

Testing the Standard Model with CP asymmetries in flavor-specific nonleptonic decays

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Motivated by recent indications that the rates of color-allowed nonleptonic channels are not in agreement with their Standard Model expectations based on QCD factorization, we investigate the potential to study CP asymmetries with these decays. In the Standard Model, these flavor-specific decays are sensitive to CP violation in $B_{(s)}^0-\bar{B}_{(s)}^0$ mixing, which is predicted with low uncertainties and can be measured precisely with semileptonic decays. Allowing beyond Standard Model (BSM) contributions to the nonleptonic decay amplitudes, we derive explicit expressions for the flavor-specific CP asymmetries in a model-independent way. We find that BSM contributions could lead to significant enhancements to the CP asymmetries. Therefore measurements of these quantities and subsequent comparison with the CP asymmetries measured with semileptonic decays have potential to identify BSM effects without relying on Standard Model predictions that might be affected by hadronic effects. In addition, we discuss the experimental prospects, and note the excellent potential for a precise determination of the CP asymmetry in $\bar{B}_s \rightarrow D_s^+ \pi^-$ decays by the LHCb experiment.

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I. INTRODUCTION

Recent theoretical investigations [1–4] have revealed a discrepancy between experimental measurements of the rates of color-allowed nonleptonic decays [5–7] and their predicted values in the Standard Model (SM), based on QCD factorization [8]. While the origin of this disagreement could be due to unaccounted-for QCD effects or maybe partly due to ultrasoft photon effects [9], there is also an enticing possibility that physics beyond the Standard Model (BSM) may be contributing. It is therefore of interest to investigate theoretically clean observables that could help to address this possibility. As we will show, the CP asymmetry in the flavor-specific decay $\bar{B}_s \rightarrow D_s^+ \pi^-$ is well suited for this purpose.

We denote the decay amplitude describing the transition of the flavor eigenstate B_q ($q = d, s$) to the final state f by \mathcal{A}_f ; for the decay of a \bar{B}_q eigenstate into f we use the

notation $\bar{\mathcal{A}}_f$. The underlying flavor changing weak quark transitions are described by the effective Hamiltonian. Thus we can write

$$\mathcal{A}_f = \langle f | \mathcal{H}_{\text{eff}} | B_q \rangle, \quad \bar{\mathcal{A}}_f = \langle f | \mathcal{H}_{\text{eff}} | \bar{B}_q \rangle, \quad (1)$$

with obvious extension to the notation for decays into the CP conjugate final states \bar{f} . A *flavor-specific decay* of the B_q meson is defined by the conditions, see e.g., Refs. [10,11],

$$\mathcal{A}_{\bar{f}} = 0 = \bar{\mathcal{A}}_f \quad (2)$$

where

$$\mathcal{A}_f \neq 0; \quad \bar{\mathcal{A}}_{\bar{f}} \neq 0. \quad (3)$$

These two conditions state that the meson B_q can decay into final state f but cannot decay into the CP conjugate final state \bar{f} and that \bar{B}_q cannot decay into f . Examples of \bar{B}_s decays that are flavor specific in the SM include semileptonic decays such as $\bar{B}_s \rightarrow D_s^+ \ell^- \bar{\nu}_\ell$ and nonleptonic decays such as $\bar{B}_s \rightarrow D_s^+ \pi^-$ and $\bar{B}_s \rightarrow K^+ \pi^-$. There

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are corresponding flavor-specific \bar{B}^0 decays to the $D^+\ell^-\bar{\nu}_\ell$, D^+K^- , and $K^-\pi^+$ final states.

Demanding further the absence of direct CP violation in the decay $B_q \rightarrow f$ we get a third condition,

$$\bar{\mathcal{A}}_{\bar{f}} = \mathcal{A}_f. \quad (4)$$

Within the SM, the semileptonic decays and the non-leptonic decays $\bar{B}_s \rightarrow D_s^+\pi^-$ and $\bar{B}^0 \rightarrow D^+K^-$ are expected to have negligible direct CP violation, while the charmless nonleptonic decays $\bar{B}_s \rightarrow K^+\pi^-$ and $\bar{B}^0 \rightarrow K^-\pi^+$ do not satisfy condition (4) [12–14].

Due to weak interactions, transitions like $\bar{B}_q \leftrightarrow B_q$ are possible via box diagrams and we define the meson mass eigenstates $|B_{q,H}\rangle$ ($H = \text{heavy}$, mass M_H^q , and decay rate Γ_H^q) and $|B_{q,L}\rangle$ ($L = \text{light}$, mass M_L^q , and decay rate Γ_L^q) as linear combinations of the flavor eigenstates:

$$|B_{q,L}\rangle = p|B_q\rangle + q|\bar{B}_q\rangle, \quad (5)$$

$$|B_{q,H}\rangle = p|B_q\rangle - q|\bar{B}_q\rangle, \quad (6)$$

with $|p|^2 + |q|^2 = 1$. The ratio of the magnitudes of the coefficients p and q , as well as the mass difference $\Delta M_q = M_H^q - M_L^q$ and the decay rate difference $\Delta\Gamma_q = \Gamma_L^q - \Gamma_H^q$ can be expressed in terms of the absorptive part Γ_{12}^q and the dispersive part M_{12}^q of the box diagrams,

$$\Delta M_q \approx 2|M_{12}^q|, \quad \Delta\Gamma_q \approx 2|\Gamma_{12}^q| \cos \phi_{12}^q, \quad (7)$$

$$\left| \frac{q}{p} \right| \approx 1 - \frac{a_{\text{fs}}^q}{2}, \quad a_{\text{fs}}^q \approx \frac{|\Gamma_{12}^q|}{|M_{12}^q|} \sin \phi_{12}^q, \quad (8)$$

with $\phi_{12}^q = \arg(-M_{12}^q/\Gamma_{12}^q)$.

To measure the a_{fs}^q parameter, which quantifies CP violation in mixing, it is necessary to study neutral mesons that mix before decaying. The general time evolution of the decay rate of neutral B_q mesons, which decay with flavor opposite to that at production, is given by (see e.g., Refs. [10,11,15])

$$\begin{aligned} \Gamma[\bar{B}_q(t) \rightarrow f] = N_f |\mathcal{A}_f|^2 \frac{(1 + |\lambda_f|^2)}{2} (1 + a_{\text{fs}}^q) e^{-\Gamma_q t} \left\{ \cosh\left(\frac{\Delta\Gamma_q t}{2}\right) - \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos(\Delta M_q t) \right. \\ \left. - \frac{2\text{Re}(\lambda_f)}{1 + |\lambda_f|^2} \sinh\left(\frac{\Delta\Gamma_q t}{2}\right) + \frac{2\text{Im}(\lambda_f)}{1 + |\lambda_f|^2} \sin(\Delta M_q t) \right\}, \end{aligned} \quad (9)$$

$$\begin{aligned} \Gamma[B_q(t) \rightarrow \bar{f}] = N_f |\bar{\mathcal{A}}_{\bar{f}}|^2 \frac{(1 + |\lambda_{\bar{f}}|^{-2})}{2} (1 - a_{\text{fs}}^q) e^{-\Gamma_q t} \left\{ \cosh\left(\frac{\Delta\Gamma_q t}{2}\right) - \frac{1 - |\lambda_{\bar{f}}|^{-2}}{1 + |\lambda_{\bar{f}}|^{-2}} \cos(\Delta M_q t) \right. \\ \left. - \frac{2\text{Re}(\frac{1}{\lambda_{\bar{f}}})}{1 + |\lambda_{\bar{f}}|^{-2}} \sinh\left(\frac{\Delta\Gamma_q t}{2}\right) + \frac{2\text{Im}(\frac{1}{\lambda_{\bar{f}}})}{1 + |\lambda_{\bar{f}}|^{-2}} \sin(\Delta M_q t) \right\}. \end{aligned} \quad (10)$$

Here $\Gamma_q = (\Gamma_L^q + \Gamma_H^q)/2$, N_f encodes a time-independent normalization factor, including phase space effects, and the quantities λ_f and $\lambda_{\bar{f}}$ are defined as

$$\lambda_f = \frac{q \bar{\mathcal{A}}_f}{p \mathcal{A}_f} \quad \text{and} \quad \lambda_{\bar{f}} = \frac{q \bar{\mathcal{A}}_{\bar{f}}}{p \mathcal{A}_{\bar{f}}}. \quad (11)$$

It should be noted that the $\bar{\mathcal{A}}_{\bar{f}}$ and $\mathcal{A}_{\bar{f}}$ terms are the product of amplitudes for the B meson decay and for decays of particles in any intermediate state. In particular, in the case of $\bar{B}_s \rightarrow D_s^+\pi^-$ and $\bar{B}^0 \rightarrow D^+K^-$ decays they include the D_s and D decay amplitudes, respectively. We assume throughout the paper that there is no CP violation in the charm meson decays. It is also possible to assume instead that there is negligible CP violation in the \bar{B}_s or \bar{B} decay, in which case measurements of asymmetries probe CP

violation in the D_s and D decays [16]. Ultimately, precise independent measurements of CP asymmetries in D_s and D decays will be needed to disentangle the two effects.

In what follows, we consider the flavor-specific CP asymmetry (often called semileptonic CP asymmetry), defined as

$$A_{\text{fs}}^q = \frac{\Gamma(\bar{B}_q(t) \rightarrow f) - \Gamma(B_q(t) \rightarrow \bar{f})}{\Gamma(\bar{B}_q(t) \rightarrow f) + \Gamma(B_q(t) \rightarrow \bar{f})}. \quad (12)$$

II. A_{fs}^q WITHIN THE SM

Within the SM we get for flavor-specific decays due to condition (2): $\lambda_f = 0 = 1/\lambda_{\bar{f}}$, the simplified time evolution

$$\Gamma[\bar{B}_q(t) \rightarrow f] = \frac{1}{2} N_f |\mathcal{A}_f|^2 (1 + a_{\text{fs}}^q) e^{-\Gamma_q t} X_q^-(t), \quad (13)$$

$$\Gamma[B_q(t) \rightarrow \bar{f}] = \frac{1}{2} N_f |\bar{\mathcal{A}}_{\bar{f}}|^2 (1 - a_{\text{fs}}^q) e^{-\Gamma_q t} X_q^-(t) \quad (14)$$

with the shorthand notation

$$X_q^\pm(t) \equiv \cosh\left(\frac{\Delta\Gamma_q t}{2}\right) \pm \cos(\Delta M_q t). \quad (15)$$

This leads to

$$A_{\text{fs}}^q = \frac{|\mathcal{A}_f|^2 (1 + a_{\text{fs}}^q) - |\bar{\mathcal{A}}_{\bar{f}}|^2 (1 - a_{\text{fs}}^q)}{|\mathcal{A}_f|^2 (1 + a_{\text{fs}}^q) + |\bar{\mathcal{A}}_{\bar{f}}|^2 (1 - a_{\text{fs}}^q)}. \quad (16)$$

Note this result for the asymmetry of time-dependent decay rates given in Eq. (9) does not depend on time. Condition (4) further gives $\bar{\mathcal{A}}_{\bar{f}} = \mathcal{A}_f$ and thus

$$A_{\text{fs}}^q = a_{\text{fs}}^q. \quad (17)$$

The SM predictions for a_{fs}^q are tiny, so that measurements of a_{fs}^q are generally considered null tests of the SM. Based on the calculations in Refs. [17–27], the most recent predictions [28]¹ are

$$\begin{aligned} a_{\text{fs}}^d &= (-4.73 \pm 0.42) \times 10^{-4}, \\ a_{\text{fs}}^s &= (2.06 \pm 0.18) \times 10^{-5}, \\ \left| \frac{\Gamma_{12}^d}{M_{12}^d} \right| &= (4.80 \pm 0.66) \times 10^{-3}, \\ \left| \frac{\Gamma_{12}^s}{M_{12}^s} \right| &= (4.82 \pm 0.64) \times 10^{-3}, \\ \phi_{12}^d &= (-98 \pm 19) \text{ mrad} = (-5.6 \pm 1.1)^\circ, \\ \phi_{12}^s &= (4.3 \pm 0.8) \text{ mrad} = (0.25 \pm 0.05)^\circ. \end{aligned} \quad (18)$$

Measurements of a_{fs}^q have so far been made almost exclusively with semileptonic final states (motivating the alternative notation a_{sl}^q). The latest world averages [30], based mainly on the results of Refs. [31–37], are

$$\begin{aligned} a_{\text{sl}}^d &= a_{\text{fs}}^d = (-21 \pm 17) \times 10^{-4}, \\ a_{\text{sl}}^s &= a_{\text{fs}}^s = (-60 \pm 280) \times 10^{-5}. \end{aligned} \quad (19)$$

¹An alternative renormalization scale setting method was recently suggested for D -mixing [29]. Applied to the case of B -mixing the new procedure would modify visibly neither $|\Gamma_{12}/M_{12}|$ nor the central values of the semileptonic CP asymmetries. It could, however, enhance the theoretical uncertainties in the SM predictions of the semileptonic CP asymmetries up to 100% of the central value. Further insight into the most appropriate renormalization scale setting procedure could be obtained by studies of NNLO-QCD corrections, which are not yet available in the literature.

The experimental precision for these quantities is expected to increase considerably. References [38,39] quote an estimated precision of $\pm 2 \times 10^{-4}$ for a_{sl}^d and $\pm 30 \times 10^{-5}$ for a_{sl}^s , achievable by the LHCb experiment with an integrated luminosity of 300 fb^{-1} . While for a_{sl}^d this approaches the precision necessary to test the SM prediction, this large data sample will still not be sufficient to observe a nonzero value at the SM expectation of a_{fs}^s . Nevertheless, significantly more precise results than currently available will provide stringent constraints on beyond SM contributions to Γ_{12}^s and M_{12}^s , as discussed below. The possibility to determine these asymmetries with flavor-specific nonleptonic decays has not been considered widely, as the lower yields available would result in considerably larger uncertainties compared to the semileptonic decay.

III. A_{fs}^q BEYOND THE SM

There are several possible ways that the quantities A_{fs}^q could be modified in the presence of new physics. We discuss these below.

A. Modification of M_{12}

General new physics effects in the dispersive part of B mixing can be parametrized as (in the convention of [21,40])

$$M_{12}^q = M_{12}^{q,\text{SM}} \cdot \Delta_q = |M_{12}^{q,\text{SM}}| \cdot |\Delta_q| e^{i(\phi_q^{M,\text{SM}} + \phi_q^\Delta)}. \quad (20)$$

Note that only the phase ϕ_{12} is physical, while $\phi_q^{M,\text{SM}}$ and ϕ_q^Δ are convention dependent. The parameters $|\Delta_q|$ are constrained to be close to unity, with around $\pm 10\%$ uncertainty, by the agreement of the experimental measurements [30,41,42] of the mass differences with the theoretical determinations via $\Delta M_q = 2|M_{12}^q|$ [26]. If one assumes Δ_q to be the only source of BSM effects, then the new phases ϕ_q^Δ are constrained by the measurements of the mixing phases $\sin 2\beta$ and $\sin 2\beta_s$ in the golden plated modes $B_d \rightarrow J/\psi K_S$ and $B_s \rightarrow J/\psi \phi$. Comparing the direct measurements of $\sin 2\beta_{(s)}$ with the values obtained by fits of the CKM matrix, e.g., CKMfitter [43] we find that ϕ_q^Δ has to be smaller than 3° . This bound could be softened by allowing for other sources of BSM effects and fine-tuned cancellations between new physics in B mixing and penguin diagrams contributing to the $b \rightarrow c\bar{c}s$ decay. But in the case of new physics only acting in M_{12} , potential sizes of a_{fs}^q could be of the order of 10^{-4} . This is considerably below the current experimental accuracy, and the possible enhancement is not large enough for an unambiguous observation at LHCb with 300 fb^{-1} .

B. Modification of Γ_{12}

The absorptive part of B mixing is in general affected by new physics as (in the convention of [40])

$$\Gamma_{12}^q = \Gamma_{12}^{q,\text{SM}} \cdot \tilde{\Delta}_q = |\Gamma_{12}^{q,\text{SM}}| \cdot |\tilde{\Delta}_q| e^{i(\phi_q^{\text{SM}} - \phi_q^{\tilde{\Delta}})}. \quad (21)$$

In this case we get constraints from the measurements of the decay rate differences $\Delta\Gamma_q$

$$\begin{aligned} \Delta\Gamma_q &= 2|\Gamma_{12}^q| \cos(\phi_q^{\tilde{\Delta}}) \\ &= 2|\Gamma_{12}^{q,\text{SM}}| \cdot |\tilde{\Delta}_q| \cos(\phi_{12}^{q,\text{SM}} + \phi_q^{\tilde{\Delta}} + \phi_q^{\tilde{\Delta}}). \end{aligned} \quad (22)$$

For $\Delta\Gamma_s$, experimental measurements [30,44–46] agree well with theory [28] with a relative theory precision of the order of 15%. Putting the origin of a potential difference of experiment and theory solely in the cosine of Eq. (22) and taking the bound on $\phi_q^{\tilde{\Delta}}$ from Sec. III A into account, then this translates into a maximal size of the new phase $\phi_s^{\tilde{\Delta}}$ of the order of 30° .² Such a sizable new phase $\phi_s^{\tilde{\Delta}}$ would lead to a strong enhancement of a_{fs}^s , close to the current experimental bound³

$$\begin{aligned} a_{\text{fs}}^q &= \frac{|\Gamma_{12}^q|}{|M_{12}^q|} \sin \phi_{12}^q \\ &= a_{\text{fs}}^{q,\text{SM}} \frac{|\tilde{\Delta}_q| \sin(\phi_{12}^{q,\text{SM}} + \phi_q^{\tilde{\Delta}} + \phi_q^{\tilde{\Delta}})}{|\Delta_q| \sin \phi_{12}^{q,\text{SM}}}. \end{aligned} \quad (23)$$

There is even more space for a possible enhancement of a_{fs}^d via BSM effects in Γ_{12}^d (see also Refs. [47,48]), since there are only relatively weak experimental constraints on $\Delta\Gamma_d$ [30,49]. This strongly motivates improved experimental measurements of a_{fs}^s and a_{fs}^d .

C. Modification of the $B \rightarrow f$ decay amplitude

As mentioned earlier, measurements of the rates of color-allowed nonleptonic decays seem to deviate significantly from SM predictions [1–4]. For the decay $\bar{B}_s \rightarrow D_s^+ \pi^-$, Ref. [2] quotes a deviation of over four standard deviations between the experimental branching fraction $(3.00 \pm 0.23) \times 10^{-3}$ [5,50] and the QCD factorization prediction $(4.42 \pm 0.21) \times 10^{-3}$. In the case of the CKM suppressed decay $\bar{B}_d \rightarrow D^+ K^-$ this deviation is even larger than five standard deviations. Commonly CKM leading, nonleptonic tree-level decays have been considered to be insensitive to new physics effects.

²There could also be some further, less pronounced enhancement due to modifications in $|\tilde{\Delta}_q|$.

³Note that Ref. [16] obtains more stringent bounds on a_{fs}^s , which we expect to be softened by a full treatment of potential BSM effects in nonleptonic tree-level decays of B mesons, as performed in Ref. [28].

However, general bounds on BSM effects in nonleptonic tree-level decays were systematically studied in Refs. [28,48,51], revealing a sizable allowed parameter space for new effects, which do not violate any theoretical or experimental bound. More recently such effects have also been investigated for the case of the decay $\bar{B}_s \rightarrow D_s^+ K^-$ [52,53]. BSM explanations have been considered in [3,54] and challenged by collider bounds in [55].

Within the SM the decays $\bar{B}_s \rightarrow D_s^+ \pi^-$ and $\bar{B}_d \rightarrow D^+ K^-$ are flavor-specific and CP conserving. Thus, using these decays to determine the asymmetries A_{fs}^q , we expect to get the tiny values a_{fs}^q . However, if BSM effects modify the decay amplitudes, the relation between A_{fs}^q and a_{fs}^q of Eq. (14) is altered. Under the presence of general new physics contributions the decay amplitude of either $B_s \rightarrow D_s^- \pi^+$ or $B_d \rightarrow D^- K^+$ can be written as

$$\begin{aligned} \mathcal{A}_f &= |\mathcal{A}_f^{\text{SM}}| e^{i\phi^{\text{SM}}} e^{i\varphi^{\text{SM}}} + |\mathcal{A}_f^{\text{BSM}}| e^{i\phi^{\text{BSM}}} e^{i\varphi^{\text{BSM}}} \\ &=: |\mathcal{A}_f^{\text{SM}}| e^{i\phi^{\text{SM}}} e^{i\varphi^{\text{SM}}} (1 + r e^{i\phi} e^{i\varphi}), \end{aligned} \quad (24)$$

with relative strong $\phi = \phi^{\text{BSM}} - \phi^{\text{SM}}$ and weak $\varphi = \varphi^{\text{BSM}} - \varphi^{\text{SM}}$ phases, and $r = |\mathcal{A}_f^{\text{BSM}}|/|\mathcal{A}_f^{\text{SM}}|$. The amplitude $\bar{\mathcal{A}}_{\bar{f}}$ for the CP conjugate process is identical to \mathcal{A}_f up to a change in the sign of φ . This allows now for direct CP violation in these decays, challenging condition (4). Nonetheless, the decays are expected to remain flavor specific, since we do not see a realistic possibility to sizably violate condition (2): e.g., at the quark level the decay $\bar{B}_s \rightarrow D_s^+ \pi^-$ looks like $b\bar{s} \rightarrow c\bar{s}\bar{u}d$, while a decay into the CP conjugate final state, triggered by an $b\bar{s} \rightarrow s\bar{c}\bar{d}u$ quark level transition would require at least dimension-nine six-quark operators. Condition (2) is also challenging to test experimentally, although this has been considered in Ref. [56]. Inserting

$$\begin{aligned} |\mathcal{A}_f|^2 &= |\mathcal{A}_f^{\text{SM}}|^2 [1 + r^2 + 2r(\cos \phi \cos \varphi - \sin \phi \sin \varphi)], \\ |\bar{\mathcal{A}}_{\bar{f}}|^2 &= |\mathcal{A}_f^{\text{SM}}|^2 [1 + r^2 + 2r(\cos \phi \cos \varphi + \sin \phi \sin \varphi)] \end{aligned} \quad (25)$$

into Eq. (13) leads to

$$\begin{aligned} A_{\text{fs}}^q &= \frac{a_{\text{fs}}^q - 2r \sin \phi \sin \varphi + 2a_{\text{fs}}^q r \cos \phi \cos \varphi + a_{\text{fs}}^q r^2}{1 + 2r \cos \phi \cos \varphi + r^2 - 2a_{\text{fs}}^q r \sin \phi \sin \varphi} \\ &\approx a_{\text{fs}}^q - A_{\text{dir}}^q, \end{aligned} \quad (26)$$

with the direct CP asymmetry $A_{\text{dir}}^q \approx 2r \sin \phi \sin \varphi$ [formally defined in the Appendix, Eq. (39)].⁴ To obtain the last expression in Eq. (26) we assume a_{fs}^q and r to be small quantities and have expanded up to leading order in these

⁴Note that a_{fs}^q is defined as an asymmetry between the final states f and \bar{f} , while A_{dir}^q is defined as an asymmetry between \bar{f} and f , hence they appear with different signs in Eq. (26).

small parameters. Allowing now for a size of $r \approx 0.1$, which is indicated by the studies in [1–4], one can get—depending on the values of the phases ϕ and φ —values of up to $|A_{\text{fs}}^q| = 0.2$, which are several orders of magnitude larger than the SM values of a_{fs}^q .

Thus, if the experimental value for $A_{\text{fs}}^s(D_s^+\pi^-)$ or $A_{\text{fs}}^d(D^+K^-)$ differs significantly from zero, with the currently achievable experimental precision, one has an unambiguous BSM signal, independent of any theory uncertainties. Moreover, the effects of BSM contributions in M_{12} and Γ_{12} , which affect a_{fs}^s , can be separated from those in the decay amplitude, which affect A_{fs}^s , if we make the assumption that there is no direct CP violation in semileptonic decays which holds to excellent accuracy within the SM (since only one decay amplitude is contributing) and to some extent also beyond the SM [57,58]. In this case $A_{\text{fs}}^s(D_s^+\pi^-) - A_{\text{fs}}^s(D_s^+\ell^-\bar{\nu}_\ell)$ gives a clean determination of $A_{\text{dir}}^s(D_s^+\pi^-)$, and likewise $A_{\text{fs}}^d(D^+K^-) - A_{\text{fs}}^d(D^+\ell^-\bar{\nu}_\ell) = -A_{\text{dir}}^d(D^+K^-)$.⁵

Neither $A_{\text{fs}}^s(D_s^+\pi^-)$ nor $A_{\text{fs}}^d(D^+K^-)$ has yet been experimentally measured. It seems likely, however, that any large asymmetry in $\bar{B}_s \rightarrow D_s^+\pi^-$ decays would have been spotted as this mode has been used for precise determinations of the B_s oscillation frequency [42] and lifetime [59], as well as being a control channel for CP violation studies in

$\bar{B}_s \rightarrow D_s^\pm K^\mp$ decays [60]. In what follows we focus on the $\bar{B}_s \rightarrow D_s^+\pi^-$ mode as this appears to have the potential for precise measurements, but experimental studies of CP violation in $\bar{B}_d \rightarrow D^+K^-$ decays are also well motivated.

IV. UNTAGGED CP ASYMMETRY

In the appendix, we present several further possible CP asymmetries that can be determined with flavor-specific decays, that have contributions from direct CP violation and/or CP violation in mixing. In that respect we will need, in addition to Eqs. (13) and (14), the decay-rate evolution for neutral B_q mesons that decay with the same flavor to that at production. Assuming the condition of Eqs. (2) and (3) is satisfied, these rates are [10,11,15]

$$\Gamma[\bar{B}_q(t) \rightarrow \bar{f}] = \frac{1}{2} N_f |\bar{\mathcal{A}}_f|^2 e^{-\Gamma_q t} X_q^+(t), \quad (27)$$

$$\Gamma[B_q(t) \rightarrow f] = \frac{1}{2} N_f |\mathcal{A}_f|^2 e^{-\Gamma_q t} X_q^+(t). \quad (28)$$

A particularly interesting observable is the untagged CP asymmetry, A_{untagged}^q , given by [15,16]

$$A_{\text{untagged}}^q = \frac{[\Gamma(\bar{B}_q(t) \rightarrow \bar{f}) + \Gamma(B_q(t) \rightarrow \bar{f})] - [\Gamma(\bar{B}_q(t) \rightarrow f) + \Gamma(B_q(t) \rightarrow f)]}{[\Gamma(\bar{B}_q(t) \rightarrow \bar{f}) + \Gamma(B_q(t) \rightarrow \bar{f})] + [\Gamma(\bar{B}_q(t) \rightarrow f) + \Gamma(B_q(t) \rightarrow f)]}. \quad (29)$$

Inserting Eq. (13), (14), (27), and (28), we obtain

$$A_{\text{untagged}}^q = \frac{|\bar{\mathcal{A}}_f|^2 [X_q^+(t) + (1 - a_{\text{fs}}^q) X_q^-(t)] - |\mathcal{A}_f|^2 [X_q^+(t) + (1 + a_{\text{fs}}^q) X_q^-(t)]}{|\bar{\mathcal{A}}_f|^2 [X_q^+(t) + (1 - a_{\text{fs}}^q) X_q^-(t)] + |\mathcal{A}_f|^2 [X_q^+(t) + (1 + a_{\text{fs}}^q) X_q^-(t)]} \quad (30)$$

$$= \frac{2r \sin \phi \sin \varphi - a_{\text{fs}}^q (1 + 2r \cos \phi \cos \varphi + r^2) Y(t)}{1 + 2r \cos \phi \cos \varphi + r^2 - 2a_{\text{fs}}^q r \sin \phi \sin \varphi Y(t)}, \quad (31)$$

with

$$Y(t) = \frac{X_q^-(t)}{X_q^+(t) + X_q^-(t)} = \frac{1}{2} \left[1 - \frac{\cos(\Delta M_q t)}{\cosh(\frac{\Delta \Gamma_q t}{2})} \right]. \quad (32)$$

Neglecting CP violation in mixing, $a_{\text{fs}}^q = 0$, we find

$$A_{\text{untagged}}^q = \frac{2r \sin \phi \sin \varphi}{1 + 2r \cos \phi \cos \varphi + r^2} = A_{\text{dir}}^q, \quad (33)$$

⁵Investigating the difference between $A_{\text{fs}}^s(D_s^+\pi^-)$ and a_{fs}^s was suggested in Ref. [16], with an emphasis on potential BSM effects in the subsequent D_s^+ decays. We concentrate here on possible BSM effects in the B_s decay itself.

while neglecting direct CP violation gives

$$A_{\text{untagged}}^q = -a_{\text{fs}}^q Y(t). \quad (34)$$

Generally, expanding everything up to linear terms in r and a_{fs}^q , we get

$$A_{\text{untagged}}^q \approx A_{\text{dir}}^q - a_{\text{fs}}^q Y(t). \quad (35)$$

In contrast to Eq. (16), this asymmetry is not independent of time. It is, however, a convenient approach with which to study B^0 decays since it allows different sources of asymmetry to be disentangled. Measurements of a_{fs}^d have been made by fitting this time-dependent untagged asymmetry, using semileptonic decays in which the contribution from A_{dir}^d is expected to vanish [34,36].

For the B_s case, it is experimentally convenient to measure the untagged asymmetry of time-integrated decay rates

$$\langle A_{\text{untagged}}^q \rangle = \frac{\int_0^\infty dt [\Gamma(\bar{B}_q(t) \rightarrow \bar{f}) + \Gamma(B_q(t) \rightarrow \bar{f})] - \int_0^\infty dt [\Gamma(\bar{B}_q(t) \rightarrow f) + \Gamma(B_q(t) \rightarrow f)]}{\int_0^\infty dt [\Gamma(\bar{B}_q(t) \rightarrow \bar{f}) + \Gamma(B_q(t) \rightarrow \bar{f})] + \int_0^\infty dt [\Gamma(\bar{B}_q(t) \rightarrow f) + \Gamma(B_q(t) \rightarrow f)]} \quad (36)$$

$$= \frac{4r \sin \phi \sin \varphi - a_{\text{fs}}^q (1 - \rho_q) (1 + 2r \cos \phi \cos \varphi + r^2)}{2(1 + 2r \cos \phi \cos \varphi + r^2 - a_{\text{fs}}^q (1 - \rho_q) r \sin \phi \sin \varphi)}, \quad (37)$$

where

$$\rho_q = \frac{\Gamma_q^2 - \frac{\Delta \Gamma_q^2}{4}}{\Gamma_q^2 + \Delta M_q^2} \quad (\rho_d \approx 0.63 \text{ and } \rho_s \approx 0.001). \quad (38)$$

Expanding again up to linear terms in r and a_{fs}^q one obtains:

$$\langle A_{\text{untagged}}^q \rangle \approx A_{\text{dir}}^q - \frac{a_{\text{fs}}^q}{2} (1 - \rho_q). \quad (39)$$

In the case of B_s decays, where the oscillation frequency is fast compared to the lifetime, the dilution factor multiplying a_{fs}^q is effectively only 0.5. Since determining $\langle A_{\text{untagged}}^q \rangle$ avoids the need to tag the flavor of the B_s meson at production, this is an experimentally attractive approach with which to measure a_{fs}^s , as quantified below. This has been exploited in existing measurements with semileptonic decays where the A_{dir}^q term is assumed to be zero [35,37,61]. The same approach is also used for measurements of direct CP violation in modes where the a_{fs}^q contribution is negligible, for example $\bar{B}^0 \rightarrow K^- \pi^+$ and $\bar{B}_s \rightarrow K^+ \pi^-$ [14]. In this case, the use of the untagged asymmetry does not cause any dilution of the sensitivity to A_{dir}^q . Note, however, that if a_{fs}^d or a_{fs}^s were as large in magnitude as 5×10^{-3} , at the extreme of their currently experimentally allowed ranges, this would according to Eq. (39) induce a correction of about $1(2.5) \times 10^{-3}$ in every A_{dir}^q measurement made with untagged B^0 (B_s) decays.

We now consider the experimental prospects for measurements of $\langle A_{\text{untagged}}^s \rangle$ in $\bar{B}_s \rightarrow D_s^+ \pi^-$ decays. The LHCb experiment appears to have by far the best prospects to determine this quantity precisely, having previously demonstrated the capability to obtain large, low-background samples in this decay channel [42]. In addition to the existing data sample, corresponding to 9 fb^{-1} of pp collision data collected in Runs 1 and 2 of the Large Hadron Collider, an additional $\approx 15 \text{ fb}^{-1}$ of data is anticipated to be recorded during Run 3 with an upgraded detector [62]. A new, fully software-implemented trigger strategy that will be utilized during Run 3 means that LHCb will benefit from enhanced efficiency for hadronic decay modes such as $\bar{B}_s \rightarrow D_s^+ \pi^-$.

Based on the yields available in the existing data [42], and the increase anticipated to be forthcoming with Run 3, we project a sensitivity to $\langle A_{\text{untagged}}^s \rangle$ in $\bar{B}_s \rightarrow D_s^+ \pi^-$ decays of

$\mathcal{O}(10^{-3})$. If systematic uncertainties can be controlled, it will be possible to further reduce this uncertainty as a total sample of up to 300 fb^{-1} is collected by LHCb through operation in subsequent LHC run periods [38]. As discussed above new physics contributions to tree-level amplitudes may modify this value from its tiny SM value to $\mathcal{O}(10^{-2})$ or above, and hence the experimental measurement will either discover or significantly constrain these BSM effects. Measurements of A_{fs}^s with semileptonic decays are expected to be even more precise, and will constrain the contribution to $\langle A_{\text{untagged}}^s \rangle$ from a_{fs}^s , assuming no direct CP violation in semileptonic decays. Indeed, the existing limits on a_{fs}^s from semileptonic measurements, which are consistent with the tiny SM expectation, are sufficient to conclude that a nonzero value of $\langle A_{\text{untagged}}^s \rangle$ in $\bar{B}_s \rightarrow D_s^+ \pi^-$ decays at the $\mathcal{O}(10^{-2})$ level would be clear evidence for BSM effects causing direct CP violation.

Experimentally, the quantity that is directly measured is

$$A_{\text{raw}} = \frac{N(D_s^+ \pi^-) - N(D_s^- \pi^+)}{N(D_s^+ \pi^-) + N(D_s^- \pi^+)} \quad (40)$$

where $N(X)$ is the total number of $B_s^0 \rightarrow X$ and $\bar{B}_s^0 \rightarrow X$ decays observed in the data. This is related to $\langle A_{\text{untagged}}^s \rangle$ by

$$\begin{aligned} \langle A_{\text{untagged}}^s \rangle &= A_{\text{raw}} - A_{\text{det}} \\ &- A_{\text{prod}} \frac{\int_{t=0}^{\infty} e^{-\Gamma_s t} \cos(\Delta M_s t) \epsilon(t) dt}{\int_{t=0}^{\infty} e^{-\Gamma_s t} \cosh(\frac{\Delta \Gamma_s t}{2}) \epsilon(t) dt} \\ &- \sum_i f_{\text{bkg}}^i \cdot A_{\text{bkg}}^i. \end{aligned} \quad (41)$$

The detector asymmetries, A_{det} , are reduced by reconstructing the D_s^\pm meson in the $D_s^\pm \rightarrow \phi \pi^\pm$ final state. The \bar{B}_s decay is then fully reconstructed in the symmetric $K^\pm K^\mp \pi^\pm \pi^\mp$ final state with the two kaons having approximately the same momentum distribution. A small detection asymmetry remains due to the momentum difference between the π^\pm originating from a \bar{B}_s versus a D_s^\pm decay, but these effects can be understood using control samples [63]. (Similarly, if reconstruction and detection asymmetries of K^\pm mesons are well understood, the whole Dalitz plot of $D_s^\pm \rightarrow K^+ K^- \pi^\pm$ decays can be used to increase the sample size.)

The $B_s^0 - \bar{B}_s^0$ production asymmetry in pp collisions with decays within the LHCb detector acceptance is denoted by

A_{prod} , and can be measured using the decay-time dependence of flavor-specific decays [64]. Due to fast B_s^0 - \bar{B}_s^0 oscillations, the impact of A_{prod} is significantly diluted by the time integral ratio in Eq. (41). The tiny residual contribution can nonetheless be calculated and corrected for. This calculation must also take into account the fact that the acceptance $\epsilon(t)$ depends on the B meson decay time, and hence enters the integrals in Eq. (41). [For completeness, the decay-time acceptance function should also be taken into account when determining Eq. (36), which impacts on the dilution factor of Eq. (39).]

The asymmetries from various sources of background decays are accounted for through A_{bkg}^i which is the asymmetry of background contribution i . Each background contribution is given a weight, f_{bkg}^i , according to its relative fraction in the data. Since the background fractions are low, the sources of background are well understood [42] and their asymmetries can be determined from control samples, this is not expected to provide a limiting systematic uncertainty.

As previously noted, by not attempting to distinguish between the mixed $\bar{B}_s(t) \rightarrow D_s^- \pi^+$ decays and the unmixed $\bar{B}_s(t) \rightarrow D_s^+ \pi^-$ decays, there is a significant gain in the statistics available to measure A_{dir}^s . This is much greater than the factor of 2 one would naively expect from the large value of ΔM_s , since one no longer requires initial state flavor tagging. LHCb has achieved a tagging efficiency for \bar{B}_s mesons of $\epsilon_{\text{tag}} \approx 80\%$ and a mistag rate of $w \approx 36\%$ [42], giving an effective tagging efficiency of $\epsilon_{\text{tag}}(1-2w)^2 \approx 6\%$. Consequently, untagged methods are highly preferable for the studies of CP asymmetries in B_s mesons discussed here.

To determine A_{dir}^d for the flavor-specific $\bar{B}^0 \rightarrow D^+ K^-$ decays it would be preferable to study the decay-time dependence of the untagged asymmetry as given in Eq. (32). Once experimental effects are taken into account it can be shown that fitting this distribution allows the separate measurement of the combinations $A_{\text{dir}}^d + A_{\text{det}} + a_{\text{fs}}^d/2$ and $A_{\text{prod}} + a_{\text{fs}}^d/2$ [36,64]. Hence it is necessary to take as an external input the value of a_{fs}^d obtained from semileptonic decays (under the assumption of no direct CP violation). The detection asymmetry A_{det} can be determined from control samples as before, although in this case with the favored $D^\pm \rightarrow K^\mp \pi^\pm \pi^\pm$ decay the final state is not symmetric so one cannot benefit from cancellations of asymmetries as in the case of $\bar{B}_s \rightarrow D_s^+ \pi^-$.

V. CONCLUSION

We have studied the CP asymmetries that can be investigated using flavor-specific decays, with particular attention to the nonleptonic decays $\bar{B}_s \rightarrow D_s^+ \pi^-$ and $\bar{B}^0 \rightarrow D^+ K^-$ that have not previously been used for this purpose.

In particular, we have derived explicit analytic expressions for the time-dependent and time-integrated flavor-specific CP asymmetries allowing BSM contributions in a model-independent way. Within the SM no direct CP violation occurs in these nonleptonic decays, and they be used to determine the flavor-specific CP asymmetry a_{fs}^q , albeit with worse precision than obtained with semileptonic decays. If new physics appears only in B mixing then semileptonic decays will still be superior in the experimental determination of the flavor-specific CP asymmetry. This changes, however, as soon as new CP violating contributions to the nonleptonic decays are allowed. In this case the tiny effects due to a_{fs}^q might be completely overshadowed by the contributions stemming from direct CP violation.

Experimentally the $\bar{B}_s \rightarrow D_s^+ \pi^-$ decay is particularly attractive, due to the large available yield, the symmetric final state, and the fact that measurements can be made without the need to determine the production flavor of the B_s^0 mesons in $\bar{B}_s \rightarrow D_s^+ \pi^-$. The untagged and time-integrated CP asymmetries depend on both a_{fs}^s and direct CP violation and are therefore sensitive to BSM effects in either. We expect a sensitivity of around one per mille for the untagged asymmetry in $\bar{B}_s \rightarrow D_s^+ \pi^-$ decays can be achieved at LHCb with Run 3 data, with improvement possible as larger data samples are collected further into the future. This will allow BSM effects causing direct CP violation in these decays to either be discovered or significantly constrained. Once the precision reaches a level that is sensitive to the Standard Model value of a_{fs}^s , one can consider the difference in A_{fs}^s values measured in $\bar{B}_s \rightarrow D_s^+ \pi^-$ and in semileptonic decays. Any significantly nonzero value of this difference would be an unambiguous signal of new physics, not relying on any theoretical estimates of nonperturbative contributions.

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APPENDIX

For completeness we present expressions for other CP asymmetries, see also Refs. [10,11,15].

1. Direct CP asymmetry

The direct CP asymmetry can be defined as the asymmetry of the decay rates for neutral B_q mesons that decay with the same flavor as at production [see Eqs. (27) and (28)], i.e.,

$$\begin{aligned}
 A_{\text{dir}}^q &= \frac{\Gamma(\bar{B}_q(t) \rightarrow \bar{f}) - \Gamma(B_q(t) \rightarrow f)}{\Gamma(\bar{B}_q(t) \rightarrow \bar{f}) + \Gamma(B_q(t) \rightarrow f)} \\
 &= \frac{|\bar{\mathcal{A}}_{\bar{f}}|^2 - |\mathcal{A}_f|^2}{|\bar{\mathcal{A}}_{\bar{f}}|^2 + |\mathcal{A}_f|^2} \\
 &= \frac{2r \sin \phi \sin \varphi}{1 + 2r \cos \phi \cos \varphi + r^2}, \\
 &\approx 2r \sin \phi \sin \varphi, \tag{A1}
 \end{aligned}$$

where the approximation is a good one for $r \ll 1$. It is simply the asymmetry of the decay amplitudes squared, and hence also equal to the asymmetry of decay rates at $t = 0$, and to the untagged CP asymmetry in the limit of negligible a_{fs}^q .

2. Indirect CP asymmetry

Indirect CP asymmetry is typically defined as

$$\begin{aligned}
 A_{\text{ind}}^q &= \frac{\Gamma(\bar{B}_q(t) \rightarrow f) - \Gamma(B_q(t) \rightarrow f)}{\Gamma(\bar{B}_q(t) \rightarrow f) + \Gamma(B_q(t) \rightarrow f)} \\
 &= -\frac{2 \cos(\Delta M_q t) - a_{\text{fs}}^q X_q^-(t)}{2 \cosh(\frac{\Delta \Gamma_q t}{2}) + a_{\text{fs}}^q X_q^-(t)}. \tag{A2}
 \end{aligned}$$

Since the definition of this asymmetry involves only one final state, for flavor-specific decays it does not depend on A_{dir}^q . The dominant contribution to this asymmetry is given by $\cos(\Delta M_q t) / \cosh(\Delta \Gamma_q t / 2)$, with a small correction proportional to a_{fs}^q .

In a similar way one may also define

$$\begin{aligned}
 \tilde{A}_{\text{ind}}^q &= \frac{\Gamma(\bar{B}_q(t) \rightarrow \bar{f}) - \Gamma(B_q(t) \rightarrow \bar{f})}{\Gamma(\bar{B}_q(t) \rightarrow \bar{f}) + \Gamma(B_q(t) \rightarrow \bar{f})} \\
 &= \frac{2 \cos(\Delta M_q t) + a_{\text{fs}}^q X_q^-(t)}{2 \cosh(\frac{\Delta \Gamma_q t}{2}) - a_{\text{fs}}^q X_q^-(t)}, \tag{A3}
 \end{aligned}$$

which gives up to an overall sign the same result as A_{ind}^q , when a_{fs}^q is replaced by $-a_{\text{fs}}^q$. Note that if we add these two asymmetries, then the leading terms cancel and the sum is proportional to a_{fs}^q :

$$A_{\text{ind}}^q + \tilde{A}_{\text{ind}}^q \approx a_{\text{fs}}^q \left[1 - \frac{\cos^2(\Delta M_q t)}{\cosh^2(\Delta \Gamma_q t / 2)} \right]. \tag{A4}$$

This provides a possibility to determine a_{fs}^q independently of, and with no assumption on, A_{dir}^q .

In addition, we consider the time-integrated indirect CP asymmetries

$$\langle A_{\text{ind}}^q \rangle = \frac{\int_0^\infty dt [\Gamma(\bar{B}_q(t) \rightarrow f) - \Gamma(B_q(t) \rightarrow f)]}{\int_0^\infty dt [\Gamma(\bar{B}_q(t) \rightarrow f) + \Gamma(B_q(t) \rightarrow f)]}, \tag{A5}$$

$$\langle \tilde{A}_{\text{ind}}^q \rangle = \frac{\int_0^\infty dt [\Gamma(\bar{B}_q(t) \rightarrow \bar{f}) - \Gamma(B_q(t) \rightarrow \bar{f})]}{\int_0^\infty dt [\Gamma(\bar{B}_q(t) \rightarrow \bar{f}) + \Gamma(B_q(t) \rightarrow \bar{f})]}. \tag{A6}$$

Using Eqs. (13), (14), (27), and (28) we obtain

$$\langle A_{\text{ind}}^q \rangle = -\frac{\rho_q - a_{\text{fs}}^q R_q}{1 + a_{\text{fs}}^q R_q}, \quad \langle \tilde{A}_{\text{ind}}^q \rangle = \frac{\rho_q + a_{\text{fs}}^q R_q}{1 - a_{\text{fs}}^q R_q} \tag{A7}$$

where ρ_q is defined in Eq. (38) and

$$R_q = \frac{\frac{\Delta \Gamma_q^2}{4} + \Delta M_q^2}{2(\Gamma_q^2 + \Delta M_q^2)}. \tag{A8}$$

The time-integrated asymmetries have a leading dependence on ρ_q and small corrections proportional to a_{fs}^q . Note that this leading term cancels in the sum of the two time-integrated CP asymmetries:

$$\langle A_{\text{ind}}^q \rangle + \langle \tilde{A}_{\text{ind}}^q \rangle \approx 2a_{\text{fs}}^q R_q (1 + \rho_q). \tag{A9}$$

Since $\Delta \Gamma_s \ll \Gamma_s \ll \Delta M_s$, one can expand further to get

$$\langle A_{\text{ind}}^s \rangle + \langle \tilde{A}_{\text{ind}}^s \rangle \approx a_{\text{fs}}^s \left(1 - \frac{\Gamma_s^4}{\Delta M_s^4} \right). \tag{A10}$$

3. Mixed CP asymmetry

We can also look at the asymmetry

$$A_{\text{mix}}^q = \frac{\Gamma(B_q(t) \rightarrow \bar{f}) - \Gamma(B_q(t) \rightarrow f)}{\Gamma(B_q(t) \rightarrow \bar{f}) + \Gamma(B_q(t) \rightarrow f)}, \tag{A11}$$

where we get for flavor-specific decays

$$\begin{aligned}
 A_{\text{mix}}^q &= \frac{|\bar{\mathcal{A}}_{\bar{f}}|^2 (1 - a_{\text{fs}}^q) X_q^-(t) - |\mathcal{A}_f|^2 X_q^+(t)}{|\bar{\mathcal{A}}_{\bar{f}}|^2 (1 - a_{\text{fs}}^q) X_q^-(t) + |\mathcal{A}_f|^2 X_q^+(t)} \\
 &= -\frac{F_q(t) - f(r, a_{\text{fs}}^q, \phi, \varphi)}{1 - f(r, a_{\text{fs}}^q, \phi, \varphi) F_q(t)}, \tag{A12}
 \end{aligned}$$

with the auxiliary functions

$$F_q(t) = \frac{\cos(\Delta M_q t)}{\cosh(\frac{\Delta \Gamma_q t}{2})}, \tag{A13}$$

$$f(r, a, \phi, \varphi) = \frac{4r \sin \phi \sin \varphi - a - 2ar(\sin \phi \sin \varphi + \cos \phi \cos \varphi) - ar^2}{2 + 4r \cos \phi \cos \varphi - a + 2r^2 - 2ar(\sin \phi \sin \varphi + \cos \phi \cos \varphi) - ar^2}. \quad (\text{A14})$$

Keeping only terms linear in a_{fs}^q and r , we arrive at

$$f(r, a_{\text{fs}}^q, \phi, \varphi) \approx A_{\text{dir}}^q - \frac{a_{\text{fs}}^q}{2}, \quad (\text{A15})$$

with A_{dir}^q given in Eq. (39). Having no new physics in the decay $\bar{B}_s \rightarrow D_s^+ \pi^-$, we get $f(0, a_{\text{fs}}^s, \phi, \varphi) = -a_{\text{fs}}^s/(2 - a_{\text{fs}}^s) \approx -a_{\text{fs}}^s/2$, while for a sizable phases ϕ and φ and for larger values of r we can neglect a_{fs}^s and get $f(r, 0, \phi, \varphi) \approx A_{\text{dir}}^q$. In the approximation of keeping only the linear terms in a_{fs}^q and r the asymmetry looks as

$$A_{\text{mix}}^q \approx -\frac{\cos(\Delta M_q t)}{\cosh(\frac{\Delta \Gamma_q t}{2})} + \left[A_{\text{dir}}^q - \frac{a_{\text{fs}}^q}{2} \right] \left[1 - \frac{\cos^2(\Delta M_q t)}{\cosh^2(\frac{\Delta \Gamma_q t}{2})} \right]. \quad (\text{A16})$$

As in the case of the indirect CP asymmetries the dominant contribution to this asymmetry is given by $\cos(\Delta M_q t)/\cosh(\Delta \Gamma_q t/2)$, but now the small corrections are proportional to r (in A_{dir}^q) and a_{fs}^q .

And again, one can define a similar asymmetry

$$\tilde{A}_{\text{mix}}^q = \frac{\Gamma(\bar{B}_q(t) \rightarrow \bar{f}) - \Gamma(\bar{B}_q(t) \rightarrow f)}{\Gamma(\bar{B}_q(t) \rightarrow \bar{f}) + \Gamma(\bar{B}_q(t) \rightarrow f)}, \quad (\text{A17})$$

for which we get

$$\begin{aligned} \tilde{A}_{\text{mix}}^q &= \frac{|\bar{\mathcal{A}}_{\bar{f}}|^2 X_q^+(t) - |\mathcal{A}_f|^2 (1 + a_{\text{fs}}^q) X_q^-(t)}{|\bar{\mathcal{A}}_{\bar{f}}|^2 X_q^+(t) + |\mathcal{A}_f|^2 (1 + a_{\text{fs}}^q) X_q^-(t)} \\ &= \frac{F_q(t) - f(r, -a_{\text{fs}}^q, \phi, -\varphi)}{1 - f(r, -a_{\text{fs}}^q, \phi, -\varphi) F_q(t)} \end{aligned} \quad (\text{A18})$$

$$\approx \frac{\cos(\Delta M_q t)}{\cosh(\frac{\Delta \Gamma_q t}{2})} + \left[A_{\text{dir}}^q - \frac{a_{\text{fs}}^q}{2} \right] \left[1 - \frac{\cos^2(\Delta M_q t)}{\cosh^2(\frac{\Delta \Gamma_q t}{2})} \right]. \quad (\text{A19})$$

One can get rid of the dominant contributions in A_{mix}^q and \tilde{A}_{mix}^q by considering the sum of the two, to obtain an observable that is directly proportional to $2r \sin \phi \sin \varphi - a_{\text{fs}}^q/2$:

$$A_{\text{mix}}^q + \tilde{A}_{\text{mix}}^q \approx 2 \left[A_{\text{dir}}^q - \frac{a_{\text{fs}}^q}{2} \right] \left[1 - \frac{\cos^2(\Delta M_q t)}{\cosh^2(\frac{\Delta \Gamma_q t}{2})} \right]. \quad (\text{A20})$$

Defining the time-integrated mixed CP asymmetries

$$\langle A_{\text{mix}}^q \rangle = \frac{\int_0^\infty dt [\Gamma(B_q(t) \rightarrow \bar{f}) - \Gamma(B_q(t) \rightarrow f)]}{\int_0^\infty dt [\Gamma(B_q(t) \rightarrow \bar{f}) + \Gamma(B_q(t) \rightarrow f)]}, \quad (\text{A21})$$

$$\langle \tilde{A}_{\text{mix}}^q \rangle = \frac{\int_0^\infty dt [\Gamma(\bar{B}_q(t) \rightarrow \bar{f}) - \Gamma(\bar{B}_q(t) \rightarrow f)]}{\int_0^\infty dt [\Gamma(\bar{B}_q(t) \rightarrow \bar{f}) + \Gamma(\bar{B}_q(t) \rightarrow f)]}, \quad (\text{A22})$$

one obtains

$$\langle A_{\text{mix}}^q \rangle = -\frac{\rho_q - f(r, a_{\text{fs}}^q, \phi, \varphi)}{1 - f(r, a_{\text{fs}}^q, \phi, \varphi) \rho_q}, \quad (\text{A23})$$

$$\langle \tilde{A}_{\text{mix}}^q \rangle = \frac{\rho_q - f(r, -a_{\text{fs}}^q, \phi, -\varphi)}{1 - f(r, -a_{\text{fs}}^q, \phi, -\varphi) \rho_q}, \quad (\text{A24})$$

where $f(r, a, \phi, \varphi)$ is given in Eq. (A14), and ρ_q in Eq. (38). For the sum of the time-integrated CP asymmetries we get

$$\langle A_{\text{mix}}^q \rangle + \langle \tilde{A}_{\text{mix}}^q \rangle \approx (2A_{\text{dir}}^q - a_{\text{fs}}^q)(1 - \rho_q^2). \quad (\text{A25})$$

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