STUDIES AND RESEARCHES - Vol. 24, 2003
Graduate School in Concrete Structures - Fratelli Pesenti
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TORSIONAL CAPACITY OF R/C BEAMS

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ABSTRACT

The torsional capacity of R/C beams is considered in this paper. On the basis of Batti and Almughrabi theory, a new general formula is proposed. Accordingly to their theory, this formula takes into account that stirrups influence the concrete torsional capacity because of their involvement in the aggregate interlock. A large number of previous test results, available in the literature (87 beams), has been considered to determine a few coefficients, by minimizing the coefficient of variation of the experimental-to-theoretical torsional capacity ratio. The obtained contributions of concrete and reinforcement on torsional capacity have both a sound physical meaning, which was not the case of the original Batti and Almughrabi's expressions. The theoretical results obtained with the proposed formula have been compared with the torsional capacities provided by other already available formulae and by some design codes. It is shown that the proposed formula is very efficient, since the computed capacities are very close to the test results and - on the whole - much closer than other well known formulae.

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1. INTRODUCTION

The first theory for calculating the ultimate strength of reinforced concrete members subjected to torsion was developed by Rausch (1929). In this theory a rectangular section member acts as a tube, in the sense that the concrete core gives no significant contributions to the ultimate strength. Each panel of the tube acts as a plane truss, where concrete struts are the diagonal members (assumed to be inclined at 45-deg with respect to the longitudinal axis), while the longitudinal reinforcement and the stirrups are the horizontal and vertical members of the truss respectively. Globally it is a spatial truss model. By imposing the equilibrium conditions, Rausch proposed the equation for the ultimate moment T_u:

$$T_u = 2\frac{A_{oh} A_t f_{yt}}{s} \tag{1}$$

where A_{oh} is the area enclosed by the centerline of the outermost closed hoops; A_t is the sectional area of one leg of a closed stirrup; f_{yt} is the yield stress of the stirrup and s is the stirrup spacing.

Rausch contribution was fundamental and still provides the basic formulation for some codes. Eq. 1, however, significantly overestimates the actual torsional strength. For this reason many scientists tried to modify Rausch's formula. Andersen (1935), first, and then many others (Cowan 1950, Hsu 1968a), introduced a "reinforcement efficiency factor", less than 1, which multiplies the coefficient 2 appearing in Eq. 1. A second approach to modify Eq. 1 was to reduce area A_{oh}: Lampert and Thurlimann (1968, 1969), for example, proposed to replace the area A_{oh} with the area bounded by the lines connecting the longitudinal bars center. Moreover they introduced a variable-angle truss model (< 45-deg) for the concrete struts.

Several design codes (ACI 2002; Eurocode 1992; Italian Code 1996) base their torsional capacity formulae on the above-mentioned simplified models.

Collins and Mitchell (1980), proposed to evaluate the depth of the concrete struts with an expression based on equilibrium equations; however, they considered as ineffective the concrete cover, with no clear reasons.

Anyway, to make the proposed theories closer to the experimental results, some arbitrary assumptions have to be introduced in all the above-mentioned cases, since the actual torsional capacity is smaller than those provided by the various theories, because of two coexisting effects. The first is a kinematical one due to the fact that concrete struts are not simply compressed, but they are subjected to bending (Lampert and Thurlimann, 1968 and

1969): the maximum compressive stress is greater than the average one. The second aspect is the softening of concrete in compression; this phenomenon was firstly observed by Robinson and Demorieux (1972): they studied the behavior, under compression, of concrete panels reinforced with transversal reinforcement placed at various angles. When a tensile force acts in the reinforcement, the concrete compressive strength reduces significantly. Vecchio and Collins (1981) proposed a stress-strain curve for the "softened" concrete, where maximum stress and the corresponding strain are scaled down with respect to the original "non-softened" curve. The fundamental parameter characterizing the model is the softening coefficient ζ (<1).

Hsu and Mo (1985) proposed a theory which at the same time takes into account equilibrium, kinematic compatibility (bending of concrete struts) and concrete softening. They set up a model which provides eight non linear equations in nine unknown quantities. Keeping one of them as a parameter, the solution of the system gives each single point (and in particular the ultimate one) of the diagram T-Θ, where Θ is the angle of torsion. The ensuing curves agree satisfactorily with experimental curves, except before first cracking, since the initial resisting mechanism is different. The solution of the non linear system is difficult. For this reason Hsu (1990) proposed a simplified model, or, alternatively, a unified simplified formula.

Another approach to study the torsional behavior of R/C beams is to model concrete members with finite elements (Ngo and Scordelis 1967, Connor and Sarne 1975). Bhatti and Almughrabi (1996) studied the problem by means of a three-dimensional nonlinear finite-element model. They were able to take into account a number of relevant phenomena usually neglected: dowel action, bond between concrete and steel and aggregate interlock. On the basis of their parametric analyses a simple formula for the ultimate torsional capacity was proposed.

In the present paper, on the basis of Bhatti and Almughrabi model, a new formula is worked out. This formula is used to compare the computed capacity with the tests on 87 beams.

The proposed-model reliability is compared also with the design codes, as well as Hsu's simplified model and approximate formula, and Bhatti and Almughrabi's model. Some fundamental features of the cited codes and models are preliminarily presented.

2. CODE PROVISIONS AND EXISTING MODELS

2.1 ACI Building Code

The American Concrete Institute Building Code (2002), makes use of the classical three-dimensional truss model, with variable inclination of concrete struts. Torsional capacity of the concrete section is

$$T_{cA} = 1.397\,\phi\,\frac{A_{oh}^2}{p_h}\sqrt{f_c} \qquad \text{(S.I. units)} \tag{2}$$

where f_c is concrete ultimate strength, p_h the length of the centerline of one closed stirrup and ϕ the strength reduction factor equal to 0.75.

Torsional capacity resisted by transverse reinforcement is

$$T_{tA} = \frac{2 A_o A_t f_{yt}}{s}\, ctg\,\theta \tag{3}$$

The inclination θ shall not be taken smaller than 30 deg, nor greater than 60 deg. It is possible to take θ equal to 45 deg for non prestressed members. A_o is the area enclosed by the centerline of the shear-flow path. If not determined by analysis, for example by means of Hsu's (1990) approximate model, A_o can be taken equal to $0.85A_{oh}$.

The torsional capacity T_{minA} for rectangular beams is defined as the minimum between the moments expressed by Eqs. 2 and 3:

$$T_{minA} = \min(T_{cA}, T_{tA}) \tag{4}$$

2.2 Eurocode 2

Like ACI Code (2002), Eurocode 2 (1992) makes use of the three-dimensional truss model with variable inclination θ. The parameter θ and the thickness of the shear flow zone t must comply with the following limitations:

$$1.0 \le ctg\ \theta \le 2.0 \tag{5}$$

$$2c \leq t \leq A/u \tag{6}$$

where A and u are the cross sectional area and the perimeter of the concrete rectangular section respectively, and c is the thickness of the concrete cover. Assuming that both transversal and longitudinal reinforcements are at yielding, the inclination θ is determined by the equilibrium condition:

$$tg^2\theta = \frac{A_t f_{ytd} u_o}{A_l f_{yld} s} \tag{7}$$

where f_{yld} and f_{tyd} are the yield design stresses, A_l is the total area of longitudinal bars and u_o is the length of the centerline of the shear flow. The torsional capacity resisted by the concrete section is

$$T_{Rd1} = \frac{2\nu f_{cd} A_o t}{cotg\theta + tg\theta} \tag{8}$$

where f_{cd} is concrete design strength, and ν is the concrete strength reduction factor:

$$\nu = 0.7\left(0.7 - \frac{f_c}{200}\right) \leq 0.35 \tag{9}$$

For rectangular sections A_o and u_o have the following expressions:

$$A_o = A - \frac{t}{2}u + t^2 \tag{10}$$

$$u_o = u - 4t \tag{11}$$

The torsional capacity resisted by transverse reinforcement is

$$T_{Rd2} = 2A_o\sqrt{\frac{A_l f_{yld}}{u_o}\frac{A_t f_{ytd}}{s}} \tag{12}$$

The torsional capacity T_{minEC2} for a rectangular beam is defined as the minimum of the moments expressed by Eqs. 8 and 12:

$$T_{minEC2} = \min\left(T_{Rd1}, T_{Rd2}\right) \tag{13}$$

2.3 Italian Code

Italian Code (D.M. 9.01.1996) uses the classical three-dimensional truss model, with 45° concrete-strut inclination. The torsional capacity resisted by each member of the truss are the following:

- Concrete struts:

$$T_{cI} = \frac{1}{2} A_2 t_d \, f_{cd} \tag{14}$$

- Longitudinal steel:

$$T_{lI} = \frac{2 A_2 A_l}{u_2} f_{lyd} \tag{15}$$

- Transversal steel:

$$T_{tI} = \frac{2 A_2 A_t}{s} f_{tyd} \tag{16}$$

where A_2 is the area bounded by a conventional centerline of the shear flow such that the thickness t_d of the shear flow zone is $b_1/6$ (b_1 is the diameter of the circle inscribed in the centerline connecting the centroids of the longitudinal bars) and u_2 is the length of the above-mentioned centerline.

The torsional capacity T_{minIC} for a rectangular beam is defined as the minimum of the moments expressed by Eqs. 14, 15 and 16:

$$T_{minIC} = \min(T_{cI}, T_{lI}, T_{tI}) \tag{17}$$

2.4 Hsu approximate models

Hsu (1990) proposed an approximate model, which simplifies the determination of the torsional capacity, with respect to his original theory (Hsu and Mo 1985). The most important hypothesis consists in setting the ratio of the concrete average stress to peak stress in the stress block equal to 0.8. This simplification provides a non linear system with three unknowns: the thickness of the shear flow zone t, the angle of inclination of the concrete struts θ and the softening coefficient ζ:

$$t = \frac{A_o \zeta^2}{u_o sin^2\theta \cos^2\theta} \tag{18}$$

$$\zeta = \frac{\frac{A_t f_{yt}}{s} + \frac{A_l f_{yl}}{u_o}}{0.80 f_c t} \tag{19}$$

$$\cos^2\theta = \frac{\frac{A_l f_{yl}}{u_o}}{\frac{A_t f_{yt}}{s} + \frac{A_l f_{yl}}{u_o}} \tag{20}$$

The first equation expresses a kinematic compatibility condition, while the second and the third are equilibrium equations. The system of Eqs. 18, 19 and 20 can easily be solved by a simple trial and error procedure: assume an initial value for t; calculate A_o and u_o by Eqs. 10 and 11; compute ζ and θ from Eqs. 19 and 20 respectively. Substituting ζ and θ into Eq. 18 gives t. If the resulting t is close enough to the initial value, then a solution is found; otherwise choose another t and repeat the cycle. Once a solution is obtained, the torsional capacity can be evaluated by the following equilibrium equation:

$$\mathrm{T_{Hit}} = 2\frac{A_t f_{yt}}{s} A_o ctg\,\theta \tag{21}$$

where the suffix “it” reminds that the value is obtained by means of an iterative process.

For design purposes, Hsu (1990) introduced further approximations to work out a simplified direct expression for the torsional capacity $\mathrm{T_u}$; in particular the thickness of the shear flow zone can be approximately estimated by

$$t = \frac{4\mathrm{T_u}}{A f_c} \tag{22}$$

and, if t is small, A_o can be evaluated from Eq. 10, neglecting the term t^2:

$$A_o = A - t\frac{u}{2} \tag{23}$$

If θ is taken equal to 45 deg, Eq. 21 (with T_u instead of T_{Hit}) and Eqs. 22 and 23 provide the torsional capacity T_H

$$T_H = \frac{2A^2 f_c f_{yt} A_t}{sAf_c + 4A_t f_{yt} u} \tag{24}$$

One should observe that ACI Code suggests the use of Eq. 21 or Eq. 24 instead of Eq. 3, when a greater accuracy is requested.

2.5 Bhatti and Almughrabi Model

Bhatti and Almughrabi (1996) showed by means of a numerical procedure that aggregate interlock along cracked sections contributes significantly to torsional capacity. This conclusion agrees with the tests made by Mattock (1968), who found that the torsional capacity of a longitudinally reinforced beam (with no stirrups), subjected to torsion and flexure, is approximately one-half the torsional capacity of the uncracked beam.

Bhatti and Almughrabi's (1996) model is based on the equations of the old ACI Building Code (1989) for rectangular sections. The torsional capacity was expressed as the sum of two contributions, the first, due to concrete (T_c) and the second, due to the reinforcement (T_s):

$$T_c = \frac{\sqrt{f_c}}{15} b^2 h \qquad \text{(S.I. units)} \qquad \text{with } b \le h \tag{25}$$

$$T_s = \alpha_t b_1 h_1 \frac{A_t f_{yt}}{s} \qquad \text{with } b_1 \le h_1 \tag{26}$$

where b_1 and h_1 are the dimensions of one closed stirrup and the efficiency factor for the reinforcement α_t has the following expression:

$$\alpha_t = \frac{1}{3}\left(2 + \frac{h_1}{b_1}\right) \tag{27}$$

Bhatti and Almughrabi modified the contributions T_c and T_s in Eqs. 25 and 26, on the basis of their numerical analyses on 7 beams. The main reasons why they did so, are:

- T_c given by Eq. 25 doesn't depend on the steel percentage ρ_s and on the aspect ratio *h/b*, while the numerical tests show that these parameters have some influence on concrete contribution. An increase of ρ_s produces an increase in aggregate interlock forces, while the larger the aspect ratio the larger the contribution to torsional strength, because of the increasing level arm concerning aggregate interlock.
- T_s as provided by Eq. 26, is a linear function of the volumetric percentage of transverse steel ρ_s, while the numerical tests suggest a second power dependence (ρ_s^2).

For these reasons, they proposed a new expression for T_c and T_s:

$$T_{c,B} = 6.643 \cdot 10^{-2} \alpha_{c,B} \beta_{c,B} b^2 h \sqrt{f_c} \qquad \text{(S.I. units)} \tag{28}$$

$$T_{s,B} = \alpha_{t,B} \beta_{s,B} \frac{b_1 h_1 A_t f_{ty}}{s} \tag{29}$$

where ρ_s, $\beta_{c,B}$, $\beta_{s,B}$ and $\alpha_{c,B}$ have the following formulations:

$$\rho_s = \frac{2(b_1 + h_1)\, A_t}{b\, h\, s} \tag{30}$$

$$\beta_{c,B} = 0.575\ \rho_s^2 - 0.486\ \rho_s + 1.238 \tag{31}$$

$$\beta_{s,B} = -\ 0.198 \rho_s^2 + 0.274\ \rho_s + 0.789 \tag{32}$$

$$\alpha_{c,B} = 0.24\ \text{h/b} + 0.62 \tag{33}$$

As for $\alpha_{t,B}$ (Eq. 27) the expression found in ACI (1989) is adopted.

The torsional capacity is:

$$T_{BA} = T_{c,B} + T_{s,B} \tag{34}$$

3. BASIS OF THE PROPOSED MODEL

The new idea of Bhatti and Almughrabi is that the volumetric percentage of stirrups ρ_s influences the ultimate capacity not only directly (since the stirrups are members of the space-truss model) but indirectly as well. This is taken into account by means of the function $\beta_{c,B}$ in Eq. 28, making the concrete contributions for the same cross-section different if steel ratios are different.

However, some comments have to be made on the functions $\beta_{s,B}$ (Eq. 32) and $\beta_{c,B}$ (Eq. 31) proposed by Bhatti and Almughrabi. The plots of these functions is shown in Fig. 1.

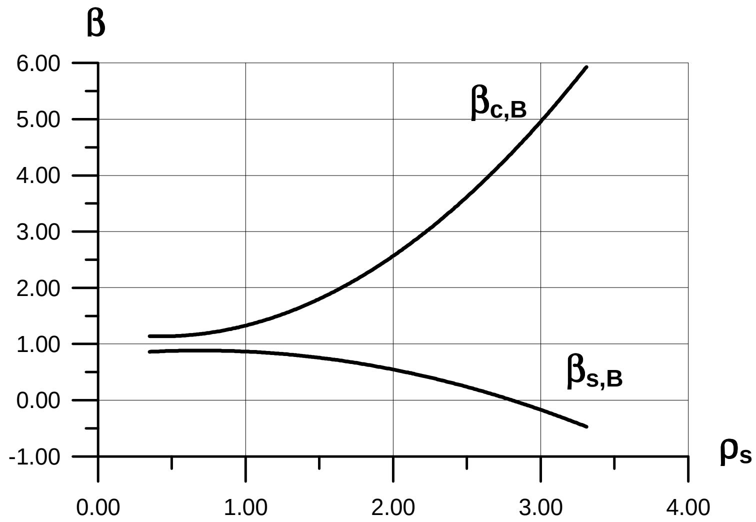

Fig. 1 – $\beta_{c,B}$ and $\beta_{s,B}$ functions versus transverse steel percentage.

Firstly, $\beta_{s,B}$ assumes negative values for ρ_s greater than 2.805. A negative contribution of the stirrups to the torsional capacity has no physical meaning.

Considering now $\beta_{c,B}$, it can be observed that the increase of concrete contribution with a positive concavity appears unreasonable. With regard to this point Hsu (1968b) showed that the cracking moment T_{cr} increases linearly with the total volume percentage reinforcement and Mattock (1968) showed that this moment for a longitudinally-reinforced beam (with no stirrups), is approximately twice as much as the concrete torsional contribution after cracking. It follows that T_c and $\beta_{c,B}$ should be linear functions of ρ_s, as T_{cr} does (Hsu 1968b). A possible concavity, if any, would be negative to explain how too much reinforcement can decrease concrete strength because of the many discontinuities the steel introduces in the concrete.

Both the unrealistic $\beta_{s,B}$ - ρ_s and $\beta_{c,B}$ - ρ_s relationships introduced by Bhatti and Almughrabi, are likely due to statistically-inconsistent tests considered by these authors: Eq. 31 is in fact a best-fit function based on 4 numerical experiments, while Eq. 32 is a best-fit expression based on 7 numerical experiments. It can be moreover observed that the coefficients appearing in each of the functions $\alpha_{c,B}$, $\beta_{c,B}$ and $\beta_{s,B}$ (Eqs. 31, 32 and 33) have been determined independently from the others. Finally the torsional capacity computed by means of Eq. 34 has been compared only with 19 experimental tests.

In the model presented in this paper, the torsional capacity is still assumed to be due to two contributions similar to those introduced by Bhatti ed Almughrabi (Eqs. 28 and 29):

$$T_c = 6.643 \cdot 10^{-2} \alpha_c \beta_c b^2 h \sqrt{f_c} \qquad \text{(S.I. units)} \qquad (35)$$

$$T_s = \alpha_t \beta_s \frac{b_1 h_1 A_t f_{ty}}{s} \qquad (36)$$

where

$$\alpha_t = k_1 + k_2\, b_1/h_1 \qquad (37)$$

$$\beta_s = k_3\, \rho_s^2 + k_4\, \rho_s + k_5 \qquad (38)$$

$$\alpha_c = k_6\, b/h + k_7 \qquad (39)$$

$$\beta_c = k_8\, \rho_s^2 + k_9 \rho_s + k_{10} \qquad (40)$$

The function α_t, which Bhatti ed Almughrabi borrowed from Hsu (1968a), is here assumed as a function to be determined. This assumption is justified by the fact that Bhatti ed Almughrabi introduced a new function α_c depending on the aspect ratio and consequently the function α_t, originally proposed by Hsu, cannot be left unchanged.

The ultimate torsional capacity is

$$T_u = T_s + T_c \qquad (41)$$

Now, in agreement with Bhatti ed Almughrabi, the transverse volume percentage ρ_s is assumed to influence both T_c and T_s. For this reason, a single statistical regression on the available data should be performed, instead of three separate statistical regressions as Bhatti and Almughrabi did. The optimization process made here minimizes the coefficient of variation (COV) of the ratios between the experimental torsional capacity T_{exp} and the proposed one $T_{u,pr}$. This process has been performed taking into account the experimental tests made on 87 rectangular beams subjected to torsion.

From the optimization process the following values have been obtained:

$k_1 = 0.366$ $\quad$ $k_2 = -0.0855$ $\quad$ $k_3 = -0.237$

$k_4 = 1.651$ $\quad$ $k_5 = -0.373$ $\quad$ $k_6 = 0.0737$

$k_7 = 0.926$ $\quad$ $k_8 = -0.914$ $\quad$ $k_9 = 2.986$

$k_{10} = 1.319$

The following formula for the ultimate torsional moment is finally proposed

$$T_{u,pr} = T_{c,pr} + T_{s,pr} \tag{42}$$

where

$$T_{c,pr} = 6.643\ 10^{-2}\ \alpha_{c,pr}\ \beta_{c,pr}\ b^2 h \sqrt{f_c} \tag{43}$$

$$T_{s,pr} = \alpha_{t,pr}\beta_{s,pr}\frac{b_1 h_1 A_t f_{yt}}{s} \tag{44}$$

with

$$\alpha_{t,pr} = 0.366 - 0.0855\ b_1/h_1 \tag{45}$$

$$\beta_{s,pr} = -0.237\ \rho_s^2 + 1.651\ \rho_s - 0.373 \tag{46}$$

$$\alpha_{c,pr} = 0.0737\ b/h + 0.926 \tag{47}$$

$$\beta_{c,pr} = -0.914\ \rho_s^2 + 2.986\rho_s + 1.319 \tag{48}$$

Before analyzing the reliability of the proposed model, the obtained functions $\beta_{s,pr}$ (Eq. 46) and $\beta_{c,pr}$ (Eq. 48) are discussed. The curves $\beta_{s,pr} - \rho_s$ and $\beta_{c,pr} - \rho_s$ are plotted in Fig. 2.

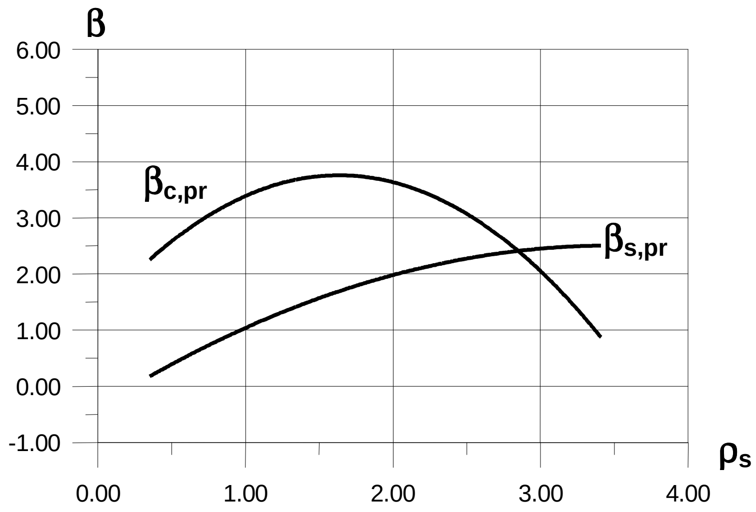

Fig. 2 - $\beta_{s,pr}$ and $\beta_{c,pr}$ functions versus transverse steel percentage.

Some physical discrepancies previously observed for the analogous Bhatti ed Almughrabi functions are no longer present. As for $\beta_{s,pr}$, it can be observed that it is always positive and increases with ρ_s.

Regarding to the function $\beta_{c,pr}$, the unexpected $\beta_{c,pr}$ decrease beyond the maximum it assumes for $\rho_s = 1.63$ needs an explanation. In order to do this, the torsional moment at first cracking T_{cr} has been considered, because of the great influence of concrete on its value. Before cracking the 3-D truss mechanism is not active. The experiments made by Hsu (1968a) are now considered, because the values of T_{cr} are reported only in his paper. The non-dimensional cracking moment t_{cr} has the following expression:

$$t_{cr} = \frac{T_{cr}}{b^2 h \sqrt{f_c}} \tag{49}$$

The possible dependence of t_{cr} on ρ_s is shown in Fig.3. The interpolating function drawn on the diagram evidences that experimental variation of t_{cr}, which is similar to that of $\beta_{c,pr}$ because of Eq. 43 and the previous discussion on T_{cr}, shows a concavity analogous to that obtained with Eq. (48).

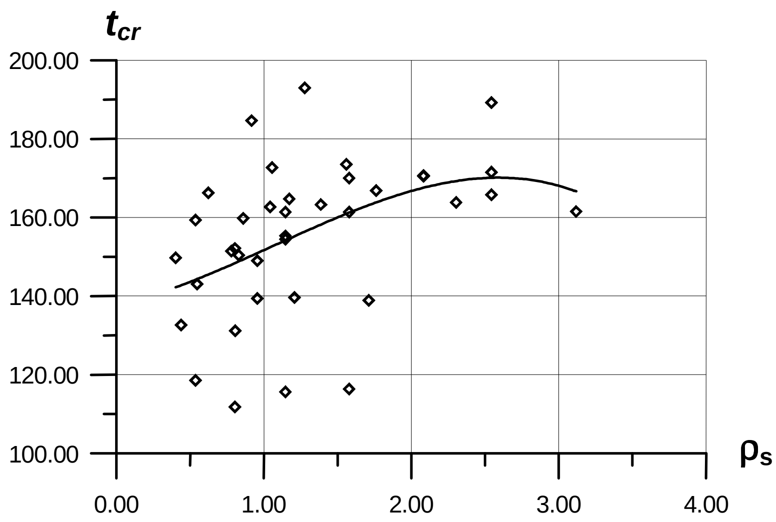

Fig. 3 – Experimental non-dimensional cracking moment versus transverse steel percentage.

4. MODEL RELIABILITY

In this chapter the results obtained by means of the proposed formula (Eq. 42) are compared with the formulae adopted by various design codes and developed by the previously-mentioned authors. The tests concerning 87 beams collected from different sources have been taken into account (Hsu 1968a; Nielsen 1983; Nielsen 1984; Narayanan and Palanjian 1986; Karayannis and Chalioris 2000; Ashour et al. 1999; Rasmussen and Baker 1995; Csikós and Hegedûs 1998).

For each beam the torsional capacity has been computed with the seven expressions mentioned earlier in this paper, and the experimental-to-computed torsional capacity ratios have been calculated. For each expression with reference to the 87 tests, the average (AVG), the standard deviation (STD) and the coefficient of variation (COV = STD/AVG) of these ratios have been computed. The experimental-to-computed torsional capacity ratio versus the volume percentage of transverse reinforcement are shown for ACI Code (2002), Eurocode 2 (1992), Italian Code (1996), Hsu (1990) iterative formulae, Hsu (1990) direct formula, Bhatti and Almughrabi (1996) formula and proposed formula in Figs. 4, 5, 6, 7, 8, 9 and 10 respectively. The AVG and COV values are also reported in the same Figures. The horizontal line through 1 in the ordinate axis represents the perfect correspondence between testing and modeling. So the closer the points to this line, the more accurate the torsional capacity prediction. The thinner the width of the strip including the points the greater the prediction uniformity.

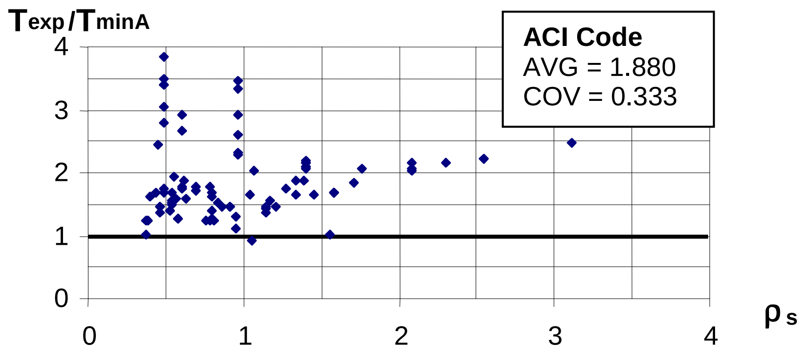

Fig. 4 - Experimental-to-computed torsional capacity ratio (ACI 2002).

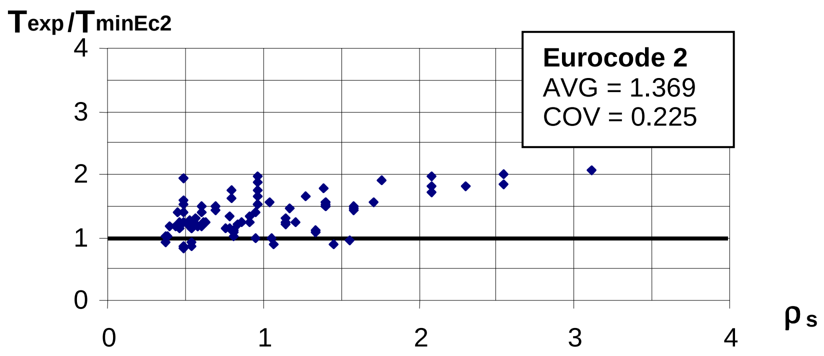

Fig. 5 - Experimental-to-computed torsional capacity ratio (Eurocode 2, 1992).

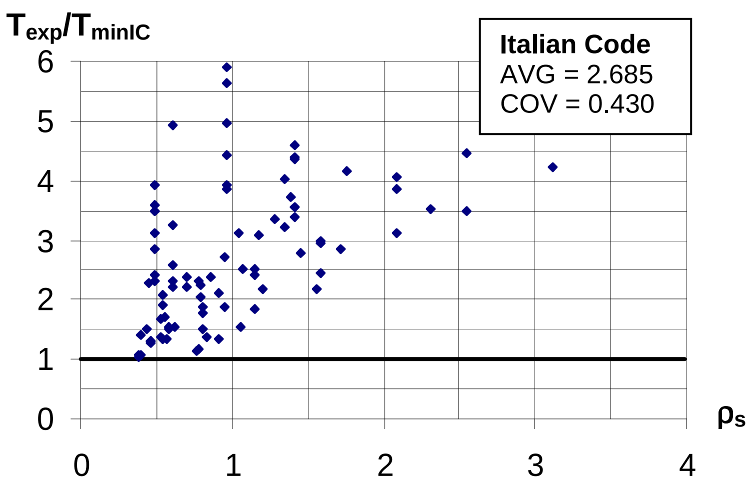

Fig. 6 - Experimental-to-computed torsional capacity ratio (Italian Code, 1996).

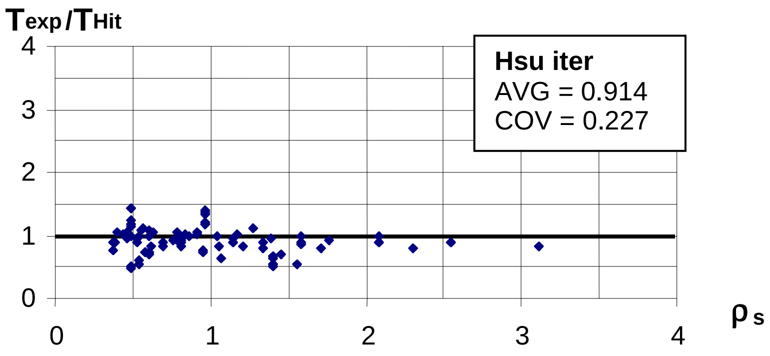

Fig. 7 - Experimental-to-computed torsional capacity ratio (Hsu, iterative method, 1990)

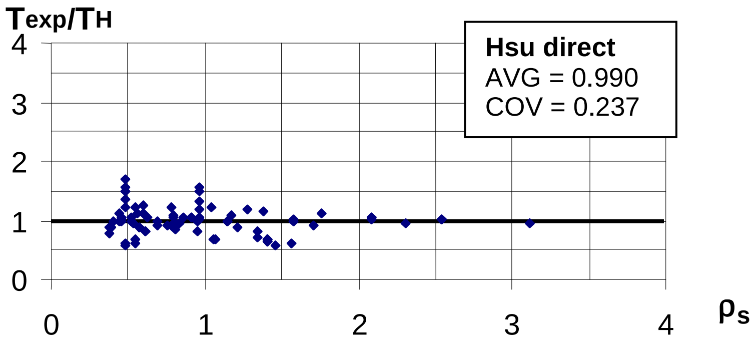

Fig. 8 - Experimental-to-computed torsional capacity ratio (Hsu, direct formula, 1990)

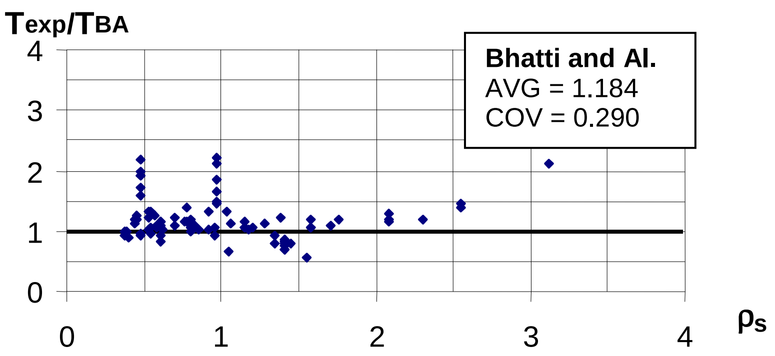

Fig. 9 - Experimental-to-computed torsional capacity ratio (Batti and Almughrabi, 1996)

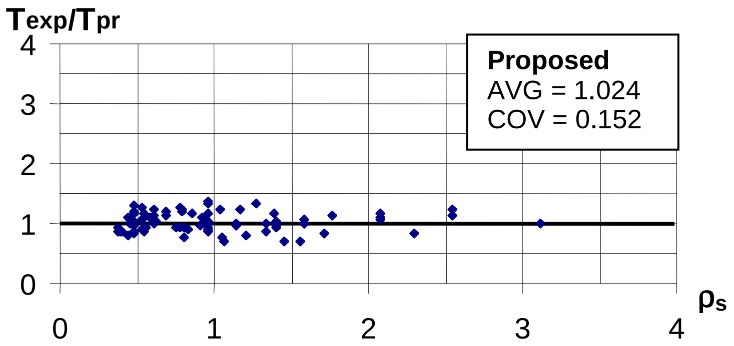

Fig. 10 - Experimental-to-computed torsional capacity ratio (proposed formula)

The average values and the coefficient of variation, are shown in Table 1. It appears that the proposed formula is the most efficient for the prediction of the torsional capacity, since it provides the lowest COV. The proposed formula is even more efficient then Hsu (1990) iterative method.

	AVG	COV
Aci Code (2002)	1.880	0.333
Eurocode (1992)	1.369	0.225
Italian Code (1996)	2.685	0.430
Hsu iter. (1990)	0.914	0.227
Hsu direct (1990)	0.990	0.237
Bhatti and Al. (1996)	1.184	0.290
Proposed formula	1.024	0.152

Tab. 1 – Comparison of results.

5. CONCLUSIONS

The many test results discussed in this paper on R/C beams with rectangular cross section subjected to pure torsion, and the proposed model lead to the following conclusions:

1. The torsional capacity of R/C beams can be predicted with good accuracy and uniformity (COV=0.152) by means of a direct expression including the transverse-steel percentage ratio.

2. The proposed expression, based on Batti and Almughrabi model, consists of two contributions, the first from the concrete and the second from the reinforcement; the formulation of both expressions have a sound physical meaning, which was not the case of the original expressions.
3. Among the design codes considered in this paper, Eurocode 2 (1992) provides the more consistent (COV=0.225) evaluation of the experimental results, while the Italian Code (1996) is extremely conservative.

ACKNOWLEDGEMENTS

The financial support by the Italian Ministry for Higher Education and Research (MIUR) is gratefully acknowledged. The contribution of Mr. Cristian Mazzero has been instrumental in collecting the test data available in the literature.

APPENDIX I - REFERENCES

ACI Committee 318 (1989). "Building Code Requirements for Reinforced Concrete" (ACI 318-89), American Concrete Institute, Detroit.

ACI Committee 318 (2002). "Building Code Requirements for Structural Concrete and Commentary" (ACI 318-02/ACI 318R-02), American Concrete Institute, Detroit.

Andersen P. (1935). “Experiments with Concrete in Torsion”, *Transactions*, ASCE, V. 100, 1935, pp. 948-983.

Ashour S. A., Samman T. A. and Radain T. A. (1999). “Torsional Behaviour of Reinforced High-Strength Concrete Beams”, *ACI Structural Journal*, Vol. 96, N° 6, pp. 1049-1058.

Bhatti M.A. and Almughrabi A. (1996). “Refined Model to Estimate Torsional Strength of Reinforced Concrete Beams”, *ACI Structural Journal*, Vol. 96, N. 6, pp. 1049-1058.

Connor J. and Sarne Y. (1975). “Nonlinear Analysis of Prestressed Concrete Reactor Vessels.” 3rd SMIRT, London.

Cowan H. J. (1950). “Elastic Theory for Torsional Strength of Rectangular Reinforced Concrete Beams.” *Magazine of Concrete Research*, London, V. 2, N. 4, July 1950, pp 3-8.

Csikós A. and Hegedûs I. (1998). “Torsion of Reinforced Concrete Beams”, 2nd Int. PhD Symposium in Civil Engineering, Budapest, pp. 1-9.

D.M. 9.01.1996. "Norme tecniche per il calcolo, l'esecuzione ed il collaudo delle strutture in cemento armato, normale e precompresso e per le strutture metalliche".

Eurocode n° 2. "Design of Concrete Structures. Part 1-1 : General Rules and Rules for Buildings". ENV 1992-1-1 Comité Européen de Normalization CEN.

Hsu Thomas T.C. (1968a). "Torsion of Structural Concrete - Behavior of Reinforced Concrete Rectangular Members", *Torsion of Structural Concrete*, SP-18, American Concrete Institute, Detroit, pp. 261-306.

Hsu Thomas T.C. (1968b). "Torsion of Structural Concrete – A Summary on Pure Torsion", *Torsion of Structural Concrete*, SP-18, American Concrete Institute, Detroit, pp. 165-178.

Hsu Thomas T.C. and Mo Y.L. (1985). "Softening of Concrete in Torsional Members - Theory and Tests", *ACI Journal, Proceedings*, Vol. 82, No. 3, May-June 1985, pp. 290-303.

Hsu Thomas T.C. (1990). "Shear Flow Zone in Torsion of Reinforced Concrete", *J. of Structural Engineering*, Vol. 116, No. 11, pp. 3206-3227.

Karayannis C. G. and Chalioris C. E. (2000). “ Influence of Steel Fibers on the Capacity of flanged Beams in Torsion”. Tech. Chron. Sci J. TCG, I, N. 2.

Lampert P. and Thurlimann B. (1968). "Torsionsversuche an Stahlbetonbalken", Bericht n. 6506-2, Institut fur Baustatik, ETH Zurich, 101 pp.

Lampert P. and Thurlimann B. (1969). "Torsion-Beige-Versuche an Stahlbetonbalken", Bericht n. 6506-3, Institut fur Baustatik, ETH Zurich, 116 pp.

Mattock A.H. (1968). "How to Design for Torsion", *Torsion of Structural Concrete*, SP-18, American Concrete Institute, Detroit, pp. 469-495.

Narayanan R. and Palanjian A. S. K. (1986). “Torsion in Beams Reinforced with Bars and Fibers”, *Journal of Structural Engineering*, ASCE, Vol. 112, N° 1, pp. 52-59.

Ngo D. and Scordelis A.C. (1967). “Finite Element Analysis of Reinforced Concrete Beams”. *ACI Journal, Proceedings* V. 64, N. 3, Mar. 1967.

Nielsen J. S. (1983). “Theoretical and Experimental Study of Concrete Beams – Especially Over-Reinforced Beams Subjected to Torsion, Part II. Experiments”, Department of Structural Engineering, The Technical University of Denmark, *Report* N° 171, 113 pp.

Nielsen J. S. (1984). “Effect of Stirrups Spacing on The Ultimate Load of Reinforced Concrete Beams Subjected to Torsion”, Department of Structural Engineering, The Technical University of Denmark, *Report* N° 180, 92 pp.

Rasmussen L. J. and Baker G. (1995). “Torsion in Reinforced Normal and High-Strength Concrete Beams-Part 1: Experimental Test Series”, *ACI Structural Journal*, Vol. 92, N° 1, pp. 56–62.

Robinson J.R. and Demorieux J.M. (1968). "Essais de traction-compression sur modèles d'ame de poutre en béton armé", *Institut de Recherches Appliquées du Béton Armé (IRABA)*, Part 1.

Robinson J.R. and Demorieux J.M. (1972). "Essais de traction-compression sur modèles d'ame de poutre en béton armé", *Institut de Recherches Appliquées du Béton Armé (IRABA)*, Part 2.

Vecchio F. and Collins M.P. (1981). "Stress-Strain Characteristics of Reinforced Concrete in Pure Shear", *Final Report, IABSE Colloquium on Advanced Mechanics of Reinforced Concrete,* International Association for Bridge and Structural Engineering, pp. 211-225, Delft.

APPENDIX II – RESISTENZA TORSIONALE DI TRAVI IN CALCESTRUZZO ARMATO A SEZIONE RETTANGOLARE

In questo lavoro viene studiata la resistenza ultima di travi in calcestruzzo armato a sezione rettangolare sottoposte a sollecitazione puramente torsionale.

Il primo fondamentale contributo scientifico nel campo della torsione per elementi in c.a. è dovuto a Rausch (1929) che estese al caso torsionale lo schema a traliccio piano già introdotto in precedenza per il taglio. Tutti i successivi miglioramenti (angolo di inclinazione dei puntoni variabile, modifica dell'impronta trasversale del traliccio) non hanno alterato nella sostanza lo schema statico descritto basato solamente sull'equilibrio, tant'è che la gran parte delle normative attualmente in vigore (tra cui ad esempio l'ACI Building Code 2002, l'Eurocode 2 1992 e il D.M. 9.1.1996) fanno ancora riferimento a questo schema.

Solo i contributi scientifici più recenti propongono modelli di comportamento che meglio adattano le previsioni teoriche ai dati sperimentali. Tali modelli tengono conto del fatto che i puntoni di calcestruzzo sono sottoposti anche ad una significativa sollecitazione flessionale e che la resistenza a compressione del calcestruzzo risulta penalizzata per la presenza delle barre d'armatura trasversali sottoposte a forze di trazione (perdita di resistenza o "softening" del calcestruzzo). Sulla base di queste considerazioni sono stati elaborati modelli molto sofisticati (Hsu and Mo 1985) che, se da un lato conducono a previsioni molto precise, dall'altro sono di scarsa utilità pratica per la loro complicata risolubilità. Per questo sono stati proposti (Hsu 1990) modelli più semplificati ed anche formule dirette per la valutazione del momento torcente ultimo, di cui la normativa statunitense (ACI Building Code 2002) consente di tener conto.

Si deve a Bhatti e ad Almughrabi (1996) l'aver formulato un modello non lineare agli elementi finiti che, in particolare, tiene conto di alcuni significativi fenomeni prima trascurati: l'effetto spinotto, l'aderenza acciaio-calcestruzzo e l'ingranamento degli inerti. Sulla base di un'analisi parametrica, essi hanno proposto una semplice formula per la resistenza torsionale ultima.

Vengono riportate le espressioni di calcolo del momento torcente previste dall'ACI Code (2002), dall'Eurocodice 2 (1993) e dalla Normativa Italiana (1996). Viene quindi riassunto il modello iterativo semplificato di Hsu (1990), nonché la sua formula diretta approssimata. Dopo aver descritto nel dettaglio il modello di Bhatti e Almughrabi (1996), vengono messe in luce alcune inconsistenze fisiche presenti nello stesso modello; tali inconsistenze riguardano in particolare le funzioni β moltiplicative dei contributi resistenti a torsione del calcestruzzo e dell'acciaio. Inoltre tale modello risulta basato su regressioni statistiche poco significative, in quanto molto modesto è il numero di campioni utilizzato. Infine, ai fini della determinazione delle funzioni β, gli stessi autori conducono regressioni statistiche separate e su campioni diversi, apparentemente senza motivo.

Il modello proposto parte dall'osservazione che il contributo resistente dovuto alle staffe non può risultare negativo e che il contributo resistente dovuto al calcestruzzo non può crescere con pendenze sempre maggiori al crescere della percentuale di staffatura. Si è ritenuto pertanto di rideterminare una delle due funzioni moltiplicative del contributo delle staffe che era stata assunta da Bhatti e Almughrabi (1996) coincidente con quella originariamente proposta da Hsu (1968a). Tale assunzione operata da Bhatti e Almughrabi non appare in realtà congruente con l'introduzione della funzione moltiplicativa del contributo del calcestruzzo legato al rapporto di forma.

E' stato quindi preso in considerazione un campione di 87 prove sperimentali condotte da diversi autori e disponibili in letteratura. Dalla regressione statistica mirata alla minimizzazione del coefficiente di variazione del rapporto tra il momento torcente resistente sperimentale e quello teorico, si è ottenuta la formulazione dei coefficienti e quindi delle quattro funzioni moltiplicative. A differenza di quelle ottenute da Bhatti e Almughrabi, le funzioni β così ottenute hanno un significato fisico. Ciò viene anche evidenziato dai risultati sperimentali relativi al momento torcente all'atto della prima fessurazione.

Per gli 87 campioni presi in considerazione vengono calcolati i momenti torcenti resistenti anche secondo l'ACI Code (2002), l'Eurocodice 2 (1993), la Normativa Italiana (1996), il modello iterativo semplificato di Hsu (1990), la formula diretta approssimata proposta da Hsu e il modello di Bhatti e Almughrabi. Il confronto tra i valori del coefficiente di variazione (COV) ottenuti per i modelli considerati evidenzia che l'espressione proposta risulta la più consistente in quanto il risultato sperimentale viene predetto nel modo più uniforme (COV=0.152).