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An integrated fuzzy-stochastic model for revenue management: The hospitality industry case

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Abstract

Revenue management aims at improving the performance of an organization by selling the right product/service to the right customer at the right time. This task is very dependent on uncontrollable external factors. In the hospitality industry, rooms of the hotel represent perishable assets and fixed capacities at the same time. Therefore, in the case of a stochastic process for customers calling in reservations prior to a particular booking date, a common problem for hotels is to devise a policy for maximizing the total expected profit conditional on the set of bookings. We propose a fuzzy model for the hotel revenue management under an uncertain and vague environment. Fuzziness of objective and constraint functions have been incorporated into a stochastic booking model considering multiple-day stays to show the effect of uncertainty on the optimal demand. By changing the relaxation parameters of the objective function, we have found a set of optimal solutions with, in most of the cases, a value of the objective function equal to the optimal solution of the stochastic model, providing several alternative optimal room allocations.

Keywords

booking system, fuzzy optimization, hospitality industry, revenue management, stochastic demand

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Revenue management (RM), or yield management, is concerned with finding the optimal inventory allocation and scheduling strategies and/or price settings so as to maximize revenue within the planning horizon.

Although RM systems have been applied in a wide range of industries, including energy, fashion retail, manufacturing, car rentals and hospitality, a substantial part of the literature is rooted in the airlines because flight companies have the longest history of development in RM.

The research started in the early 1970s with the seat inventory control rule proposed by Littlewood (1972) for airlines offering fare products that mixed discount and higher fare passengers in the same aircraft compartments. With the expected marginal seat revenue heuristic, Belobaba (1989) extended the Littlewood's one-period model with two fare classes to the case of multiple fares. Overbooking models (Chatwin, 1998), procedures for finding the booking limit (the limit on the number of items that can be sold at a cheaper price) that maximizes the company's expected revenue (Belobaba and Weatherford, 1996; Bodily and Weatherford, 1995; Weatherford et al., 1993) and models with customers diversion (Pfeifer, 1989) appeared in the literature shortly after. Weatherford (1998), McGill and van Ryzin (1999), Pak and Piersma (2002) and Talluri and van Ryzin (2004) provided an overview of RM applications in airline seat inventory control.

From its origins, RM has grown to its current status of mainstream business practice because control of costs is an important task to succeed in the current intense competition but it is the RM the task most dependent on uncontrollable external factors. Moreover, the more a company offers perishable products/services or operates with fixed capacities, the more revenues are sensitive to such factors.

Several articles provide a review of the literature on RM problems addressed in a variety of industries. Chiang et al. (2007) and McGill and van Ryzin (1999), for instance, discussed a list of published articles, books, conference proceedings, working articles, industrial technical reports and graduate theses and provided a wide bibliography of works in RM up to 1999 and since 1999, respectively. Pullman and Rodgers (2010) focused on the RM approaches that have gained significant worldwide adoption in the accommodation industry. In this domain, most RM theories deal with the uncertainty of the environment using statistical forecasting methods and mathematical optimization techniques. Weatherford and Kimes (2003) provided a comparison of such forecasting methods. Bitran and Mondschein (1995) and Weatherford (1995) used simulations to test their heuristics for whether or not to accept a reservation request. Baker (1994) extended the above studies and compared them with his heuristic models for overbooking and allocation. Lai and Ng (2005) and Liu et al. (2008) proposed a network optimization model for hotel RM in a stochastic programming formulation so as to capture the randomness of the unknown demand (unknown number of arrivals and unknown length of stays).

In a wide range of tourism studies, uncertainty, imprecision and other ambiguities have been modelled by applying the fuzzy programming approach. Accurate forecasting of tourism arrivals (Chou et al., 2010; Hadavandi et al., 2011; Huarng et al., 2007; Lee et al., 2012; Wang, 2004), sustainable development of tourism (Stojanovic, 2011) and electronic tourism (e-tourism) (Hamedi and Jafari, 2011) are just a few successful examples. A fuzzy approach for the hotel revenue optimization is instead still missing in the literature.

In this work, we incorporate fuzziness of the objective and constraint functions into the stochastic model proposed by Liu et al. (2008) to show the effect of uncertainty on the optimal demand. Indeed, in the case of a stochastic process for customers calling for reservations before a particular booking date, a problem for hotels is to devise a policy for booking the set of customers that maximizes the total expected profit (Badinelli, 2000).

We assume that the hotel has only one type of room to allocate but the unit rate per room may be different during every booking period and every reservation may cover several days.

We have found a set of optimal solutions with the value of the objective function being equal. Each optimal solution embodies a different degree of uncertainty and subjectivity in the measurement of the expected demand. Therefore, our model provides several alternative optimal room allocations to which the decision maker (DM) can refer to face the real customers' demand.

This article is organized as follows: In section 'Properties of the accommodation industry', properties of the accommodation industry are discussed. The fuzzy approach is introduced in section 'The fuzzy approach'. The mathematical models with notations and parameters used in this article will be introduced in section 'Stochastic programming'. The illustrative examples with model settings and computational results are given in section 'Experimental analysis'. The final section concludes and gives some future research recommendations.

Properties of the accommodation industry

Hotel management has the following characteristics:

- Short-term costs are largely fixed, and variable costs per user are small. Thus, in most situations, it is sufficient to seek for booking policies that maximize revenues.
- Booking decisions are repeated millions of times per year. Thus a risk-neutral approach is justified, although there is a lower risk in accepting a current booking request than in waiting for later possible bookings.
- Rooms in the hotel represent perishable assets (they cannot be stored for future sale) and fixed capacities at the same time: A room left empty in a hotel for a night represents a revenue loss for the company management.
- The best practice is to fulfil the requests of highly profitable guests as much as possible. Yet, it is generally necessary to allow for less profitable guests in order to prevent rooms from remaining vacant.
- Advance booking is allowed (and thus cancellations, no-shows and overbooking problems exist).
- Capacity is usually fixed and the cost of instant expansion is very high.
- The management faces a network capacity control problem when customers require a sequence of nights at the hotel.

The fuzzy approach

Fuzzy linear programming (FLP) is one of the most popular decision-making approaches based on the fuzzy set theory. Since Zadeh's (1965) pioneer work, FLP has been widely applied in many disciplines, such as operational research, management science, control theory and artificial intelligence.

Some early works include Bellman and Zadeh (1970), Negoita and Sularia (1976), Tanaka et al. (1973) and Zimmermann (1974).

Among the approaches for solving FLP, the method proposed by Zimmermann is the most often used when the objective function and/or the right-hand sides of the constraints are fuzzy. Indeed, as several literature (Lai and Hwang, 1992; Zimmermann, 2001) has pointed out, the Zimmermann's approach has the advantage of few assumptions and easy computation compared with alternative fuzzy methods (Safi et al., 2007).

The Zimmermann's approach

Starting from a standard linear programming problem:

$$\begin{array}{ll} & \max\limits_{x} \; z = c^{\mathrm{T}}x \\ \text{s.t.} & \\ & Ax \le b \\ & x \ge 0 \end{array}, \tag{1}$$

let us assume that the objective function and the right-hand side coefficient in the constraints are vague because of imprecise human evaluations, inconsistent or incomplete evidence, natural language to be modelled and so on.

For such a case, Zimmermann (1976) proposed a model that included both fuzzy objective and constraints:

$$\begin{array}{ll} & \widetilde{\max\limits_{x}} \; z = c^{\mathrm{T}}x \\ \text{s.t.} & \\ & Ax \lesssim b \\ & x \ge 0 \end{array}, \tag{2}$$

where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$, $x \in \mathbb{R}^n$; $\widetilde{\max\limits_{x}}$ and $\lesssim$ denote the relaxed or fuzzy version of the ordinary max and $\le$ operators, respectively.

If the DM can establish an aspiration level, b_0, he wants to achieve as much as possible and if the constraints of the model can be slightly violated, without causing unfeasibility of the solution, then model (2) can be written as follows (Zimmermann, 2001):

$$\begin{array}{ll} & \text{find } x \\ \text{s.t.} & \\ & c^{\mathrm{T}}x \gtrsim b_0 \\ & Ax \lesssim b \\ & x \ge 0 \end{array}. \tag{3}$$

For treating fuzzy inequalities, Zimmermann proposed linear membership function:

$$\tilde{\mathrm{A}}^0(x) = \begin{cases} 1 & \text{if} c^{\mathrm{T}}x > b_0 \\ 1 - \dfrac{b_0 - c^{\mathrm{T}}x}{p_0} & \text{if} b_0 - p_0 \le c^{\mathrm{T}}x \le b_0 \\ 0 & \text{if} c^{\mathrm{T}}x \le b_0 - p_0 \end{cases}, \tag{4}$$

$$\tilde{\mathrm{A}}^i(x) = \begin{cases} 1 & \text{if} (Ax)_i < b_i \\ 1 - \dfrac{(Ax)_i - b_i}{p_i} & \text{if} b_i \le (Ax)_i \le b_i + p_i \\ 0 & \text{if} (Ax)_i > b_i + p_i \end{cases}, \tag{5}$$

where for $i = 1, 2, \ldots, m$, $(Ax)_i$ is the ith row of Ax, b_i is the ith element of b and p_i is a constant expressing the admissible violation of the ith inequality. p_0 is the admissible violation of the objective function. b_0 and p_i are subjectively chosen by the DM or calculated following Zimmermann (1978) and Werners (1987).

Using the max–min operator of Bellman and Zadeh (1970), the optimal solution of model (3) can be found solving the linear programming problem:

$$\begin{array}{ll} & \max \mu \\ \text{s.t.} & \\ & \mu \leq \tilde{\mathrm{A}}^i(x) \\ & x \geq 0 \end{array}, \tag{6}$$

where $i = 0, 1, \ldots, m$ and $\mu \in [0, 1]$.

Stochastic programming

Stochastic programming is an approach for modelling optimization problems that involve uncertainty. It dates back to the 1950s (Dantzig, 1955) as an extension of linear programming to problems with uncertain parameters in the constraints or in the objective function.[1] In fact, real-world problems almost regularly include parameters which are unknown at the time a decision should be made.

Since it is impossible to resolve the uncertainty fully, the best way to make decisions under an uncertain environment is to study uncertainty first and then include it into the model. Uncertainty is usually characterized by a probability distribution on the parameters and it can range in detail from a few scenarios (possible outcomes of the data) to specific and precise joint probability distributions. Stochastic programming models take advantage of the fact that probability distributions governing the data are known or can be estimated from historical data. Thus, it is possible to replace the unknown variables by their best point estimator using, for instance, their expected value.

Often these models apply to settings in which decisions are made repeatedly in essentially the same circumstances, and the aim is to find a solution feasible for all (or almost all) possible parameter choices, which optimizes a given objective function. To say it differently, the purpose of such models is to come up with a decision that will perform well on average.

In these circumstances, the optimization problem (1) could be written as:

$$\begin{array}{ll} & \max\limits_x z = c(\omega)^{\mathrm{T}} x \\ \text{s.t.} & \\ & A(\omega)x \leq b(\omega) \\ & x \geq 0 \end{array}, \tag{7}$$

where $c(\omega)$, $A(\omega)$ and $b(\omega)$ are model parameters, $\omega \in \Omega$ is a random parameter associated with random data and Ω is the space of events.

When a simple stochastic representation of uncertainty is not feasible or viable, a more recent decision framework is robust optimization. Instead of seeking to immunize the solution in some probabilistic sense to stochastic uncertainty, with robust optimization, the DM constructs a solution that is feasible for any realization of the uncertainty in a given set.

Robust optimization is the approach followed to obtain the stochastic programming model described in the next session.

The stochastic model

In this study, we refer to the stochastic model presented in Liu et al (2008) for hotel RM with multiple-day stays:

$$
\begin{aligned}
&\max z_{tot} = \sum_{s=1}^{S} p_s \sum_{t=1}^{T}\sum_{i=1}^{I} r_{tis}x_{ti} - \lambda \sum_{s=1}^{S} p_s y_s - \sum_{s=1}^{S} p_s \sum_{t=1}^{T}\sum_{i=1}^{I} w_{ti}\, q_{tis} \\
&\text{s.t.} \\
&\sum_{t=1}^{T}\sum_{i=1}^{I} x_{ti} \le C \\
&\sum_{t=1}^{T}\sum_{i=1}^{I} r_{tis}\, x_{ti} - \sum_{s=1}^{S} p_s \sum_{t=1}^{T}\sum_{i=1}^{I} r_{tis}\, x_{ti} + y_s \ge 0 \\
&x_{ti} - q_{tis} \ge d_{tis} \\
&x_{ti} \le \max_{s=1,\ldots,S}\{d_{tis}\} \\
&x_{ti}, q_{tis} \ge 0 \text{ and integers} \\
&y_s \ge 0 \\
&t = 1, \ldots, T; i = 1, \ldots, I; s = 1, \ldots, S
\end{aligned}
\qquad (8)
$$

where t is the time index, T is the number of periods in advance of the booking dates ($t = T$ is the start of the booking horizon and $t = 1$ is the walk-in day and represents the end of the reservation period), i is the index for the number of days the customer will stay in the hotel, I is the maximum number of such days, s is the index associated with the total number S of scenarios, C is the room capacity of the hotel, x_{ti} are the decision variables representing the number of rooms to be booked out during period t for i days, y_s are variables representing the expected losses, q_{tis} are slack variables representing unmet demands, w_{ti} are the penalty factors for demand constraint violations, d_{tis} is the demand during period t for i days in scenario s, r_{tis} is the unit rate for each room night per booking made in period t for i days, under scenario s, p_s are the probabilities for each scenario, with $\sum_{s=1}^{S} p_s = 1$, and λ is the risk-aversion factor for the DM.

In model (8), the first term of the objective function is the expected revenue of the hotel, the second term measures the revenue risk and the third component is a penalty term for the expected constraints violation. The first constraint states that the actual number of reservations cannot exceed the total capacity, the second and third constraints come from a linearization method used to obtain a linear programming model, and the fourth constraint puts an upper bound on the number of rooms booked that cannot exceed the maximum demand over all the scenarios.[2]

The fuzzy-stochastic model

In this section, we develop a fuzzy-stochastic approach to systematically quantify both probabilistic and fuzzy uncertainties associated with customers' demand and RM. In particular, uncertainty is incorporated into our model assuming that the DM can refer to a set of scenarios associated with realizations of customers' demand (as in Liu et al., 2008), while the possible violation of the DM's revenue aspiration level, z^*_{tot}, is modelled with fuzzy uncertainties.

The integrated fuzzy-stochastic RM model we propose is described below:

$$
\begin{aligned}
&\max_{\mu\in[0,1]} \mu \\
\text{s.t.}\quad &\\
&\mu \geq 1 + \frac{z_{tot} - z^*_{tot}}{z^*_{tot} z_{lo}} \\
&\mu \leq 1 - \frac{z_{tot} - z^*_{tot}}{z^*_{tot} z_{up}} \\
&\mathrm{z_{tot}} = \sum_{s=1}^{S} p_s \sum_{t=1}^{T}\sum_{i=1}^{I} r_{tis}x_{ti} - \lambda \sum_{s=1}^{S} p_s y_s - \sum_{s=1}^{S} p_s \sum_{t=1}^{T}\sum_{i=1}^{I} w_{ti}q_{tis} \\
&\sum_{t=1}^{T}\sum_{i=1}^{I} x_{ti} = C \\
&\sum_{t=1}^{T}\sum_{i=1}^{I} r_{tis}x_{ti} - \sum_{s=1}^{S} p_s \sum_{t=1}^{T}\sum_{i=1}^{I} r_{tis}x_{ti} + y_s \geq 0 \\
&\mu \leq 1 - \frac{x_{ti} - \ q_{tis} - d_{tis}}{d_{tis}l_{ti}} \\
&\mu \leq 1 - \frac{x_{ti} - \max_{s=1,\dots,S}\{d_{tis}\}}{\max_{s=1,\dots,S}\{d_{tis}\} l_{ti}} \\
&0 \leq \mu \leq 1 \\
&x_{ti},\ q_{tis} \geq 0 \text{ and integers} \\
&y_s \geq 0 \\
&t = 1, \dots, T; i = 1, \dots, I; s = 1, \dots, S
\end{aligned}
\tag{9}
$$

where μ is the fuzzy optimization variable; l_{ti} are parameters used for the admissible fuzzy violation of the inequality constraints; z^*_{tot} is the optimal value of the objective function used as the aspiration level for the DM, and z_{lo} and z_{up} are parameters used to calculate the fuzzy violation allowed on the left and right of the objective function. In particular, the inequality constraints of model (9) involving the fuzzy optimization variable μ are in a form that recalls the Zimmerman's linear membership functions shown in models (4) and (5) for the objective function and the inequality constraints.

The model here formulated offers a valuable tool for systematically quantifying various uncertainties in RM, and it also provides a realistic support for reservation-related decisions.

Experimental analysis

Models settings

We consider the same settings as in Liu et al (2008) for the three scenarios example with different unit room rate per scenario. Therefore, in the proposed model, $T = 5$, $I = 6$, and $S = 3$. Demands under each scenario are listed in Table 1. Table 2 shows the value of the unit rates r_{tis}, while the

Table 1. Demands per scenario (d_{tis}).

		Demand					
Scenarios	T\I	1	2	3	4	5	6
1	1	20	13	12	7	4	2
	2	10	15	12	9	5	3
	3	3	14	15	10	5	2
	4	2	11	13	12	6	2
	5	2	11	12	8	6	2
2	1	15	12	10	6	4	2
	2	6	13	12	9	5	3
	3	3	10	12	10	5	2
	4	2	10	10	12	6	2
	5	2	9	10	5	3	2
3	1	10	10	8	6	4	2
	2	6	10	10	9	5	3
	3	3	11	8	7	5	2
	4	2	9	10	9	6	2
	5	1	8	9	5	3	2

Table 2. Unit room rates per scenario (r_{tis}).

T\S	1	2	3
1	0.82	0.84	0.86
2	0.80	0.82	0.84
3	0.78	0.80	0.82
4	0.76	0.78	0.80
5	0.74	0.76	0.78

Table 3. Fuzzy violation of the demand inequalities (l_{ti}).

T\I	1	2	3	4	5	6
1	0.60	0.50	0.30	0.20	0.10	0.05
2	0.50	0.40	0.20	0.10	0.05	0.02
3	0.45	0.35	0.18	0.13	0.04	0.01
4	0.40	0.30	0.15	0.10	0.03	0.01
5	0.30	0.20	0.10	0.08	0.02	0.01

penalty factors for demand constraint violation, w_{ti}, and the risk-aversion factor, λ, have been set to the value of 1. Table 3 shows the values of l_{ti}.

Computational results

The optimal solution for both the stochastic and the fuzzy programming models is shown in Tables 4 and 5, respectively. In particular, Table 5 shows the optimal demands when the fuzzy

Table 4. Optimal solution for the stochastic model.

T\I	1	2	3	4	5	6
1	0	12	12	7	4	2
2	0	12	12	9	5	3
3	0	10	15	10	5	2
4	0	10	13	12	6	2
5	0	9	12	8	6	2

Table 5. Optimal solution for the fuzzy model ($z_{up} = z_{lo} = 0.001$).

T\I	1	2	3	4	5	6
1	0	12	12	7	4	2
2	0	12	12	9	5	3
3	3	10	15	10	5	2
4	0	6	13	12	6	2
5	2	8	12	8	6	2

violation of the objective inequality is so negligible ($z_{lo} = z_{up} = 0.001$) that the two models can be considered equivalent. In spite of this, the optimization converges to a different solution with the same value of the objective function, meaning that the fuzzy relaxation of constraints is sufficient to move the optimal demands into the solutions space. It is worth noting that the fuzzy optimal demands also include the walk-in day (Table 5, first column) that the stochastic results exclude.

We have run a total of 121 fuzzy optimizations changing the value of both the parameters z_{lo} and z_{up} in the interval (0.0, 0.5), with increment step of 0.05 (the actual sequence of values was 0.001, 0.05, 0.10, 0.15, … , 0.50). For each run, we have found a feasible optimal solution with, in most of the cases, a value of the objective function equal to that of the stochastic counterpart. An optimal demand with a higher objective value has never been found, instead. Figure 1 shows the value of the objective function in the fuzzy model for each run.

The low dependency of the value of the objective function from the fuzzy violations is confirmed by the multiple linear regression shown in Table 6, with a RMSE $= 0.084$, F-test $= 5.023$ and p value $= 0.008$.

Figure 2 displays the value of μ for each combination of z_{lo} and z_{up}. Almost all the combinations give a value of μ equal to 1, meaning that the fuzzy violations work properly. Moreover, as shown in Figure 3, the frequencies of the (μ, z_{tot}) pair confirm that the fuzzy model provides the higher optimal value of the objective function with $\mu = 1$ in most of the cases.

In order to evaluate the efficiency of the optimal fuzzy solution with respect to the stochastic one, we have used the semi-absolute distance:

$$\bar{d} = \sum_{s=1}^{S} p_s \sum_{t=1}^{T} \sum_{i=1}^{I} \max(0, d_{tis} - x_{ti}^*),$$

where x_{ti}^* is the optimal solution. The lower the value of $\bar{d}$, the higher the average fulfilment of the demands. Assuming that the stochastic model has a value of $\bar{d} = 0.17$, Figure 4 shows that, in most of the cases, the fuzzy violations provide better values of $\bar{d}$. Some descriptive statistics are shown in Table 7.

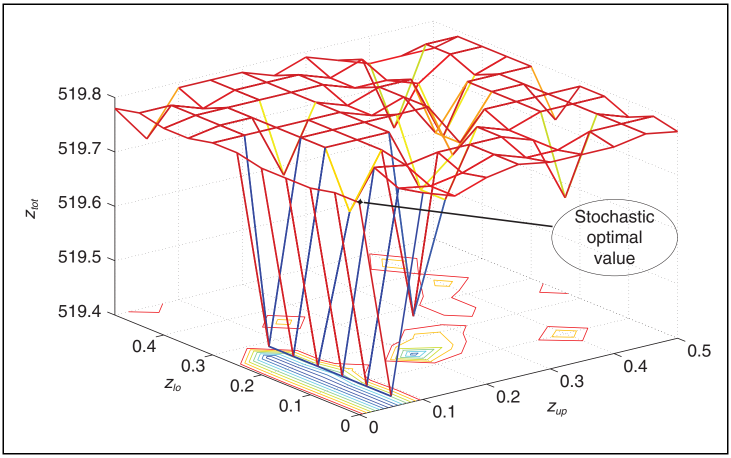

Figure 1. Mesh and contour plot of z_{up}, z_{lo} versus z_{tot}.

Table 6. Multiple linear regression.

z_{tot}	Coefficient	Standard error	t	Pr > t	[95% CI]
z_{up}	0.125	0.049	2.570	0.011	[0.028, 0.221]
z_{lo}	0.090	0.049	1.860	0.066	[−0.006, 0.186]
Constant	519.699	0.019		0.000	[519.662, 519.737]

Notes: CI = confidence interval.

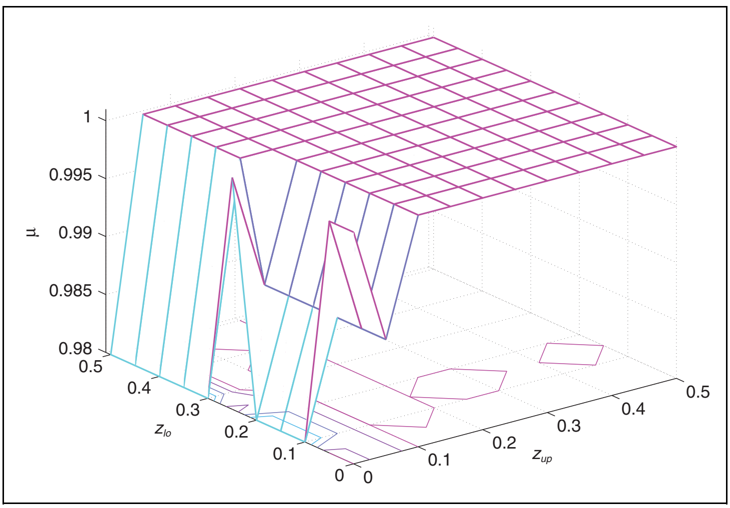

Figure 2. Mesh and contour plot of z_{up}, z_{lo} versus μ.

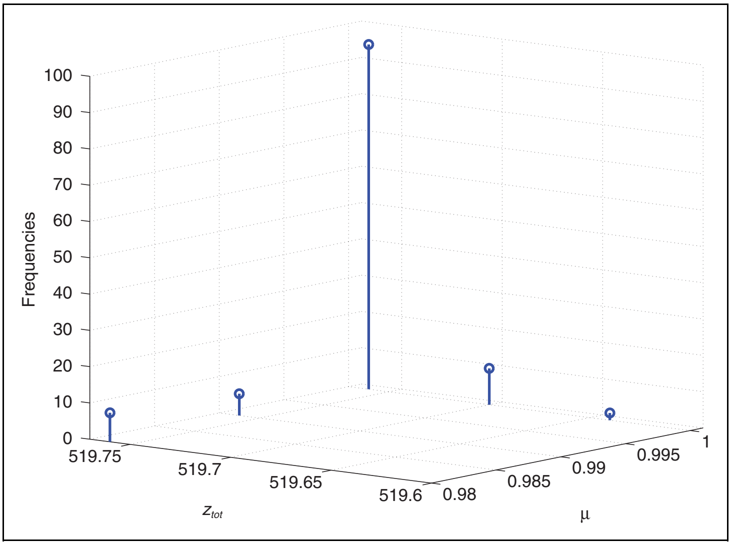

Figure 3. Frequencies of μ versus z_{tot}.

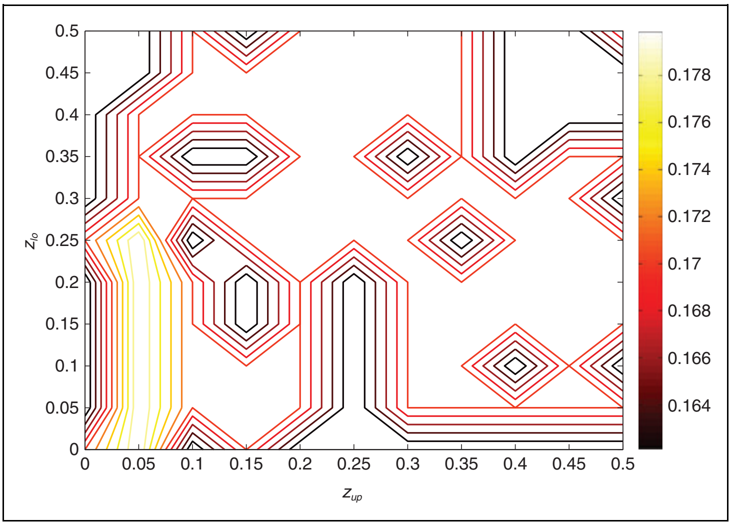

Figure 4. Contour plot of z_{up}, z_{lo} versus $\bar{d}$.

Table 7. Descriptive statistics.

	μ	z_{tot}	$\bar{d}$
Mean	0.998	519.753	0.167
SD	0.005	0.087	0.006
min	0.980	519.420	0.160
max	1.000	519.790	0.180

Conclusions

In this article, we discuss the effects of incorporating fuzziness of objective and constraints functions into a pure stochastic model. Starting from the model of Liu et al (2008), our approach provides several alternative optimal room allocations with the value of the objective function equal to that of the stochastic counterpart. An optimal demand with a higher objective value has never been found, instead.

In most of our optimization runs, we got a value of μ equal to 1, meaning that fuzzy violations work properly, and a higher value for the efficiency of the optimal fuzzy solution, as measured by the $\bar{d}$ index. Moreover, when the relaxation of constraints is so negligible that the stochastic and the fuzzy models can be considered equivalent, we are able to provide an alternative optimal solution including the walk-in day that the stochastic results exclude.

The fuzzy-stochastic approach developed in this study offers a valuable tool for systematically quantifying various uncertainties in RM, and it also provides more realistic support for reservation-related decisions.

The contribution of this study is two-fold. First, it offers a set of Pareto-optimal solutions to which the DM can refer to face the real customers' demand. Second, the fuzzy revenue optimization model we propose demonstrates to be a useful environment to capture both uncertainty and imprecision in solving the optimal allocation of rooms for profit maximization.

Overbooking, cancellation, no-show and early checkout are well-known practices that introduce new challenges for hotel managers. They are all worthy topics in hotel RM and are left for future research.

Declaration of conflicting interests

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Notes

1. The interested reader can refer to several textbooks (Birge and Louveaux, 1997; Huber, 1981; Kall and Wallace, 1994; Prékopa, 1995) and references therein for a more comprehensive picture of stochastic programming.
2. The interested reader can refer to Liu et al. (2008) for model details.

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