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USE OF THEORY OF TRUNCATED DISTRIBUTION FOR EVALUATION OF RISK OF FINANCIAL OPERATIONS

Summary. The use of the truncated distributions is considered as a Maxwell distribution for determination of risk coefficients by conducting the financial operations. The mathematical expectations are calculated for the used distributions, the sizes of risk coefficients are certained.

Keywords: risk, financial risk, truncated distributing, Maxwell distribution, mathematical expectation of the truncated distributing.

Анотація. Розглянуто застосування зрізаного розподілу типу розподілу Максвелла для визначення коефіцієнтів ризику від проведення фінансових операції. Обчислено математичні сподівання для використаних розподілів, визначено величину коефіцієнтів ризику.

Ключові слова: ризик, фінансовий ризик, зрізаний розподіл, розподіл Максвелла, математичне сподівання зрізаного розподілу.

Аннотация. Рассмотрено применение усечённых распределений типа распределения Максвелла для определения коэффициентов риска при проведения финансовых операций. Вычислены математические ожидания для использованных распределений, определены величины коэффициентов риска.

Ключевые слова: риск, финансовый риск, усеченное распределение, распределение Максвелла, математическая ожидание усеченного распределения.

Introduction. The reliable functioning of economic entity is necessary pre-condition of steady economic development of the state. Concerning the activity of economic entities happened in the conditions of vagueness and financial risk, the most problem in system of management of enterprise finances are questions related to their management. It is known that authenticity of estimation of risks depends on those methods and receptions which use economic entity for their determination, and possibility to manage – from exactness of estimation of size of these risks. Consequently question tying up with exactness of evaluation of risks were and remain the issue of the day of present days.

Currently applying probabilistic methods to determine the level of financial risks is widespread. Details of the methodological approach discussed in the works [1,2,3,4,5]. Formally, it can be summarized as follows. Suppose that the result of any transaction or event associated with it, is a random variable defined on the interval I_1: ($-\infty < A \le X \le B < \infty$). It means that determine interval of possible values of the random variable X can be finite or infinite. On the same interval is the function and density distribution of the random variable *X*. Based on the economic substance problem on interval I_1 selected interval I_2: ($-\infty < A \le u \le X \le w \le B < \infty$) is $I_2 \subset I_1$. You should determine the probability *P* of random event that means that a random variable is $X \in I_2$, to define the math expectation and standard deviation of a random variable *X* in this case, to determine the value of a predetermined function of the random argument *X* - ratio of expected losses.

Let's consider the economic maintenance of these tasks more in detail. For example, such task is considered in works [1,2,3].

Possibly, that along abscise axis, which has economic maintenance of size of possible losses as a result of some financial operation the areas of possible and impermissible levels of risk are selected. We must find the expectation and variance of the level of risk. In the works [2,3] is proposed to calculate the coefficient of expected loss Kz as the ratio

$$Kz = \frac{(M_z^-)}{(M_z^-) + (M_z^+)} \quad , \tag{1}$$

where: M_z^- and M_z^+ – conditional mathematical expectation negative (-) and favorable (+) results.

In works [4,5] it is proposed the coefficient of bank risk for crediting the group of enterprises, which is determined according to the formula:

$$Kr = -\frac{\int_0^{1+r} f(x)(x-(1+r))dx}{\int_{1+r}^{\infty} f(x)(x-(1+r))dx}, \tag{2}$$

where r - the interest rate of the bank, x - amount of bank revenue on one monetary unit cost, f(x) - density distribution of the random variable X - profitability of enterprises. In these same studies is proposed to determine the hazard ratio for the expected profit of the bank in the form:

$$Kc = -\frac{\int_0^{c(1+r)} f(x)(x-c(1+r))dx}{\int_{c(1+r)}^{\infty} f(x)(x-c(1+r))dx} \quad , \tag{3}$$

where: c - sum of credit; *g(x)* is a function of closeness of probability of size of liquid assets which are at disposal of enterprises. In the quoted works there are tasks in a general view. For the concrete distributing the results of coefficients determination (1)-(3) are unknown to the authors of this work

Literature analysis. Formally, the expressions (1)-(3) are united that the distributions, which are given, belong to the class of truncated distributions. The features of truncated distributions and their main differences from classical distributions are described in [6,7,8].

Let's suppose that on the interval (u, w), may be infinite, is given the distribution function F(x) and density function f(x) of the random variable X.

It is necessary at a certain interval (u, w) and a given function F(x) and f(x) to find the function G(x), g(x), the math expectation $M[x_{u,w}]$ and dispersion $D[x_{u,w}]$. In the work [6], which was published in the original language in 1952, it was posed and solved the problem of determining the distribution functions and density of the random variable (BB) X in the conditions of normal source distribution and cutting interval $x_0 \le x < \infty$. In the work [7], published in 1962, but performed about in the same period of work [6] it is presented a general theory of the problem solution. In the work [8] it's presented the numerical solution of the problem of determining the truncated normal distribution characteristics, provided that the interval has following view $0 \le x < \infty$.

According to the work [7] let's show the basic stages of problem solution.

Assume that bilateral cut on the interval $A \le u \le w \le B$ distribution is given as follows:

$$G(x) = \begin{cases} 0, & if \quad x \notin (u,w) \\ CF(x), & if \quad x \in (u,w) \end{cases} \tag{4}$$

The density distribution in this case:

$$g(x) = \begin{cases} 0, & if \quad x \notin (u,w) \\ cf(x), & if \quad x \in (u,w) \end{cases} \tag{5}$$

Thus:

$$A \le u \le x \le w \le B . \tag{6}$$

The normalizing factor C, which is present in equations (1) and (2), are calculated as follows:

$$C = \int_{u}^{w} f(x)dx = 1 , \tag{7}$$

or

$$C = [F(w) - F(u)]^{-1} . \tag{8}$$

Function of truncated distribution:

$$G(x) = \begin{cases} 0, & if \quad x \notin (u,w) \\ C\int_{u}^{x} f(x)dx = C[F(x) - F(u)], & if \quad x \in (u,w) \end{cases} \tag{9}$$

A mathematical expectation is determined with the formula:

$$M[X_{u,w}] = C\int_{u}^{w} xf(x)dx , \tag{10}$$

second initial moment:

$$\alpha_{u,w}^{2} = C\int_{u}^{w} x^{2}f(x)dx . \tag{11}$$

In this case the dispersion makes:

$$D[X_{u,w}] = \alpha_{(u,w)}^{2} - \left(M[X_{u,w}]\right)^{2} . \tag{12}$$

Problem definition. To define the coefficients (1)-(3) for given in the work [2] distribution, this belongs to the distribution families of Maxwell and according to authors' opinion of this work can be used as distributing level of losses risk-level:

$$f(x) = \frac{4x^2}{b^3\sqrt{\pi}} e^{-x^2/b^2} . \tag{13}$$

Its view is given on the figure 1, also given in the work [2].

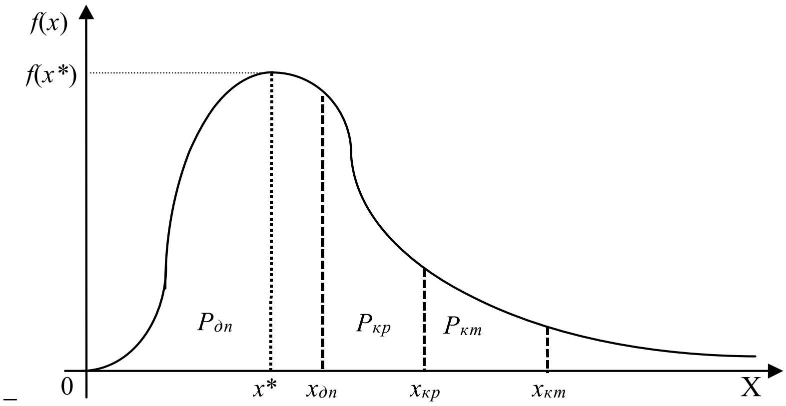

Figure 1. The curve of density of distribution losses probability.

On this figure are taken following symbols:

x^* - a point that corresponds to the most probable (modal) level of losses;

$x_{дп}$ - a point that corresponds to the potential losses which sizes are equal to the expected value (estimated) profit. The points $x = 0$ and $x = x_{дп}$ define the limits of acceptable risk zone;

$x_{кр}$ - a point that corresponds to the loss which sizes are equal to the full value of the estimated amount of proceeds. The points $x = x_{дп}$ and $x = x_{кр}$ define the limits of the critical areas of risk;

$x_{кт}$ - a point that corresponds to the loss which sizes are equal value of all entrepreneur property. The points $x = x_{кр}$ and $x = x_{кт}$ define the zone of catastrophic risk. Distribution parameter $b = x^*$. Determination of the ratio of expected losses (1) we will do in two steps.

In the first step we define the numerical characteristics of the risk level if $0 < X \le x_{дп}$.. Other words, we assume that the possible X values are within the acceptable risk level. On the second step we will define the numerical descriptions of risk level, if $x_{дп} < X \le \infty$.

For solving of each formulated tasks it is necessary to define the function of distributing of probability of *F(x),* using equality (13).

Then:

$$F(x) = \int_0^x \frac{4x^2}{b^3\sqrt{\pi}} e^{-\left(\frac{x}{b}\right)^2} dx = erf\left(\tfrac{x}{b}\right) - \frac{2x\exp(-\frac{x}{b})^2}{b\sqrt{\pi}} . \tag{14}$$

It is accepted in expression (14), that *erf(x)* - is the function of errors related to the function of normal distribution $\Phi(x)$ if:

$$erf\left(\frac{x}{b}\right) = 2\Phi\left(\sqrt{2\frac{x}{b}} - 1\right) \cdot \tag{15}$$

For a task 1 let's define the limits of existence interval of truncated distribution $I_1 : 0 \le X \le x_{äï}$. Without limitation of generalization let's accept, that $x_{äï} = u$.
A rationing multiplier (8) is for an interval $0 \le X \le u$:

$$C_1 = (F(u) - F(0))^{-1} = (F(u))^{-1} = \left(erf(\tfrac{u}{b}) - \frac{2u\exp(-\frac{u}{b})^2}{b\sqrt{\pi}} \right)^{-1}. \tag{16}$$

Let's define in this case a mathematical expectation for density function (13):

$$M[X_{0,u}] = C_1 \int_0^u xf(x)dx = C_1 \left(\frac{2}{\sqrt{\pi}} b - \frac{e^{-u^2/b^2}(b^2 + u^2)}{b} \right). \tag{17}$$

After implementation of corresponding transformations we get, that:

$$M[X_{0,u}] = \frac{2(b^2 \exp\left(\frac{u}{b^2}\right) - b^2 - u^2)}{\sqrt{\pi} b \exp(\frac{u}{b})^2 erf(\frac{u}{b}) - 2u}. \tag{18}$$

On the second step let's define the numerical descriptions of risk level, if $x_{\partial n} < X \le \infty$.

For the solving of the second part of task let's define at first the mathematical expectation of random variable X, which is defined on an interval $x_{\partial n} \le X < \infty$ or $u \le X < \infty$ and has distribution function (14). Normalizing factor in this case takes the following form:

$$C_2 = (F(\infty) - F(u))^{-1} = (1 - \left(erf(\tfrac{u}{b}) - \frac{2u\exp(-\frac{u}{b})^2}{b\sqrt{\pi}} \right))^{-1}. \tag{19}$$

Let's calculate a value

$$M[X_{u,\infty}] = C_2 \int_u^\infty xf(x)dx = C_2 \, \frac{2e^{-u^2/b^2}(b^2 + u^2)}{\sqrt{2\pi} \cdot b} . \tag{20}$$

After implementation of corresponding transformations we get, that:

$$M[X_{u,\infty}] = \frac{2\pi\left(b^2 + u^2\right)}{2u - \sqrt{\pi} \cdot b \cdot \exp(\frac{u}{b})^2 (erf(\frac{u}{b}) - 1)}. \tag{21}$$

Let's define the coefficient of expected losses Kz on an interval (u,∞) as a relation:

$$Kz = \frac{(M_z^-)}{(M_z^-) + (M_z^+)} = \frac{M\left[X_{u,\infty}\right]}{M\left[X_{u,\infty}\right] + M\left[X_{0,u}\right]}. \tag{22}$$

After corresponding transformations we get, that:

$$K_z = \frac{be^{u^2/b^2}\left(\sqrt{\pi}erf\left(\frac{u}{b}\right)+2b\right)-2\left(b^2+u(u+1)\right)}{\sqrt{\pi}be^{u^2/b^2}erf\left(\frac{u}{b}\right)-2u} . \qquad (23)$$

For completion of this research let's define the mathematical expectations of the level of possible losses in the frame $x_{\hat{e}\eth} \leq X \leq x_{\hat{e}\grave{o}}$, that's in the area of catastrophic risk. Without limitation of generalization level of received solution we accept, that $x_{кр}$ $=u,$ $x_{кт}$ =w. According to described method let's define a normalizing multiplier:

$$C_3 = (F(w)-F(u))^{-1} = \frac{\sqrt{\pi}b\exp((u^2+w^2)/b^2)}{2u\exp(\frac{w^2}{b^2})-\frac{u^2}{b^2}\left[\sqrt{\pi}b\exp(\frac{w^2}{b^2})(erf(\frac{u}{b})-erf(\frac{u}{b})+2w\right]} \qquad (24)$$

Then a mathematical expectation we define as follows:

$$M\,[X_{u,w}] = C_3\int_u^w xf(x)dx = C_3\cdot\frac{2}{b\sqrt{\pi}}\left[\left(e^{-u^2/b^2}(b^2+u^2)-e^{-w^2/b^2}(b^2+w^2)\right)\right]. \qquad (25)$$

In the form (25) the final type of formula is not resulted because of its largeness, but numeral value M $[X_{u,w}]$ can be certain, taking into account equality (24) effortlessly.

For determination of risk coefficients certained by equalizations (2) and (3) let's consider the auxiliary expression:

$$I = -\frac{\int_0^a u(x)(x-a)dx}{\int_a^\infty u(x)(x-a)dx}. \qquad (26)$$

The density function *u(x)* is set by a condition (13), consequently:

$$u(x) = \frac{4x^2}{b^3\sqrt{\pi}}e^{-x^2/b^2}.$$

Let's consider an integral:

$$I_1 = \int_0^a \frac{4x^2}{b^3\sqrt{\pi}}e^{-x^2/b^2}(x-a)dx. \qquad (27)$$

Executing the integration we get, that:

$$I_1 = -\tfrac{1}{\sqrt{\pi}}(2be^{-(a/b)^2}+(a\sqrt{\pi}erf(\tfrac{a}{b})-2b), \qquad (28)$$

in its turn :

$$I_2 = \int_a^{\infty} \frac{4x^2}{b^3\sqrt{\pi}} e^{-x^2/b^2}(x-a)dx\ , \qquad (29)$$

then:

$$I_2 = -asign(b) + \frac{2be^{-a^{2/b\wedge 2}}}{\sqrt{\pi}} + aerf(\frac{a}{b}). \qquad (30)$$

We remind that the function is follows:

$$sign(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x,0 \end{cases}.$$

Because of task *b>0,* then I_2 will be in such view:

$$I_2 = -a + \frac{2be^{-a^2/b^2}}{\sqrt{\pi}} + aerf\ (\frac{a}{b}). \qquad (31)$$

Dividing the expression (28) into the expression (31) and taking into account a sign before a relation (26) we will get its expression (26) as follows:

$$I = \frac{\exp(\frac{a^2}{b^2})\left[a\sqrt{\pi}erf(\frac{a}{b}) - 2b\right] + 2b}{a\sqrt{\pi}\exp(\frac{a^2}{b^2})\left[erf(\frac{a}{b}) - 1\right] + 2b} \qquad (32)$$

Accepting in expression (32) $a = 1 + r$ or $a = c(1 + r)$ we will get accordingly the value of coefficients (2) or (3).

Conclusions.

1. To determine the coefficients of expected loss, bank risk in lending of enterprises group, the risk for expected bank profit it is offered to use the truncated distributions of random variables.
2. For the truncated distribution as Maxwell distribution there are evidently received the expressions for calculating the coefficients of expected loss, bank risk of in lending of enterprises group, risk for expected bank profits.

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