Analysis of a Strip Loop Antenna Located on the Surface of an Open Cylindrical Waveguide Filled with a Resonant Magnetoplasma

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***Abstract*—The electrodynamic characteristics of a circular loop antenna located on the surface of an open waveguide in the form of an axially magnetized plasma column are studied using the integral equation method. The current distribution and the input impedance of the antenna excited by a time-harmonic external voltage are obtained in closed form for the case where the plasma inside the column is resonant.**

I. Introduction

The characteristics of loop antennas immersed in a magnetoplasma have been studied extensively in many works (see, e.g., [1] and references therein). In the past decade, enhanced interest has been shown in the characteristics of antennas operated in the presence of magnetic-field-aligned cylindrical plasma structures capable of guiding the excited electromagnetic waves [2]. Recently, the problem of a loop antenna located on the surface of a plasma column in free space has been solved in the case of a nonresonant magnetoplasma inside the column [3]. In this work, we consider a strip loop antenna located on the surface of a circular column filled with a resonant magnetoplasma. By resonant magnetoplasma, we mean a cold collisionless magnetized plasma in which the refractive index of one of the characteristic waves tends to infinity at a certain angle between the wave vector and an external dc magnetic field [2].

II. Formulation of the Problem and Basic Equations

Consider an antenna having the form of an infinitesimally thin, perfectly conducting narrow strip of half-width d coiled into a circular loop of radius a ($d \ll a$). The antenna is located coaxially on the surface of a uniform circular plasma column placed in free space and aligned with an external dc magnetic field $\boldsymbol{B}_0$ (see Fig. 1) which is parallel to the z axis of a cylindrical coordinate system (ρ, ϕ, z). The medium inside the column is described by a general dielectric tensor with the following nonzero elements: $\varepsilon_{\rho\rho} = \varepsilon_{\phi\phi} = \epsilon_0\varepsilon$, $\varepsilon_{\rho\phi} = -\varepsilon_{\phi\rho} = -i\epsilon_0 g$, and $\varepsilon_{zz} = \epsilon_0\eta$ (here, ϵ_0 is the permittivity of free space). The elements ε, g, and η of the plasma dielectric tensor are functions of the angular frequency ω, and expressions for them can be found elsewhere [2].

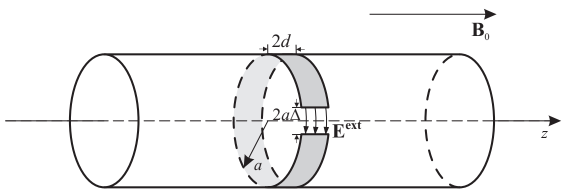

Fig. 1. Geometry of the problem.

The antenna is excited by a time-harmonic ($\sim \exp(i\omega t)$) voltage which creates an electric field with the only nonzero component $E_\phi^{\rm ext}$ in a narrow angular interval (gap) $|\phi - \phi_0| \le \Delta \ll \pi$ on the surface of the strip (i.e., at $\rho = a$ and $|z| < d$):

$$E_\phi^{\rm ext}(a,\phi,z) = \frac{V_0}{2a\Delta}[U(\phi-\phi_0+\Delta) - U(\phi-\phi_0-\Delta)] \times [U(z+d) - U(z-d)]\,. \qquad (1)$$

Here, V_0 is an amplitude of the given voltage, U is a Heaviside function, and Δ is the angular half-width of the gap centered at $\phi = \phi_0$. The excitation field $E_\phi^{\rm ext}$ is represented as

$$E_\phi^{\rm ext} = \sum_{m=-\infty}^{\infty} A_m \exp(-im\phi), \qquad (2)$$

where

$$A_m = \frac{V_0}{2\pi a}\frac{\sin(m\Delta)}{m\Delta}\exp(im\phi_0). \qquad (3)$$

The density $\boldsymbol{J}$ of the electric current excited on the antenna by the field (1) can be sought as

$$\boldsymbol{J} = \boldsymbol{\phi}_0 I(\phi, z)\delta(\rho - a), \tag{4}$$

where $|z| < d$, δ is a Dirac function, and $I(\phi, z)$ is the surface current density which admits the following representation:

$$I(\phi, z) = \sum_{m=-\infty}^{\infty} I_m(z)\exp(-im\phi). \tag{5}$$

To find $I(\phi, z)$, we express the azimuthal (E_ϕ) and longitudinal (E_z) components of the electric field excited by the current (4) in terms of unknown quantities $I_m(z)$ and then use the boundary conditions on the surface of the plasma column ($\rho = a$ and $-\infty < z < \infty$) along with the boundary conditions on the antenna surface ($\rho = a$ and $|z| < d$):

$$E_\phi + E_\phi^{\rm ext} = 0, \;\; E_z = 0. \tag{6}$$

The described procedure makes it possible to obtain integral equations for the above-mentioned unknown quantities and then reduce the problem of the antenna current distribution to solving the corresponding integral equations.

The field excited by the current with the surface density of Eq. (5) can be obtained in a standard way [2]. To do this, the fields inside and outside the plasma column are expressed in terms of cylindrical functions with the appropriate arguments. On the column surface, the fields should satisfy the boundary conditions, which consist in the continuity of the tangential components E_ϕ, E_z, and H_ϕ at the boundary $\rho = a$. As for the H_z component, it is continuous at this boundary if $|z| > d$, and undergoes a jump corresponding to the surface current (4) if $|z| < d$. As a result, omitting some algebra, we get the following expressions for the azimuthal and longitudinal components of the antenna-excited electric field at $\rho = a$:

$$\begin{bmatrix} E_\phi(a,\phi,z) \\ E_z(a,\phi,z) \end{bmatrix} = \sum_{m=-\infty}^{\infty} \exp(-im\phi) \times \int_{-d}^{d} \begin{bmatrix} K_m(z-z') \\ k_m(z-z') \end{bmatrix} I_m(z')dz'. \tag{7}$$

Here,

$$\begin{aligned} K_m(\zeta) &= \sum_n \frac{2\pi a}{N_{m,n}} E^2_{\phi;m,n}(a)\exp\left(-ik_0 p_{m,n}|\zeta|\right) \\ &+ \frac{ik_0}{2\pi}\int_0^\infty \frac{q}{p(q)} \sum_{l=1}^{2}\sum_{k=1}^{2} \frac{B^{(l)}_{mk}}{\Delta^{(l)}_m}\bigg[J_{m+1}(Q_k) \\ &+ \alpha_k m \frac{J_m(Q_k)}{Q_k}\bigg]\exp(-ik_0 p(q)|\zeta|)dq, \end{aligned} \tag{8}$$

$$\begin{aligned} k_m(\zeta) &= \operatorname{sgn}\zeta\bigg\{\sum_n \frac{2\pi a}{N_{m,n}} E_{\phi;m,n}(a)E_{z;m,n}(a) \\ &\times\exp\left(-ik_0 p_{m,n}|\zeta|\right) + \frac{i}{2\pi a\eta}\int_0^\infty \frac{q}{p(q)} \sum_{l=1}^{2}\sum_{k=1}^{2}\frac{B^{(l)}_{mk}}{\Delta^{(l)}_m} \\ &\times n_k Q_k J_m(Q_k)\exp(-ik_0 p(q)|\zeta|)dq\bigg\}, \end{aligned} \tag{9}$$

where J_m is a Bessel function of the first kind of order m, $k_0 = \omega/c$ is the wave number in free space, $E_{\phi;m,n}(\rho)$ and $E_{z;m,n}(\rho)$ are functions describing the distributions over the transverse coordinate ρ of the azimuthal and longitudinal electric-field components of eigenmodes that are guided by the column and have the azimuthal and radial indices m and n, respectively ($m = 0, \pm1, \pm2, ...$ and $n = 1, 2, ...$), $N_{m,n}$ are the norms of the eigenmodes, $p_{m,n}$ are the eigenmode propagation constants normalized to k_0, and $p(q) = (1-q^2)^{1/2}$ is the normalized propagation constant of the characteristic wave of free space for the transverse wave number $q = k_\perp/k_0$ (it is assumed that $\operatorname{Im} p(q) < 0$). Expressions for the fields and norms of the eigenmodes supported by a magnetized plasma column as well as their dispersion properties are discussed in [4]. Other quantities in Eqs. (8) and (9) are written as

$$\begin{aligned} B^{(l)}_{mk} &= -(-1)^k Z_0 \frac{k_0 a}{Q_k J_m(Q_k)}\bigg[\eta n_{k-v}\hat{J}^{(k-v)}_m \mathcal{H}^{(l)}_m \\ &+\eta p\frac{m}{Q^2}J^{(k-v)}_m - n_{k-v}\left(\mathcal{H}^{(l)}_m\right)^2 + p^2\frac{m^2}{Q^4}n_{k-v}\bigg], \\ \Delta^{(l)}_m &= (-1)^l\bigg\{n_2\left[\eta J^{(1)}_m\hat{J}^{(2)}_m - \left(J^{(1)}_m + \eta\hat{J}^{(2)}_m\right)\mathcal{H}^{(l)}_m\right] \\ &-n_1\left[\eta\hat{J}^{(1)}_m J^{(2)}_m - \left(J^{(2)}_m + \eta\hat{J}^{(1)}_m\right)\mathcal{H}^{(l)}_m\right] \\ &+(n_2-n_1)\left[\left(\mathcal{H}^{(l)}_m\right)^2 - p^2\frac{m^2}{Q^4}\right] \\ &+\eta p\frac{m}{Q^2}\left[J^{(1)}_m - J^{(2)}_m + \hat{J}^{(1)}_m - \hat{J}^{(2)}_m\right]\bigg\}, \\ n_k &= -\frac{\varepsilon}{p(q)g}\left[p^2(q) + q_k^2(p(q)) + \frac{g^2}{\varepsilon} - \varepsilon\right], \\ Q_k &= k_0 a q_k(p(q)),\; l=1,2,\; k=1,2,\; v=(-1)^k, \end{aligned} \tag{10}$$

where

$$J^{(k)}_m = \frac{J_{m+1}(Q_k)}{Q_k J_m(Q_k)} + m\frac{\alpha_k}{Q_k^2},\;\; \hat{J}^{(k)}_m = \frac{J_{m+1}(Q_k)}{Q_k J_m(Q_k)} - m\frac{\beta_k}{Q_k^2},$$

$$\mathcal{H}^{(l)}_m = \frac{H^{(l)}_{m+1}(Q)}{QH^{(l)}_m(Q)} - \frac{m}{Q^2},\;\; Q = k_0 a q,$$

$$\alpha_k = -1 + \frac{p^2(q) + q_k^2(p(q)) - \varepsilon}{g},\;\; \beta_k = 1 + \frac{p(q)}{n_k},$$

$$\begin{aligned} q_k(p) &= \frac{1}{\sqrt{2}}\bigg\{\varepsilon - \frac{g^2}{\varepsilon} + \eta - \left(\frac{\eta}{\varepsilon}+1\right)p^2 \\ &- \left(\frac{\eta}{\varepsilon}-1\right)(-1)^k\left[\left(p^2-P_b^2\right)\left(p^2-P_c^2\right)\right]^{1/2}\bigg\}^{1/2}, \\ P_{b,c} &= \bigg\{\varepsilon - (\eta+\varepsilon)\frac{g^2}{(\eta-\varepsilon)^2} + \frac{2\chi_{b,c}}{(\eta-\varepsilon)^2} \\ &\times\left[\varepsilon g^2\eta\left(g^2-(\eta-\varepsilon)^2\right)\right]^{1/2}\bigg\}^{1/2}, \end{aligned} \tag{11}$$

$\chi_b = -\chi_c = -1$, Z_0 is the impedance of free space, and $H^{(1)}_m$ and $H^{(2)}_m$ are Hankel functions of the first and second kinds, respectively. In the above formulas, q_1 and q_2 are the transverse wave numbers of two characteristic waves of a magnetoplasma for $p = p(q)$. The propagation constants $p = p_{m,n}$ of eigenmodes of the plasma column are found as roots of the equation $\Delta_m^{(2)}(p) = 0$.

Using the boundary conditions (6) for the tangential components of the electric field on the antenna surface and allowing for Eqs. (2), (3), and (7), we can obtain the integral equations

$$\int_{-d}^{d} \begin{bmatrix} K_m(z-z') \\ k_m(z-z') \end{bmatrix} I_m(z')dz' = \begin{bmatrix} -A_m \\ 0 \end{bmatrix} \qquad (12)$$

for the complex amplitudes of the angular harmonics $I_m(z)$ of the surface current density, where $|z| < d$.

The forthcoming analysis is determined by whether the plasma inside the column is resonant or nonresonant, in which cases either the condition $\operatorname{sgn}\varepsilon \neq \operatorname{sgn}\eta$ or the condition $\operatorname{sgn}\varepsilon = \operatorname{sgn}\eta$ holds. In this paper, we restrict ourselves to consideration of a resonant magnetoplasma in the case where $\varepsilon > 0$ and $\eta < 0$, which is of interest for many important applications [2].

III. Solution of the Integral Equations

The behavior of solutions of the obtained integral equations is determined by the properties of their kernels. We can represent the kernels $K_m(\zeta)$ and $k_m(\zeta)$ as the sums of singular ($K_m^{(s)}$ and $k_m^{(s)}$) and regular ($K_m^{(r)}$ and $k_m^{(r)}$) parts. The singular parts tend to infinity for $\zeta \to 0$, whereas the regular parts remain finite in this limit and can be taken at $\zeta = 0$ if the antenna is so narrow that the following conditions take place:

$$d \ll a, \;\; d \ll a|\eta/\varepsilon|^{1/2}, \;\; k_0 d \ll 1, \\ (k_0 d)^2 \max\{|\varepsilon|, |g|, |\eta|\} \ll 1. \qquad (13)$$

The singular part of $K_m(\zeta)$ can be written as

$$K_m^{(s)}(\zeta) = iZ_0 \left(-\frac{k_0^2 a}{2} \int_0^\infty J_{m+1}^2(k_0 a q) \exp(-k_0 q|\zeta|) dq + \frac{m^2}{\pi k_0 a^2} \frac{1}{|\varepsilon\eta|} \int_0^\infty \frac{\mathcal{I}_m^2(k_0 a q)}{q U_m(q)} \exp\left(-k_0 \sqrt{\frac{\varepsilon}{|\eta|}} q|\zeta|\right) dq \right) + \sum_{n=n^*}^{\infty} \frac{2\pi a}{N_{m,n}} E_{\phi;m,n}^2(a) \exp\left(-ik_0 p_{m,n}|\zeta|\right). \qquad (14)$$

Here, $U_m(q) = \mathcal{I}_{m+1}^2(k_0 a q) + |\varepsilon\eta|^{-1}\mathcal{I}_m^2(k_0 a q)$, where $\mathcal{I}_m$ is a modified Bessel function of the first kind of order m. The quantity n^* in Eq. (14) is a certain large positive integer such the propagation constants of eigenmodes with the radial indices $n > n^*$ can be approximated as $p_{m,n} = \mu_n^{(m+1)} \sqrt{-\varepsilon/\eta}\, \xi (k_0 a)^{-1}$, where $\mu_n^{(m+1)}$ is the nth zero of the Bessel function J_{m+1} and ξ is a certain quantity depending on η. It can be shown that in the case $|\eta| \gg 1$, $\xi \simeq 1$.

Under conditions (13), the singular part (14) of the kernel $K_m(\zeta)$ in the limit $\zeta \to 0$ can be represented in the form

$$K_m^{(s)}(\zeta) = -iZ_0 \frac{k_0}{2\pi} \left(\frac{1}{\alpha_m} \ln \frac{|\zeta|}{2a} - \gamma_m \right), \qquad (15)$$

where

$$\alpha_m = i(k_0 a)^2 \varepsilon_{\text{eff}} [m^2 - i(k_0 a)^2 \varepsilon_{\text{eff}}]^{-1}, \\ \varepsilon_{\text{eff}} = \frac{\sqrt{|\varepsilon\eta|}(|\varepsilon\eta| + 1)}{2i\sqrt{|\varepsilon\eta|} - 4(|\varepsilon\eta| + 1)}. \qquad (16)$$

It can similarly be shown under the same conditions that $k_m^{(s)}(\zeta) = mC_m/\zeta$ for $\zeta \to 0$ and $k_m^{(r)}(0) = 0$. The coefficients γ_m and C_m in the above formulas are independent of ζ and are found numerically for each m.

The derived representations of the kernels make it possible to obtain the following solution for $I_m(z)$, which satisfies both equations in (12) simultaneously:

$$I_m(z) = \frac{2i}{Z_0 k_0 \sqrt{d^2 - z^2}} \frac{A_m \alpha_m}{\ln(4a/d) - S_m}, \qquad (17)$$

where $S_m = \alpha_m[-\gamma_m + 2\pi i K_m^{(r)}(0)/(Z_0 k_0)]$. Substituting (17) into (5) yields $I(\phi, z)$. Integrating $I(\phi, z)$ over z between $-d$ and d, we obtain the total antenna current $I_\Sigma(\phi)$.

Note that in a general case, the summation over m in the current representation can be made only numerically. A closed-form expression for the current distribution can be derived if the strip is so narrow that the inequality $\ln(4a/d) \gg |S_m|$ is valid. Then, under the additional condition $d \ll 2a\Delta \ll a$, we can neglect the quantities S_m and make steps similar to those performed in [1]. As a result, we deduce

$$I_\Sigma(\phi) = -\frac{iV_0 \pi h}{Z_0 k_0 \ln(4a/d)} \frac{\cos[(\pi - \phi + \phi_0)ha]}{\sin(\pi h a)}, \qquad (18)$$

where $0 \le \phi - \phi_0 \le \pi$ and $h = k_0(1+i)\sqrt{\varepsilon_{\text{eff}}/2}$.

IV. Numerical Results

Using the above-described approach, we have calculated the current distribution and the input impedance of the antenna for some cases of interest. Calculations have been performed for the following values of parameters which can easily be realized under laboratory conditions: the plasma density inside the column is equal to $N = 10^{13}$ cm^{-3}, the external dc magnetic field $B_0 = 800$ G, and the angular frequency $\omega = 1.7 \times 10^8$ s^{-1}. The chosen values correspond to the case of a resonant plasma, for which the diagonal elements of the dielectric tensor have the opposite signs: $\varepsilon = 1.62 \times 10^2$ and $\eta = -1.1 \times 10^6$. It is assumed that the midpoint of the region to which the given voltage is supplied has the azimuthal coordinate $\phi_0 = 0$, the antenna radius a coinciding with the column radius is equal to $a = 2.5$ cm, $d/a = 0.02$, and $\Delta = 0.05$ rad.

The behavior of the current distribution is notably determined by the presence of eigenmodes guided by the plasma column, which can support an infinite number of propagating eigenmodes whose contribution to the kernels of Eq. (12) is singular. As an example, Fig. 2 shows the components of the fields $\boldsymbol{E}_{m,n}(\rho)$ and $\boldsymbol{H}_{m,n}(\rho)$ of an eigenmode with the azimuthal index $m = 1$ and the radial index $n = 18$ as functions of the radial coordinate ρ for the above-mentioned parameters of the column. The relative propagation constant of this eigenmode amounts to $p_{m,n} = 52.26$. It is seen that the mode fields has both large- and fine-scale oscillations over the radius. It is interesting to mention that the number of fine-scale oscillations increases with increasing radial index.

The absolute value $|I_\Sigma(\phi)|$, normalized to its maximum $|I_{\Sigma\max}|$, and the phase $\theta(\phi) = \arctan(\operatorname{Im} I_\Sigma(\phi)/\operatorname{Re} I_\Sigma(\phi))$

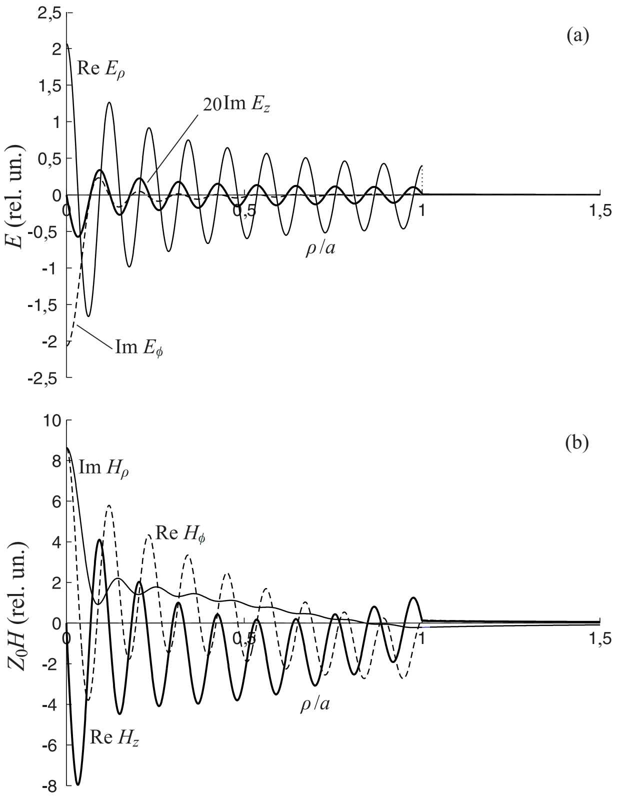

Fig. 2. Fields (a) $\boldsymbol{E}_{m,n}(\rho)$ and (b) $\boldsymbol{H}_{m,n}(\rho)$ of an eigenmode with the indices $m = 1$ and $n = 18$ for $N = 10^{13}$ cm^{-3}, $B_0 = 800$ G, $\omega = 1.7 \times 10^8$ s^{-1}, and $a = 2.5$ cm.

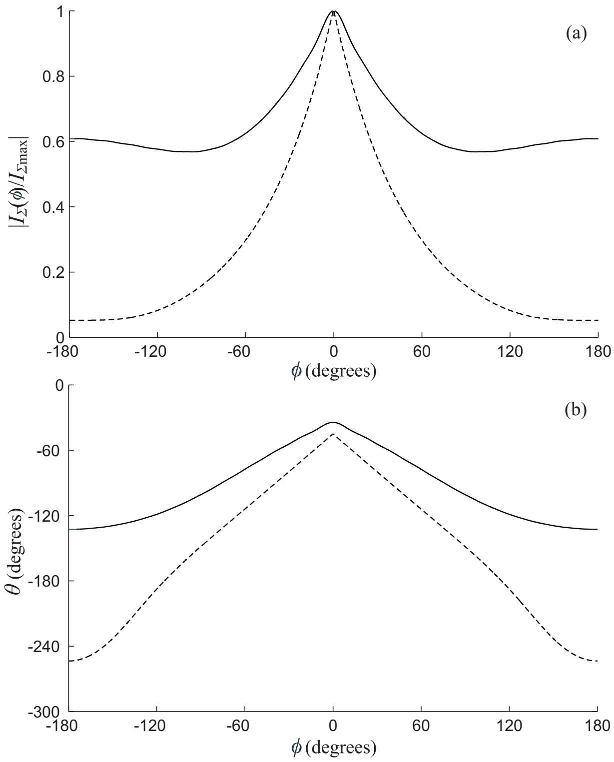

Fig. 3. Normalized amplitude (a) and phase (b) of the antenna current as functions of the angle ϕ if the antenna is located on the surface of a plasma column (solid line) and in a homogeneous magnetoplasma (dashed line) for $a = 2.5$ cm, $d/a = 0.02$, $\Delta = 0.05$ rad, $\phi_0 = 0$, $N = 10^{13}$ cm^{-3}, $B_0 = 800$ G, and $\omega = 1.7 \times 10^8$ s^{-1}.

of the antenna current are shown in Fig. 3 as functions of the angle ϕ. In addition, Fig. 3 shows the current distribution of the same antenna located in a homogeneous magnetoplasma the parameters of which coincide with those of the plasma medium inside the column. It is evident that in this case, the dependence $|I_\Sigma(\phi)/I_{\Sigma\max}|$ for the antenna located on the surface of the plasma column qualitatively resembles the corresponding dependence for a loop antenna in a homogeneous magnetoplasma. If the antenna of the same radius were located in free space, it would have a quasi-uniform current distribution. The presence of the plasma column evidently leads to an essentially different current distribution of the antenna.

The input impedance $Z = V_0/I_\Sigma(\phi_0) = R + iX$ of the antenna depends on the parameters of the problem in a fairly complex way. For the parameters chosen above, it amounts to $Z = (8.36 + i7.95)\ \Omega$, and the value of the radiation resistance R is almost completely determined by the eigenmodes of the plasma waveguide.

V. Conclusion

In this paper, we have obtained the solution to the problem of the current distribution and the input impedance of a strip loop antenna located on the surface of an axially magnetized plasma column in free space and operated in the resonant frequency band of a magnetoplasma. The found solution describes the distribution of the surface-current density both along and across the strip and makes it possible to study the electrodynamic characteristics of such an antenna as functions of its parameters as well as the parameters of the plasma column.

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