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Shear Strength Degradation due to Flexural Ductility Demand in R.C. Elements.

***Abstract*: A proposal is formulated that allows to evaluate the residual shear strength of reinforced concrete columns and beams for an assigned flexural ductility demand by limiting the range of the deviation angle between the inclinations of the yield θ and the crack β lines. In order to take into account the degradation due to cyclic loads, the reduction of the range of the deviation angle is related to the value of cinematic ductility**

***Keywords*— shear strength, ductility, degradation, cyclic loads.**

I. Introduction

Modern approaches to structural analysis in seismic areas aim to evaluate the system's capacity related to large inelastic displacement, i.e. corresponding to a large ductility demand. Unfortunately, the classical formulations for assessment of shear strength of reinforced concrete elements are independent of the deformation undergone, leading to overestimation of shear capacity when large ductility demand occurs. This issue assumes a key role when the seismic capacity of existing structures is evaluated by pushover analysis.

Several studies [1,2] have suggested to solve these drawbacks on the basis of smeared cracking non-linear models [3-5]. However, despite their success in modeling several structural type behaviors, they do not appear suitable to handily provide relationships required for designers or to be implemented in software for seismic analysis of whole structure.

In the proposed paper, it is observed that the models included in the present Eurocode for static action are derived by using the stress fields approach, and they render possible an ample variation of the angle θ of inclination of the concrete stress field, which is, in general, different from the inclination β of the first cracking surface. When large deformation and cyclic actions of wide intensity occur, the progressive roughness reduction limits the range of variation of θ, preventing the development of directions of yielding lines with a slope different from the first cracking one, β.

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In this context a new proposal is formulated that allows to evaluate the residual shear strength of reinforced concrete (RC) beams, columns and bridge piers for an assigned flexural ductility demand by limiting the range of the deviation angle between the inclinations of the yield θ and the crack line β. In order to take into account the degradation due to cyclic load, the reduction of the range of the deviation angle is linked to the value of cinematic ductility

II. Models for shear strength

A. *Classical model for cyclic actions*

In 1996, Priestley and Benzoni [6] developed a model to take into account the reduction in shear strength due to the ductility demand, which provides close agreement with tests on simple RC members. In this model the shear strength of a member is obtained as the sum of three different contributions due to transverse reinforcement, compressed concrete and axial load, respectively. Thus, in a rectangular cross-section the shear strength V_{Rd} can be evaluated as follows

$$V_{sd} \le \frac{A_{sw}}{s_w}\cdot 0.9d\cdot\frac{f_{yk}}{\gamma_s}\cdot ctg\beta + k_p(\mu_\theta)\cdot 0.9d\cdot b_w\cdot\frac{\sqrt{f_{ck}}}{\gamma_c} + N_{sd}\cdot\tan\alpha \tag{1}$$

where b_w, d and x are the cross-section dimensions and the neutral axis depth; A_{swt} and s_w the steel stirrup cross-section area and spacing, respectively; f_{ck} and f_{yk} are the characteristic strength of the steel and compressed concrete; γ_c and γ_s the safety coefficients for concrete and steel, respectively; $k(\mu)$ the strength degradation coefficient (Fig. 1), β the slope of the first cracking assumed equal to 30° in the columns and 45° in the beams, and N_{sd} the design axial force. The role of the terms N_{sd} tan α and more details on the model can be found in the papers [7,8].

B. *Stress field model for static N-M-V force*

When RC elements are simultaneously loaded by axial force N, bending moment M and shear force V, the stress distribution in the cross-section is complex; thus an analytical model based on plastic theory [9-15] and able to predict the stress distribution cannot easily be derived. On the basis of the stress-field approach proposed by Bach et al. [16], Recupero *et al.* [17,18] proposed a model in which the tensile longitudinal reinforcement and the compressed chord are modeled by element with zero length, having geometrical shape depending

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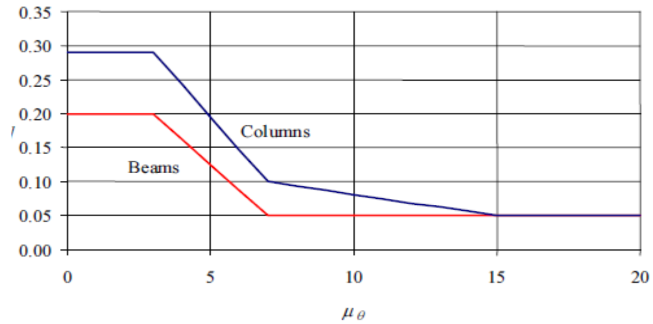
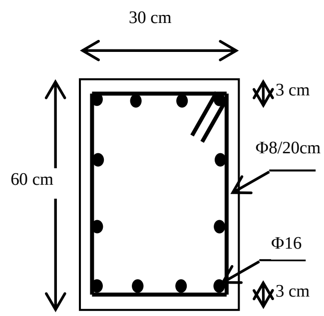
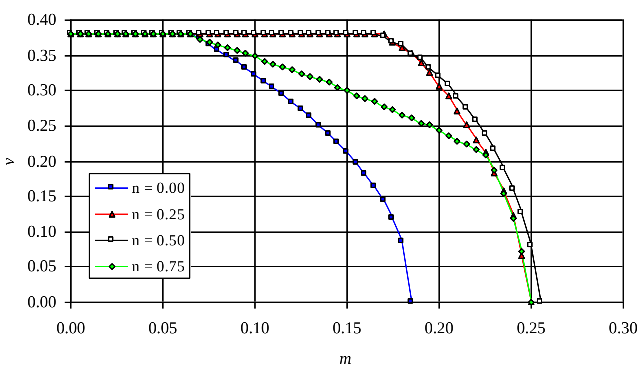


Figure 1. Shear strength degradation coeffcient for beams and columns [6]

on the cross section shape. The model has been used for evaluation of M-N-V internal force interaction strength domain [8,17,18] as shown in non dimensional form in Fig. 2b for the rectangular section in Fig. 2a, made of concrete with characteristic concrete cylindrical strength f_{ck}=25 MPa and steel yielding strength f_{yk}=450 MPa. The domain can be evaluated once the slopes β and θ of the first cracking and the compressed concrete stress field respectively are calculated. In the following section it will be shown that a suitable reduction of the variation range of the compressed concrete stress field θ is able to reproduce the results of the classical truss model.



a)



b)

Figure 2. a) cross section; b) dimensionless shear ν vs. bending moment *m* interation strength domain for different values of specific axial force *n*

III. Strength domain for Flexural Ductility demand

A procedure that limiting the range of variation of the deviation δ of the concrete stress field from the first cracking slope β (δ=θ–β) is shown, related to the geometrical and mechanical characteristic of the element and the amplitude of plastic deformation undergone under the effect of the seismic actions. The procedure enables the evaluation of the internal force interaction reduced strength domain under seismic actions. In the stress field model, the difference of the slope of the yield surface in comparison to that of the cracking surface is partly generated by the effects of aggregate interlock, which avoid slips along cracks, and is a function of the roughness of the crack sides in contact. When the maximum deformations and/or the accumulated damage due to small amplitude of cyclic actions increase, the roughness of the sliding surfaces is reduced. Thus, the range of the deviation angle δ is limited. The proposed model assumes a limit value of the angle δ that should depends on a measure of the damage generated by the combined effects of amplitude of maximum flexural ductility demand and cumulated effect of cyclic action, i.e. on a damage index that should include both the two aforementioned contributes. As an example, the Park and Ang index [19] appears to be a suitable damage index for governing the limitation of the deviation angle δ.

However here, due to the lack of adequate amount of experimental data for investigating the effect of cyclic action, the limit value of the deviation angle δ is linked to the maximum value of the flexural ductility demand by reproducing the results of the Priestly and Benzoni model [6].

Firstly, aiming at stressing how such an assumption modifies the strength domains of RC members, the effects of the progressive reductions of the deviation angle δ on *N-M-V* domains are shown for the section shown in Fig.1a, i.e. by setting the values of θ=β–δ. The normalized strength domains are shown in Fig. 2b for four limit values: *ctg* θ = 2.5 (θ≈22°), *ctg* θ = 2 (θ ≈ 26), *ctg* θ = 1.5 (θ ≈ 34) and *ctg* θ = 1 (θ = 45) and for four normalized axial force values $n = N_{sd}/(f_{ck}\, A_c) = 0$, 0.25, 0.50 and 0.75. Figs. 3 show that the progressive reduction in the yield surface inclination (angle θ) causes a major reduction in the maximum shear strength; by contrast, it does not have any influence on the ultimate bending moment. The domains show also that for large values of the axial force, the reduction of the shear strength is large for small values of the bending moment and any value of the stress field inclination also. In order to characterize the relation between angular deviation δ and flexural ductility demand on the basis of the indications provided by Priestley and Benzoni [6], it is observed that the limit of the yield surface inclination influences the horizontal line of the strength domain corresponding to small values of the bending moment, for which the failure of the structural element is reached by attainment of shear strength, that in non dimensional and dimensional form read respectively:

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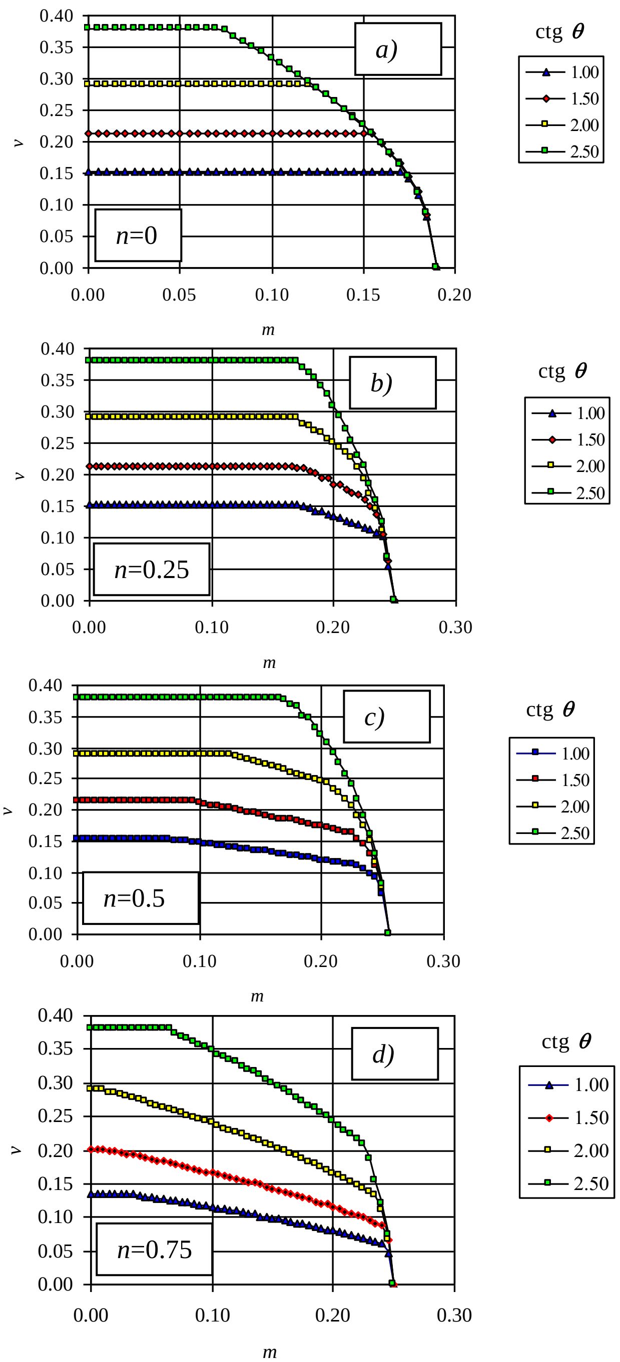

Figure 3. Interation strength domain as function of the compressed concrete stress field slopeand different axial force values

$$\frac{\tau}{f_{ck2}} = \omega_w \cdot [ctg\theta]_{max} \qquad (2)$$

$$V_{sd} = \frac{A_w}{s} \cdot z_3 \cdot f_{yd} \cdot ctg\theta \qquad (3)$$

where f_{ck2} is the concrete compressive strength reduced for the presence of transversal stress Eq. 3 represents the shear strength related to the capacityof the tranversal reinforcement.

The shear stress τ and the mechanical ratio of transverse reinforcement ω_w read:

$$\tau = \frac{V_{sd}}{b_w \cdot z_3} \qquad (4)$$

$$\omega_w = \frac{A_w \cdot f_{yk}}{s \cdot b_w \cdot f_{ck2}} \qquad (5)$$

where z_3 is the depth of the cross section part that have to carries the shear stress [17,18.]

Taking into account that, when the shear capacity are evaluated for a stubby column by the stress field approach, the arch action [N_{sd} tan α] must be added also in Eq.3, Eq. (6) is able to reproduce the shear strength degradation predicted by Eq. (1) if the degradition is assmed to be due to the limitation of the deviation angle θ, i.e. *[cotg θ]$_{max}$*. limitation. In order to achieve this results, Eq.(1) and Eq.(3) are matched, as follows:

$$\frac{A_w}{s} \cdot z_3 \cdot f_{yd} \cdot [ctg\,\theta(\mu)]_{max} = \frac{A_w}{s} \cdot z \cdot f_{yd} \cdot ctg\beta + k(\mu) \cdot z \cdot b_w \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \qquad (6)$$

where the dependence of the value of the slope of the compressed concrete stress filed on the value of the curvature ductility demand μ is emphasized, and the circumstance that the maximun shear strength is obtained when the maximun value of *[cotgθ]* is chosen, has been retained. The non-dimensional form of Eq. (6) is

$$\omega_w \cdot \frac{z_3}{z} \cdot \left[ctg\theta(\mu)\right]_{max} = \omega_w \cdot ctg\beta + k(\mu) \cdot \frac{\sqrt{f_{ck}}}{f_{ck2}} \qquad (7)$$

By setting

$$\varsigma = \frac{z_3}{z} \quad ; \qquad \varphi = \frac{\sqrt{f_{ck}}}{f_{ck2}} \qquad (8,9)$$

Eq. (7) is rewritten in the following form

$$\left[ctg\theta(\mu)\right]_{max} = \frac{ctg\beta}{\varsigma} + k(\mu) \cdot \frac{\varphi}{\varsigma \cdot \omega_w} \qquad (10)$$

by which the value of the corresponding angle can be easily derived.

$$\bar{\theta}(\mu) = arctan\left[\left(\frac{ctg\beta}{\varsigma} + k(\mu) \cdot \frac{\varphi}{\varsigma \cdot \omega_w}\right)^{-1}\right] \qquad (11)$$

Eq. (11) can be used either for columns or for beams, i.e. for element with or without axial force. When columns are considered, the degradation coefficient of shear strength provided by the concrete k(μ) depicted in Fig. 1 are considered. It was derived by Priestley and Benzoni [6] by

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regression of experimental risults, assuming $\beta = 30°$ (cot β = 1.732) and z=0.9 d = 0.9 (H-c). In order to solve Eq. (9) the hypothesis that shear collapse occurs when axial force and bending moment action on the section are able to yield the longitudinal reinforcement at the compression and tension chords are assumed. Thus, the axial force is carried on by the compressed concrete only, and the dimensionless neutral axis depth is

$$\xi = \frac{x}{H} = \frac{N_{sd}}{H \cdot b_w \cdot f_{ck}} = n \qquad (12)$$

The depth z_3 of the cross section part that have to carry the shear stress [17,18] is evaluated as:

$$z_3 = H - 2 \cdot c - x = = H \cdot \left(1 - \xi - 2 \cdot \frac{c}{H}\right) \leq H \cdot \left(1 - 4 \cdot \frac{c}{H}\right) \qquad (13)$$

where c is the cover. The nondimensional form of Eq.(13) reads

$$\varsigma = \frac{z_3}{z} = \frac{\left(1 - \xi - 2 \cdot c/H\right)}{0.9 \cdot \left(1 - c/H\right)} \qquad (14)$$

If the value of non dimensional cover is assumed c/H=0.05, the non dimensional form of the section depth charged to carry the shear action is

$$\varsigma \cong \frac{(0.9 - \xi)}{0.855} \cong 1.05 - 1.17 \cdot n \qquad (15)$$

Substitution of Eq.(15) in Eq.(11) provvides:

$$\bar{\theta}(\mu) = arctan\left[\left(\frac{ctg\beta}{1.05 - 1.17 \cdot n} + k(\mu) \cdot \frac{\varphi}{(1.05 - 1.17 \cdot n) \cdot \omega_w}\right)^{-1}\right] \qquad (16)$$

The range of validity of Eq.(16) is restricted the value of non dimensional axial force $n \leq 0.9$, that comprises almost all the cases of design of structure in seismica area, where $n \leq 0.65$ is suggested by the code.

When beams are considered, in Eq. (11) β = 45° ($ctg\,\beta = 1$), $z \cong 0.9 \cdot d = 0.9 \cdot (H - c)$ and the curve of $k(\mu)$ that pertain to the beams in Figure 1 are assumed. Thus, when significative amount of longitudinal reinforcement are placed at the compressed chord, the following expression of non dimensional cross section depth z_3/z carrying the shear stress is obtained:

$$\varsigma = \frac{z_3}{z} \cong \frac{H}{z} \cdot \left(1 - 4 \cdot \frac{c}{H}\right) = \frac{\left(1 - 4 \cdot c/H\right)}{0.9 \cdot \left(1 - c/H\right)} \qquad (17)$$

By assuming c/H=0.06, substitution of Eq.(17) in Eq.(11) provvides:

$$\bar{\theta}(\mu) \geq arctan\left[\left(\frac{1}{0.9} + k(\mu) \cdot \frac{\varphi}{0.9 \cdot \omega_w}\right)^{-1}\right] \qquad (18)$$

IV. Numerical analysis

Some numerical analyses were carried out, in order to show how different geometrical and mechanical parameters and ductility demands can reduce the range of variability of the inclination of the stress fields of compressed concrete. In Fig. 4 the range of the minimum slope $\bar{\theta}$ for concrete columns with a C25/30 concrete subjected to non dimensional axial force n=0.2 is depicted versus the mechanical ratio of transverse reinforcement ω_w for different values of the flexural ductility demand μ. For gravity loads, corresponding to ductility demand value μ=1, the minimum slope of concrete stress field is comprised in the range 16.92° < $\bar{\theta}$ <23° (3.29> $ctg\,\bar{\theta}$ >2.35) when 0.1 < ω_w < 0.5. Thus, the angular deviation δ with respect to the first cracking slope β=30° assumed in [6] is comprised in the range 7° < δ < 13.1. A wider deviation is required for small mechanical ratios of stirrups. In the presence of low stirrup density, the maximum values of shear are attained when the inclination of the stress fields is reduced in order to allow a larger number of stirrup legs to cross into the yield line, where the equilibrium is imposed. With the increases in the required ductility, the range of $\bar{\theta}$ is reduced; for $\mu \geq$ 15 and for the different values of themechanical percentage of stirrups the range 23.03° < $\bar{\theta}$ < 24.8° (2.32 > $[\text{ctg}\theta]_{max}$ > 2.16) was found to be admissible. Thus, large ductility demand strongly penalizes the shear strength of elements with a small amount of stirrups, while the reduction is small for members with a large

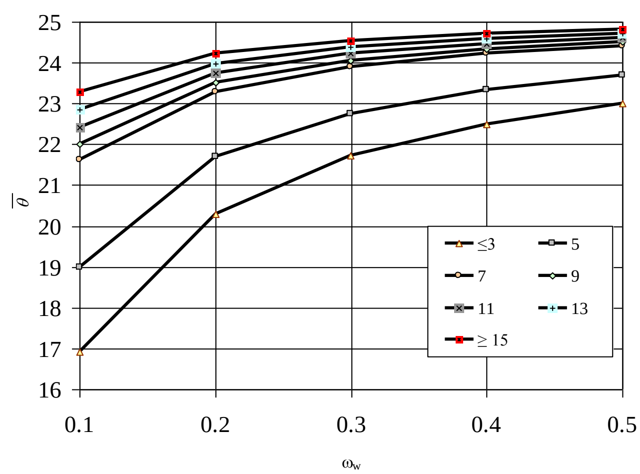

Figure 4. Columns (n=0.2): $\bar{\theta}$ Slope vs. trensversal reinforcement mechanical ratio for different values of flexural ductility demand

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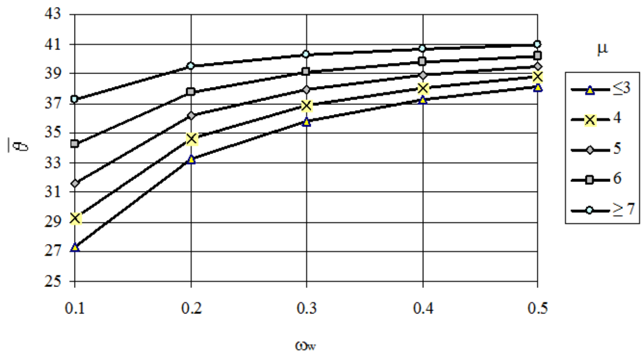

Figure 5. Beams (n=0.2): $\bar{\theta}$ Slope vs. trensversal reinforcement mechanical ratio for different values of flexural ductility demand

mechanical ratio of transversal reinforcement A similar numerical analysis is performed for beams made of the same C25/30 concrete. The curve in Fig. 5 show the variation of the minimum compressed field slope $\bar{\theta}$ in the same range of transversal reinforcement mechanical ratio used for column analysis, i.e. $0.1 < \omega_w < 0.5$. As the results of the shape of the curve of $k(\mu)$ in Fig. 1 for the beams, constant values of for $\mu > 7$ are obtained. The curves show that the slope of concrete stress field for static action ($\mu \leq 1$) is comprised in the range (27.3° < $\bar{\theta}$ <38.1)° (1.93> $ctg\,\bar{\theta}$ >1.27). Taking into account that for the beam first cracking slope β=45° was assumed, the deviation angles δ are comprised in the range 6.9°> δ >17.7°, they are highly dependent on the values of the transversal reinforcement ratio ω_w. When the ductility demand increases, a sharp increment of the $\bar{\theta}$ slope is required, that for μ values of 7 or larger, reduces the $\bar{\theta}$ range as 37.2° < $\bar{\theta}$ < 41°, and the stress field slope approaches the first cracking slope, i.e. θ~β=45° ($ctg\,\theta$~1). This limitation produce a noticeable reduction of the strength domain (Fig. 3a), proving that in the beams a large ductility demand produces a large reduction of the shear strength. By comparison of Fig. 4 and Fig. 5 the larger reduction of shear capacity in the beams (having a first cracking slope β=45°) with respect to that of the column (β=30°) can be recognized.

V. Conclusions

A modification of a stress field model for the prediction of shear strength degradation in reinforced concrete elements when large flexural ductility demand are required, is proposed. The model enables the evaluation of the internal force interaction domain, which amplitude is reduced by a limitation of the angle deviation of the concrete stress field form the angle of first cracking. The limitation of the deviation as a function of the flexural ductility demand is here derived from the model of Priestly and Benzoni [6]. Further investigation are required to link the deviation angle limitation to damage index in order to take into account the effects of cyclic action due to seismic excitation.

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