

Predicting tire/pavement noise impact reduction using numerical simulation and experimental data for open graded asphalt mixture

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ABSTRACT: The environmental impact of noise from roads and highways traffic is relevant in urban and rural areas. The use of open-graded asphalt pavements reduces significantly the noise, entrapping the acoustic waves inside the porous structure of the material. In this paper we propose a simulation approach in order to predict the acoustic properties of the asphalt mixture from geometrical and topological indicators. In detail we have generated, using a Random Sequential Adsorption model, synthetic samples starting from the same grading and bitumen contents of real samples manufactured in laboratory. We have measured the acoustic adsorption coefficient of the real samples and we have investigated the correlation between this coefficient and some numerical indicators extracted from the simulated samples. Dimension and content of voids seem to be the most significant indicators for predicting acoustic properties of HMA. These correlations, that seem to be very promising, are useful in order to optimize the design of HMA in the perspective of minimizing noise impact.

KEY WORDS: open graded asphalt mixture, tire/pavement noise, absorption coefficient, random sequential adsorption, noise reduction.

1. INTRODUCTION

Traffic noise is a growing concern for both public health and economy of each country, especially in urban areas. Studies indicated that 30% of European Union (EU) citizens are exposed to traffic noise exceeding the World Health Organization (WHO) recommended acceptable level. In addition, a 1dB increase in noise results in a 1% decrease in house prices near the busy roads [1]. In fact, the traffic noise impacts on communities are escalating worldwide due to increasing traffic volume and development near the highway facilities [2].

The three main sources of roadway traffic noise are vehicle engine (power train), aerodynamics and tire-pavement interaction. The tire-pavement interaction was found to be actually the major contributor to traffic noise [3]. This provides a window for noise reduction by the pavement itself [4,5].

In general it is very challenging to investigate the acoustic characteristics of the pavement because there are various levels of uncertainty related to the propagation of the sound wave in such a heterogeneous medium (reflection, refraction and diffraction). In fact the acoustic impedance largely depends on the system of interconnected voids on the surface (i.e., pavement surface type (porous or non-porous) and the pavement surface texture) [6]. An absorptive surface prevents effective reflection of the sound energy produced due to the interaction of pavement and vehicle tire and helps to reduce the roadside noise.

In this paper, we propose a simulation approach in order to predict and to check the acoustic properties of the asphalt mixture from geometrical and topological indicators of the mixture.

At this regard, using a Random Sequential Adsorption model, we have generated by simulation synthetic HMA samples reproducing real samples having the same grading and bitumen content.

We manufactured real samples by means of a gyratory compactor and we have considered three level of compaction through different numbers of gyrations, which are consistent to the initial mixture compaction (N_{des}), the compaction approximately after half of the service life (N_{int}) and the compaction at the end of the service life (N_{max}).

For each samples we have measured through a stationary waves device (Kundt's tube) the absorption coefficient α . This coefficient has been related to some numerical indicators, derived from the simulation process. Inversely, simulating the HMA samples is possible to extract the mentioned indicators and predict the absorption coefficient in order to propose a procedure to help the design and the optimization of the mixture.

2. THE SOURCE OF TIRE/PAVEMENT NOISE AND NOISE REDUCTION

Tire/pavement noise is generated by several sub-sources [7]. At the interface, several mechanisms create energy which is eventually radiated as sound. These will be referred to as source generation mechanism:

- tread impact: the tire tread blocks travel around the tire as the tire turns. At the entrance of the interface between the tire and the pavement(referred to as the contact patch) an impact occurs as the tread hits the pavement. The tread impact can be compared to a small rubber hammer hitting the pavement. This impact causes vibration of the tire carcass. If both the tread block and the pavement can be made resilient, the energy created by this impact can be reduced;
- air pumping: within the contact patch, the passages and the grooves in the tire are compressed and distorted. The air entrained in these passages will be compressed and pumped in and out of the passages. Because of air compression effects and air pumping, aerodynamically generated sound is created;
- slip-stick: within the contact patch the tread blocks transfer tractive forces from the tire to the pavement for acceleration or braking. In addition, due to the distortion of the tire carcass in the contact patch, the tread block/pavement interface experiences significant horizontal forces. If these forces exceed the limits of friction, the tread block will slip briefly and then re-stick to the pavement. This action of slipping and sticking can happen quite rapidly and will generate both noise and vibration;
- stick-snap (adhesion): the contact between the tread block and the pavement causes adhesion between them. When the tread block exits the contact patch, the adhesive force holds the tread block. The release of the tread block causes both sound energy and vibration of the tire carcass;

There are also characteristics of the tire/pavement interface that cause that energy to be converted to sound and radiated efficiently. These characteristics will be referred to as sound amplifying mechanism:

- horn effect: the geometry of the tire above the pavement is natural horn. Sound create by any source mechanism near throat of the horn will be enhanced by the horn;
- organ pipes and Helmholtz resonators: the tread passage of the tire in the contact patch take on shapes of acoustical systems that amplify sound generation;
- carcass vibration: the vibration energy created at the tire/pavement interface is enhanced by the response of the tire carcass. Vibrational waves propagate in the tread band, which is the structural element of the tire located adjacent to the tread blocks. These waves create sound which is radiated from the tire carcass. In addition, the tire carcass sidewalls near the contact patch vibrate and radiate sound;
- internal acoustic resonance: The air inside the tire, which is used to inflate the tire, is also excited by the excitation of the tire. At certain frequencies associated with the natural frequency of the toroidal enclosure inside the tire, the air inside the tire will resonate. The response of the air inside the tire is sufficient for these resonances to be audible.

Starting from these sources and amplifying mechanisms, the most effective way to achieve lower noise levels is to reduce the amount of sound generated. In general open-graded asphalt pavement represents an effective solution [8] in order to reduce potential noise for road surface (figure 1). By means of experimental surveys, using both in field [9] and in laboratory [10] measurements, some traffic noise prediction models were performed elsewhere [11,12]. In particular there are three characteristics, which are suitable to describe the acoustical behavior of the road surface:

- surface roughness,
- porosity,
- elasticity.

These parameters influence the excitation of tyre vibrations, air-pumping and sound radiation from tyres. All three characteristics can be qualified and quantified by a set of parameters. They influence the acoustical behaviour of a road surface independently from each other. These parameters are: surface roughness (Roughness depth - Roughness wavelength - Shape – Air/flow resistance - Skid resistance); porosity (Layer thickness – Air void content – Air flow resistance – Tortuosity); and elasticity (Stiffness – Loss factor) of the pavement [14].

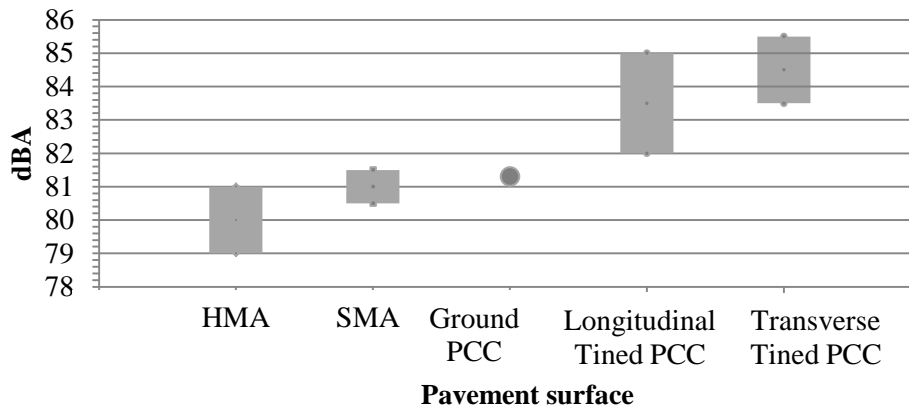


Figure 1. Exterior noise test results for Wisconsin pavement surfaces [13].

However it is very difficult to find reliable correlation between these characteristics and the acoustic properties by analytical models.

In order to overcome these problems, in this paper we try to find, under a numerical approach, a method to assess the acoustic features and to solve the optimization problem of mix design. In particular the dependence of the acoustic parameters by the geometrical and topological indicators is evaluated as well as characteristics of the pavement, using a full numerical model.

2. NUMERICAL MODEL

The simulation model for the numerical generation of asphalt sample is based on Random Sequential Adsorption (RSA). RSA is a simple but fundamental problem in statistical physics. Objects are added randomly, one at a time, to a d-dimensional space. They must not overlap with previously added objects. In the present case [15] the space is two-dimensional, why simulating a cross section of the sample, or three-dimensional, why simulating fully the sample. As the process of adding objects is proceeding it becomes more and more difficult to find regions where the objects can find a new place. Theoretically when no further addition is possible the process is reaching the so called “jamming limit” [16].

The single particle of aggregate is approximated here by a sphere. The diameter of the sphere is an approximation of the specimen size. Assuming that the asphalt production is a random process, the RSA can be accepted as a good approximation of the mixture forming. In general it is evident that the spherical approximation is not always very realistic for this purpose, but in the case of porous HMA mix this approximation has been validated at a first stage [15]. Eventually the model could be upgraded considering also different shapes for the particles if needed. Similar examples are discussed in the literature (hyper-spheres, ellipses, rectangles, parallel squares) [17]. The case of spheres is well and diffusely discussed in the literature [18] if the particles have the same diameter. The case of different sizes has been rarely investigated. However some applications and studies can be found in the literature about the binary [19] and polydisperse mixtures [20]. Here the case of a polydisperse mixture, two phase (solid, air), is simulated according to the sizes distribution of the aggregate. The spheres have diameters from a minimum value δ_{\min} to a maximum value δ_{\max} . The method is here applied to simulate real porous hot-mix asphalt samples, with the dimension and the shape of a cylinder Φ 0.15 m and 0.05 m high. The RSA algorithm selects one diameter (D) of a single sphere at random and one point (x,y,z) within the three-dimensional domain, that is the position of the center of the sphere. The selection of the diameter is made within the range of possible diameters ($\delta_{\min} \leq D \leq \delta_{\max}$). The final set of selected diameters must accord to the distribution of aggregate sizes. At this scope the expected grading is divided in mutually exclusive but exhaustive classes (i.e. class i is from diameter

D_i to diameter D_{i+1}). Within each class a set of diameters is extracted at random in a way that the final distribution of particles all over the classes accords to the real grading of aggregates. Each sphere inserted in the domain is labelled with a number that points to that sphere in a list where all the inserted spheres are associated to the coordinates of the position of the center and to the diameter. Basically it facilitates the process of checking for overlapping. In the tests we did not attempt to reach the jamming limit because the real samples are not compacted at the maximum theoretical limit. Otherwise the simulation is stopped when the rate between solid, as sum of the particles volumes, and the volume of voids tends to the real rate. Finally the distribution of spheres sizes is checked, to verify the consistency of the simulation respect to the real grading.

Two different approaches have been adopted. A complete three dimensional approach and a simplified two-dimensional approach. A 2D sample is shown in figure 2.

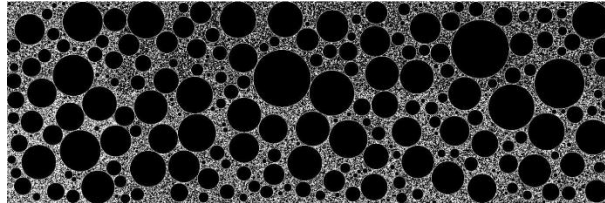


Figure 2. Example of two dimensional generation of asphalt sample.

It has been verified [13] that this second approach (2D) gives results that are strictly correlated to the three-dimensional approach and it can be considered absolutely more efficient, under a computational point of view, if a full and accurate generation of the real sample is not required.

According to one aggregate grading and one bitumen content it is possible to generate how many different samples how the different random seeds are. This casual procedure simulates the real procedure of sample making. It is well known that the laboratory procedure to make samples produces, from the same initial conditions, different results, also under rigorous standards and methods. This is the reason why, in the laboratory tests, a number of samples is required to calculate the average values of mechanical or hydraulic properties. For obvious reasons of time and cost this number of samples is always very limited (generally four samples). The numerical simulation makes it possible to generate a great amount of virtual samples. Over this great amount of samples it is possible to extract more stable and representative averages, following a Monte Carlo procedure [21,22].

3. ACOUSTIC PROPERTIES OF HMA

In order to assess the acoustic properties of HMA, as described previously, we manufactured samples by means of a gyratory compactor and we have considered three different steps, as mentioned, in terms of number of gyrations. They are consistent to the initial mixture compaction (N_{des}), the compaction approximately after half of the service life (N_{int}) and the compaction at the end of the service life (N_{max}).

In particular four grading curves have been selected, as shown in figure 3. Two different percentage of modified bitumen have been used, 5.5% for G70 and G75 and 5% for G80 and G85, to prepare three samples for each grading curve and number of gyrations, for a total of 36 samples.

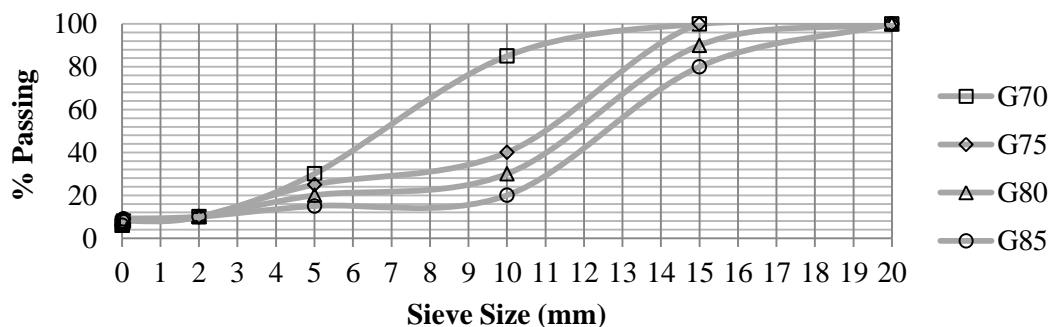


Figure 3. Selected grading curve.

For each of the 36 samples we have measured the acoustic characteristics by the determination of absorption coefficient α , defined as the ratio between the intensity of the absorbed acoustic wave and the intensity of the incident wave. This parameter is measured through a stationary waves device: Kundt's tube (figure 4). This tube has an internal diameter of 100mm [23].

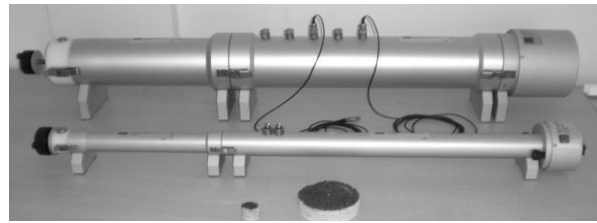


Figure 4. Stationary wave device – Kundt's Tube.

The measuring equipment consists of a computer for acquisition and data processing and a tube. At the two extreme parts of the tube there are respectively positioned: a speaker, that generates a tone of known frequency, and the sample (figure 4).

The signal generated from the speaker is a white noise, that has a frequency distribution almost constant in the range for the measurement. The stationary waves, generated from the speaker, propagate in the tube as plane waves which invest the sample and part of them are reflected. The overlap of the reflected waves and the incident waves return the typical phenomenon of stationary wave within the tube. More in detail the absorption coefficient, as described previously, is obtained as a function of the various frequencies, by determining a coefficient of reflection, obtained by measuring the sound pressure in two fixed locations (microphones) and by calculating, consequently, the response of the sample to the incident wave. Figure 5 shows the measured absorption coefficient during the life cycle of the open-graded samples.

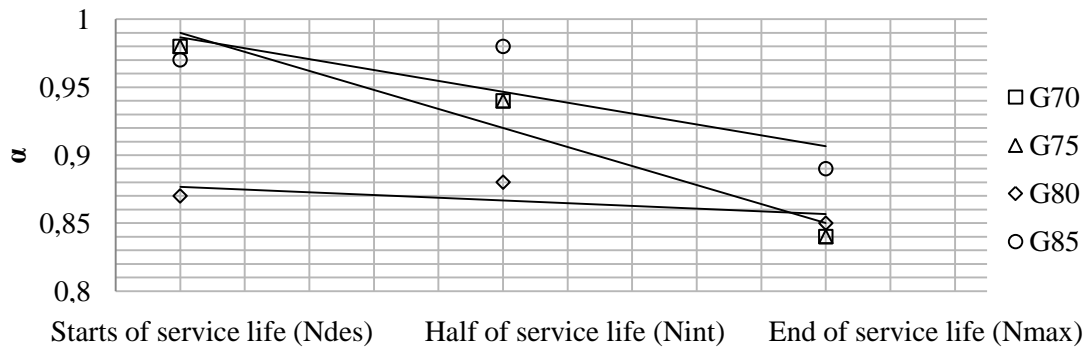


Figure 5. Absorption coefficient (α) versus service life.

The plot position shows a good correlation between the absorption coefficient α and the voids content (%), decreasing with the increase of the number of gyrations.

This correlation is showed in figure 6:

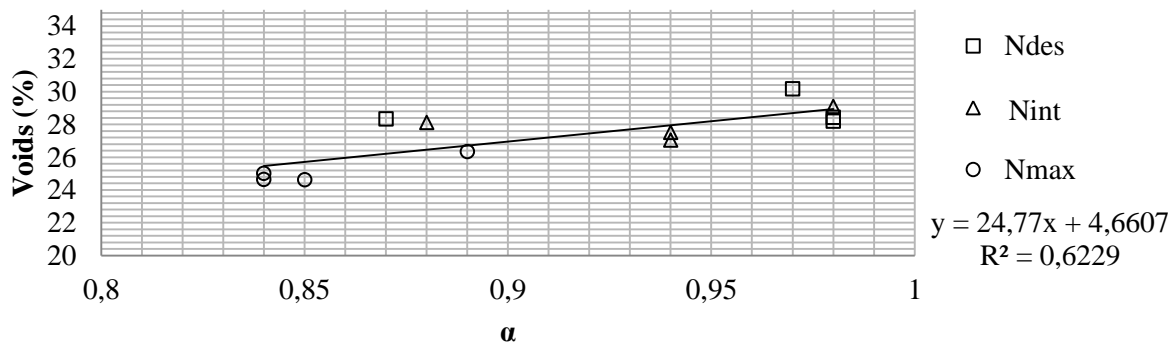


Figure 6. Absorption coefficient (α) versus voids content.

4. THE NUMERICAL INDICATORS

Once the set of samples have been simulated starting from the grading of aggregates and from the bitumen content, according to [8], it is possible to extract the average values, over the 36 simulated samples, of some indicators representing geometrical and topological characteristics of the mixtures. The first step is the definition of these indicators .

Here 15 different indicators, according to [8], are examined. These indicators have been selected considering the main factors that are expected to play significant role in the acoustic behavior of the mixture.

Table 2 lists all the 15 indicators.

A graphical and conceptual framework is given in Figure 7. In Figure 7 the equations for computing the values of the indicators are shown.

Table 2. The indicators extracted from simulated samples.

N°	Indicator	Definition
1	I_1	Expected value of bitumen film thickness z_i
2	I_2	Number of contacts among all the particles (there is a contact between two particles only if the distance d_i in Figure is less than d_{min})
3	I_3	Number of contacts among big particles, a big particle is a particle with diameter $D_i > D_{min}$
4	I_4	Number of contacts among little particles, a little particle is a particle with diameter $D_i < D_{min}$
5	I_5	Number of contacts between big and little particles ($I_3 + I_4$)
6	I_6	Number of contacts Type I among particles, the contact type I (thin bitumen film) is a contact between two particles with a bitumen thickness z_i thinner than z_{min}
7	I_7	Number of contacts Type II among particles, the contact type II (thick bitumen film) is a contact between two particles with a bitumen thickness z_i thicker than z_{min}
8	I_8	Expected value of the distance d_i among contacts
9	I_9	Expected value of the distance d_i among contacts weighted by the sum of particles radii
10	I_{10}	Expected value of the distance d_i among contacts Type I
11	I_{11}	Expected value of the distance d_i among contacts among contacts Type II
12	I_{12}	Number of interspace between particles (there is an interspace between two particles only if the distance d_i in Figure is greater than d_{min})
13	I_{13}	Expected value of the specific surface of particles
14	I_{14}	Expected value of the specific surface of particles and bitumen film
15	I_{15}	The rate between the number of contacts (I_2) and the number of total particles

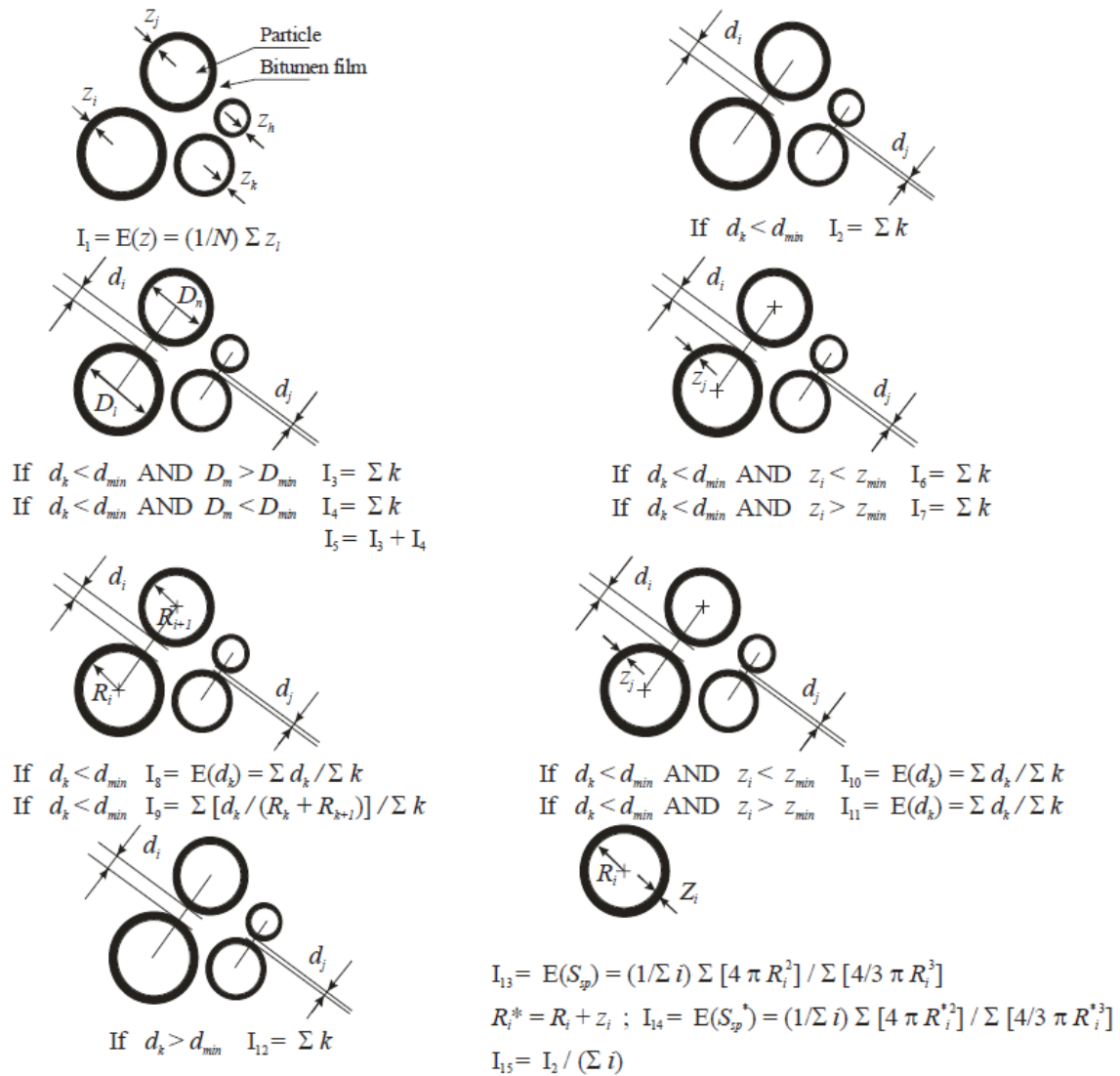


Figure 7. Graphical and conceptual framework of the indicators.

5. CORRELATION

The in depth analysis of all the indicators shows a high correlation between the number of contacts per particle, extracted from the synthetic HMA sample (ratio between the number of contacts and the number of particles included in the reference domain, I_{15}) and the number of voids (%) of the real samples, as the number of gyrations ($N_{des} - N_{ini} - N_{max}$) increases.

This correlation is shown in figure 8:

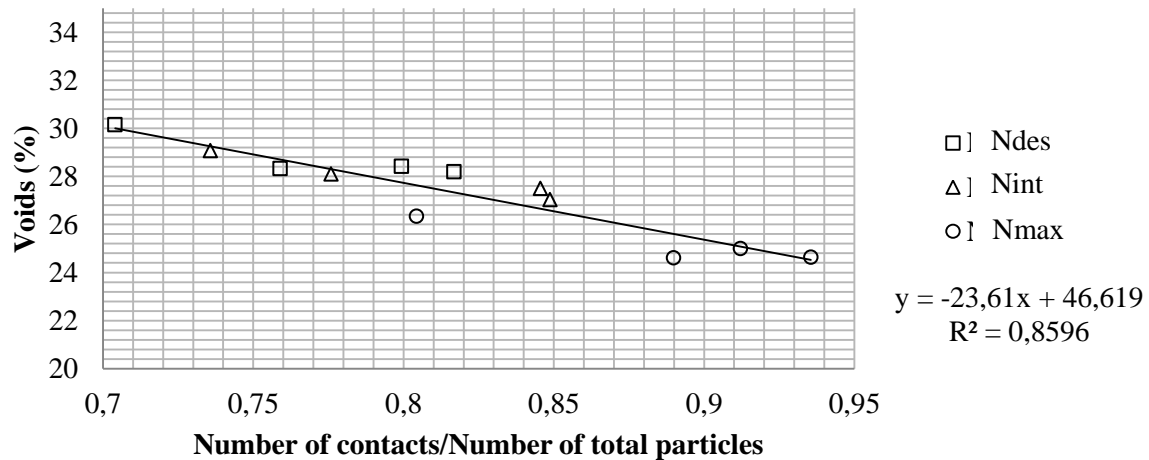


Figure 8. Ratio between number of contacts and number of particles versus voids content.

As expected, the figure shows a decreasing trend of the voids content as the number of gyrations increases. Accordingly, the increasing of gyrations cycles produces an increasing of the ratio between the number of contacts and the number of particles.

Moreover, if we consider the acoustic absorption coefficient (α), the correlation with the rate of contacts per particle is weaker (figure 9). This is reasonably due by the various levels of uncertainty as synthetically described above (grain size characteristics, dimension and distribution of voids), that produces a not negligible dispersion in the plot position.

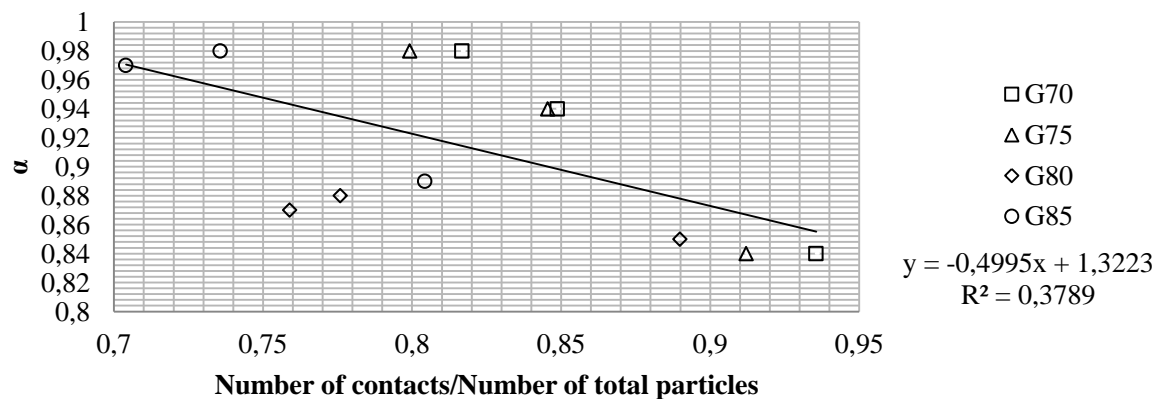


Figure 9. Ratio between number of contacts and number of particles versus absorption coefficient (α).

In order to qualitatively explain the increasing dispersion of data a conceptual model is here proposed.

Let us assume a rectangular domain in which some particles are regularly inserted (figure 10). At time $t_0=0$ in the domain we have a voids content V_0 and the absorption coefficient is α_0 (figure 10). This coefficient is given by the rate between the amplitude of the wave received beyond the particles and the amplitude of the wave at the acoustic source. It is possible to calculate α_0 using numerical simulation following the 2d applet created by the

physics department of the university of Modena and Reggio Emilia and Modena institute of nanoscience (figure 10).

In order to simulate the compaction process, let us assume that the particles at the time $t=t_1$ are closer in the domain and the voids content is V_1 (figure 10). After compaction the absorption coefficient decreases to α_1 as computed by simulation (figure 10).

Acoustic Frequency = 500-1000 Hz
 Maximum dimension of voids = 25 mm

Pressure = 0.2 Pa
 Minimum dimension of voids = 2.5 mm

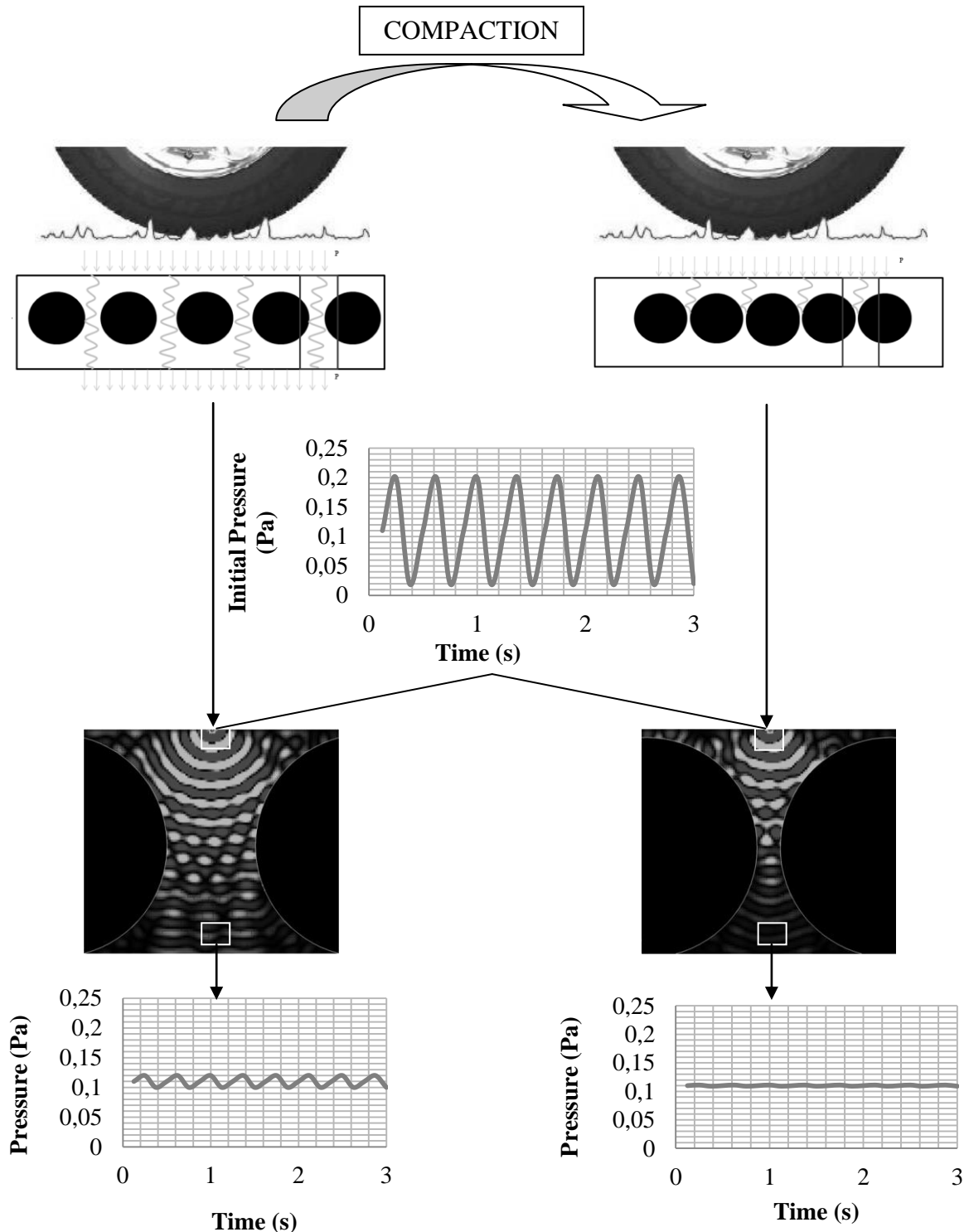


Figure 10. Conceptual model to explain the increasing dispersion of data.

Following this conceptual over-simplified model, it is possible to evaluate as the acoustic absorption coefficient is expected to decrease by a progressive compaction. Under a quantitative point of view, figure 11 shows as the adsorption coefficient decreases as the voids content decreases.

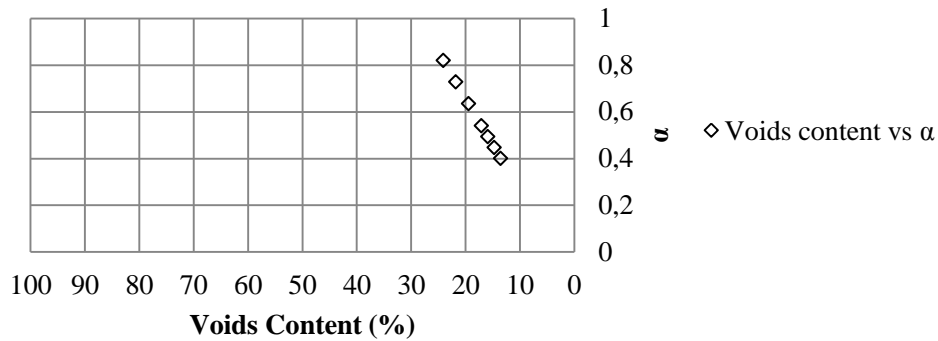


Figure 11. Voids content versus absorption coefficient (α).

It is clear that the absorption coefficient is reduced much more rapidly than the voids ratio, that decreases much slowly.

In a more complex and realistic model, where dimension and size distribution of particles are very variable, the absorption coefficient decreasing can not be predicted easily, but an analogous trend is reasonably expected. More in depth it is expected that coarser is the grading smoother is the decreasing trend of the absorption coefficient, because the average final (after compaction) dimension of the voids is greater, while finer is the grading greater is the decreasing gradient for the absorption coefficient.

This assumption is qualitatively confirmed by the results, as it is shown in figure 12, where the four materials, that have different grading curves, are compared. Here the angular coefficients of the regression linear are mainly greater for finer grading and lower for coarser.

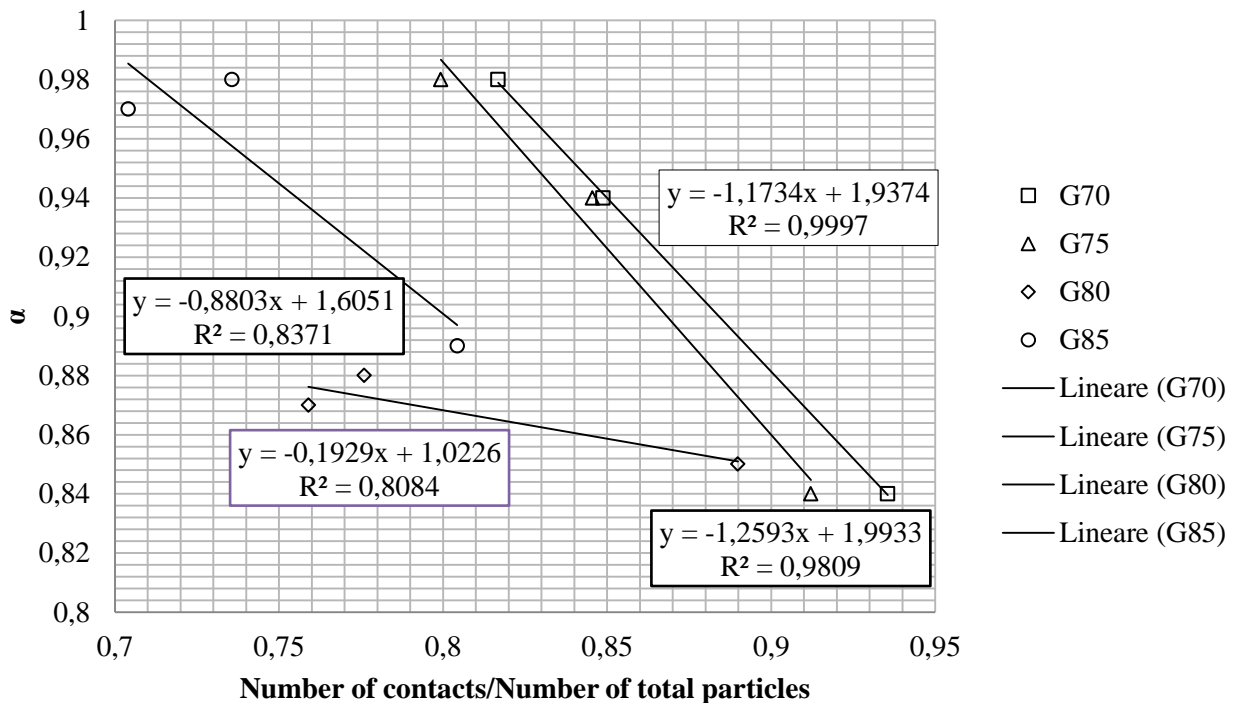


Figure 5. Ratio between number of contacts and number of particles versus α for each grading curve.

It is possible to write in a general form the correlation equation between the absorption coefficient and the number of gyratory cycles, or life time, as it follows:

$$\alpha = \omega t + \alpha_0 \quad (1)$$

where:

α = absorption coefficient;

ω = coefficient of regression (function of the grain size distribution);

t = time related to the gyratory cycles and finally to the ratio between number of contacts and the number of inserted particles ($N_{des} - N_{ini} - N_{max}$);

α_0 = absorption coefficient at time t_0 (function of the grain size distribution).

The coefficient of regression is expected to be a function of the particle size distribution, as well as the absorption coefficient at time t_0 .

6. CONCLUSIONS

In today's society traffic noise is a serious problem because it can be considered an environmental pollution that lowers the standard of living. In particular annoyance and associated negative effects often reduce the quality of life. In addition traffic noise may also impact on health and human behaviors. Then it is one of the most frequent cause of public opposition to new constructions.

Numerous researches have indicated that it is possible to build pavement surfaces that will reduce the level of tire/pavement generation noise generated on roadways. In fact use of open-graded mixes to reduce noise levels could potentially induce an economic and environmental improvement by reducing significantly the noise from tire/pavement interaction, entrapping the acoustic waves inside the porous structure of the material and dissipating energy by reflections.

However it is very difficult to predict the acoustic properties of the mixes, because it is a function of many variables related to the physical characteristics and to the microstructure (distribution, dimension and interconnection of voids) of the sample and to the propagation of the wave which make the evaluation very uncertain.

In this paper a numerical model is proposed to simulate open-graded sample in order to predict the acoustical features of HMA.

In particular number of contacts and dimension of voids seem to be the most significant indicators for predicting acoustic properties (absorption coefficient) of HMA. More in depth the correlation between the absorption coefficient and density of contacts appears as a function of grain size distribution and as a function of the increasing number of gyrations ($N_{des} - N_{ini} - N_{max}$), that represents three different temporal steps of the lyfe cycle of the pavement.

It is possible to write in a general form the correlation equation between the absorption coefficient and the life time (number of gyratory cycles). This correlation, that seems to be very promising, is very useful in order to optimize and to support the design of HMA to minimize noise impact. However additional experimental validations have to be carried out, in order to reach the needed reliability and a stable procedure.

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