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ERROR AND UNCERTAINTY ANALYSIS OF RESIDUAL STRESS EVALUATION BY USINGTHE RING-CORE METHOD

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Sommario

Il metodo della cava anulare o Ring-Core Method è un metodo meccanico utilizzato per l'analisi delle tensioni residue in componenti meccanici. Sia per tensioni residue uniformi che per tensioni residue variabili nello spessore del componente esaminato, il metodo da luogo in genere a risultati accurati sebbene allo stato attuale, contrariamente a quanto accade per altri metodi come il più famoso metodo del foro, l'utilizzatore non dispone di appropriate procedure per la correzione di eventuali errori sistematici né per la stima della incertezza dei risultati, dovuta agli errori casuali. Allo scopo di superare questi inconvenienti, attraverso una sistematica analisi delle principali grandezze di influenza, nel presente lavoro sono proposte delle appropriate procedure per la correzione degli errori dovuti alle principali grandezze di influenza, nonché per la valutazione della incertezza delle tensioni residue principali calcolate e del relativo orientamento. L'applicazione pratica delle procedure proposte consente altresì all'utilizzatore di evidenziare l'entità degli errori e della incertezza introdotta da ciascuna delle grandezze di influenza, indicando cosi possibili setup sperimentali che consentono di minimizzare errori ed incertezza.

Abstract

The Ring-Core Method is a technique used for the experimental analysis of the residual stresses in mechanical components. For uniform and non-uniform residual stresses estimation, the use of the method leads in general to accurate results but, unfortunately at present the user does not have appropriate procedures to correct the obtained results from systematic errors as well as to estimate the uncertainty due to random errors. In order to overcome such drawbacks, in the present work, the procedures for the correction of the effects of the main error sources and for the stress uncertainty estimation, are proposed. The practical application of such procedures allow the user to highlight the relative magnitude of the error and stress uncertainty associated with the main influence parameters.

Parole chiave: Ring-core method, residual stresses, uncertainty estimation.

1. INTRODUCTION

The Ring-Core Method (RCM) is a semi-destructive method used for the residual stress (RS) evaluationin mechanical components [1-4]. Respect to the more known Hole Drilling Method (HDM), it is characterized by a higher stress relaxation that allows the user to reach higher depth from the surface of the analyzed component and, in general, leads to a lower error's sensitivity. Although various contributions have been published in literature, the Ring-Core method has still not been standardized. In practice, the computational approach used for the evaluation of non-uniform RS by the RCM is the same as used for the HDM and reported in the well-knownASTM E837-13a standard

[5], but the experimental conditions are different because the geometrical variation introduced by using a proper annular cutter is very different from the HDM and, consequently, it is characterized by different influence parameters from which the accuracy of the RS evaluation depends.
Although the use of the RCM is increased in recent years, especially in Europe also thank to new modern equipment commercially available, only few research works devoted to the error and uncertainty analysis [6-8] have been reported in literature. Therefore, the aim of this work is to give a contribution to the evaluation of the uncertainty of the RS estimated by the RCM, by summarizing all the factors influencing the accuracy of calculated RS and, subsequently, accomplishing an appropriate procedure to the correction of the main systematic errors as well as to the stress uncertainty estimation. Obviously, such a procedure allows the user to increase the accuracy of the computed RS as well as to obtain a reliable estimation of their uncertainty. In fact, as it is well known, a reliable uncertainty estimation can be carried out only after the systematic errors are properly detected and corrected.
Exploiting the similarities between RCM and HDM, the work has been carried out by considering all the literature [9-11] on the estimation of the uncertainty of the RS computed by the HDM, with particular reference to the recent works ofScafidiet al. [12] and Schajerand Altus [13].

2. PROCEDURE FOR THE RESIDUAL STRESS UNCERTAINTY ESTIMATION

A general procedure for the correction of the main errors affecting the RS evaluation, as well as for the RS uncertainty estimation by considering the main mutually independent influence parameters, is well described in Oettel´s work [9] and successive works [10, 14].
As occur in most practical cases, the RS is not directly measurable, but depends on various parameters that can be computed or measured by the user. Indicating by Y the unknown RS and assuming that it depends on N independent variables $X_1, X_2,\ldots,X_N$, i.e.:

$$Y = f(X_1, X_2, \ldots, X_N) \quad (1)$$

then, in general,the vector ($X_1, X_2,\ldots,X_N$) is constituted by some variables whose values and uncertainties are directly determined during the measurement process, and other variables whose values and uncertainties are transferred to the measurement procedure from external sources (manufacturer´s specifications, data provided by calibration and from other certificates, etc.). Consequently, the uncertainty of Y is determined indirectly from known and estimated causes. In accordance to the ISO/IEC GUIDE 98-3:2008, the estimated value of $Y$is usually represented in the following way:

$$Y = y \pm U, \quad (2)$$

where y is the test (or measurement) mean result, U is the so called expanded uncertainty associated with y. Such expanded uncertainty is obtained by multiplying the standard uncertainty $u_c(y)$ by a proper coverage factor k, i.e:

$$U = k \cdot u_c(y) \quad (3)$$

Commonly it is used k=2, that for a normal distribution corresponds to a coverage probability, p, of approximately 95%.
In case of the RS evaluation by a mechanical method, as RCM or HDM, due to the impossibility to repeat the measure under the same conditions, y represent the values obtained from the direct RS evaluation, after correction of the systematic errors due to the various influence parameters.

3. INFLUENCE PARAMETERS

In the present work the determination of the main parameters (error sources) that influences the RS evaluation by using the RCM,is carried out by considering the use of a modern automaticequipment as that commercialized by the SINT Technology [15],that in general usesstrain gage rosette type HBM - RY51 [16] or similar. In detail, an accurate analysis of the actual experimental conditions shows that the RS measurement is influenced by the following main parameters:

1. residual stresses induced by the milling cutter;
2. core-rosette eccentricity;
3. plasticity effects due tostress concentration at the notch bottom;
4. core axis inclination (with respect to the normal to the component surface);
5. temperature variations of the zone close to the strain gage, due to the milling;
6. effects of the slope/radius at the notch bottom;
7. zero depth offset (due to a wrong mill initial positioning);

As it occurs in a generic mechanical method, in the RCM the residual stresses induced by the cutter, depend on the milling procedure, on the material type and on the particular material thermal treatment (hardening, quenching etc.). In detail, thanks to the relative distance between the strain gages and the core surface,if amilling procedure is used, then the experimental evidence shows that in general the residual stresses induced by machining are characterized by relatively low mean values. Obviously, since the magnitude of such induced stress is strongly related to material characteristics, proper analyses should be carried out if particular materials or treatments are considered
As it is well known in literature [3, 6, 15, 17], one of the main advantage of the RCM respect with the HDM is its low sensitivity to the rosette eccentricity. In detail, in [15] the authors claimed that the use of modern devices equipped with automatic alignment system, allows the user to make negligible such an error. Moreover, in [7] Zuccarello has found that for eccentricity equals to 1 mm the maximum errors of σ_{max} and σ_{min} are 10% and 20% respectively, whereas in the more common case in which the maximum eccentricity is less than 0.4mm such errors are less than 1.5% and 3.0%. Also in Ref.[6, 8] by considering the particular case of a biaxial uniform RS distribution, the authors have found that for a coarse eccentricity error of 0.5 mm in both vertical and horizontal axis, the RS error falls in the range 2.0 – 0.4 % up to depths of 2 mm, whereas it falls in the range -0.5 ÷ -5.9% for depths in the range 2-4 mm. However, the same authors state that the use of an automatic device that is equipped with a proper microscope to center the cutter, allows the user to limit the eccentricity to values of about 0.1mm.
In accordance with the ASTM E837-13a standard, using the HDM it is possible to obtain satisfactory evaluations of RS, if its maximum value is less than 80% the material yield stress. Taking into account that the RCM is characterized by lower stress concentration effects and therefore by lower plasticity effects at the groove bottom, then such a limitation can be extended to the RCM. In more detail, according the work of Petrucci and Zuccarello [18], in presence of biaxial RS distributions, the maximum error of the computed RS is negligible for actual RS up to 60% the material yield stress, whereas for RS up to 70%, 80%, 90% the maximum errors occurs at the first steps (up to 1 mm) and can be equal to about 10%, 20% and 33% respectively; furthermore, errors having similar modulus but negative sign, occur at depths of about 3-3.5 mm.
Systematical analysis of the inclination of the core obtained by using an automatic equipment as that manufactured by SINT Technology [15], has shown that such an error is in general very low, in practice less than 1°. No study, covering the effect of the possible inclination of the core axis on computed RS, has been carried out until now.
Considering the influence of the temperature, the experimental evidence shows that due to the milling process the temperature of the zone close to the strain gage rosette can increase up to about 5°C; such a value is also been confirmed by the manufacturer SINT Technology [15]. However, the thermal effects on the relaxed strain measurement are commonly negligible when self-compensated rosette is installed on a free component.

Since the numerical simulations used to compute the influence functions assume a plane bottom of the groove, the actual experimental measurements can be affected by significant errors if the actual cutter geometry leads to different shape of the notch bottom. Systematical analyses of the groove bottom profile given by the use of a standard cutter (see Fig. 1) shows that the bottom is inclined toward the outside of about 5° in order to improve the cutting condition. In principle such a geometry influences the actual stress relaxation and it should be considered for a very accurate RS evaluation. According to previous numerical studies, at the first milling steps such an error (5° inclination) can lead to RS errors up to 30%; the error is reduced to 2% at 5 mm depth.

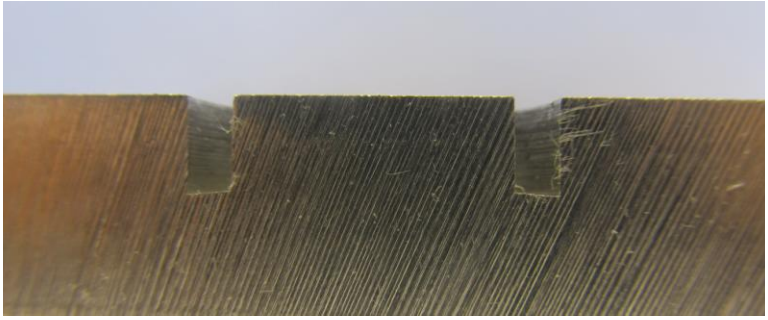

Figure1: Groove profile with inclined bottom, obtained by using a standard cutter.

The measurement of the groove depth can be affected by significant errors due to possible zero depth offset (end mill that does not touch correctly the component surface). In general, due to the shape of the end mill and/or to the limited electric contact area [1], a systematic error of about 0.01 mm can occur also when the initial position of the end mill is relieved by monitoring the electric contact. Although such a result has been observed in the HDM, it can be extended to the RCM that use in practice the same device to monitor the electric contact between cutter and metallic surface of the examined component.

4. CORRECTION OF THE MAIN ERRORS AFFECTING THE RS

In the following, a procedure to correct the main errors affecting the RS analyzed by the RCM, is proposed. It considers the application of the RCM under the following general assumptions:

1. Minimal dimension of the tested component bigger than: 30mm for thickness, 50mm for width and 50mm for length;
2. Calculation of the RS distribution by using the Integral Method, as exposed on ASTM E837-13a;
3. RS distribution through the core-depth evaluated by using 8 optimized steps, as suggested in [20].

In detail, assumption (1) is used to avoid significant boundary effects that influence the stress relaxation [2]; in this cases appropriate numerical simulationsare necessary to calculate the correct influence functions. Assumption (2) refers to a standard procedure widely used by the users, whereas assumption (3) leads to optimal condition that permits to minimize the influence of the main experimental errors.
Considering the local thermal effects due to machining, it is possible to state that the heating of the core surface during the milling procedure leads to an apparent thermal strain, also when self-compensated strain gages are used.
For an accurate evaluation the residual thermal effect of the self-compensated rosette can be considered, by using the characteristic thermal curve provided by the manufacturer; as an example for the HBM RY51 the residual thermal effect can be obtained by computing the difference of the values acquired by the following polynomial at the final temperature T_f and at the initial temperature T_i:

$$P(T) = -12{,}48 + 1{,}51T - 5.10^{-2}T^2 + 2{,}29.10^{-4}T^3 \quad (4)$$

Denoting with $\varepsilon_{T(SC)}$ such a contribution, i.e. $\varepsilon_{T(SC)}= P(T_f)-P(T_i)$, we have the following relation:

$$\varepsilon_T= \varepsilon_{T(SC)} \tag{5}$$

Moreover, in the generic case in which a strain gage rosette self-compensated for a material having expansion coefficient equal to α_{Sm} is installed on a generic component having expansion coefficientα_S , then the thermal strain due to the expansion coefficient mismatch is added to the mean strain and it follows:

$$\varepsilon_T = \varepsilon_{T(SC)} + (\alpha_S - \alpha_{Sm})\Delta T \tag{6}$$

As an example, installing a rosette self-compensated for steel (α_{Sm}=10.8 ppm) to a component made by a different alloy having $\alpha_s=13$ ppm and considering, in accordance to the experimental evidence a maximum $\Delta T = 5°C$ (from 20 °C to 25 °C), then equation (5) and equation (6) provide:

$$\varepsilon_T = +(13 - 10.8) \cdot 5 + \varepsilon_{T(SC)} = 11 - 1.95 = 9{,}05\ \mu\text{m/m} \tag{7}$$

The strain values corrected from the effects of the local thermal variations are expressed from the measured values:

$$\varepsilon_{mi} = \varepsilon_{mi}^{meas} - \varepsilon_T = \varepsilon_{mi}^{meas} - 9{,}05 \tag{8}$$

The computed values of the apparent thermal strain show that, as occur often in the strain measurement, the apparent thermal strain can become quite high; for this reason a good practice in the RS analysis by mechanical methods that use ER (HDM or RCM), is to wait a sufficient time after the incremental depth, to obtain the thermal stabilization of the measured relaxed strain.
As above mentioned, if a modern device equipped by a proper optical centering system, is used, then the core-rosette eccentricity can be considered negligible and no correction has to be carried out.
The strain correction related to the bottom inclination strongly depends on the cutter shape, therefore each cutter requires its own error determination.
After the strain corrections, in accordance with the ASTM standard, the three components p_i, q_i and t_i can be computed by combining the corrected relaxed strains ε_{mi} as:

$$p_i = \frac{\varepsilon_{ci} + \varepsilon_{ai}}{2}, \tag{9}$$

$$q_i = \frac{\varepsilon_{ci} - \varepsilon_{ai}}{2}, \tag{10}$$

$$t_i = \frac{\varepsilon_{ci} + \varepsilon_{ai} - 2\varepsilon_{bi}}{2}, \tag{11}$$

where $m=a$, b, c (measuring grid of the rosette), $i=1,2,\ldots N$ (step number).
The influence coefficients a_{ij} and b_{ij} (j is the layer number at i-th step) are obtained by numerical simulations carried out by considering an equi-biaxial (σ_{is}) and a shear (σ_{iu}) stress state respectively, by following simple formulas [21]:

$$a_{ij} = \frac{\varepsilon_{aij}}{\sigma_i} \tag{12}$$

$$b_{ij} = \frac{\varepsilon_{aij}}{\sigma_i} \tag{13}$$

Taking into account the zero depth offset z_0, then the measured depths z_i^{meas} has to be corrected by the simple formula:

$$z_i = z_i^{meas} - z_0 \tag{14}$$

Moreover, considering the dependence on the material properties, equations (12) and (13) are transformed to:

$$A_{ij} = \frac{E}{1+\mu} . a_{ij} \tag{15}$$

$$B_{ij} = E \cdot b_{ij} \tag{16}$$

In accordance to the ASTM standard, the hydrostatic (P_i) and shear (Q_i, T_i) residual stress components at each step are computed by using the following relationships:

$$P_i = \frac{1}{A_{ii}} \left(\frac{E.p_i}{1+\mu} - \sum_{j=1}^{i-1} A_{ij} . P_j \right) \tag{17}$$

$$Q_i = \frac{1}{B_{ii}} \left(E.q_i - \sum_{j=1}^{i-1} B_{ij} . Q_j \right) \tag{18}$$

$$T_i = \frac{1}{B_{ii}} \left(E.t_i - \sum_{j=1}^{i-1} B_{ij} . T_j \right) \tag{19}$$

Finally, the principal residual stresses and the relative orientation β are computed as:

$$\sigma_{1,2i} = P_i \pm \sqrt{Q_i^2 + T_i^2} \tag{20}$$

$$\beta_i = \frac{1}{2} \arctan \left(\frac{-T_i}{-Q_i} \right) \tag{21}$$

The principal residual stresses $\sigma_{1i,2i}$ are in general influenced by two error sources: the plasticity effects due to stress concentration at the bottom of the groove and the stresses σ_{ind} induced by machining.
It is to be noted that the induced stresses are equi-biaxial and do not influence β.Therefore, by assuming that the stresses induced by the machining do not vary with depth (common condition), the principal RSs can be corrected by using the simple formula:

$$\sigma_{c1,2i} = \sigma_{1,2i} - \sigma_{1,2ind} \tag{22}$$

Considering the influence of the possible plasticity at the notch bottom, due to the lack of a general relationship between actual RS level and relative error on the computed RS, then by extending the suggestion of the ASTM E837-13a standard, this work consider only RS evaluations with actual maximum RS level less than 80% the yield stress.
Finally, because its small values the RS error due to the core axis inclination error can be neglected in the correction of the main systematical errors.

5. RESIDUAL STRESS UNCERTAINTY EVALUATION

For a correct and a complete residual stress uncertainty evaluation, the user must identify all the possible sources of uncertainty that can influence (directly or indirectly) the measurements. In general, the uncertainty source list cannot be identified comprehensively beforehand, as it is associated with the particular test procedure and apparatus used. This means that the values should be updated each time a particular test parameter changes (for example the amplifier, the cutter shape etc.).
However, in general, the list of the typical sources of uncertainty includes the following parameters:

1. strain gage calibration factor K;
2. rosette diameter D;
3. accuracy $u(W)$ of the strain measurement device;
4. apparent thermal strain ε_T, due to the milling;
5. core diameter D_0;
6. notch thickness s;
7. milled depths h_i;
8. core-rosette eccentricity$e$;
9. zero depth offset z_0;
10. core axis inclinationα_i:
11. stress σ_{ind} induced by the milling;
12. plasticity effect at the groove bottom;
13. Young modulus $E$of the tested component;
14. Poisson ratio $\mu$of the tested component;
15. surface curvature $R_s$of the tested component.

In accordance with ISO/IEC GUIDE 98-3:2008 [14], by assuming that all the influence parameters are not mutually dependent, the uncertainty propagation law is given by the following general formula:

$$u_c^2(y) = \sum_{k=1}^{N} c_k^2 . u^2(x_k), \tag{23}$$

where y and x_k(k =1,…N) are the computed parametersand the relative influence factors respectively; $u_c(y)$and $u(x_k)$ are the corresponding uncertainties. The constants c_k are the sensitivity coefficients. In the case in which the analytical relationship between y and x_k(k=1,…N) is known, the sensitivity coefficient is given by following differential relationship:

$$c_k = \frac{\partial y}{\partial x_k} \tag{24}$$

In many cases the calculation required to obtain the sensitivity coefficients by partial differentiation can be a lengthy process, particularly when there are many contributions and uncertainty estimates are needed for particular ranges of values. Obviously, if the functional relationship for a particular measurement is not known, the sensitivity coefficients may be obtained experimentally.
Firstly, it is necessary to compute the uncertainty $u_{ci}\,(\varepsilon_{mi}^{meas})$ of the i-th measured strain.Taking into account the basic formula of the strain gauge technique, i.e.:

$$\varepsilon_{mi}^{meas} = \frac{4}{k} \cdot \frac{\Delta V_i}{V} \qquad (i=a,b,c) \tag{25}$$

being ΔV_i the voltage variation of the *i-th* (i=a,b,c) strain gauge bridge and V the supply voltage, it follows that by applying equation (23) and considering all the influence parameters on the measured strains, the following formula can be written:

$$u_c^2(\varepsilon_{mi}^{meas}) = 4 * \left(\frac{\varepsilon_{mi}^{meas}}{k}\right)^2 . u^2(k) + \left(\frac{\varepsilon_{mi}^{meas}}{\frac{\Delta V_i}{V}}\right)^2 . u^2\left(\frac{\Delta V_i}{V}\right) + u^2(e) \qquad (i=a,b,c) \tag{26}$$

In eq. (23) $u(e)$ the uncertainty of the rosette-core eccentricity.
As an example, if a strain gage rosette type HBM RY51S/350 (having k = 2.13 and Ro =350 Ω) is used and a uniform uncertainty distribution is considered, then according the producer it follows:

$$u(k) = \pm 1\% k/\sqrt{3} = 0.0213/\sqrt{3} = 0{,}0123 \tag{27}$$

$$u\left(\frac{\Delta V_i}{V}\right) = \pm 1\% \frac{\Delta V_i}{V}/\sqrt{3} = \pm 1\%\ 350/\sqrt{3} = 2{,}021 \tag{28}$$

Also, if an automatic strain reader with accuracy class 0.05 (type Quantum X of HBM) is used for strain data reading, then commonly it follows:

$$u(W)=0.5\left[\frac{\mu m}{m}\right] \qquad (29)$$

In normal experimental condition, by using a modern device for the RCM (as that manufactured by SINT Technology [15)] the core-rosette eccentricity is less than 0.1mm and the uncertainty contribution $u(e)$ can be neglected.
From the uncertainty of the measured strains the uncertainty of the strain corrected from the error due to the local thermal effects is obtained by applying equation (23) to equation (8) and taking account the equation (7), i.e.:

$$u_c^2(\varepsilon_{mi}) = u_c^2(\varepsilon_{mi}^{meas}) + u_c^2(\varepsilon_T) \qquad (i=a,b,c) \qquad (30)$$

Considering that commonly temperature changes of about ΔT=5°C occurs in the experimental practice, it follows:

$$u_c^2(\varepsilon_{mi}) = u_c^2(\varepsilon_{mi}^{meas}) - 9{,}05 \qquad (i=a,b,c) \qquad (31)$$

After the uncertainty of the strains is estimated, the uncertainties $u_c(p_i)$, $u_c(q_i)$ and $u_c(t_i)$ of the strain components can be computed from the uncertainty $u_{ci}(\varepsilon_{mi})$ of the corrected measured strains by applying equation (23) to equations (9-11).

$$u_c^2(p_i) = \frac{1}{4}u_c^2(\varepsilon_{ci}) + \frac{1}{4}u_c^2(\varepsilon_{ai}) \qquad (32)$$

$$u_c^2(q_i) = \frac{1}{4}u_c^2(\varepsilon_{ci}) + \frac{1}{4}u_c^2(\varepsilon_{ai}) \qquad (33)$$

$$u_c^2(t_i) = \frac{1}{4}u_c^2(\varepsilon_{ci}) + \frac{1}{4}u_c^2(\varepsilon_{ai}) + u_c^2(\varepsilon_{bi}) \qquad (34)$$

To determine the uncertainty of the influence coefficients a_{ij} and b_{ij}(j =1,..i; i= 1,…N) the following main influence parameters have to be considered:

- Poisson's ratio μ
- groove depth measurement h_i
- component's surface curvature radius R_S
- ratio D_o/s between core diameter and groove thickness
- zero depth offset z_o

By applying eq.(23) the uncertainty of each influence coefficient is given by the summation of the contribution due to each above mentioned influence parameter, i.e.:

$$u_c^2(a_{ij}) = u^2\left(a_{ij}^{\mu}\right) + u^2\left(a_{ij}^{h_i}\right) + u^2\left(a_{ij}^{R_S}\right) + u^2\left(a_{ij}^{D_0/s}\right) + u^2\left(a_{ij}^{z_0}\right) \qquad (35)$$

$$u_c^2(b_{ij}) = u^2\left(b_{ij}^{\mu}\right) + u^2\left(b_{ij}^{h_i}\right) + u^2\left(b_{ij}^{R_S}\right) + u^2\left(b_{ij}^{D_0/s}\right) + u^2\left(b_{ij}^{z_0}\right) \qquad (36)$$

Since the analytical relationship between the influence coefficients and the relative main influence parameters is not knownand each influence coefficient is computed by numerical simulations for fixed values of the influence parameter, then each uncertainty contribution that appears into eq.(35) and (36) can be determined by proper numerical simulations carried out by varying each influence parameter in the range defined by its mean value and its typical uncertainty. As an example, if the RCM is used for the RS analysis of components made by steel or aluminum, having Poisson ratio $\mu \approx 0.3$, then taking into account that the typical uncertainty of the material Poisson´s ratio is ±*3%*, the evaluation of the uncertainty contributions $u\left(a_{ij}^{\mu}\right)$ and $u\left(b_{ij}^{\mu}\right)$ can be carried out by computing the variations of a_{ij} and b_{ij} that occur when the Poisson's ratio varies in the range 0.291-0.309. The uncertainty contributions $u\left(a_{ij}^{\mu}\right)$ and $u\left(b_{ij}^{\mu}\right)$ so computed, are reported in Appendix 1.

Using a similar approach, Appendix 2 shows the uncertainty contributions $u\left(a_{ij}^{h_i}\right)$ and $u\left(b_{ij}^{h_i}\right)$ computed by considering a typical uncertainty of h_i of about 0.01 mm (95% confidence level); the experimental evidence has shown that such an uncertainty value is commonly obtained by using proper automatic systems as that produced by SINT Technology [15], whereas higher uncertainty occurs if the groove depth is measured with common devices.

Considering the influence of the radius of curvature (R_S) of the component surface, the strain gage rosette manufacturer declares that the influence of the surface curvature is in general negligible for radius higher than 2-3 m. Based on the Civin research [6] for R_S=1250 mm the maximum strain deviations is 4.6%. Therefore, accurate RS analysis on component with curved surface having $R_S \leq 1$ m, requires proper numerical simulations to determine the correct influence coefficients.

Also, the uncertainty contributions $u\left(a_{ij}^{D_0/s}\right)$ and $u\left(b_{ij}^{D_0/s}\right)$ of the non-dimensional geometrical parameter (D_o/s) can be computed by considering that for the usual value D_o = 14 mm its typical deviation is about ±0.1 mm, whereas for the common value of the annular groove thickness s=2 mm its typical deviation is ±0.05 mm. Proper numerical simulation carried out by considering such deviations have permitted to compute the detected uncertainty contributions $u\left(a_{ij}^{D_0/s}\right)$ and $u\left(b_{ij}^{D_0/s}\right)$ reported in Appendix 3.

Finally, the uncertainty contributions $u\left(a_{ij}^{z_0}\right)$ and $u\left(b_{ij}^{z_0}\right)$ due to the uncertainty of the zero depth offset (z_0) was obtained by considering the maximum error of 0.01 mm and have been reported in Appendix 4.

The analysis of the maximum uncertainty contributions reported in Appendix 1, 2, 3 and 4 shows that the lower values corresponds to the Poisson's ratio followed by the D_o/s ratio, then by the groove depth h_i and the zero depth offset z_0 that is therefore the parameter that exhibits the maximum influence on the influence coefficients a_{ij} and b_{ij} (j =1,..i; i = 1,…N).

After the evaluation of the uncertainties $u(a_{ij})$ and $u(b_{ij})$ by using eq.(35) and (36), the uncertainties of the material dependent influence coefficients A_{ij} and B_{ij} (j =1,..i; i = 1,…N) given by eq.(15) and (16), can be obtained immediately by applying equation (23) to these last equations; it follows:

$$u_c^2(A_{ij}) = \left(\frac{a_{ij}}{1+\mu}\right)^2 u^2(E) + \left(-\frac{Ea_{ij}}{(1+\mu)^2}\right)^2 u^2(\mu) + \left(\frac{E}{1+\mu}\right)^2 u^2(a_{ij}) \tag{37}$$

$$u_c^2(B_{ij}) = b_{ij}^2 u^2(E) + E^2 u^2(b_{ij}) \tag{38}$$

Like the Poisson's ratio, the typical uncertainty of the Young modulus is about ±3%.

Also, after the evaluation of the uncertainty of the influence coefficients by using eqs.(37) and (38), by applying equation (23) to equations (17-19) the uncertainties of the three stress components are given by using the following formulas:

$$u_c^2(P_i) = \left(\frac{1}{A_{ii}^2}\cdot\frac{E.p_i}{1+\mu} - \sum_{j=1}^{i-1} A_{ij}\,.\,P_j\right)^2 .\,u_c^2(A_{ii}) + \left(\frac{1}{A_{ii}}\cdot\frac{p_i}{1+\mu}\right)^2 .u_c^2(E) + \left(\frac{1}{A_{ii}}\cdot\frac{E.p_i}{1+2\mu+\mu^2}\right)^2 u_c^2(\mu) +$$
$$+ \left(\frac{1}{A_{ii}}\cdot\frac{E}{1+\mu}\right)^2 u_c^2(p_i) + \sum_{j=1}^{i-1}\left(\frac{P_j}{A_{ii}}\right)^2 .u_c^2(A_{ij}) + \sum_{j=1}^{i-1}\left(\frac{A_{ij}}{A_{ii}}\right)^2 .u_c^2(P_j) \tag{39}$$

$$u_c^2(Q_i)=\left(\frac{1}{B_{ii}^2}E.q_i-\sum_{j=1}^{i-1}B_{ij}\,.\,Q_j\right)^2 u_c^2(B_{ii})+\left(\frac{q_i}{B_{ii}}\right)^2 u_c^2(E)+\sum_{j=1}^{i-1}\left(\frac{Q_j}{B_{ii}}\right)^2 u_c^2(B_{ij})+\sum_{j=1}^{i-1}\left(\frac{B_{ij}}{B_{ii}}\right)^2 u_c^2(Q_j)+ + \left(\frac{E}{B_{ii}}\right)^2 u_c^2(q_i) \tag{40}$$

$$u_c^2(T_i)=\left(\frac{1}{B_{ii}^2}E.t_i-\sum_{j=1}^{i-1}B_{ij}\,.\,Q_j\right)^2 .u_c^2(B_{ii})+\left(\frac{t_i}{B_{ii}}\right)^2 u_c^2(E)+\sum_{j=1}^{i-1}\left(\frac{T_j}{B_{ii}}\right)^2 u_c^2(B_{ij})+\sum_{j=1}^{i-1}\left(\frac{B_{ij}}{B_{ii}}\right)^2 u_c^2(T_j)+ + \left(\frac{E}{B_{ii}}\right)^2 u_c^2(t_i) \tag{41}$$

It is to be noted that although eqs.(39)-(41) are quite laborious, their solution requires only the results obtained by the previous formulas.
Finally, by applying equation (23) to equation (20) the following formula for the uncertainty of the computed principal stresses is obtained:

$$u_c^2(\sigma_{1,2i})=u_c^2(P_i)+\left(\frac{Q_i}{\sqrt{Q_i^2+T_i^2}}\right)^2 u_c^2(Q_i)+\left(\frac{T_i}{\sqrt{Q_i^2+T_i^2}}\right)^2 u_c^2(T_i) \tag{42}$$

Taking into account the correction of the calculated residual stresses from the stresses induced by the milling and the effects of core axis inclination, the final uncertainty of the residual stresses is obtained:

$$u_c^2(\sigma_{c1,2i})=u_c^2(\sigma_{1,2i})+u_c^2(\sigma_{1,2ind})+u_c^2(\sigma_{incl}) \tag{43}$$

As above mentioned, the extended uncertainty $U_{1,2i}$ of the principal residual stresses can be estimated by considering a normal distribution corresponding to a coverage probability, p, of approximately 95% ($k=2$), so that the principal residual stresses at *i-th* step are given by :

$$\sigma_{r1,2i}=\sigma_{c1,2i}\pm U_{1,2i}=\sigma_{c1,2i}\pm 2u_c(\sigma_{c1,2i}) \tag{44}$$

The uncertainty of the β angle is calculated by applying equation (23) to equation (21), i.e.:

$$u_c^2(\beta_i)=\left(\frac{1}{2}\cdot\frac{Q_i}{Q_i^2+T_i^2}\right)^2 u_c^2(T_i)+\left(\frac{1}{2}\cdot\frac{T_i}{Q_i^2+T_i^2}\right)^2 u_c^2(Q_i) \tag{45}$$

The corresponding extended uncertainty is calculatedby using the same approach, i.e.:

$$\beta_{ri}=\beta_i\pm U_{\beta i}=\beta_i\pm 2u_c(\beta_i) \tag{46}$$

6. CONCLUSIONS

In the present work, by a systematic analysis of the main error sources, the procedure to correct the main errors affecting the RS computed by the RCM, as well as a procedure to evaluate the RS uncertainty due to the main influence parameters, are proposed.
When the analytical relationship between the RS and the influence parameters was not known, the sensitivity coefficient involved in the calculation of the various uncertainty contributions have been determined by proper numerical simulations performed by varying the considered parameter in the corresponding range defined by it mean value and its uncertainty interval.
Such proposed procedures allow the user to correct the main errors on the computed principal RS and their orientation, as well as to estimate their uncertainty by propagating the effects of the various influence parameters.

In order to highlight the actual values of the error and of the uncertainty associated with the main influence parameters, practical applications of the proposed procedures for various experimental conditions and RS distributions are in progress.

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APPENDIX 1

	h_i[mm]	$u(a_{ij}^{\mu})[\cdot 10^{-9}]$							
i=1	0,6	1,68							
i=2	1,05	3,04	1,45						
i=3	1,45	4,03	2,25	1,29					
i=4	1,85	4,77	2,89	1,96	1,24				
i=5	2,3	5,47	3,44	2,51	1,88	1,34			
i=6	2,8	6,03	3,84	2,91	2,37	1,99	1,28		
i=7	3,5	6,52	4,19	3,24	2,75	2,52	2,01	1,50	
i=8	5	6,87	4,50	3,54	3,08	2,90	2,55	2,47	2,01
		j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8

	h_i[mm]	$u(b_{ij}^{\mu})$ $[\cdot 10^{-9}]$							
i=1	0,6	1,98							
i=2	1,05	2,81	1,52						
i=3	1,45	2,92	1,98	1,26					
i=4	1,85	2,56	1,98	1,55	1,10				
i=5	2,3	2,12	1,80	1,57	1,38	1,19			
i=6	2,8	2,76	1,31	1,30	1,29	1,36	1,12		
i=7	3,5	2,15	0,88	0,75	0,92	1,18	1,30	1,84	
i=8	5	2,51	0,94	1,17	0,26	0,87	0,55	1,93	2,81
		j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8

APPENDIX 2

	h_i[mm]	$u\left(a_{ij}^{h_i}\right)[\cdot 10^{-9}]$							
i=1	0,6	4,26							
i=2	1,05	7,56	5,67						
i=3	1,45	10,73	7,06	3,88					
i=4	1,85	13,07	8,77	6,52	3,62				
i=5	2,3	12,98	9,42	6,88	5,16	2,08			
i=6	2,8	12,35	9,22	6,98	4,75	2,57	0,31		
i=7	3,5	10,73	8,14	6,22	3,77	1,02	1,42	3,24	
i=8	5	9,34	7,18	5,29	3,11	1,08	1,24	5,61	9,97
		j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8

	h_i[mm]	$u\left(b_{ij}^{h_i}\right)[\cdot 10^{-9}]$							
i=1	0,6	3,25							
i=2	1,05	7,07	5,57						
i=3	1,45	10,32	5,61	2,97					
i=4	1,85	14,99	8,57	5,83	2,81				
i=5	2,3	19,11	11,54	6,78	4,52	2,58			
i=6	2,8	18,37	11,38	8,23	5,24	2,03	0,18		

i=7	3,5	19,05	12,20	9,05	5,47	1,80	0,92	2,86	
i=8	5	19,86	12,95	9,83	5,71	1,86	0,81	4,72	9,27
		j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8

APPENDIX 3

	h_i[mm]	$u\left(a_{ij}^{D_{o/s}}\right)$ [$\cdot 10^{-9}$]							
i=1	0,6	3,74							
i=2	1,05	4,10	2,93						
i=3	1,45	4,67	2,63	3,43					
i=4	1,85	5,61	4,85	2,22	1,25				
i=5	2,3	6,12	3,59	2,86	1,76	2,02			
i=6	2,8	5,49	2,90	1,90	1,11	0,82	0,93		
i=7	3,5	5,23	3,06	1,88	1,12	0,78	1,22	2,12	
i=8	5	4,95	2,64	1,72	0,90	0,22	0,96	2,68	4,46
		j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8

	h_i[mm]	$u\left(b_{ij}^{D_{o/s}}\right)$ [$\cdot 10^{-9}$]							
i=1	0,6	3,16							
i=2	1,05	3,98	1,92						
i=3	1,45	5,26	2,77	2,29					
i=4	1,85	6,79	3,53	2,80	2,01				
i=5	2,3	8,82	4,69	3,27	2,37	1,82			
i=6	2,8	9,27	4,58	3,20	2,00	1,35	1,48		
i=7	3,5	8,96	5,10	3,08	2,33	1,83	0,75	2,10	
i=8	5	9,62	5,40	3,63	2,47	1,54	0,84	2,14	3,87
		j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8

APPENDIX 4

	h_i[mm]	$u\left(a_{ij}^{z_o}\right)$ [$\cdot 10^{-9}$]							
i=1	0,6	22,98							
i=2	1,05	30,66	0,60						
i=3	1,45	37,54	7,04	4,60					
i=4	1,85	37,98	0,89	1,25	1,12				
i=5	2,3	41,42	1,46	1,73	1,86	1,22			
i=6	2,8	43,09	2,44	2,40	2,44	2,20	2,28		
i=7	3,5	43,46	4,40	4,49	4,48	5,19	5,19	5,32	
i=8	5	44,63	5,18	4,14	4,79	5,35	5,62	6,25	6,27
		j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8

	h_i[mm]	$u\left(b_{ij}^{z_o}\right)$ [$\cdot 10^{-9}$]							
i=1	0,6	14,47							

i=2	1,05	21,93	0,71						
i=3	1,45	28,35	1,01	1,57					
i=4	1,85	34,78	1,61	0,47	0,52				
i=5	2,3	43,19	0,78	0,35	0,96	0,63			
i=6	2,8	48,54	1,87	1,40	1,77	2,21	1,59		
i=7	3,5	55,62	3,70	3,47	2,72	2,73	2,78	2,41	
i=8	5	65,86	6,49	4,81	5,01	4,85	5,29	6,26	6,77
		j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8