_i) \to \{\emptyset\} \cup (\mathcal{N} - \mathcal{I}_k)$ where $\mathcal{I}_k$ is the set of suppliers that have been investigated before round k. Define $\mathcal{I}_k^\pi$ for all $k = 1, \ldots, \hat{N} + 1$ and Π the same way as in §3, and define $S^\pi(\mathbf{c}) := \mathcal{I}_{\hat{N}+1}^\pi(\mathbf{c})$ for all $\mathbf{c} \in C$. The dependency of $S^\pi(\mathbf{c})$ on π and $\mathbf{c}$ will be suppressed for notational simplicity.
- **Allocation rule A.** $A(\mathbf{c}, \mathbf{t}^S) := [A_i(\mathbf{c}, \mathbf{t}^S)] \in [0,1]^N$, where $A_i(\mathbf{c}, \mathbf{t}^S)$ is the probability that supplier i wins the contract, given the bid vector $\mathbf{c}$ and the revealed cost-reductions $\{t_i : i \in S(\mathbf{c})\}$.
- **Payment rule P.** $P(\mathbf{c}, \mathbf{t}^S) := [P_i(\mathbf{c}, \mathbf{t}^S)] \in \mathbb{R}^N$, where $P_i(\mathbf{c}, \mathbf{t}^S)$ is the expected payment from the buyer to supplier i, given the bid vector $\mathbf{c}$ and the revealed cost reductions $\{t_i : i \in S(\mathbf{c})\}$.

We now prove that OPT is the optimal three-step direct mechanism, and is hence an optimal mechanism among the class of dynamic mechanisms we consider in this paper. To that end, let $\mathcal{M} := (\pi, A, P)$ denote a three-step direct mechanism. We denote by $\hat{U}_i(z_i; c_i, \mathcal{M})$ supplier i's expected profit under $\mathcal{M}$ when his cost is c_i and he bids z_i in step 1, and other suppliers bid their true costs in step 1; i.e., $\hat{U}_i(z_i; c_i, \mathcal{M})$ equals the expected payment to him minus his updated cost (cost minus cost-reduction if he is investigated) when he wins the contract (we suppress the dependence on $\mathcal{M}$ for notational simplicity when there is no confusion):

$$\hat{U}_i(z_i; c_i) = \mathbf{E}_{\mathbf{c}_{-i}, \mathbf{t}} \left[P_i(z_i, \mathbf{c_{-i}}, \mathbf{t}^{S^\pi(z_i, \mathbf{c}_{-i})}) - A_i(z_i, \mathbf{c_{-i}}, \mathbf{t}^{S^\pi(z_i, \mathbf{c}_{-i})})(c_i - \mathbf{1}_{[i \in S^\pi(z_i, \mathbf{c_{-i}})]} t_i) \right]. \tag{A.4}$$

Let $U_i(c_i) := \hat{U}_i(c_i; c_i)$. Then, supplier i's individual rationality, $\mathbf{IR}^i$, and incentive compatibility constraints, $\mathbf{IC}^i$, can be formulated as follows: for all $c_i \in C_i$,

$$(\mathbf{IC}^i) \quad U_i(c_i) = \max_{z_i \in C_i} \hat{U}_i(z_i; c_i), \quad (\mathbf{IR}^i) \quad U_i(c_i) \geq 0.$$

Hence, the mechanism design problem can be formulated as the following mathematical program:

$$\min_{\mathcal{M}=(\pi, A, P)} \sum_{i=1}^N \mathbf{E} \left[P_i(\mathbf{c}, \mathbf{t}^{S^\pi(\mathbf{c})}) + \mathbf{1}_{[i \in S^\pi(\mathbf{c})]} d \right] \tag{A.5}$$

$$s.t. \quad \sum_{i=1}^N A_i(\mathbf{c}, \mathbf{t}^{S^\pi(\mathbf{c})}) = 1, A_i(\mathbf{c}, \mathbf{t}^{S^\pi(\mathbf{c})}) \geq 0, \text{ for all } i = 1, \ldots, N \tag{A.6}$$

$$(\mathbf{IR}^i) \text{ and } (\mathbf{IC}^i) \text{ hold for all } i = 1, \ldots, N. \tag{A.7}$$

Following a standard derivation (see Myerson (1981)), (A.7) is equivalent to the following conditions:

$$\mathbf{E}_{\mathbf{c}_{-i},\mathbf{t}}[A_i(\mathbf{c},\mathbf{t}^{S^\pi(\mathbf{c})})] \text{ is nonincreasing in } c_i \text{ for all } i\in\mathcal{N} \text{ and for all } c_i\in C_i, \tag{A.8}$$

$$\mathbf{E}_{\mathbf{c}_{-i},\mathbf{t}}\left[P_i(\mathbf{c},\mathbf{t}^{S^\pi(\mathbf{c})})\right] = U_i(\bar{c}_i) + \mathbf{E}_{\mathbf{c}_{-i},\mathbf{t}}\left[A_i(\mathbf{c},\mathbf{t}^{S^\pi(\mathbf{c})})(c_i - \mathbf{1}_{[i\in S^\pi(\mathbf{c})]}t_i) + \int_{c_i}^{\bar{c}_i} A_i(z,\mathbf{c}_{-i},\mathbf{t}^{S^\pi(z,\mathbf{c}_{-i})})dz\right], \tag{A.9}$$

$$U_i(\bar{c}_i)\geq 0, \text{for all } i\in\mathcal{N}. \tag{A.10}$$

By plugging (A.9) into (A.5) and some standard algebraic manipulation, the objective function becomes:

$$\sum_{i=1}^N \mathbf{E}\left[P_i(\mathbf{c},\mathbf{t}^{S^\pi(\mathbf{c})}) + \mathbf{1}_{[i\in S^\pi(\mathbf{c})]}d\right] = \sum_{i=1}^N U_i(\bar{c}_i) + \sum_{i=1}^N \mathbf{E}_{\mathbf{c},\mathbf{t}}\left[A_i(\mathbf{c},\mathbf{t}^{S^\pi(\mathbf{c})})\left(\psi_i(c_i) - \mathbf{1}_{[i\in S^\pi(\mathbf{c})]}t_i\right) + \mathbf{1}_{[i\in S^\pi(\mathbf{c})]}d\right].$$

Because $U_i(\bar{c}_i)$ is simply a constant, the cost-minimizing buyer will set $U_i(\bar{c}_i)=0$ for all i so as to minimize costs while still satisfying (A.10). Then, the buyer's *mechanism design problem* (MD) can be written as:

$$MD: \min_{\mathcal{M}=(\pi,A,P)} \sum_{i=1}^N \mathbf{E}_{\mathbf{c},\mathbf{t}}\left[A_i(\mathbf{c},\mathbf{t}^{S^\pi(\mathbf{c})})\left(\psi_i(c_i) - \mathbf{1}_{[i\in S^\pi(\mathbf{c})]}t_i\right)\right] + \sum_{i=1}^N \mathbf{E}_{\mathbf{c},\mathbf{t}}\left[\mathbf{1}_{[i\in S^\pi(\mathbf{c})]}d\right] \tag{A.11}$$

$$s.t. \quad \text{The constraints (A.6), (A.8), and (A.9) hold.} \tag{A.12}$$

To solve MD, we first relax (A.8) in MD and consider the resulting *Relaxed Mechanism Design problem* (RMD). Note that, given the set of investigated suppliers $S^\pi(\mathbf{c})$ and their cost-reductions $\mathbf{t}^{S^\pi(\mathbf{c})}$, it is optimal to award the contract to the supplier with the lowest updated virtual cost. Indeed, the second summation term in (A.11) is fixed given $S^\pi(\mathbf{c})$, so the buyer should minimize the first summation by assigning largest weight (greatest probability) to the supplier with lowest updated virtual cost. Therefore $A_i^*(\mathbf{c},\mathbf{t}^{S^\pi(\mathbf{c})})=1$ if supplier i has the lowest updated virtual cost $\psi_i(c_i)-\mathbf{1}_{[i\in S^\pi(\mathbf{c})]}t_i$, and $A_i^*(\mathbf{c},\mathbf{t}^{S^\pi(\mathbf{c})})=0$ otherwise. (In case of a tie, randomly pick one of the suppliers with the lowest updated virtual cost and allocates the contract to him.) Note that the payment function below (after integration over C_{-i} and T) satisfies constraint (A.9):

$$P_i^*(\mathbf{c},\mathbf{t}^{S^\pi(\mathbf{c})}) = A_i^*(\mathbf{c},\mathbf{t}^{S^\pi(\mathbf{c})})(c_i - \mathbf{1}_{[i\in S^\pi(\mathbf{c})]}t_i) + \int_{c_i}^{\bar{c}_i} \mathbf{E}_{\mathbf{t}^{\overline{S^\pi(\mathbf{c})}}}[A_i^*(z_i,\mathbf{c}_{-i},\mathbf{t}^{S^\pi(z_i,\mathbf{c}_{-i})})|\mathbf{t}^{S^\pi(\mathbf{c})}]\,dz_i, \tag{A.13}$$

where the expectation of the integrand is taken with respect to all the t_i such that $i\in\overline{S^\pi(\mathbf{c})}$.

So far, we have characterized the optimal allocation rule and optimal payment rule given an investigation policy π for RMD. To fully characterize the optimal solution of RMD and show that it is feasible (and thus optimal) to MD, we need to solve for the optimal investigation policy

$$\pi^* = \arg\min_\pi \mathbf{E}_{\mathbf{c},\mathbf{t}}\left[\min_{i\in\mathcal{N}}\{\psi_i(c_i) - \mathbf{1}_{[i\in S^\pi(\mathbf{c})]}t_i\} + |S^\pi(\mathbf{c})|d\right] \tag{A.14}$$

(note that (A.14) is obtained by plugging in the allocation and payment rules) and verify that under $\mathsf{OPT} := (\pi^*, A^*, P^*)$, (A.8) holds. Note that (A.14) is the same as (1) if we replace $S^*(\mathbf{c})$ and $\psi_i(c_i)$ by $S^*(\mathbf{b})$ and b_i respectively. Hence, the index rule defined in Theorem 1 is optimal to RMD if we replace μ_i and u_k by $\hat{\mu}_i$ and $\hat{u}_k$. Moreover, the comparative statics statements in Theorem 1 parts (b) and (c) also hold if we replace $b_i, b_j, \kappa(.)$ by $c_i, c_j, \hat{\kappa}(.)$. This comparative statics result has two implications. First, it implies that (A.8) holds, so the investigation policy, the allocation and payment rules derived above constitute an optimal solution to MD. Second, it also implies that, conditioning on $\mathbf{t}^{S^*(\mathbf{c})}$, for any $z_i\geq c_i$ and any $\mathbf{t}^{\overline{S^*(\mathbf{c})}}$, $A_i^*(z_i,\mathbf{c}_{-i},\mathbf{t}^{S^*(z_i,\mathbf{c}_{-i})})\leq A_i^*(\mathbf{c},\mathbf{t}^{S^*(\mathbf{c})})$. Hence, $A_i^*(\mathbf{c},\mathbf{t}^{S^*(\mathbf{c})})=0$ implies that $P_i^*(\mathbf{c},\mathbf{t}^{S^*(\mathbf{c})}) = \int_{c_i}^{\bar{c}_i}\mathbf{E}_{\mathbf{t}^{\bar{S}^*(\mathbf{c})}}[A_i^*(z_i,\mathbf{c}_{-i},\mathbf{t}^{S^*(z_i,\mathbf{c}_{-i})})|\mathbf{t}^{S^*(\mathbf{c})}]\,dz_i = 0$. Thus, the payment rule is equivalent to paying *zero* to the losing suppliers but paying the winning supplier i according to (7). $\square$

Proof of Theorem 5 Part (a). Note that in BIA, given the suppliers' bids, the buyer's investigation problem is the same as in the base model except that, since the buyer only retains a fraction $1-\eta$ of the identified cost-savings, the buyer needs to use the random variable $(1-\eta)t_i$ instead of t_i for suppliers' cost-savings. The proof follows directly from Theorem 1 by replacing t_i with $(1-\eta)t_i$.

Next, before proving part (b) and (c), we first leverage the buyer's optimal investigation rule to characterize the equilibrium in BAI, and then characterize equilibrium in BIA.

BIA equilibrium analysis. Due to the buyer's optimal investigation policy, we can formulate the suppliers' utility functions and formulate the equilibrium conditions similar to the base model: Supplier i's winning probability in BIA when he bids b_i and his competitors bid $\mathbf{b}_{-i}$, which we denote by $W_i(b_i, \mathbf{b}_{-i})$, has the following expression:

$$W_i(b_i, \mathbf{b}_{-i}) = \begin{cases} \bar{\omega}_i(\mathbf{b}), & \forall i : \kappa(i) > \bar{N} \\ \int_{T_i} \underline{\omega}_i(\mathbf{b}, t_i) dG_i(t_i), & \forall i : \kappa(i) \le \bar{N} \end{cases},$$

where $\bar{N}$ is defined in Theorem 5 part (a), and $\bar{\omega}_i$ and $\underline{\omega}_i$ are defined as:

$$\bar{\omega}_i(\mathbf{b}) = \left[\prod_{j:\kappa(j)\le \bar{N}} G_{j,\eta}(b_j - b_i) \right] \left[\prod_{j:\kappa(j)>\bar{N}, j\ne i} \mathbf{1}_{[b_i < b_j]} \right],$$

$$\underline{\omega}_i(\mathbf{b}, t_i) = \left[\prod_{j:\kappa(j)\le \bar{N}, j\ne i} G_{j,\eta}(b_j - b_i + \tau_i \wedge (1-\eta)t_i)^{\mathbf{1}[\mu_j < \mu_i]} G_{j,\eta}(b_j - b_i + (1-\eta)t_i)^{\mathbf{1}[\mu_i \le \mu_j < b_i - (1-\eta)t_i]} \right] \cdot \left[\prod_{j:\kappa(j)>\bar{N}} \mathbf{1}_{[b_i - (1-\eta)t_i < b_j]} \right].$$

Note that one can verify that $W_i(b_i, \mathbf{b}_{-i})$ is nonincreasing in b_i. Suppose that each supplier i, anticipating that the buyer will use the optimal investigation policy, follows a bidding strategy $\beta_i : C_i \to B_i$. Denote by $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_N)$ the bidding strategy profile of all suppliers. Hence, supplier i's expected profit when his cost is c_i and he bids b_i and all other suppliers bid according to $\boldsymbol{\beta}_{-i}$ is

$$u_i^{\boldsymbol{\beta}_{-i}}(b_i; c_i) = \mathbf{E}_{\mathbf{c}_{-i}, \mathbf{t}} \left[(b_i - c_i) W_i(b_i, \boldsymbol{\beta}_{-i}(\mathbf{c}_{-i})) \right] + \mathbf{E}_{\mathbf{c}_{-i}, \mathbf{t}} \left[\mathbf{1}_{[i \in S^\pi(b_i, \boldsymbol{\beta}_{-i}(\mathbf{c}_{-i}))]} t_i \eta \underline{\omega}_i(b_i, \boldsymbol{\beta}_{-i}(\mathbf{c}_{-i}), t_i) \right], \quad (A.15)$$

where S^π is the random set of investigated suppliers induced by the buyer's optimal investigation rule, and the second term is the fraction of identified savings the buyer shares with supplier i. A strategy profile $\tilde{\boldsymbol{\beta}}$ forms an equilibrium if

$$U_i(c_i) := u_i^{\tilde{\boldsymbol{\beta}}_{-i}}(\tilde{\boldsymbol{\beta}}_i(c_i); c_i) = \max_{b_i \in B_i} u_i^{\tilde{\boldsymbol{\beta}}_{-i}}(b_i; c_i)$$

for all $i \in \mathcal{N}$ and $c_i \in C_i$. Note that this means that the suppliers places the bid which maximizes his expected payoff. Note that the second term in (A.15) is independent of c_i, so the proof of the existence of nondecreasing equilibrium follows from the same argument as in the proof of the base model in Lemma 1.

BAI equilibrium analysis. The equilibrium analysis is similar to the base model except that in Award step, since the buyer only captures a fraction $(1-\eta)$ of the cost-reduction, she would investigate the winning supplier, say i, when $(1-\eta)\Delta_i^t > d$. Thus, in the Award step, the buyer would award the contract to the supplier with the lowest expected payment $b_i - r_i(\eta)$ where $r_i(\eta) := \max\{0, (1-\eta)\Delta_i^t - d\}$. Note that if supplier i wins the contract when he bids b_i and c_i, his profit equals $b_i - c_i$ if $r_i(\eta) = 0$, and equals $b_i - c_i + \eta\Delta_i^t$ if $r_i(\eta) > 0$. Thus, in Bid step, suppose there exists a Bayesian Nash equilibrium where supplier j bids according to $\tilde{\beta}_j(c_j)$, then: $\tilde{\beta}_i(c_i) = \arg\max_{b_i \in B_i} u_i(b_i; c_i)$ where $u_i(b_i; c_i) = (b_i - c_i + \eta\Delta_i^t \mathbf{1}_{[r_i(\eta)>0]})\mathbf{P}_{\mathbf{c}_{-i}}(b_i - r_i(\eta) \le \tilde{\beta}_j(c_j) - r_j(\eta), \forall j \ne i)$. We now construct bidding functions $\{\tilde{\beta}_i\}_{i\in\mathcal{N}}$ below and verify that they form an equilibrium. Note that the equilibrium bidding functions $\{\check{\beta}_i\}_{i\in\mathcal{N}}$ of a first-price sealed bid auction in which suppliers' costs are $\check{c}_i = c_i - (\eta\Delta_i^t \mathbf{1}_{[r_i(\eta)>0]} + r_i(\eta))$ for all i must satisfy the following:

$$\begin{aligned}
\check{\beta}_i(\check{c}_i) &\in \arg\max_{\check{b}_i \in [\underline{c}_i - r_i(\eta), \bar{c}_i - r_i(\eta)]} (\check{b}_i - \check{c}_i)\mathbf{P}(\check{b}_i \le \check{\beta}_j(\check{c}_j), \forall j \ne i) \\
&= \arg\max_{\check{b}_i \in [\underline{c}_i - r_i(\eta), \bar{c}_i - r_i(\eta)]} (\check{b}_i + \eta\Delta_i^t \mathbf{1}_{[r_i(\eta)>0]} + r_i(\eta) - \check{c}_i - \eta\Delta_i^t \mathbf{1}_{[r_i(\eta)>0]} - r_i(\eta))\mathbf{P}(\check{b}_i \le \check{\beta}_j(\check{c}_j), \forall j \ne i).
\end{aligned}$$

Let $\tilde{\beta}_i(c_i) := \check{\beta}_i(c_i - \eta\Delta_i^t \mathbf{1}_{[r_i(\eta)>0]} - r_i(\eta)) + r_i(\eta)$ for all i, then the equation above implies that

$$\begin{aligned}
\tilde{\beta}_i(c_i) &= \check{\beta}_i(c_i - \eta\Delta_i^t \mathbf{1}_{[r_i(\eta)>0]} - r_i(\eta)) + r_i(\eta) = \check{\beta}_i(\check{c}_i) + r_i(\eta) \\
&\in \arg\max_{b_i \in [\underline{c}_i, \bar{c}_i]} (b_i - c_i + \eta\Delta_i^t \mathbf{1}_{[r_i(\eta)>0]})\mathbf{P}(b_i - r_i(\eta) \le \tilde{\beta}_j(c_j) - r_j(\eta), \forall j \ne i) \\
&= \arg\max_{b_i \in B_i} (b_i - c_i + \eta\Delta_i^t \mathbf{1}_{[r_i(\eta)>0]})\mathbf{P}(b_i - r_i(\eta) \le \tilde{\beta}_j(c_j) - r_j(\eta), \forall j \ne i),
\end{aligned}$$

where the inclusion follows by a change of variable $b_i := \check{b}_i + r_i(\eta)$, and the last equality follows since any $b_i \in [0, \underline{c}_i)$ is suboptimal. Thus, $\{\tilde{\beta}_i\}_{i\in\mathcal{N}}$ forms an equilibrium.

Part (b). By the envelope theorem, for any mechanism $\mathcal{M}$, supplier i's expected profit equals $U^{\mathcal{M}} = U_i^{\mathcal{M}}(\bar{c}_i) + \int_{c_i}^{\bar{c}_i} \phi_i^{\mathcal{M}}(y)dy$, where $\phi_i^{\mathcal{M}}(y)$ is supplier i's equilibrium chance of winning when his cost is y. Note that when suppliers are ex ante symmetric, by the equilibrium analysis of BAI, the highest cost type wins the contract with probability zero, so $U_i^{\mathsf{BAI}}(\bar{c}) = 0$. In contrast, in BIA, if the highest cost type bids $\bar{c}_i$, his equilibrium utility is nonnegative, so $U_i^{\mathsf{BIA}}(\bar{c}) \ge 0$. The result follows by the same argument as in the proof of Theorem 2.

Part (c). The proof follows by a similar argument as in Theorem 3. Specifically, replace $\Delta^t, \underline{t}, t_1, \bar{t}$ and ϵ_i^t by $(1-\eta)\Delta^t, (1-\eta)\underline{t}, (1-\eta)t_1, (1-\eta)\bar{t}$ and $(1-\eta)\epsilon_i^t$ everywhere in the proof of Theorem 3 except for the notation in the argument of the assumption $\Xi^c < \Delta^c + \bar{c} - \Delta^t - \bar{t}$. $\square$

Proof of Theorem 6 Part (a). Since an increasing equilibrium $\{\beta_i\}_{i\in\mathcal{N}}$ exists, the buyer would infer supplier i true cost when he bids b_i as $\hat{c}_i = \beta_i^{-1}(b_i)$, which means that supplier i's cost-reduction's average equals $\alpha(\beta_i^{-1}(b_i) - \mathbf{E}[c_i]) + \Delta_i^t$ instead of Δ_i^t as in the base model. Thus the proof follows from Theorem 1 by replacing Δ_i^t by $\alpha(\beta_i^{-1}(b_i) - \mathbf{E}[c_i]) + \Delta_i^t$.

Part (c). The proof that Theorem 3 part (a) generalizes in the extension follows the same argument as in Theorem 3 except that we replace Δ^t by $\min_{c_1 \in C_1}\{\alpha(c_1 - \mathbf{E}[c_1])\} + \Delta^t$ and Assumption 2 by Assumption 4. The proof that Theorem 3 part (b) generalizes in the extension follows a similar argument as in Theorem 3 but with the following changes. The bounds for TC(BIA) and TC(BIA) are adjusted as follows

$$\mathrm{TC}(\mathsf{BIA}) = \mathbf{E}\left[\min_{i\in\mathcal{N}}\{\tilde{\beta}_i^{\mathsf{BIA}}(c_i) - [\alpha(c_i - \mathbf{E}[c_i]) + \Delta^t + \sigma\epsilon_i^t]\}\right] \le \mathbf{E}\left[\min_{i\in\mathcal{N}}\{\bar{c} + \Delta^c - \min_{c_i \in C_i}\{\alpha(c_i - \mathbf{E}[c_i])\} - \Delta^t - \sigma\epsilon_i^t\}\right]$$

$$= \bar{c} + \Delta^c - \min_{c_1 \in C_1}\{\alpha(c_1 - \mathbf{E}[c_1])\} - \Delta^t - \sigma\mathbf{E}\left[\max_{i\in\mathcal{N}} \epsilon_i^t\right];$$

$$\mathrm{TC}(\mathsf{BAI}) = \mathbf{E}\left[\min_{i\in\mathcal{N}}\{\tilde{\beta}_i^{\mathsf{BAI}}(c_i) - \alpha(c_i - \mathbf{E}[c_i]) - \Delta^t\}\right] \geq \underline{c} + \Delta^c - \max_{c_1\in C_1}\{\alpha(c_1 - \mathbf{E}[c_1])\} - \Delta^t,$$

As a consequence, (A.3) is adjusted as follows

$$\mathrm{TC}(\mathsf{BIA}) - \mathrm{TC}(\mathsf{BAI}) \leq \Xi^c + \max_{c_1\in C_1}\{\alpha c_1\} - \min_{c_1\in C_1}\{\alpha c_1\} - \sigma\mathbf{E}\left[\max_{i=1,\ldots,N} \epsilon_i^t\right] = \Xi^c(1+|\alpha|) - \sigma\mathbf{E}\left[\max_{i=1,\ldots,N}\epsilon_i^t\right],$$

which gives rise to the condition that $\Xi^c < \frac{\bar{t}}{1+|\alpha|}$. Finally, the assumption $\Xi^c < \Delta^c + \bar{c} - \max_{c_1\in C_1}\{\alpha(c_1 - \mathbf{E}[c_1])\} - \Delta^t - \bar{t}$ is equivalent to $\Delta^c + \underline{c} - \max_{c_1\in C_1}\{\alpha(c_1 - \mathbf{E}[c_1])\} - \Delta^t - \bar{t} > 0$, which ensures the existence of $N^\star$ and $\sigma^\star(.)$ so that for all $N \geq N^\star$, there exist σ such that it is large enough to satisfy the condition $\sigma \geq \sigma^\star(N)$ but not so large that results in negative updated cost or negative cost-reductions. □

Part (b). We first show that when suppliers are ex ante symmetric and an increasing equilibrium exists in BAI, the contract winner is the supplier with the lowest cost. To see that, let β denote the symmetric increasing equilibrium. Then, for each supplier i with bid $b_i \in \mathcal{B}$, the buyer can calculate the expected procurement cost if supplier i wins the contract as

$$\lambda(b_i) := b_i - \mathbf{E}_{\hat{c}_i}[\alpha(\hat{c}_i - \mathbf{E}[c_i]) + \Delta^t] = b_i - \alpha\beta^{-1}(b_i) + \alpha\mathbf{E}[c_i] - \Delta^t,$$

where the first equality follows since Assumption 4 implies that it is optimal for the buyer to investigate the contract winner in BAI. Thus, given suppliers' bids, it is optimal for the buyer to allocate the contract to the supplier with lowest expected payment, i.e., $i \in \arg\min_i \lambda(b_i)$. Note that this implies that $\lambda(\beta(c_i))$ must be increasing in c_i. Otherwise, suppose there exists $c < c'$ such that $\lambda(\beta(c)) \geq \lambda(\beta(c'))$, then the cost type c has incentive to mimic c' because that would result in higher profit margin upon wining (since β is increasing) and a weakly larger winning probability (since winning probability is fully determined by supplier's expected procurement cost). Since $\lambda(\beta(c_i))$ is increasing and the winner is the one with the lowest expected procurement cost, the lowest cost type wins the contract in BAI. Thus, following a similar argument as in the proof of Theorem 2, a supplier's ex ante profit in BAI equals

$$\frac{1}{N}\sum_{i=1}^N \mathbf{E}[U_i^{\mathsf{BAI}}(c_i)] = \frac{1}{N}\mathbf{E}\left[\min_{1\leq i\leq N}\frac{F(c_i)}{f(c_i)}\right].$$

Next, consider BIA. Under the assumption in the statement of the theorem, there exists an increasing equilibrium bidding function β. Let $w(b_i, c_i)$ denote the winning probability for a supplier i who bids b_i but with cost c_i and all other suppliers bid according to β and the buyer conducts optimal investigation based on the belief that all suppliers bid according to β. Then, similar to the base model, we have that

$$w(b_i,c_i) := \mathbf{E}_{\mathbf{c}_{-i}}[W_i^\beta(b_i,\beta(\mathbf{c}_{-i});c_i)], \text{ where } W_i^\beta(b_i,\mathbf{b}_{-i};c_i) = \begin{cases} \bar{\omega}_i^\beta(\mathbf{b}), & \forall i: \kappa(i) > \bar{N} \\ \int_{\epsilon_i^t} \underline{\omega}_i^\beta(\mathbf{b},\epsilon_i^t;c_i)dG(\epsilon_i^t), & \forall i:\kappa(i) \leq \bar{N} \end{cases} \quad (A.16)$$

where $\bar{N}$ is defined in Theorem 6 part (a), and $\bar{\omega}_i^\beta$ and $\underline{\omega}_i^\beta$ are defined as:

$$\bar{\omega}_i^\beta(\mathbf{b}) = \prod_{j:\kappa(j)\leq\bar{N}} G(\lambda(b_j) - b_i) \times \prod_{j:\kappa(j)>\bar{N}, j\neq i} \mathbf{1}_{[b_i<b_j]}, \quad (A.17)$$

$$\begin{aligned} \underline{\omega}_i^\beta(\mathbf{b},\epsilon_i^t;c_i) = &\prod_{j:\kappa(j)\leq\bar{N}, j\neq i} \Big\{ G\left(\lambda(b_j) - \mu_i(b_i) \vee (b_i - \alpha c_i + \alpha\mathbf{E}[c_i] - \Delta^t - \epsilon_i^t)\right)^{\mathbf{1}_{[\mu_i(b_i)>\mu_j(b_j)]}} \\ &\times\ G\left(\lambda(b_j) - (b_i - \alpha c_i + \alpha\mathbf{E}[c_i] - \Delta^t - \epsilon_i^t)\right)^{\mathbf{1}_{[\mu_i(b_i)<\mu_j(b_j)<b_i-\alpha c_i+\alpha\mathbf{E}[c_i]-\Delta^t-\epsilon_i^t]}} \Big\} \\ &\times \prod_{j:\kappa(j)>\bar{N}} \mathbf{1}_{[b_i-\alpha c_i+\alpha\mathbf{E}[c_i]-\Delta^t-\epsilon_i^t<b_j]}. \end{aligned} \quad (A.18)$$

We note that $w(b_i, c_i)$ not only depends on b_i as the buyer infers the supplier's cost type and calculates the index based on b_i, but also depends on c_i since the actual cost-reduction realization explicitly depends on c_i due to correlation. Moreover, it can be shown that $w(b_i, c_i)$ is always decreasing in b_i, and is increasing in c_i if $\alpha > 0$, decreasing in c_i is $\alpha < 0$ and independent of c_i if $\alpha = 0$. Let $U_i^{\mathsf{BIA}}(c_i)$ denote supplier i's expected profit in the equilibrium where everyone bids according to β and when his cost is c_i,

$$U_i^{\mathsf{BIA}}(c_i) = \min_{b_i \in B}(b_i - c_i)w(b_i, c_i).$$

Since $w(b, c)$ is differentiable w.r.t. c, by the envelop theorem (Milgrom and Segal 2002),

$$\frac{d}{dc_i}U_i^{\mathsf{BIA}}(c_i) = -w(\beta(c_i), c_i) + (\beta(c_i) - c_i)\frac{\partial w(\beta(c_i), c_i)}{\partial c}.$$

Note that the highest cost type always bids $\bar{c} + \Delta^c$, so $U_i^{\mathsf{BIA}}(\bar{c} + \Delta^c) = 0$. Thus,

$$\begin{aligned} U_i^{\mathsf{BIA}}(c_i) &= U_i^{\mathsf{BIA}}(\bar{c} + \Delta^c) - \int_{c_i}^{\bar{c}+\Delta^c} dU_i^{\mathsf{BIA}}(c_i) = -\int_{c_i}^{\bar{c}+\Delta^c} dU_i^{\mathsf{BIA}}(x) \\ &= \int_{c_i}^{\bar{c}+\Delta^c} w(\beta(x), x)dx - \int_{c_i}^{\bar{c}+\Delta^c} (\beta(x) - x)\frac{\partial w(\beta(x), x)}{\partial c}dx. \end{aligned}$$

Now, consider positive and non-positive correlations separately. If $\alpha \le 0$, then $w(b, c)$ is nonincreasing in c. Note also that in equilibrium $\beta(c_i) - c_i \ge 0$. Thus, we conclude that $U_i^{\mathsf{BIA}}(c_i) \ge \int_{c_i}^{\bar{c}+\Delta^c} w(\beta(x), x)dx$, so

$$\begin{aligned} \frac{1}{N}\sum_{i=1}^{N}\mathbf{E}[U_i^{\mathsf{BIA}}(c_i)] &\ge \frac{1}{N}\sum_{i=1}^{N}\int_{\underline{c}+\Delta^c}^{\bar{c}+\Delta^c}\int_{c_i}^{\bar{c}+\Delta^c} w(\beta(x), x)dx\, dF_i(c_i) \\ &= \frac{1}{N}\mathbf{E}\left[\sum_{i=1}^{N}\frac{F(c_i)}{f(c_i)}\mathbf{1}_{[i \text{ wins in } \mathsf{BIA}]}\right] > \frac{1}{N}\mathbf{E}\left[\min_{1\le i\le N}\frac{F(c_i)}{f(c_i)}\right] = \frac{1}{N}\sum_{i=1}^{N}\mathbf{E}[U_i^{\mathsf{BAI}}(c_i)], \end{aligned}$$

where the inequality holds since in BIA, due to cost-reduction, there is a positive probability that the winner is not the one with the lowest cost type. Hence, the stated result holds when $\alpha \le 0$.

Next, consider $\alpha > 0$. Note that in equilibrium, $\beta(c_i) - c_i \in [0, \bar{c} - \underline{c}]$. Note that

$$\begin{aligned} w(b_i, c_i) &= \mathbf{E}_{\mathbf{c}_{-i},\mathbf{t}}\left[W_i(b_i, \beta(\mathbf{c}_{-i}), \mathbf{t})\right] \\ &= \mathbf{P}(\kappa(i) > \bar{N})\mathbf{E}_{\mathbf{c}_{-i}}\left[\bar{\omega}_i(b_i, \beta(\mathbf{c}_{-i}))|\,\kappa(i) > \bar{N}\right] + \\ &\quad\ \mathbf{P}(\kappa(i) \le \bar{N})\mathbf{E}_{\mathbf{c}_{-i},t_i}\left[\underline{\omega}_i(b_i, \beta(\mathbf{c}_{-i}), t_i)|\,\kappa(i) \le \bar{N}\right], \end{aligned}$$

where only the the second conditional expectation depends on c_i, i.e.,

$$\mathbf{E}_{\mathbf{c}_{-i},t_i}\left[\underline{\omega}_i(b_i, \beta(\mathbf{c}_{-i}), t_i)|\,\kappa(i) \le \bar{N}\right] = \mathbf{E}_{\mathbf{c}_{-i},\epsilon_i}\left[\underline{\omega}_i(b_i, \beta(\mathbf{c}_{-i}), \alpha c_i - \alpha\mathbf{E}[c_i] + \Delta^t + \epsilon_i)|\,\kappa(i) \le \bar{N}\right].$$

Note also that the event $\{\kappa(i) \le \bar{N}\}$ is independent of c_i, so we have that:

$$\frac{\partial}{\partial c}w(\beta(c_i), c_i) = \alpha\mathbf{P}(\kappa(i) \le \bar{N})\mathbf{E}_{\mathbf{c}_{-i},\epsilon_i}\left[\frac{\partial}{\partial t_i}\underline{\omega}_i(b_i, \beta(\mathbf{c}_{-i}), \alpha c_i - \alpha\mathbf{E}[c_i] + \Delta^t + \epsilon_i)\middle|\,\kappa(i) \le \bar{N}\right] \le \alpha K$$

for some positive constant K, where the inequality follows since the expectation and the probability are both bounded. Hence,

$$\begin{aligned} U_i^{\mathsf{BIA}}(c_i) &= \int_{c_i}^{\bar{c}+\Delta^c} w(\beta(x), x)dx - \int_{c_i}^{\bar{c}+\Delta^c} (\beta(x) - x)\frac{\partial w(\beta(x), x)}{\partial c}dx \\ &\ge \int_{c_i}^{\bar{c}+\Delta^c} w(\beta(x), x)dx - \alpha K(\bar{c} - \underline{c})(\bar{c} - c_i). \end{aligned}$$

As a result, we have that

$$\begin{aligned}
&\sum_{i=1}^{N}\mathbf{E}[U_i^{\mathsf{BIA}}(c_i)]-\sum_{i=1}^{N}\mathbf{E}[U_i^{\mathsf{BAI}}(c_i)]\\
&\quad\geq \mathbf{E}\left[\sum_{i=1}^{N}\frac{F(c_i)}{f(c_i)}\mathbf{1}_{[i\text{ wins in }\mathsf{BIA}]}\right]-\mathbf{E}\left[\min_{1\leq i\leq N}\frac{F(c_i)}{f(c_i)}\right]-\int_{\underline{c}+\Delta^c}^{\bar{c}+\Delta^c}\alpha K(\bar{c}-\underline{c})(\bar{c}-c_i)dF(c_i)\\
&\quad= \mathbf{E}\left[\sum_{i=1}^{N}\frac{F(c_i)}{f(c_i)}\mathbf{1}_{[i\text{ wins in }\mathsf{BIA}]}\right]-\mathbf{E}\left[\min_{1\leq i\leq N}\frac{F(c_i)}{f(c_i)}\right]-\alpha K(\bar{c}-\underline{c})(\bar{c}-\mathbf{E}[c_i]).
\end{aligned}$$

We state a technical result whose proof is deferred to the end of the current proof.

Lemma A.2. *There exist $\Delta>0$ and $\check{\alpha}>0$ such that for all $\alpha\in[0,\check{\alpha}]$,*

$$\mathbf{E}\left[\sum_{i=1}^{N}\frac{F(c_i)}{f(c_i)}\mathbf{1}_{[i\text{ wins in }\mathsf{BIA}]}\right]-\mathbf{E}\left[\min_{1\leq i\leq N}\frac{F(c_i)}{f(c_i)}\right]>\Delta.$$

By Lemma A.2, there exists a positive constant $\tilde{\alpha}:=\min\{\check{\alpha},\Delta/[K(\bar{c}-\underline{c})(\bar{c}-\mathbf{E}[c_i])]\}$ such that

$$\frac{1}{N}\sum_{i}^{N}\mathbf{E}[U_i^{\mathsf{BIA}}(c_i)]-\frac{1}{N}\sum_{i=1}^{N}\mathbf{E}[U_i^{\mathsf{BAI}}(c_i)]>\frac{1}{N}\left[\Delta-\hat{\alpha}K(\bar{c}-\underline{c})(\bar{c}-\mathbf{E}[c_i])\right]\geq 0,$$

for all $\alpha\in[0,\tilde{\alpha}]$, which completes the proof. □

Next we prove the technical lemma below.

Proof of Lemma A.2 Note that on all sample path $\mathbf{c}=(c_1,\ldots,c_N)$, we can define $h(\mathbf{c}):=\sum_{i=1}^{N}\frac{F(c_i)}{f(c_i)}\mathbf{1}_{[\ i\text{ wins in }\mathsf{BIA}]}$ and $g(\mathbf{c}):=\sum_{i=1}^{N}\frac{F(c_i)}{f(c_i)}\mathbf{1}_{[i\text{ wins in }\mathsf{BAI}]}$. Then we have

$$h(\mathbf{c})\geq\min_{1\leq i\leq N}\frac{F(c_i)}{f(c_i)}=g(\mathbf{c}).$$

Let ϵ be a positive constant defined later and consider sample paths such that $c_1\in[\bar{c}+\Delta^c-3\epsilon,\bar{c}+\Delta^c-2\epsilon]$ and $c_i\in[\bar{c}+\Delta^c-\epsilon,\bar{c}+\Delta^c]$ for $i\neq 1$. Let $A(\epsilon)$ denote the collection of these sample paths such that supplier 1 does not win in BIA. Then we have that

$$(h(\mathbf{c})-g(\mathbf{c}))\mathbf{1}_{A(\epsilon)}\;\geq\;\left(\frac{F(\bar{c}-\epsilon)}{f(\bar{c}-\epsilon)}-\frac{F(\bar{c}-2\epsilon)}{f(\bar{c}-2\epsilon)}\right)\mathbf{1}_{A(\epsilon)}.$$

Thus, we have that by law of total expectation

$$\begin{aligned}
\mathbf{E}\left[h(\mathbf{c})-g(\mathbf{c})\right]\;&=\;\mathbf{E}\left[h(\mathbf{c})-g(\mathbf{c})|A(\epsilon)\right]\mathbf{P}(A(\epsilon))+\mathbf{E}\left[h(\mathbf{c})-g(\mathbf{c})|A^c(\epsilon)\right]\mathbf{P}(A^c(\epsilon))\\
&\geq\;\mathbf{E}\left[h(\mathbf{c})-g(\mathbf{c})|A(\epsilon)\right]\mathbf{P}(A(\epsilon))\geq\left(\frac{F(\bar{c}-\epsilon)}{f(\bar{c}-\epsilon)}-\frac{F(\bar{c}-2\epsilon)}{f(\bar{c}-2\epsilon)}\right)\mathbf{P}(A(\epsilon)).
\end{aligned}$$

Finally, note that in equilibrium in BIA, $b_1\geq\bar{c}+\Delta^c-3\epsilon$ and $b_i\leq\bar{c}+\Delta^c$ for all $i\neq 1$. Furthermore, irrespective of the bidding function, the buyer would infer suppliers' cost to be some value between $[\underline{c},\bar{c}]$. Thus we conclude that $\mu_1\geq\bar{c}+\Delta^c-3\epsilon-\alpha(\bar{c}+\Delta^c-\mathbf{E}[c_i])-\Delta^t-\tau$ and $\mu_i\leq\bar{c}+\Delta^c-\alpha(\underline{c}+\Delta^c-\mathbf{E}[c_i])-\Delta^t-\tau$. Hence, in equilibrium in BIA, when $\bar{c}+\Delta^c-3\epsilon\leq c_1\leq\bar{c}+\Delta^c-2\epsilon$ and $\bar{c}+\Delta^c-\epsilon\leq c_i\leq\bar{c}+\Delta^c$ for $i\neq 1$, we have that for all $i\neq 1$

$$\mu_i-\mu_1\leq 3\epsilon+\alpha(\bar{c}-\underline{c}).$$

Let $K(\epsilon) := (F(\bar{c}-2\epsilon) - F(\bar{c}-3\epsilon))(1-F(\bar{c}-\epsilon))^{N-1}$, and let $B(\epsilon) := \{\mathbf{c} : c_1 \in [\bar{c}+\Delta^c - 3\epsilon, \bar{c}+\Delta^c - 2\epsilon], c_i \in [\bar{c}+\Delta^c-\epsilon, \bar{c}+\Delta^c], \forall i \neq 1\}$, then

$$\begin{aligned}
\mathbf{P}(A(\epsilon)) &\geq K(\epsilon)\mathbf{P}(b_1 - t_1 \geq \mu_2, b_1 - t_1 > b_2 - t_2 | B(\epsilon)) \\
&= K(\epsilon)\mathbf{P}(\mu_1 + \tau - \epsilon_1 \geq \mu_2, \mu_1 + \tau - \epsilon_1 > \mu_2 + \tau - \epsilon_2 | B(\epsilon)) \\
&= K(\epsilon)\mathbf{P}(\tau - \epsilon_1 \geq \mu_2 - \mu_1, \epsilon_2 - \epsilon_1 > \mu_2 - \mu_1 | B(\epsilon)) \\
&\geq K(\epsilon)\mathbf{P}(\min\{\tau-\epsilon_1, \epsilon_2 - \epsilon_1\} > 3\epsilon + \alpha(\bar{c} - \underline{c})).
\end{aligned}$$

Thus, there exists some constant $\check{\alpha} > 0$ and some constant $\epsilon > 0$ such that for all $\alpha \in [0, \check{\alpha}]$, $\mathbf{P}(A(\epsilon)) > 0$. The result then follows by letting $\Delta = \mathbf{P}(A(\epsilon))(F(\bar{c}-\epsilon)/f(\bar{c}-\epsilon) - F(\bar{c}-2\epsilon)/f(\bar{c}-2\epsilon))$. □

Proof of Theorem 7 Part (a1). Following a similar argument as in Step 1 in the proof of Theorem 1, it is never optimal to investigate suppliers who are ranked higher than $\bar{N}$. Define $\Psi_p(\mathcal{I}) := \mathbf{E_t}[\min_i\{b_i - \mathbf{1}_{[i\in\mathcal{I}]}t_i\}] + |\mathcal{I}|d$. We first claim that there exists *an* optimal solution $S_p^*(\mathbf{b}) \in \{\mathcal{I}_j\}_{j=0}^{\bar{N}}$. We prove the result by contradiction. Suppose that the claim is false, then for given $\mathbf{b}$ there exists a set $\mathcal{I}$, $|\mathcal{I}| \leq \bar{N}$, that satisfies the following: (†) there exist some $j,k \in \mathcal{N}$ such that $\iota(j) \in \mathcal{I}$, $\iota(k) \notin \mathcal{I}$, and $b_{\iota(k)} - \Delta^t_{\iota(k)} < b_{\iota(j)} - \Delta^t_{\iota(j)}$, and (††) $\Psi_p(\mathcal{I}) < \Psi_p(\mathcal{I}_j)$ for all $j = 0, \ldots, \bar{N}$. Let $\mathcal{I}' = \mathcal{I} \cup \{\iota(k)\} - \{\iota(j)\}$. Then

$$\begin{aligned}
\Psi_p(\mathcal{I}) &= \mathbf{E_t}\left[\min\left\{\min_{i\in\mathcal{I}-\{\iota(j)\}}\{b_i - t_i\}, \min_{i\notin\mathcal{I}\cup\{\iota(k)\}}\{b_i\}, b_{\iota(j)} - t_{\iota(j)}, b_{\iota(k)}\right\}\right] + |\mathcal{I}|d \\
&\geq \mathbf{E_t}\left[\min\left\{\min_{i\in\mathcal{I}-\{\iota(j)\}}\{b_i - t_i\}, \min_{i\notin\mathcal{I}\cup\{\iota(k)\}}\{b_i\}, b_{\iota(j)}, b_{\iota(k)} - t_{\iota(k)}\right\}\right] + |\mathcal{I}|d = \Psi_p(\mathcal{I}'),
\end{aligned}$$

where the inequality holds since $\min\{b_{\iota(j)} - t_{\iota(j)}, b_{\iota(k)}\} \geq_{s.t.} \min\{b_{\iota(k)} - t_{\iota(k)}, b_{\iota(j)}\}$. If $\mathcal{I}' \in \{\mathcal{I}_j\}_{j=0}^{\bar{N}}$, we immediately get a contradiction with (††). If $\mathcal{I}' \notin \{\mathcal{I}_j\}_{j=0}^{\bar{N}}$, then there exist some $j', k' \in \mathcal{N}$ such that $\iota(j') \in \mathcal{I}'$, $\iota(k') \notin \mathcal{I}'$, and $b_{\iota(k')} - \Delta_{\iota(k')} < b_{\iota(j')} - \Delta_{\iota(j')}$. Note that $\mathcal{N}$ is a finite set; we can then repeat the same procedure again and eventually get a contradiction with (††). Hence, we can restrict our attention to $\{\mathcal{I}_j\}_{j=0}^{\bar{N}}$ in search for an optimal solution. Let $v_j := \min\{\min_{1\leq i\leq j-1}\{b_{\iota(i)} - t_{\iota(i)}\}, \min_{j\leq i\leq N}\{b_{\iota(i)}\}\}$. Note that on every sample path $\mathbf{t}$, v_j is nonincreasing in j and is independent of $t_{\iota(j)}$. So the difference

$$\Psi_p(\mathcal{I}_j) - \Psi_p(\mathcal{I}_{j-1}) = d - \mathbf{E_t}\left[v_j - \min\left\{v_j, b_{\iota(j)} - t_{\iota(j)}\right\}\right] = d - \mathbf{E_t}\left[\max\left\{0, v_j - (b_{\iota(j)} - \Delta^t_{\iota(j)}) + \epsilon^t_{\iota(j)}\right\}\right]$$

is nondecreasing in j. (This is because the expectation is nonincreasing in j due to the fact that $v_j - (b_{\iota(j)} - \Delta^t_{\iota(j)}) + \epsilon^t_{\iota(j)}$ is stochastically nonincreasing in j and the function $\max\{0, x\}$ is nondecreasing in x.) Hence, the optimal parallel search set $S_p^*(\mathbf{b})$ equals $\mathcal{I}_{j^*(\mathbf{b})}$ where $j^*(\mathbf{b}) = \max\{j \leq \bar{N} : \Psi_p(\mathcal{I}_j) - \Psi_p(\mathcal{I}_{j-1}) < 0\}$. Moreover, note that $\Psi_p(\mathcal{I}_j) - \Psi_p(\mathcal{I}_{j-1})$ is nondecreasing in $b_{\iota(j)}$. This means that there exists a threshold $\tau_{\iota(j)}(\mathbf{b}_{-\iota(j)})$ such that $j^*(\mathbf{b}) = \max\{j \leq \bar{N} : b_{\iota(j)} < \tau_{\iota(j)}(\mathbf{b}_{-\iota(j)})\}$ and $\tau = \tau_{\iota(j)}(\mathbf{b}_{-\iota(j)})$ is the solution to the following equation:

$$\mathbf{E_t}\left[\min\left\{\tilde{v}_j, \tau\right\} - \min\left\{\tilde{v}_j, \tau - t_{\iota(j)}\right\}\right] = d,$$

where $\tilde{v}_j := \min\{\min_{1\leq i\leq j-1}\{b_{\iota(i)} - t_{\iota(i)}\}, \min_{j+1\leq i\leq N}\{b_{\iota(i)}\}\}$. The desired result follows.

Part (a2). Note that $\mathbf{1}_{[j\in S_p^*(\mathbf{b})]}=1$ if and only if $\kappa(j)\le\bar{N}$ and $b_j<\tau_j(\mathbf{b}_{-j})$. As b_j decreases, j's ranking $\kappa(j)$ weakly decreases. As a result, $\tilde{v}_{\kappa(j)}:=\min\{\min_{i:\kappa(i)<\kappa(j)}\{b_i-t_i\},\min_{i:\kappa(i)>\kappa(j)}\{b_i\}\}$ becomes stochastically larger. Note that $\tau=\tau_j(\mathbf{b}_{-j})$ is the solution to the following equality

$$\mathbf{E}_{\mathbf{t}}\left[\min\{\tilde{v}_{\kappa(j)},\tau\}-\min\{\tilde{v}_{\kappa(j)},\tau-t_j\}\right]=d.$$

Hence, as $\tilde{v}_{\kappa(j)}$ becomes stochastically larger, $\tau_j(\mathbf{b}_{-j})$ increases as well. In sum, as b_j decreases, $\tau_j(\mathbf{b}_{-j})$ increases, so $\mathbf{1}_{[j\in S_p^*(\mathbf{b})]}$ weakly increases and supplier j is more likely to be investigated. If Δ_j^t increases, following a similar argument above, one can show that $\tilde{v}_{\kappa(j)}$ becomes stochastically larger. In addition, t_j also becomes stochastically larger. Hence, $\tau_j(\mathbf{b}_{-j})$ increases whereas b_j remains unchanged, and $\mathbf{1}_{[j\in S_p^*(\mathbf{b})]}$ weakly increases.

Now consider any $i\neq j$. Suppose b_i increases. Then $\kappa(j)$ decreases. Similar to the argument for changes of b_j, $\mathbf{1}_{[j\in S_p^*(\mathbf{b})]}$ weakly increases. If Δ_i^t decreases, $\kappa(j)$ decreases and $\tilde{v}_{\kappa(j)}$ becomes stochastically larger. Hence $\tau_j(\mathbf{b}_{-j})$ increases whereas b_j remains the same. We then conclude that $\mathbf{1}_{[j\in S_p^*(\mathbf{b})]}$ weakly increases.

Finally, if d decreases, $\kappa(j)$ does not change, but $\tau_j(\mathbf{b}_{-j})$ increases. Since b_j remains unchanged, we then conclude that $\mathbf{1}_{[j\in S_p^*(\mathbf{b})]}$ weakly increases.

Next, before moving on to prove part (b) and (c), we first conduct an equilibrium analysis of BIA_p. Note that as we have established in part (a), when $i\in S_p^*(\mathbf{b})$, supplier i wins if his updated bid is lower than the updated bids of all other suppliers in $S_p^*(\mathbf{b})$, *and* lower than the bids of all the suppliers not in $S_p^*(\mathbf{b})$; when $i\notin S_p^*(\mathbf{b})$, supplier i wins if his bid is lower than the updated bids of all the suppliers in $S_p^*(\mathbf{b})$, *and* lower than the bids of all other suppliers in $\overline{S_p^*(\mathbf{b})}$. Thus, the winning probability of supplier i with cost c_i and bids b_i and other suppliers bid $\mathbf{b}_{-i}$ equals:

$$W_i(b_i,\mathbf{b}_{-i}):=\begin{cases}\int_{T_i}\left(\prod_{j\in S_p^*(\mathbf{b})-\{i\}}G_j(b_j-b_i+t_i)\right)\left(\prod_{j\in\overline{S_p^*(\mathbf{b})}}\mathbf{1}_{[b_j>b_i-t_i]}\right)dG_i(t_i), & \forall i\in S_p^*(\mathbf{b})\\ \prod_{j\in S_p^*(\mathbf{b})}G_j(b_j-b_i)\prod_{j\in\overline{S_p^*(\mathbf{b})}-\{i\}}\mathbf{1}_{[b_j>b_i]}, & \forall i\in\overline{S_p^*(\mathbf{b})}\end{cases}\tag{A.19}$$

Note also that $W_i(b_i,\mathbf{b}_{-i})$ is nonincreasing in b_i; so the existence of equilibrium follows from a SCC argument as in the proof of Lemma 1. The characterization of $\tilde{\boldsymbol{\beta}}$ follows a similar argument as in the proof of Lemma 1.

Part (b). The proof follows by replacing BIA by BIA_p in the proof of Theorem 2.

Part (c). The proof follows by replacing BIA by BIA_p in the proof of Theorem 3. □

Appendix A.2: Derivation of the winning probability in BIA

To derive $W_i(b_i,\mathbf{b}_{-i})$, consider two cases. If $\kappa(i)>\bar{N}$, supplier i will never be investigated (by Theorem 1). Hence, supplier i wins if and only if $b_i\le\min_{j\neq i}\{b_j-\mathbf{1}_{[j\in S^*(\mathbf{b})]}t_j\}$. Note that for all j such that $\kappa(j)>\bar{N}$, $\mathbf{1}_{[j\in S^*(\mathbf{b})]}=0$; note also that if supplier i wins, then for any j such that $\kappa(j)\le\bar{N}$, $\mu_j(b_j)\le\mu_i(b_i)=b_i-\tau_i\le b_i\le\min_{j\neq i}\{b_j-\mathbf{1}_{[j\in S^*(\mathbf{b})]}t_j\}$, so $\mathbf{1}_{[j\in S^*(\mathbf{b})]}=1$. Hence, when $\kappa(i)>\bar{N}$, supplier i wins with probability:

$$\bar{\omega}_i(\mathbf{b})=\left[\prod_{j:\kappa(j)\le\bar{N}}G_j(b_j-b_i)\right]\left[\prod_{j:\kappa(j)>\bar{N},j\neq i}\mathbf{1}_{[b_i<b_j]}\right].$$

If $\kappa(i) \leq \bar{N}$, then supplier i wins if and only if his updated bid is no higher than any other supplier j. Recall that by the definition of $\tilde{N}$, $\tau_i > 0$ since $\kappa(i) \leq \bar{N} \leq \tilde{N}$. This means that supplier i wins only if he is investigated: Otherwise, the lowest updated bid among other suppliers must be no lower than the supplier i's initial bid (in order for supplier i to win) and lower than supplier i's index (so that supplier i will not be investigated under the optimal investigation policy), which contradicts with the fact that supplier i's index should be strictly lower than his initial bid (i.e., $\tau_i > 0$). This observation allows us to derive $W_i(b_i, \mathbf{b}_{-i})$ by conditioning on t_i. For any supplier i such that $\kappa(i) \leq \bar{N}$, let $\underline{\omega}_i(\mathbf{b}, t_i)$ denote supplier i's winning probability conditioning on his cost reduction being equal to t_i. Take any supplier $j \neq i$. If $\kappa(j) > \bar{N}$, supplier j will never be investigated; so supplier i's updated bid undercuts supplier j's if and only if $b_i - t_i < b_j$. If $\kappa(j) \leq \bar{N}$, supplier j may be investigated. In this case, there are two scenarios. If $\mu_j < \mu_i$, the buyer will investigate supplier j before supplier i. Therefore, supplier i's updated bid undercuts supplier j's if and only if $b_j - t_j \geq \mu_i (= b_i - \tau_i)$ and $b_j - t_j \geq b_i - t_i$, which happens with probability $G_j(b_j - b_i + \tau_i \wedge t_i)$. If $\mu_j \geq \mu_i$ and supplier j is investigated, he must be investigated *after* supplier i. Note that supplier i can lose the contract to supplier j only if supplier j is investigated (as discussed above), i.e., $\mu_j < b_i - t_i$; therefore, we only need to consider these suppliers. For any of these suppliers j, supplier i's updated bid undercuts theirs if $b_j - t_j > b_i - t_i$ which happens with probability $G_j(b_j - b_i + t_i)$. Hence,

$$\underline{\omega}_i(\mathbf{b}, t_i) = \left[\prod_{j:\kappa(j)\leq\bar{N}, j\neq i} G_j(b_j - b_i + \tau_i \wedge t_i)^{\mathbf{1}_{[\mu_j < \mu_i]}} G_j(b_j - b_i + t_i)^{\mathbf{1}_{[\mu_i \leq \mu_j < b_i - t_i]}}\right]\left[\prod_{j:\kappa(j)>\bar{N}} \mathbf{1}_{[b_i - t_i < b_j]}\right].$$

Therefore, (A.15) holds.

Appendix A.3: Derivation of the payment rule of OPT

Recall that $\hat{\imath}(k)$ denotes the supplier with the k^{th} lowest index and $\hat{\kappa}(i)$ denotes supplier i's ranking. For notational simplicity, we do not explicit write the dependency of $\hat{\kappa}(.)$ and $\hat{\imath}(.)$ on suppliers' reported cost; in the remainder of Appendix A.3, $\hat{\kappa}(.)$ and $\hat{\imath}(.)$ are associated with the suppliers' ranking when they all truthfully report their cost $\mathbf{c}$. Similarly, we suppress the dependency of $\tilde{N}, \bar{N}, S^*$ on suppliers' reported cost. Unless noted otherwise, $\tilde{N}, \bar{N}, S^*$ should be understood as when all suppliers truthfully report their cost $\mathbf{c}$. Then, the last term in (7) has the following expression:

$$\mathsf{Markup}_i(\mathbf{c}, \mathbf{t}^{S^*}) = \begin{cases} \omega_i^1(\mathbf{c}, \mathbf{t}^{S^*}), & \text{if } \hat{\kappa}(i) > \bar{N} \\ \omega_i^{2a}(\mathbf{c}, \mathbf{t}^{S^*}) + \omega_i^{2b}(\mathbf{c}, \mathbf{t}^{S^*}), & \text{if } \hat{\kappa}(i) \leq \bar{N} \end{cases} \tag{A.20}$$

where ω_i^1, ω_i^{2a} and ω_i^{2b} are defined as:

$$\begin{aligned}
\omega_i^1(\mathbf{c}, \mathbf{t}^{S^*}) &= \sum_{l=0}^{\bar{N}-|S^*|} \int_{\tilde{c}_i(l)}^{\tilde{c}_i(l+1)} \left(\mathbf{1}_{[\psi_i(z) \leq \psi_{\hat{\imath}(j)} - t_{\hat{\imath}(j)}, \forall j \leq |S^*|]}\right) \left(\prod_{j=1}^{l} G_{\hat{\imath}(|S^*|+j)}(\psi_{\hat{\imath}(|S^*|+j)} - \psi_i(z))\right) \\
&\qquad \left(\mathbf{1}_{[\psi_i(z) \leq \psi_{\hat{\imath}(j)}, \forall j > |S^*|+l]}\right) dz \\
\omega_i^{2a}(\mathbf{c}, \mathbf{t}^{S^*}) &= \int_{\hat{\mu}_i^{-1}(\hat{\mu}_{\iota(\bar{N}+1)})}^{\bar{c}_i} \left(\mathbf{1}_{[\psi_i(z) \leq \psi_{\hat{\imath}(j)} - t_{\hat{\imath}(j)}, \forall j \leq |S^*|, \hat{\imath}(j) \neq i]}\right) \left(\prod_{j=|S^*|+1}^{\bar{N}+1} G_{\hat{\imath}(j)}(\psi_{\hat{\imath}(j)} - \psi_i(z))\right) \\
&\qquad \left(\mathbf{1}_{[\psi_i(z) \leq \psi_{\hat{\imath}(j)}, \forall j > \bar{N}+1]}\right) dz \\
\omega_i^{2b}(\mathbf{c}, \mathbf{t}^{S^*}) &= \sum_{l=0}^{\bar{N}-|S^*|} \int_{\hat{c}_i(l)}^{\hat{c}_i(l+1)} \left(\mathbf{1}_{[(\psi_i(z) - t_i) \vee \hat{\mu}_i(z) \leq \psi_{\hat{\imath}(j)} - t_{\hat{\imath}(j)}, \forall j \leq |S^*|, \hat{\imath}(j) \neq i]}\right)
\end{aligned}$$

$$\left(\prod_{j=|S^*|+1}^{|S^*|+l} G_{\hat{\iota}(j)}(\psi_{\hat{\iota}(j)} - \hat{\mu}_i(z) \vee (\psi_i(z) - t_i))\right)$$
$$\left(\prod_{j=|S^*|+l+1}^{\bar{N}} \left[G_{\hat{\iota}(j)}(\psi_{\hat{\iota}(j)} - \psi_i(z) + t_i)\right]^{\mathbf{1}_{[\psi_i(z)-t_i \geq \hat{\mu}_{\hat{\iota}(j)}]}} \mathbf{1}_{[\psi_i(z)-t_i \leq \psi_{\hat{\iota}(j)}]}{}^{\mathbf{1}_{[\psi_i(z)-t_i < \hat{\mu}_{\hat{\iota}(j)}]}}\right)$$
$$\left(\prod_{j=\bar{N}+1}^{N} \mathbf{1}_{[\psi_i(z)-t_i \leq \psi_{\hat{\iota}(j)}]}\right) dz,$$

where the thresholds in the integrals are defined as follows: $\tilde{c}_i(0) := c_i$, $\tilde{c}_i(k) := \psi_i^{-1}(\hat{\mu}_{\hat{\iota}(|S^*(\mathbf{c})+k|)})$ for all $1 \leq k \leq \bar{N} - |S^*(\mathbf{c})|$, and $\tilde{c}_i(\bar{N} - |S^*(\mathbf{c})| + 1) := \bar{c}_i$; $\hat{c}_i(0) = c_i$, $\hat{c}_i(k) = \hat{\mu}_i^{-1}(\hat{\mu}_{\hat{\iota}(|S^*(\mathbf{c})|+k)})$ for all $1 \leq k \leq \bar{N} - |S^*(\mathbf{c})| + 1$. We now provide the derivation of the expression above which will reveal supplier i's bidding incentive. Note that depending on supplier i's ranking, there are two cases analyzed below by conditioning on $\mathbf{t}^{S^*(\mathbf{c})}$.

Case 1: $\hat{\kappa}(i) > \bar{N}$**.** In this case, supplier i will never be investigated if he inflates his cost; moreover, the buyer will always investigate suppliers in $S^*(\mathbf{c})$ and maybe more. Hence, if supplier i bids $z_i \geq c_i$, we can classify the other suppliers into three groups: (i) suppliers in $S^*(\mathbf{c})$, (ii) suppliers in $S^*(z_i, \mathbf{c}_{-i}) - S^*(\mathbf{c})$, and (iii) suppliers in $\overline{S^*(z_i, \mathbf{c}_{-i})}$. Hence, supplier i wins if and only if the following hold: The updated virtual costs of suppliers in (i) are all higher than supplier i's virtual cost, the cost reductions of suppliers in (ii) are not too large so that their updated virtual costs are all higher than supplier 1's virtual cost, and the virtual costs of suppliers in (iii) are all higher than supplier i's virtual cost. Hence, for i such that $\hat{\kappa}(i) > \bar{N}$, based on the number of additional suppliers that will be investigated $l = |S^*(z_i, \mathbf{c}_{-i}) - S^*(\mathbf{c})|$ (this also explains the definition of the thresholds $\tilde{c}_i(k)$ above) when supplier i reports z_i, $\mathsf{Markup}_i(\mathbf{c}, \mathbf{t}^{S^*})$ equals $\omega_i^1(\mathbf{c}, \mathbf{t}^{S^*})$.

Case 2: $\hat{\kappa}(i) \leq \bar{N}$**.** Note that in this case, for supplier i to win when all suppliers report truthfully, supplier i needs to be investigated, i.e., $i \in S^*(\mathbf{c})$; so conditioning on $\mathbf{t}^{S^*(\mathbf{c})}$, t_i is determined. (Otherwise, if supplier i wins and is not investigated, then $\hat{\mu}_i(c_i) \leq \psi_i(c_i) \leq \min_{j \neq i}\{\psi_j(c_j) - \mathbf{1}_{[j \in S^*(z_i, \mathbf{c}_{-i})]} t_j\}$; this means that supplier i should be investigated, leading to a contradiction.) In this case, when supplier i inflates his cost to $z_i \geq c_i$, there are two scenarios depending on how much he inflates.

Scenario (a): Supplier i inflates his cost so high that he is no longer ranked among the lowest $\bar{N}(z_i, \mathbf{c}_{-i})$ suppliers. Then the buyer will not investigate supplier i but may investigate other suppliers who are not in $S^*(\mathbf{c})$. Note that if supplier i still wins, then his virtual cost $\psi_i(z_i)$ is no higher than all other suppliers' updated virtual cost; this means that the buyer will investigate exactly $\bar{N}(z_i, \mathbf{c}_{-i})$ suppliers. Note that $\bar{N}(z_i, \mathbf{c}_{-i}) = \tilde{N} \wedge (\hat{N} + 1)$ in Scenario (a) where supplier i is not investigated when he reports z_i. We can classify the *other* suppliers into three groups: (i) suppliers in $S^*(\mathbf{c}) - \{i\}$, (ii) suppliers in $S^*(z_i, \mathbf{c}_{-i}) - S^*(\mathbf{c})$ which correspond to those suppliers who are in $\overline{S^*(\mathbf{c})}$ but whose indices are no higher than $\tilde{N} \wedge (\hat{N} + 1)$ (based on the ranking when suppliers report $\mathbf{c}$), and (iii) suppliers in $\bar{S}_s^*(z_i, \mathbf{c}_{-i}) - \{i\}$ which correspond to those suppliers whose rankings (based on the ranking when suppliers report $\mathbf{c}$) are strictly higher than $\tilde{N} \wedge (\hat{N} + 1)$. Hence, supplier i wins if and only if the following hold: The updated virtual costs of suppliers in (i) are higher than supplier i's virtual cost, the cost-reductions of suppliers in (ii) are not too large so that their updated virtual costs are higher than supplier i's virtual cost, and the virtual costs of suppliers in (iii) are higher than the supplier i's virtual cost. Thus, the portion of $\mathsf{Markup}_i(\mathbf{c}, \mathbf{t}^{S^*})$ that comes from scenario (a) equals $\omega_i^{2a}(\mathbf{c}, \mathbf{t}^{S^*})$.

Scenario (b): supplier i does not inflate his cost too much, and he still remains to be one of the $\bar{N}$ suppliers with the lowest indices. First note that in this scenario he wins only if he is one of the investigated suppliers, i.e., $i \in S^*(z_i, \mathbf{c}_{-i})$. (Otherwise, if supplier i wins and is not investigated, then $\hat{\mu}_i(z_i) \leq \psi_i(z_i) \leq \min_{j \neq i}\{\psi_j(c_j) - \mathbf{1}_{[j \in S^*(z_i, \mathbf{c}_{-i})]} t_j\}$; so by the optimal investigation policy, the buyer should investigate supplier i, leading to a contradiction.) Note that given $\mathbf{t}^{S^*(\mathbf{c})}$, the buyer will always investigate suppliers in the set $S^*(\mathbf{c}) - \{i\}$ and may investigate more suppliers if supplier i inflates his true cost to $z_i \geq c_i$. Hence, when supplier i reports $z_i \geq c_i$, we can classify the other suppliers into four groups: (i) suppliers in $S^*(\mathbf{c}) - \{i\}$, (ii) suppliers in $S^*(z_i, \mathbf{c}_{-i}) - S^*(\mathbf{c})$ who are investigated before supplier i, (iii) suppliers in $S^*(z_i, \mathbf{c}_{-i}) - S^*(\mathbf{c})$ who are investigated after supplier i, (iv) suppliers in $\overline{S^*(z_i, \mathbf{c}_{-i})}$. Hence, supplier i wins if and only if the following hold: The updated virtual costs of suppliers in (i) are higher than supplier i's updated virtual cost and his index, the cost reductions of suppliers in (ii) are not too large so that supplier i is still investigated and supplier i's updated virtual cost is lower, the cost reductions of suppliers in (iii) are not too large so that supplier i's updated virtual cost is lower, and the virtual costs of suppliers in (iv) are higher than supplier i's updated virtual cost. Then, based on the number of additional suppliers (i.e., those not in $S^*(\mathbf{c})$) that will be investigated before supplier i (this also explains the definition of the thresholds $\hat{c}_i(k)$ above) when supplier i reports z_i, the portion of $\mathsf{Markup}_i(\mathbf{c}, \mathbf{t}^{S^*})$ that comes from scenario (b) equals $\omega_i^{2b}(\mathbf{c}, \mathbf{t}^{S^*})$.

Combining the above leads to (A.20). As is evident from our analysis, inflating one's true cost not only would directly make his virtual cost (and hence his updated virtual cost) less competitive, but may also de-prioritize his investigation which could decrease his chance to be investigated if prior investigations reveal large cost reductions for other suppliers. Therefore, Markup_i can be interpreted as the amount that the winning supplier i could inflate his bid but still win the contract.