MR3191427 Naralenkov, Kirill M., A Lusin type measurability property for vector-valued functions. *J. Math. Anal. Appl.* 417 (2014), no. 1, 293307. 28A20

In the paper under review the author introduces the notion of Riemann measurability for vector-valued functions, generalizing the classical Lusin condition, which is equivalent to the Lebesgue measurability for real valued functions.

Let X be a Banach space, let $f : [a, b] \to X$ and let E be a measurable subset of $[a, b]$. The function f is said to be Riemann measurable on E if for each $\varepsilon > 0$ there exist a closed set $F \subset E$ with $\lambda(E \setminus F) < 0$ (where λ is the Lebesgue measure) and a positive number δ such that

$$\|\sum_{k=1}^{K}\{f(t_k) - f(t'_k)\} \cdot \lambda(I_k)\| < \varepsilon$$

whenever $\{I_k\}_{k=1}^{K}$ is a finite collection of pairwise non-overlapping intervals with $\max_{1\le k\le K} \lambda(I_k) < \delta$ and $t_k,\ t'_k \in I_k \bigcap F$.

The Riemann measurability is more relevant to Riemann type integration theory, such as those of McShane and Henstock, rather than the classical notion of Bochner or scalar measurability. In particular the author studies the relationship between the Riemann measurability and the $\mathfrak{M}$ and the $\mathfrak{H}$ integrals that are obtained if we assume that the gauge in the definitions of McShane and Henstock integral can be chosen Lebesgue measurable.

The main results are the following

- *If $f : [a, b] \to X$ is $\mathfrak{H}$-integrable on a measurable subset E of $[a, b]$, then f is Riemann measurable on E.*
- *If $f : [a, b] \to X$ is both bounded and Riemann measurable on a measurable subset E of $[a, b]$, then f is $\mathfrak{M}$-integrable on E.*
- *If $f : [a, b] \to X$ is both Riemann measurable and McShane (Henstock) integrable on a measurable subset E of $[a, b]$, then f is $\mathfrak{M}$-integrable ($\mathfrak{H}$-integrable) on E.*
- *Suppose X separable. If $f : [a, b] \to X$ is McShane (Henstock) integrable, then f is $\mathfrak{M}$-integrable ($\mathfrak{H}$-integrable.)*

The author concludes the paper with the following open problem: for which families of non-separable Banach spaces does the McShane (or even the Pettis) integrability imply Riemann measurability?

Reviewed by (*L. Di Piazza*)