OPTIMAL STATISTICAL ARBITRAGE: A MODEL SPECIFICATION ANALYSIS ON ISEQ EQUITY DATA

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ABSTRACT

A comprehensive empirical analysis of the novel optimal statistical arbitrage trading model of Bertram (2010) is performed on a dataset of stocks quoted on the Irish Stock Exchange. Evidence of significant errors on average in the key measures underlying the trading model is presented, reflecting the mis-specification of the underlying Gaussian Ornstein–Uhlenbeck (OU) process. Overestimation of the expected return per unit time measure and underestimation of the expected trade cycle time measure are most notable. It is further shown that the Bertram (2010) trading model is more robust to high mean reversion and/or volatility parameter estimates compared to two benchmark models based on the exact and approximate first-hitting time densities of Linetsky (2004) for an OU process.

INTRODUCTION

Statistical arbitrage trading strategies are commonly applied in industry to exploit the long-term statistical relationships that often exist between assets, with *pairs trading* being one of the most well-known applications. Cointegration techniques are generally used to formally establish the statistical relationships upon which various trading strategies may be designed. One of the key considerations in such strategies is the optimal choice of entry and exit levels for the trades. A directly related risk is the stochastic nature of the trade cycle time (i.e. the time between entering, exiting and subsequently re-entering a trade) for such trades.

A number of alternative studies in the area of statistical arbitrage trading have been conducted to date. Many of the papers focus on the design of statistical arbitrage trading rules and the resulting performance when applied to empirical data. These include Burgess (1999, 2000), Trapletti, Geyer and Leisch (2002), Vidyamurthy (2004), Whistler (2004), Elliott,Van Der Hoek and Malcolm (2005), Gatev, Goetzmann and Rouwenhorst (2006) and Do, Faff and Hamza (2006). Andrade, di Petro and Seasholes (2005), Papadakis and Wysocki (2007) and Do and Faff (2009) contribute to the literature by means of providing independent verification of the trading rule proposed by Gatev et al. (2006) and examining the sustainability of profits. Aldridge (2009), Bowen, Hutchinson and O'Sullivan (2010) and Dunis, Giorgini, Laws and Rudy (2010) consider the application of statistical arbitrage trading on high-frequency data. Kanamura, Rachev and Fabozzi (2010) use the approximate first-time hitting density formulation of Linetsky (2004) to develop a total profit model for pairs trading. Other papers of interest include Shleifer and Vishny (1997), Hogan, Jarrow, Teo and Warachka (2004) and Lin, McRae and Gulati (2006).

However, few of these papers deal directly with the issue of optimal entry and exit trading levels in the presence of stochastic trade cycle times. Vidyamurthy (2004) proposes an optimal entry level given by the maximum point on a profitability profile constructed as the product of probability estimates – obtained from counting the number of times each candidate entry level is exceeded – and the associated absolute profit levels. Vidyamurthy (2004) further proposes an exit level that lies through the long-run equilibrium level and of equal distance away as the entry level; this is to account for any potential trade *slippage*. Elliott et al. (2005) use first-passage time theory on the standard Ornstein–Uhlenbeck (OU) process to develop a framework for calculating the expected trade cycle time of a statistical arbitrage strategy, along with symmetric entry and exit boundaries. Do et al. (2006) similarly consider statistical arbitrage trading under an OU framework, drawing on asset pricing theory to inform the underlying statistical applications.

In contrast to the above papers, Bertram (2010) presents a novel approach to the issue of optimal statistical arbitrage trading. Specifically, modelling a given spread series as a mean-reverting OU process, analytic solutions are derived that allow for the optimal entry and exit levels to be determined through maximising either (i) the expected return *per unit time* or (ii) the associated *per unit time* Sharpe ratio. Considering the expected return and Sharpe ratio on a per unit time basis is a very important innovation of Bertram (2010). Statistical arbitrage trading strategies with defined entry and exit levels offer deterministic (log-) returns. However, uncertainty lies in the stochastic trade cycle times associated with such trades. The expected return per unit time is defined as the ratio of the deterministic return to the expected trade cycle time, with the definition of the Sharpe ratio following in a similar way. This normalisation explicitly accounts for the different deterministic returns and expected trade cycle times associated with alternative statistical arbitrage trading strategies defined by alternative entry and exit levels. Hence, the normalisation allows for consistent cross-comparison of alternative statistical arbitrage trading strategies.

The analytic solutions provide significant computational efficiencies, which are of particular advantage for the implementation of statistical arbitrage trading at a high frequency level. However, the underlying OU process being Gaussian only allows for a normal distribution for changes in the spread series. Whereas this allows for analytic solutions to determining optimal entry and exit levels, empirical data does not conform with the assumption of normality. This study contributes to the literature by means of performing a comprehensive model specification analysis of the optimal statistical arbitrage trading model of Bertram (2010) – herein referred to as the Bertram trading model – on a set of cointegrated pairs identified from the current (July 2010) listing of ISEQ stocks on the Irish Stock Exchange. The empirical analysis allows for the identification and quantification of model mis-specification errors in the Bertram trading model. That is, it looks to investigate the mis-specification error introduced in using a Gaussian OU process to describe non-Gaussian empirical cointegration spread series.

The paper further contributes by means of unifying the optimal statistical arbitrage trading criteria set out in the Bertram trading model with the first-hitting time density framework of Linetsky (2004) in order to develop two alternative trading models for benchmark purposes. The benchmark trading models are based on the *exact* and *approximate* first-hitting time density formulations of Linetsky (2004), defined under the OU model specification. The benchmark trading models provide insight into the superior stability of the Bertram trading model for cases where the speed of mean reversion and/or volatility parameters of the OU process are particularly high. The analysis, in presenting results for individual pairs, also helps to understand the sensitivity of the Bertram trading model to variations in the key parameters of the OU process. Furthermore, in implementing the benchmark trading model based on the approximate first-hitting time density of Linetsky (2004), the error introduced as a result of the approximation is investigated and quantified in the context of a trading application.

The remainder of the paper is organised as follows. The next section provides an overview of the optimal statistical arbitrage trading model of Bertram (2010) and discusses the benchmark models based on the first-hitting time density approach of Linetsky (2004). The following section discusses the ISEQ stock data and presents the cointegration and OU fitting results for the set of stock pairings used. The fourth and fifth sections respectively present the empirical analysis in the cases of maximising the expected return per unit time and maximising the Sharpe ratio. The final section concludes.

OPTIMAL STATISTICAL ARBITRAGE TRADING

Bertram (2010) approaches the issue of optimal statistical arbitrage trading by first assuming that the spread on two given asset log-prices, denoted s_t, is described by the following OU process:

$$ds_t = -\alpha s_t dt + \sigma dW_t \tag{1}$$

with α, $\sigma < 0$ and W_t a Wiener process. This model by construction allows for mean reversion of the spread process s_t about a long-run mean level of zero, where the speed of the mean reversion is given by α.[2] The Wiener process, W_t, drives the randomness in the process, where, by definition, changes in the Wiener process,

dW_t, are normally distributed with mean zero and variance dt. The volatility parameter σ is a scaling parameter, which scales this variance to $\sigma^2 dt$. It is the normality of the Wiener that implies the normality of the spread process s_t.

Defining the entry and exit levels of the trading strategy by a and m respectively, a complete trade cycle is the time taken for the spread process to transition from the entry level a to the exit level m and then return back to the entry level a. Formally, the trade cycle time is defined as follows:

$$\mathrm{T} \equiv \mathrm{T}_{a \to m} + \mathrm{T}_{m \to a}$$

where $\mathrm{T}_{a\to m}$ is the time to transition from a to m and $\mathrm{T}_{m\to a}$ is the time to transition from m to a, and the independence of the two times follows from the Markovian property of the OU process. So, T is a random variable representing the complete trade cycle time for the statistical arbitrage trading strategy.

Given relative transaction costs c, the total log-return from one complete trade cycle is given by $r(a,m,c) \equiv m - a - c$. That is, the log-return is given as the difference between the exit level and entry level. Important to note is that this log-return is deterministic and known in advance, whereas the associated trade cycle time is stochastic, as already discussed. So, the time it takes to achieve this deterministic log-return is random and unknown in advance. In this context, Bertram (2010) proposes the concept of the expected return per unit time as follows:

$$\xi(a,m,c) \equiv \frac{r(a,m,c)}{E(\mathrm{T})}$$

where $E(\mathrm{T}) = E(\mathrm{T}_{a\to m}) + E(\mathrm{T}_{m\to a})$. That is, $\xi(a,m,c)$ is the ratio of the deterministic log-return to the expected trade cycle time. This normalisation explicitly accounts for the different deterministic log-returns and expected trade cycle times associated with alternative choices of the entry and exit levels. Therefore, the normalisation allows for consistent cross-comparison of the alternative statistical arbitrage trading strategies. Bertram (2010) further proposes a variance of return per unit time measure as follows:

$$\varsigma(a,m,c) \equiv \frac{r^2(a,m,c)V(\mathrm{T})}{E^3(\mathrm{T})}$$

where $V(\mathrm{T}) = V(\mathrm{T}_{a\to m}) + V(\mathrm{T}_{m\to a})$ is the variance of the trade cycle time.

Following a transformation of the OU process in Equation 1 to a dimensionless system, and drawing on the first-passage time theory of Thomas (1975), Sato (1977) and Ricciardi and Sato (1988), Bertram (2010) derives analytic expressions for $E(\mathrm{T})$, $V(\mathrm{T})$, $\xi(a,m,c)$ and $\varsigma(a,m,c)$. These analytic expressions involve standard mathematical tools; namely the imaginary error function, the gamma function and the digamma function. With these analytic results in place, it is shown that the optimal entry and exit levels a^* and m^* may be derived by maximising the expected return per unit time, $\xi(a,m,c)$. Solving for the optimal entry and exit levels a^* and m^*

is straightforward and, furthermore, it is shown that they are symmetrically positioned about the long-run mean level.

Bertram (2010) further develops a second approach, whereby the optimal entry and exit levels are determined by means of maximising the associated per unit time Sharpe ratio. For this, the per unit time Sharpe ratio is defined as follows:

$$S(a,m,c,r_f) \equiv \frac{\xi(a,m,c) - \dfrac{r_f}{E(\mathrm{T})}}{\sigma}$$

where r_f is the risk-free rate of interest. Solving for the optimal entry and exit levels a^* and m^* is straightforward and again they are shown to be symmetric about the long-run mean level. For technical details on any of the above, the interested reader is directed to the paper of Bertram (2010).

Benchmark Models

This section presents a unification of the optimal trading criteria (i.e. maximisation of expected return per unit time, $\xi(a,m,c)$, or Sharpe ratio, $S(a,m,c,r_f)$) proposed by Bertram (2010) and the first-hitting time density approach of Linetsky (2004). In so doing, an alternative statistical arbitrage trading model is presented that serves as a benchmark for the Bertram trading model. Linetsky (2004) considers the more general OU process:

$$ds_t = \alpha(\mu - s_t)dt + \sigma dW_t \tag{2}$$

The only difference between this specification and that of Equation 1 is that the process s_t mean reverts around the long-run mean level μ, which is not necessarily zero. Under this process, Linetsky (2004) considers the associated first-hitting time density for the movement of the process from a given point to another defined point. The first-hitting time density describes the probability distribution for the stochastic time it takes to move between the two points and allows one to calculate, for instance, the expected time of this movement. In the context of this study, the two points may be considered to be the entry and exit levels of the statistical arbitrage trading model. Linetsky (2004) derives both exact and approximate solutions for the first-hitting time density. The exact formulation involves the Hermite function, which is a standard mathematical tool, whereas the approximate formulation involves nothing more complex than the cosine function. For technical details the interested reader is directed to the paper of Linetsky (2004).

Armed with these first-hitting time density formulations, calculation of $E(\mathrm{T})$, $V(\mathrm{T})$, $\xi(a,m,c)$ and $\varsigma(a,m,c)$ is straightforward. Whereas Bertram (2010) provides analytic expressions from which to determine the optimal entry and exit levels a^* and m^*, the approach here requires calculating either $\xi(a,m,c)$ or $S(a,m,c,r_f)$ as required over a grid of potential entry and exit levels and extracting the optimal levels from the results. For the purposes of the empirical analysis to follow, both the exact and approximate approaches will be implemented as benchmark models for the Bertram model. Further to this, implementation of the approximate approach provides an opportunity to investigate the effect of the error introduced by the approximation in the context of a trading application. For the exposition to follow, the benchmark models will be referred to as the Linetsky Exact and Linetsky Approximate models.

DATA AND PRELIMINARY STATISTICAL RESULTS

Drawing from the current (July 2010) ISEQ stock listing, 32 stocks in total are examined with end-of-day mid-quoted price data spanning the sample period 21 July 2000 to 23 July 2010.[3,4] The 32 stocks chosen are those for which time series are available over the full sample period, which is deemed a sufficiently long period to test for cointegration and, more importantly, to comprehensively test the Bertram trading model as described below. The ISEQ stocks not considered are those for which the date of first listing succeeds 21 July 2000. The majority of these stocks (all bar two) actually have listing dates from 2006 onwards and so the associated time series are not deemed sufficiently long for the empirical testing.

All price series are first tested for stationarity using the standard augmented Dickey–Fuller (Dickey and Fuller, 1981) test and, hence, one stock is dropped from the sample data set for failing to reject the null hypothesis of stationarity. From the remaining 31 stocks, the residual-based cointegration test of Engle and Granger (1987) is performed on the resulting 465 stock pairings, which assumes the following linear model:

$$s_{2,t} = \gamma + \beta s_{1,t} + e_t$$

where $s_{1,t}$ and $s_{2,t}$ are the log-prices of the two assets in the pair, and the resulting cointegration spread series is defined such that $s_t \equiv s_{2,t} - \beta s_{1,t}$. Using a 1 per cent significance level criterion, cointegration is established between 37 pairs of stocks in total, made up of the 22 stocks listed in Table 1. Use of the high 1 per cent significance level serves to reduce the number of pairings examined for this particular study, allowing for results for individual pairs to be more easily reported for the perusal of the reader. It is the individual results that provide insights into the sensitivity of the Bertram trading model to variations in the key parameters of the OU process. In practice, of course, a 5 per cent significance level may be deemed acceptable to establish cointegration in the price spread. Indeed, cointegration is recognised as a strong statistical test and so may not identify weaker forms of predictability that offer trading opportunities. Burgess (1999) discusses this point and proposes an alternative variance ratio test approach to establishing predictability. Table 2 presents the cointegration pairs, along with the associated *t*-statistics from the cointegration testing. Descriptive statistics for the resulting spread series are also presented.

With the cointegrated pairs identified, the next stage of analysis fits the general OU process in Equation 2 to each spread series. Table 2 presents the estimated parameters $\hat{\alpha}$, $\hat{\mu}$ and $\hat{\sigma}$ for each of the 37 pairings. These estimates will be used in

TABLE 1: ISEQ STOCKS IN PAIRINGS

Reuters Instrument Code	Name
ALBK	Allied Irish Banks
AMNX	Aminex
ARYN	Aryzta
BKIR	Bank of Ireland
DQ5	CPL Resources
CRH	CRH
DQ7	Donegal Creameries
DGO	Dragon Oil
FBD	FBD Holdings
GRF	Grafton Group
GNC	Greencore Group
INME	Independent News & Media
IPM	Irish Life & Permanent
JEV	Kenmare Resources
KYGa	Kerry Group 'A'
KSP	Kingspan Group
MCI	McInerney Holdings
ORM	Ormonde Mining
OVG	Ovoca Gold
PACC	Prime Actvie Capital
RDMX	Readymix
UDG	United Drug

the application of the Bertram, Linetsky Exact and Linetsky Approximate trading models in the forthcoming sections. From the descriptive statistics for the spread series reported in Table 2, it is clearly evident that the OU process, being Gaussian, is inadequate to capture the non-normal asymmetric and leptokurtic features of the spread series data. The next section investigates the error introduced as a result of this model mis-specification within the trading models.

TABLE 2: COINTEGRATION AND OU MODEL FITTING RESULTS

Pairing	*t*-stat	Standard Deviation	Skewness	Kurtosis	$\hat{\alpha}$	$\hat{\mu}$	$\hat{\sigma}$
ALBK–AMNX	-4.03	0.36	-0.35	2.97	4.18	-2.46	1.01
ALBK–BKIR	-4.32	0.17	0.65	6.12	4.62	0.64	0.53
ALBK–GNC	-4.51	0.16	0.14	3.07	3.64	-0.02	0.43
ALBK–INME	-5.13	0.28	-0.87	4.94	3.96	0.46	0.75

(*Continued*)

TABLE 2: (*CONTINUED*)

Pairing	*t*-stat	Standard Deviation	Skewness	Kurtosis	$\hat{\alpha}$	$\hat{\mu}$	$\hat{\sigma}$
ALBK–IPM	-5.20	0.26	0.48	6.21	4.04	-0.64	0.73
ALBK–OVG	-5.75	0.35	0.56	3.71	9.52	-1.70	1.49
ALBK–RDMX	-6.34	0.20	-0.38	6.31	7.48	-1.82	0.76
AMNX–BKIR	-4.14	0.35	-0.35	3.04	4.40	-2.18	1.02
AMNX–INME	-3.95	0.35	-0.54	3.02	4.16	-2.29	1.01
AMNX–IPM	-4.34	0.35	-0.39	2.85	4.75	-2.82	1.05
AMNX–OVG	-6.39	0.33	-0.18	2.67	12.86	0.58	1.66
ARYN–KYGa	-4.46	0.09	-0.01	3.11	4.27	1.18	0.26
BKIR–GNC	-4.62	0.16	0.68	3.19	4.34	0.26	0.46
BKIR–IPM	-6.97	0.19	0.10	5.07	7.32	-1.27	0.71
BKIR–OVG	-5.95	0.34	0.79	4.08	10.15	-1.41	1.50
BKIR–RDMX	-4.81	0.25	-0.12	7.94	5.09	-1.25	0.78
DQ5–CRH	-4.44	0.13	0.25	3.22	3.92	2.82	0.37
CRH–JEV	-4.58	0.14	0.09	3.61	4.76	3.36	0.41
CRH–KSP	-5.22	0.10	0.09	2.85	5.26	2.33	0.33
CRH–ORM	-3.88	0.17	0.19	2.76	3.23	3.57	0.43
DQ7–UDG	-4.40	0.18	0.29	2.66	4.25	-0.03	0.51
DGO–KYGa	-3.94	0.10	-0.43	2.77	3.07	2.81	0.25
FBD–OVG	-4.81	0.40	-0.05	2.63	7.13	-2.13	1.48
GRF–INME	-3.98	0.48	-0.29	3.15	5.16	2.49	1.55
GRF–OVG	-5.39	0.38	0.13	3.04	9.49	-0.24	1.64
GNC–IMP	-5.10	0.15	0.83	3.71	5.04	-0.36	0.48
GNC–OVG	-4.32	0.29	-0.31	2.13	5.56	1.35	0.94
INME–IMP	-4.21	0.31	-1.22	5.75	2.76	1.01	0.70
INME–OVG	-5.88	0.32	-0.21	3.20	10.80	-1.56	1.48
INME–PACC	-4.73	0.36	-0.01	2.26	6.93	-1.78	1.34
INME–RDMX	-4.37	0.25	-0.90	3.48	4.16	-1.46	0.70
IPM–OVG	-5.82	0.35	0.73	3.52	9.65	-2.06	1.50
IPM–RDMX	-4.91	0.29	-0.82	6.91	4.61	-2.38	0.88
MCI–OVG	-4.07	0.49	0.37	2.90	4.87	-0.49	1.48
ORM–UDG	-5.26	0.33	0.17	3.64	5.86	-4.28	1.07
OVG–PACC	-5.39	0.38	0.13	3.04	9.49	-0.24	1.64
OVG–RDMX	-3.99	0.55	-0.23	1.96	4.93	0.82	1.70

Note: The reported *t*-stats result from the residual-based cointegration test of Engle and Granger (1987). $\hat{\alpha}$, $\hat{\mu}$ and $\hat{\sigma}$ are the estimated speed of mean reversion, long-run mean and volatility parameter estimates for the general OU process.

EMPIRICAL ANALYSIS: MAXIMISING EXPECTED RETURN

This section presents an empirical analysis of the optimal Bertram trading model, along with the optimal benchmark Linetsky Exact and Linetsky Approximate models, where the expected return per unit time is maximised.[5] For each stock pairing under each trading model, the optimal entry and exit levels a^* and m^* are determined. Using the empirical spread series, sample counterparts to $E(\mathrm{T})$, $V(\mathrm{T})$, $\xi(a,m,c)$ and $\varsigma(a,m,c)$ are calculated, herein denoted $\bar{\mathrm{T}}$, $\hat{V}$, $\hat{\xi}$, and $\hat{\varsigma}$ respectively. Complete trade cycles $a^* \rightarrow m^* \rightarrow a^*$ are identified and the associated trade cycle times recorded. For this, and given the discrete daily frequency structure of the data, each occurrence of the spread series crossing over either the optimal a^* or m^* level is first identified and then interpolation is used as required to assign an associated time (as a fraction of a year). For each spread series j = 1,…37, and given the sampled trade cycle times $\{\mathrm{T}_i^j\}$, i = 1,…,n_j,[6] the sample measures are defined as follows:

$$\bar{\mathrm{T}}^j = \frac{1}{n_j}\sum_{i=1}^{n_j}\mathrm{T}_i^j$$

$$\hat{V}^j = \frac{1}{n}\sum_{i=1}^{n_j}(\mathrm{T}_i^j - \bar{\mathrm{T}})^2$$

$$\hat{\xi}^j = (m_j^* - a_j^* - c)\big/\bar{\mathrm{T}}^j$$

$$\hat{\varsigma}^j = (m_j^* - a_j^* - c)^2\,\hat{V}^j\big/(\bar{\mathrm{T}}^j)^3$$

Table 3 presents the optimal entry and exit levels relative to the estimated long-run mean parameters. The errors between the model and corresponding empirical expected return per unit time and expected trade cycle time measures are also presented for each model. Table 4 provides the actual expected return per unit time and expected trade cycle time measures under each model. To conserve space, the variance of return per unit time and variance of trade cycle time measures are not reported, but are available upon request.

An important first observation to make is that for the Linetsky models, no results are reported for a number of pairings (highlighted with the symbol x). In these cases, the Linetsky models are found to exhibit an instability that is not experienced by the Bertram model. Specifically, the Linetsky models are found to generate implausible expected return values, resulting directly from the excessively small (i.e. close to zero) expected trade cycle time estimates. From the fitted OU model parameters, these pairings can be seen to correspond to speed of mean reversion and/or volatility parameter estimates that are particularly high relative to the other pairings.

For the Bertram trading model, the optimal entry and exit levels are, by construction, symmetric about the long-run mean level. The results of the Linetsky Exact and Linetsky Approximate models support this, showing symmetry for all pairings for which valid results are achieved. The trading models generally overestimate the expected return per unit time relative to the empirical data. The mean error for the Bertram model across all 37 pairings is significant at 46.19 per cent. Across only the valid pairings for the Linetsky models, mean errors for the Bertram, Linetsky Exact and Linetsky Approximate models are again significant at 28.39 per cent, 27.65 per cent and 28.11 per cent respectively. Underlying these errors is the underestimation of expected trade cycle times relative to the empirical data. Across all 37 pairings, the mean error in the trade cycle time for the Bertram model is approximately four and a half months, at -0.3813 years. For the valid pairings

TABLE 3: MODEL-EMPIRICAL MEASURE ERRORS: MAXIMISED EXPECTED RETURN

	$a^*/\hat{\mu}$			$m^*/\hat{\mu}$			$\xi^j - \hat{\xi}^j$			$E(T^j) - \bar{T}^j$		
Pairing	**B**	**LE**	**LA**	**B**	**LE**	**LA**	**B**	**LE**	**LA**	**B**	**LE**	**LA**
ALBK–AMNX	1.023	1.029	1.044	0.977	0.971	0.956	0.29	0.29	0.25	-0.20	-0.25	-0.27
ALBK–BKIR	0.945	0.948	0.922	1.055	1.052	1.078	0.28	0.28	0.29	-1.97	-1.80	-3.39
ALBK–GNC	3.247	3.075	4.113	-1.247	-1.075	-2.113	0.13	0.13	0.15	-0.39	-0.37	-0.65
ALBK–INME	0.898	0.881	0.821	1.103	1.120	1.179	0.33	0.31	0.30	-0.86	-0.84	-0.89
ALBK–IPM	1.073	1.082	1.122	0.927	0.918	0.878	0.29	0.29	0.21	-0.55	-0.59	-0.38
ALBK–OVG	1.033	×	×	0.967	×	×	1.00	×	×	-0.31	×	×
ALBK–RDMX	1.021	×	×	0.979	×	×	0.36	×	×	-0.23	×	×
AMNX–BKIR	1.026	1.033	1.049	0.974	0.967	0.951	0.34	0.33	0.27	-0.25	-0.30	-0.27
AMNX–INME	1.025	1.031	1.046	0.975	0.969	0.954	0.40	0.40	0.38	-0.45	-0.58	-0.66
AMNX–IPM	1.020	1.025	1.038	0.980	0.975	0.963	0.47	0.44	0.39	-0.47	-0.48	-0.46
AMNX–OVG	0.907	×	×	1.093	×	×	1.20	×	×	-0.17	×	×
ARYN–KYGa	0.981	0.978	0.978	1.019	1.022	1.022	0.03	0.05	0.06	-0.07	-0.19	-0.20
BKIR–GNC	0.872	0.880	0.820	1.128	1.120	1.180	0.15	0.16	0.18	-0.31	-0.36	-0.73
BKIR–IPM	1.030	×	×	0.970	×	×	0.36	×	×	-0.30	×	×
BKIR–OVG	1.039	×	×	0.961	×	×	1.08	×	×	-0.36	×	×
BKIR–RDMX	1.036	1.039	1.059	0.964	0.961	0.941	0.40	0.38	0.40	-0.79	-0.75	-1.11
DQ5–CRH	0.990	0.991	0.986	1.011	1.009	1.014	0.07	0.04	0.04	-0.15	-0.07	-0.10
CRH–JEV	0.991	0.992	0.988	1.009	1.008	1.012	0.15	0.15	0.17	-0.33	-0.34	-0.57
CRH–KSP	0.989	0.987	0.987	1.011	1.013	1.013	0.11	0.08	0.09	-0.28	-0.20	-0.21
CRH–ORM	0.990	0.991	0.986	1.010	1.010	1.014	0.09	0.10	0.09	-0.23	-0.26	-0.32

(Continued)

TABLE 3: (*CONTINUED*)

	$a^*/\hat{\mu}$			$m^*/\hat{\mu}$			$\xi^j - \hat{\xi}^j$			$E(\mathrm{T}^j) - \overline{\mathrm{T}}^j$		
Pairing	**B**	**LE**	**LA**	**B**	**LE**	**LA**	**B**	**LE**	**LA**	**B**	**LE**	**LA**
DQ7–UDG	2.106	2.133	2.700	-0.106	-0.133	-0.700	0.18	0.18	0.17	-0.41	-0.42	-0.50
DGO–KYGa	0.991	0.990	0.990	1.009	1.010	1.010	0.01	0.00	0.00	-0.03	0.00	-0.02
FBD–OVG	1.029	×	×	0.971	×	×	0.91	×	×	-0.52	×	×
GRF–INME	0.972	0.962	0.962	1.028	1.039	1.039	0.61	0.61	0.64	-0.24	-0.32	-0.32
GRF–OVG	1.247	×	×	0.753	×	×	0.99	×	×	-0.20	×	×
GNC–IMP	1.090	1.082	1.123	0.910	0.918	0.877	0.15	0.17	0.18	-0.23	-0.25	-0.43
GNC–OVG	0.964	×	×	1.037	×	×	0.31	×	×	-0.16	×	×
INME–IMP	0.949	0.939	0.908	1.051	1.061	1.092	0.19	0.20	0.20	-0.42	-0.61	-0.78
INME–OVG	1.034	×	×	0.966	×	×	1.00	×	×	-0.22	×	×
INME–PACC	1.033	×	×	0.967	×	×	0.79	×	×	-0.48	×	×
INME–RDMX	1.031	1.034	1.051	0.969	0.966	0.949	0.21	0.20	0.26	-0.26	-0.24	-0.63
IPM–OVG	1.027	×	×	0.973	×	×	0.97	×	×	-0.26	×	×
IPM–RDMX	1.021	1.025	1.037	0.979	0.976	0.963	0.37	0.37	0.47	-0.45	-0.55	-2.03
MCI–OVG	1.142	1.099	1.199	0.858	0.901	0.801	0.69	0.72	0.70	-0.47	-0.42	-0.62
ORM–UDG	1.012	×	×	0.988	×	×	0.31	×	×	-0.11	×	×
OVG–PACC	1.247	×	×	0.753	×	×	0.99	×	×	-0.20	×	×
OVG–RDMX	0.907	0.934	0.867	1.093	1.066	1.133	0.89	0.74	0.85	-0.79	-0.24	-0.73

Note: $a^*/\hat{\mu}$ is the ratio of the optimal entry level a^* to the estimated long-run mean level $\hat{\mu}$. $m^*/\hat{\mu}$ is the ratio of the optimal exit level m^* to the estimated long-run mean level $\hat{\mu}$. The error $\xi^j - \hat{\xi}^j$ is the difference between the expected return per unit time calculated under a given trading model and estimated from the empirical data. The error $E(\mathrm{T}^j) - \overline{\mathrm{T}}^j$ is the difference between the expected trade cycle time calculated under a given trading model and estimated from the empirical data. B, LE and LA denote the Bertram, Linetsky Exact and Linetsky Approximate trading models respectively.

TABLE 4: TRADING MODEL MEASURES

	Maximising Expected Return						Maximising Sharpe Ratio								
	ξ^j			$E(T^j)$			S^j			ξ^j			$E(T^j)$		
Pairing	**B**	**LE**	**LA**	**B**	**LE**	**LA**	**B**	**LE**	**LA**	**B**	**LE**	**LA**	**B**	**LE**	**LA**
ALBK–AMNX	0.57	0.57	0.59	0.20	0.25	0.36	1.50	1.49	1.49	0.56	0.58	0.59	0.58	0.61	0.48
ALBK–BKIR	0.31	0.31	0.32	0.23	0.21	0.31	1.43	1.43	1.42	0.30	0.32	0.32	0.69	0.64	0.64
ALBK–GNC	0.23	0.23	0.24	0.29	0.27	0.40	1.25	1.25	1.25	0.22	0.23	0.23	0.90	0.81	0.81
ALBK–INME	0.41	0.41	0.42	0.23	0.27	0.39	1.41	1.41	1.41	0.40	0.42	0.42	0.68	0.66	0.66
ALBK–IPM	0.41	0.41	0.42	0.23	0.25	0.37	1.42	1.42	1.41	0.40	0.42	0.42	0.67	0.62	0.62
ALBK–OVG	1.28	×	×	0.09	×	×	2.25	2.25	2.24	1.24	1.29	1.31	0.26	0.27	0.21
ALBK–RDMX	0.57	×	×	0.13	×	×	1.86	1.86	1.85	0.55	0.57	0.57	0.41	0.41	0.41
AMNX–BKIR	0.60	0.60	0.61	0.19	0.24	0.35	1.53	1.53	1.53	0.58	0.60	0.61	0.55	0.59	0.46
AMNX–INME	0.57	0.57	0.59	0.20	0.24	0.36	1.49	1.49	1.49	0.56	0.58	0.59	0.58	0.60	0.48
AMNX–IPM	0.64	0.64	0.66	0.17	0.22	0.32	1.59	1.59	1.58	0.62	0.65	0.65	0.51	0.54	0.43
AMNX–OVG	1.65	×	×	0.07	×	×	2.61	2.60	2.59	1.61	1.68	1.68	0.19	0.20	0.20
ARYN–KYGa	0.14	0.14	0.15	0.31	0.35	0.34	1.18	1.18	1.17	0.13	0.14	0.14	1.01	0.97	0.97
BKIR–GNC	0.26	0.26	0.27	0.25	0.23	0.34	1.36	1.36	1.35	0.25	0.26	0.26	0.77	0.70	0.70
BKIR–IPM	0.53	×	×	0.14	×	×	1.83	1.83	1.81	0.51	0.54	0.54	0.42	0.41	0.41
BKIR–OVG	1.33	×	×	0.08	×	×	2.32	2.32	2.31	1.29	1.34	1.36	0.24	0.25	0.20
BKIR–RDMX	0.49	0.49	0.50	0.18	0.20	0.29	1.58	1.58	1.58	0.47	0.50	0.50	0.54	0.50	0.50
DQ5–CRH	0.20	0.20	0.21	0.29	0.26	0.37	1.25	1.25	1.24	0.19	0.20	0.20	0.91	0.92	0.84
CRH–JEV	0.25	0.25	0.26	0.24	0.21	0.31	1.39	1.38	1.37	0.24	0.25	0.25	0.75	0.76	0.64
CRH–KSP	0.20	0.20	0.21	0.24	0.29	0.28	1.36	1.36	1.35	0.19	0.20	0.20	0.77	0.69	0.69
CRH–ORM	0.21	0.21	0.22	0.32	0.31	0.46	1.19	1.19	1.18	0.20	0.21	0.21	1.00	0.94	0.94

(*Continued*)

TABLE 4: (*CONTINUED*)

	Maximising Expected Return						Maximising Sharpe Ratio								
	ξ^j			$E(T^j)$			S^j			ξ^j			$E(T^j)$		
Pairing	**B**	**LE**	**LA**	**B**	**LE**	**LA**	**B**	**LE**	**LA**	**B**	**LE**	**LA**	**B**	**LE**	**LA**
DQ7–UDG	0.29	0.29	0.30	0.24	0.25	0.36	1.38	1.38	1.37	0.28	0.29	0.29	0.74	0.75	0.75
DGO–KYGa	0.12	0.12	0.12	0.41	0.49	0.47	1.04	1.04	1.03	0.11	0.12	0.12	1.32	1.34	1.14
FBD–OVG	1.10	×	×	0.11	×	×	1.98	1.97	1.97	1.07	1.12	1.12	0.32	0.28	0.28
GRF–INME	0.98	0.98	1.01	0.14	0.19	0.19	1.71	1.71	1.71	0.96	1.01	1.01	0.41	0.38	0.38
GRF–OVG	1.41	×	×	0.08	×	×	2.27	2.27	2.26	1.37	1.44	1.44	0.25	0.21	0.21
GNC–IMP	0.30	0.30	0.31	0.22	0.20	0.29	1.46	1.46	1.45	0.28	0.30	0.30	0.67	0.59	0.59
GNC–OVG	0.61	×	×	0.16	×	×	1.68	1.68	1.68	0.60	0.62	0.62	0.47	0.46	0.46
INME–IMP	0.32	0.32	0.33	0.31	0.38	0.55	1.20	1.20	1.19	0.31	0.33	0.33	0.93	0.94	0.94
INME–OVG	1.36	×	×	0.08	×	×	2.38	2.38	2.37	1.32	1.38	1.38	0.23	0.24	0.24
INME–PACC	0.98	×	×	0.12	×	×	1.93	1.93	1.92	0.96	1.00	1.00	0.34	0.36	0.29
INME–RDMX	0.40	0.40	0.41	0.22	0.25	0.36	1.43	1.43	1.42	0.38	0.40	0.40	0.67	0.61	0.61
IPM–OVG	1.30	×	×	0.09	×	×	2.27	2.26	2.26	1.26	1.31	1.33	0.25	0.27	0.21
IPM–RDMX	0.53	0.53	0.54	0.19	0.22	0.32	1.54	1.54	1.53	0.51	0.53	0.53	0.56	0.54	0.54
MCI–OVG	0.91	0.91	0.93	0.15	0.11	0.21	1.66	1.66	1.66	0.89	0.93	0.93	0.44	0.42	0.42
ORM–UDG	0.72	×	×	0.15	×	×	1.75	1.75	1.74	0.70	0.73	0.73	0.43	0.46	0.36
OVG–PACC	1.41	×	×	0.08	×	×	2.27	2.27	2.26	1.37	1.44	1.44	0.25	0.21	0.21
OVG–RDMX	1.05	1.05	1.08	0.14	0.10	0.20	1.69	1.69	1.69	1.03	1.08	1.08	0.41	0.40	0.40

Note: ξ^j is the expected return per unit time calculated under a given trading model. $E(T^j)$ is the expected trade cycle time calculated under a given trading model. S^j is the per unit time Sharpe ratio calculated under a given trading model. B, LE and LA denote the Bertram, Linetsky Exact and Linetsky Approximate trading models respectively.

under the Linetsky models, the mean trade cycle time errors show underestimation of almost six months for both the Bertram and Linetsky Exact models, at -0.4414 and -0.4337 respectively, and in excess of six months for the Linetsky Approximate model, at -0.6788. On a case-by-case basis, the lowest errors generally correspond to those pairings with spread series that are close to Gaussian, in particular those that exhibit kurtosis close to 3. As expected, the closer the spread series is described by a Gaussian distribution, the smaller the model mis-specification error that is introduced.

To conclude, it is worth making some final comments on the variance of return per unit time and the variance of trade cycle time, where we focus on only those pairings with valid results under the Linetsky models. The mean variance of return per unit time is 0.0771, 0.0772 and 0.0831, for the Bertram, Linetsky Exact and Linetsky Approximate models respectively, with corresponding mean errors of 0.0584, 0.0543 and 0.0611. For the Bertram model, the mean variance of trade cycle time is 0.0813, with a mean error of -0.5875. For the Linetsky Exact and Approximate models, the mean variances of trade cycle time are higher relative to the Bertram model at 0.0878 and 0.3415 respectively, with errors of -0.5037 and -0.6838.

In summary, the trading models examined show the following common attributes: overestimation of the expected return per unit time; underestimation of the expected trade cycle time; overestimation of the variance of return per unit time; and underestimation of the variance of trade cycle time.

EMPIRICAL ANALYSIS: MAXIMISING SHARPE RATIO

Similar to the previous section, an empirical analysis of the statistical arbitrage trading models is performed whereby the optimal entry and exit levels are determined this time by means of maximising the Sharpe ratio. The empirical counterparts to the expected trade cycle time, variance of trade cycle time, expected return per unit time and variance of return per unit time measures are calculated as outlined previously. In addition to these, an empirical counterpart to the Sharpe ratio is defined as follows:

$$\hat{S}^j \equiv (m_j^* - a_j^* - c - r_f)\sqrt{\frac{\bar{\bar{\mathrm{T}}}^j}{(m_j^* - a_j^* - c)^2 \hat{V}^j}}$$

For ease of the analysis to follow, the risk-free rate of interest r_f is set equal to the average three-month composite EURIBOR over the full sample period of 2.9966 per cent. Table 5 presents the optimal entry and exit levels, along with the model-empirical measure errors. Table 4 again provides the actual expected return per unit time and expected trade cycle time measures under each model. The variance of return per unit time and variance of trade cycle time measures are again not reported in order to conserve on space.

In contrast to the previous section, the Linetsky models do not show instability for any of the 37 pairings. This likely reflects the fact that, overall, the reported

TABLE 5: MODEL-EMPIRICAL MEASURE ERRORS: MAXIMISED SHARPE RATIO

	$a^*/\hat{\mu}$			$m^*/\hat{\mu}$			$S^j - \hat{S}^j$			$\xi^j - \hat{\xi}^j$			$E(\mathrm{T}^j) - \overline{\mathrm{T}}^j$		
Pairing	**B**	**LE**	**LA**	**B**	**LE**	**LA**	**B**	**LE**	**LA**	**B**	**LE**	**LA**	**B**	**LE**	**LA**
ALBK–AMNX	1.066	1.073	1.058	0.934	0.927	0.942	-0.17	-0.11	-0.44	0.20	0.17	0.27	-0.31	-0.26	-0.42
ALBK–BKIR	0.839	0.843	0.843	1.161	1.157	1.157	-3.35	-3.41	-3.42	0.27	0.29	0.29	-6.61	-6.79	-6.79
ALBK–GNC	7.540	7.226	7.226	-5.540	-5.226	-5.226	0.14	0.15	0.15	0.10	0.12	0.12	-0.73	-0.95	-0.95
ALBK–INME	0.706	0.701	0.701	1.294	1.299	1.299	0.31	0.23	0.22	0.17	0.16	0.16	-0.48	-0.40	-0.40
ALBK–IPM	1.209	1.204	1.204	0.791	0.796	0.796	-0.04	0.01	0.01	0.22	0.24	0.24	-0.86	-0.84	-0.84
ALBK–OVG	1.094	1.103	1.083	0.906	0.897	0.917	1.34	1.33	1.36	1.04	1.07	1.13	-1.31	-1.31	-1.38
ALBK–RDMX	1.061	1.064	1.064	0.939	0.936	0.936	1.19	1.20	1.19	0.47	0.48	0.48	-2.31	-2.18	-2.18
AMNX–BKIR	1.074	1.081	1.065	0.926	0.919	0.935	-0.05	-0.23	0.03	0.23	0.24	0.25	-0.35	-0.39	-0.32
AMNX–INME	1.071	1.077	1.062	0.929	0.923	0.938	-0.62	-0.50	-0.56	0.28	0.29	0.35	-0.58	-0.62	-0.69
AMNX–IPM	1.057	1.062	1.050	0.943	0.938	0.950	0.06	-0.02	0.05	0.28	0.29	0.37	-0.42	-0.43	-0.55
AMNX–OVG	0.734	0.715	0.715	1.266	1.285	1.285	0.78	0.78	0.77	1.03	1.09	1.09	-0.34	-0.36	-0.36
ARYN–KYGa	0.943	0.942	0.942	1.058	1.058	1.058	0.21	0.21	0.19	0.04	0.05	0.05	-0.41	-0.46	-0.46
BKIR–GNC	0.628	0.640	0.640	1.373	1.361	1.361	0.38	0.38	0.37	0.14	0.16	0.16	-1.02	-1.06	-1.06
BKIR–IPM	1.086	1.088	1.088	0.914	0.912	0.912	0.74	0.55	0.54	0.37	0.41	0.41	-1.07	-1.34	-1.34
BKIR–OVG	1.111	1.122	1.098	0.889	0.878	0.902	1.47	1.41	1.48	1.09	1.14	1.16	-1.33	-1.46	-1.21
BKIR–RDMX	1.103	1.099	1.099	0.897	0.902	0.902	0.72	0.72	0.71	0.41	0.44	0.44	-3.54	-3.60	-3.60
DQ5–CRH	0.969	0.968	0.972	1.031	1.033	1.033	0.05	-0.24	0.06	0.02	0.02	0.04	-0.12	-0.09	-0.18
CRH–JEV	0.974	0.972	0.976	1.026	1.028	1.024	0.01	-0.05	-0.11	0.12	0.13	0.13	-0.73	-0.78	-0.74
CRH–KSP	0.968	0.970	0.970	1.032	1.030	1.030	0.41	0.49	0.47	0.11	0.12	0.12	-0.99	-1.00	-1.00
CRH–ORM	0.972	0.972	0.972	1.029	1.028	1.028	0.26	0.26	0.25	0.10	0.11	0.11	-1.00	-1.05	-1.05

(Continued)

TABLE 5: (*CONTINUED*)

	$a^*/\hat{\mu}$			$m^*/\hat{\mu}$			$S^j - \hat{S}^j$			$\xi^j - \hat{\xi}^j$			$E(\mathrm{T}^j) - \overline{\mathrm{T}}^j$		
Pairing	**B**	**LE**	**LA**	**B**	**LE**	**LA**	**B**	**LE**	**LA**	**B**	**LE**	**LA**	**B**	**LE**	**LA**
DQ7–UDG	4.208	4.400	4.400	-2.208	-2.400	-2.400	0.31	-0.22	-0.23	0.18	0.21	0.21	-1.32	-2.02	-2.02
DGO–KYGa	0.974	0.972	0.976	1.026	1.028	1.024	-0.38	-0.26	-0.26	-0.02	-0.01	0.01	0.19	0.15	-0.06
FBD–OVG	1.082	1.075	1.075	0.918	0.925	0.925	0.93	0.98	0.98	0.95	1.01	1.01	-2.49	-2.50	-2.50
GRF–INME	0.920	0.923	0.923	1.080	1.077	1.077	0.53	0.52	0.52	0.57	0.63	0.63	-0.61	-0.62	-0.62
GRF–OVG	1.701	1.630	1.630	0.299	0.370	0.370	1.22	1.23	1.22	0.93	1.03	1.03	-0.51	-0.53	-0.53
GNC–IMP	1.262	1.246	1.246	0.738	0.754	0.754	0.29	0.28	0.27	0.17	0.19	0.19	-1.01	-0.97	-0.97
GNC–OVG	0.896	0.894	0.894	1.104	1.106	1.106	0.89	0.86	0.85	0.53	0.56	0.56	-3.90	-4.05	-4.05
INME–IMP	0.855	0.847	0.847	1.145	1.153	1.153	0.06	0.02	0.01	0.15	0.17	0.17	-0.88	-0.98	-0.98
INME–OVG	1.098	1.104	1.104	0.902	0.896	0.896	1.44	1.43	1.42	1.19	1.25	1.25	-2.11	-2.26	-2.26
INME–PACC	1.093	1.102	1.082	0.907	0.898	0.918	0.81	0.73	0.83	0.68	0.73	0.75	-0.85	-1.01	-0.86
INME–RDMX	1.088	1.085	1.085	0.912	0.915	0.915	0.88	0.89	0.88	0.25	0.28	0.28	-1.26	-1.40	-1.40
IPM–OVG	1.077	1.086	1.068	0.923	0.915	0.932	1.37	1.34	1.40	1.11	1.16	1.10	-1.82	-1.97	-1.05
IPM–RDMX	1.060	1.061	1.061	0.940	0.939	0.939	0.52	0.50	0.50	0.41	0.43	0.43	-2.22	-2.27	-2.27
MCI–OVG	1.400	1.398	1.398	0.600	0.602	0.602	0.42	0.43	0.42	0.74	0.78	0.78	-2.20	-2.26	-2.26
ORM–UDG	1.035	1.039	1.031	0.965	0.961	0.969	0.41	0.29	0.43	0.25	0.27	0.33	-0.24	-0.27	-0.30
OVG–PACC	1.701	1.630	1.630	0.299	0.370	0.370	1.22	1.23	1.22	0.93	1.03	1.03	-0.51	-0.53	-0.53
OVG–RDMX	0.739	0.734	0.734	1.261	1.266	1.266	0.83	0.84	0.84	0.93	0.97	0.97	-3.63	-3.60	-3.60

Note: $a^*/\hat{\mu}$ is the ratio of optimal entry level a^* to the estimated long-run mean level $\hat{\mu}$. $m^*/\hat{\mu}$ is the ratio of optimal exit level m^* to the estimated long-run mean level $\hat{\mu}$. The error $S^j - \hat{S}^j$ is the difference between the per unit time Sharpe ratio calculated under a given trading model and estimated from the empirical data. The error $\xi^j - \hat{\xi}^j$ is the difference between the expected return per unit time calculated under a given trading model and estimated from the empirical data. The error $E(\mathrm{T}^j) - \overline{\mathrm{T}}^j$ is the difference between the expected trade cycle time calculated under a given trading model and estimated from the empirical data. B denotes the Bertram trading model, LE denotes the Linetsky Exact trading model and LA denotes the Linetsky Approximate trading model.

optimal entry and exit levels for each pairing represent a wider range around the long-run mean level compared to maximisation of expected return per unit time and, hence, the associated expected trade cycle time is much longer. Indeed, the mean expected trade cycle times for the Bertram, Linetsky Exact and Linetsky Approximate models respectively are 0.5709, 0.5553 and 0.5226 years. Despite the greater absolute returns on offer from the wider optimal entry and exit levels, it is particularly interesting to note that the longer expected trade cycle times lead to expected return per unit time measures that are quite comparable to the previous section. So on a per unit time basis, there appears to be marginal difference between the trading strategy based on either maximisation of expected return per unit time or Sharpe ratio. Further to this, the mean error in the expected return per unit time relative to the empirical data is 44.92 per cent, 47.80 per cent and 48.89 per cent for the Bertram, Linetsky Exact and Linetsky Approximate models respectively. Again, the trading models significantly overestimate the expected return relative to the empirical data, reflecting significant underestimation of the expected trade cycle time with mean errors of -1.3474, -1.4175 and -1.3994 years.

On the associated Sharpe ratio measures, the trading models show evidence of both overestimation and underestimation relative to the empirical data, with varying degrees of magnitude. Overall, the mean error is positive at 0.4214, 0.3848 and 0.3952 for the Bertram, Linetsky Exact and Linetsky Approximate models respectively. The mean variance of return per unit time is 0.1258, 0.1389 and 0.1375 for the Bertram, Linetsky Exact and Linetsky Approximate models respectively, with corresponding mean errors of 0.0950, 0.1099 and 0.1094. The mean variance of trade cycle time is 0.2101, 0.2085 and 0.1932 for the Bertram, Linetsky Exact and Linetsky Approximate models respectively, with corresponding mean errors of -1.2718, -1.2618 and -1.2511.

In summary, the trading models examined show the following common attributes: overestimation of the expected return per unit time; underestimation of the expected trade cycle time; overestimation of the variance of return per unit time; and underestimation of the variance of trade cycle time. These observations are similar to the last section. Furthermore, the trading models all show mixed results for the Sharpe ratio, with evidence of either over- or underestimation.

CONCLUSION

This study presents a comprehensive model specification analysis of the Bertram (2010) optimal statistical arbitrage trading model on quoted ISEQ stocks. A number of key contributions to the literature are made. Firstly, the empirical analysis allows for the identification and quantification of model mis-specification errors in the Bertram trading model. That is, it looks to investigate the mis-specification error introduced in using a Gaussian OU process to describe non-Gaussian empirical cointegration spread series. Significant errors are reported on average in the key measures underlying the trading model. In particular, for both maximisation of the expected return per unit time and maximisation of the Sharpe ratio, it is found that the trading model generally overestimates the expected return per unit time and underestimates the expected trade cycle time relative to the empirical data. Errors in the Sharpe ratio show evidence of both overestimation and underestimation. In general, and as expected, the closer the data is to normal, the better the trading model performs.

Secondly, the study contributes by means of unifying the optimal statistical arbitrage trading criteria set out in the Bertram trading model with the first-hitting time density framework of Linetsky (2004) in order to develop two alternative trading models for benchmark purposes. Comparison of the Bertram trading model against these benchmark models shows the former to be much more robust to high mean reversion and/or volatility parameter estimates. The analysis further helps to understand the sensitivity of the Bertram trading model to variations in the key parameters of the OU process by reporting results for individual pairs. Thirdly, in implementing the benchmark trading model based on the approximate first-hitting time density of Linetsky (2004), the error introduced as a result of the approximation is investigated and quantified in the context of a trading application.

The key advantage of the Bertram trading model is that the analytic approach to determining optimal entry and exit levels provides significant computational efficiencies, which is of particular advantage for the implementation of statistical arbitrage trading at a high-frequency level. However, given that the underlying OU process only allows for a normal distribution for changes in the spread series, model mis-specification error is a feature when applied to non-normal empirical data. Therefore, for practitioners, there is a tradeoff to be made between the computational efficiencies that the Bertram trading model offers and the error that it introduces. This study shows that the errors in the key underlying measures using ISEQ data are significant on average. However, on an individual pair basis, the closer the spread series is to normal, the less the error will be in general. It is advisable that practitioners be cognisant of the model mis-specification error issue when using the Bertram trading model and where possible to comprehensively backtest any statistical arbitrage trading strategy based on the resulting optimal entry and exit levels.

Finally, informed by the model specification insights of this study, a formal trading strategy validation would significantly extend the literature. Examination of the performance of the Bertram trading model against alternative statistical arbitrage models, in addition to alternative trading strategies (e.g. technical rules), would be of particular interest to practitioners. However, such analysis would need to proceed while controlling for data snooping through the use of appropriate techniques, such as the reality check bootstrap of White (2000) and the superior predictive ability test of Hansen (2005).

ENDNOTES

1 The author would like to thank Professor Ciarán Ó hÓgartaigh and the two anonymous referees involved in the review process, whose comments and feedback greatly improved the paper.

2 The zero mean assumption does not present any issue in practice. The optimal entry and exit levels obtained can be easily translated to account for a non-zero mean in empirical data.

3 Data were obtained using the equity price database available via the Thomson Reuters Xtra 3000 platform. A full listing of the 32 stocks is available from the author upon request.

4 It has been pointed out by one of the anonymous referees that including only stocks which are live at the end of July 2010 introduces survivor bias. The author would like to thank the referee for pointing this out. Including dead stocks would need to be coupled with extended analysis of structural change effects in any cointegration relationships identified. For a given pairing that includes a stock that ultimately delists or ceases trading, any long-term statistical relationship that exists is likely to undergo some form of structural change or may indeed break down entirely during the lead-up period. Such analysis is defered for future research.

5 For the implementation of the trading models in this section and the next, the transaction costs parameter is arbitrarily set at a negligible level of ten basis points. As the objective is to investigate errors between the models and empirical data, it is only necessary to apply the transaction parameter consistently. In practice, of course, transaction costs are an essential consideration.

6 A key assumption made in the calculation of sampled trade cycle times is that the spread may be transacted at the entry and exit levels exactly. In practice, of course one or more of the assets underlying the spread may be illiquid and so it may not be possible to transact immediately once the entry and exit levels are reached. The author would like to thank one of the anonymous referees for raising this issue.

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