An Euler-Bernoulli beam element with lumped plasticity applied on RC framed structures

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Most of existing reinforced concrete structures suffer due to corrosion of steel and concrete degradation. In many cases existing structures reveal to be inadequate to absorb the expected seismic load and need to be rehabilitated according to the in force code. In the worst case some structures have not been designed to absorb horizontal actions.
The rehabilitation process begins with the complete knowledge of its geometrical configuration and the evaluation of the vulnerability of the structure to seismic loads. This analysis permits to identify critical zones and to establish focused strengthening actions. A comparison between the behavior of the structure in the current and in the future configurations determines the goodness of adopted intervention techniques.
The evaluation of the vulnerability of an RC structure to seismic loads can be done by performing nonlinear finite element analyses.
In literature, three different approaches have been tuned to simulate the elastoplastic behavior of a beam/column element: lumped elastoplasticity models, distributed nonlinearity models, fiber models. Lumped models consider the constitutive nonlinearity concentrated at a section level of a frame element, usually employing nonlinear springs at the ends of beam/column elements. Distributed nonlinearity models average the nonlinearity over a finite element by considering the possibility to form plastic hinges at different evaluation points of the element and calculating weighted integrals of the section responses. Fiber models subdivide a section with a large number of finite elements and nonlinearity is related to the stress-strain relationship of a single finite element. Within lumped models, commercial finite element programs contemplate the possibility to develop plasticity at the two ends of the beam only. In the particular cases where plasticity concentrates in points different than the ends of the beam, it computationally comes in the need to proceed with a re-meshing of the model or in the definition of multiple elements before running the analysis. In the first case, it results in an increased computational cost of the analysis. In the second case, a less precision of the response is obtained especially when the exact position of the plastic hinge is not a-priori known.
The present work is devoted to the implementation of a new elastoplastic 3D Euler-Bernoulli beam element including slope discontinuities, in the framework of lumped elastoplasticity models. In the new finite element, plastic hinges can appear at any position of the beam, theoretically in a priori not-established number. Multiple slope discontinuities are included in the analysis through a non uniform bending stiffness of the beam, making use of the Dirac-delta function [1-2]. Fictitious springs, with a stiffness variable according to the level of plasticity in the section, transfer the correct bending moment in correspondence of plastic hinges.

The nonlinear behavior of the hinge is defined in the framework of a thermo-dynamically consistent elastoplastic theory [3-4]. Associated flow rules are derived in the classical manner adopting a convex activation domain known in literature and experimentally calibrated for reinforced concrete sections. The activation domain is similar to the one suggested by the Italian seismic code [5-6]. It is given in a M_y-M_z bending moment reference system for a fixed axial force. An elastoplastic behavior is assumed for section curvatures, while deformations in the axial and shear directions are assumed elastic.
The elastoplastic frame element is introduced in a finite element analysis program to run nonlinear simulations on 2D and 3D framed structures. To this end, state equations and flow rules are rewritten in a discrete manner to solve the single iteration of the Newton-Raphson procedure. A classic elastic predictor phase is followed by a plastic corrector phase in the case of activation of the inelastic phenomena. The corrector phase is based on the evaluation of return bending moments by employing the closest point projection method, in order to satisfy the loading-unloading conditions (Kuhn-Tucker relations).
The formation of one or more hinges inside a finite element modifies the distribution of internal forces and its stiffness matrix. As a consequence, the global stiffness matrix is continuously modified at each plastic load step until it becomes singular.
Numerical examples are furnished as validation tests of the program. The efficiency of the proposed model is demonstrated comparing the results with those available in literature.

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