An approach to the concept of statistical distribution: a pedagogical path based on Guided Inquiry

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Abstract

This paper describes a teaching approach to the concept of distribution that uses a specific activity related to the field of statistical mechanics. The concept of velocity distribution of a particle system is dealt with using an Inquiry Based approach involving an experimental examination of Maxwell's Distribution. Some outcomes of a teaching experiment are described.

Keywords: Teaching methods and strategies, statistical distribution, Inquiry Based Science Education

Introduction

Mathematical representations are relevant to scientifically describe and explain phenomena, but very often students have difficulty in getting their uses and limits. Many research results have highlighted how much the understanding of the distribution concept in physics can be influenced by the conceptual difficulties connected with its mathematical representation [1,2]. In his classical work "Language of art", Goodman [3] outlines that the mathematical representations concerning drawing and interpreting graphs involve the using/understanding of a ''symbol scheme correlated with a field of reference''.

The distribution concept is a fundamental component of statistical thinking. It can be seen as a lens through which the variability that exists in various phenomena can be looked at with greater clarity [4].

Here, we propose a learning approach to the concept of distribution in physics based on laboratory and modelling activities aimed at overcoming the above-mentioned difficulties. For this purpose, a specific activity, related to the field of statistical mechanics, is proposed, in which the concept of distribution of velocity/energy of a particle system is dealt with by using an Inquiry Based [5] experimental analysis of Maxwell's distribution.

Inquiry learning environments can be organised in different ways [6,7] in relation with the spectrum of inquiry levels that accounts for the gradual shift of the focus of research control from the teacher to the student. In particular, a Guided Inquiry Approach [6] is intended as an approach where the teacher provides students with the research questions, concerning a problematic situation, and students are requested to design the procedures to conduct an appropriate investigation. Students are encouraged to plan and carry out their laboratory activities by formulating hypotheses, collecting and analysing data, for the purpose of gaining a more meaningful understanding of the proposed problematic situation.

Our learning environment (the Workshop - W) is based on the methodology of the Guided Inquiry Approach and uses experiments and models, previously published [8], for the determination of the velocity distribution of the electron gas emitted by a metal as a result of thermionic effect.

In the following section we report the main characteristics of the workshop. Then, we describe the characteristics of our teaching experiment and some preliminary result of the assessment procedure. In the last section findings and limits of our research are outlined and suggestions are proposed for the improvement of the workshop methodology.

Workshop

The W inquiry context involved the development of methods (experimental and theoretical) aimed at analysing the velocities/energies of a system of particles. As a consequence, the main idea to develop concerned a method to filter particles having a well-defined velocity/energy (or within in a given interval). In the specific context of our research, the idea of "electron velocity filtering" by means of electric fields was the starting point, because it can naturally introduce students to the concept of a distribution of frequencies of particles having velocities in a given interval. The proposed experiments analyse the distribution of energies of thermionic electrons and are discussed in detail in Battaglia et.al [8].

The workshop was developed in different stages characterized by different inquiry procedures. According to Windschitl's et.al [9] theoretical framework, the different stages focus on different aspects of scientific reasoning and inquiry procedures.

1. The first stage involved the presentation of the inquiry context to the students and the definition of the main problem by setting the broad parameters to be investigated. The problem of identifying methods that can emphasize the distribution of some variables (energy, velocity, etc.) among particles of a system (atom, molecules, electron) was analysed. The most popular students' ideas and proposals were discussed by the whole class and difficulties in performing experiments making evident the various characteristics were pointed out. The instructor illustrated several scientific publications facing the problem of experimental determination of the velocity distribution of a system of atoms/molecules.

Some historical papers have been described [10,11] and the research was oriented toward the following problem:

"To establish the characteristics of the electrons gas emitted by thermionic effect through the typical variables of statistical mechanics."

2. The second phase was aimed at summarizing what students knew and what they should know. Class discussion was oriented at identifying and making explicit their representations about the problem and at defining experimental situations able to represent the problem. The awareness of the necessary additional information allowed students to make explicit their research question:

"How it is possible to realise an electron velocity analyser able to show how many electrons have velocities in given intervals?"

3. The third stage begun with a discussion of the different approaches proposed by students aimed at making explicit the design problems of such electron velocity analyser.

4. In the fourth stage students were divided into groups of up to 4 people. The groups worked independently, by designing the measuring apparatus and using the tools (worksheets, circuit diagrams, instructions for fitting procedures) provided by the teacher at their request. Each group was responsible of the experimental setting, as well as of the choice of the different kinds of measurements to be performed.

5. In the fifth stage students were mainly involved in data analysis. After a great group discussion, during which each group compared its apparatus and scheduled measurements with the ones proposed by the others groups, all the students understood the need to perform

a set of measurement maintaining constant the temperature of emitted electrons (and, consequently, the filament temperature), although many students were not fully able to explicitly and formally see the relationship between the values of current and the ranges of electron velocities. Each group independently performed three sets of experiments by fixing, for each set, the filament temperature and varying the retarding anode potential from zero to values that allow a current in the range of sensitivity of our micro-ammeter.

6. The final stage was devoted at pointing out arguments for appropriate explicative model building [12,13]. The starting point was a great group discussion aimed at the formalization of a possible model that would put in relation the current with the electron velocity distribution. By comparing the results of the different groups the limits of the model became explicit.

Methodology

The W experimentation was carried out at the Faculty of Engineering of University of Palermo from March to May 2012 by one of the authors (O. R. B.) and involved 20 hours of students' activities. 43 students, that already completed the curricular mathematical and numerical calculation courses, as well as the general physics courses, participated to the W. In such courses they already faced the problem of statistical distributions, although only from a theoretical point of view.

During the W designing phase, the authors informally interviewed four lectures/professors teaching physics at the engineering faculty, with the aim of collecting information about the typical difficulties encountered by undergraduates on the learning of the distribution concept, during the attendance of a traditional lecture-based physics course. They recognised that this "one-way" teaching approach hardly permits to receive a direct feedback of the students' learning during the course. They identified the greatest learning difficulty of students in their ability to apply their theoretical background of knowledge to the understanding and solution of practical/experimental situations.

The problematic situation proposed at the beginning of our W was really new for our student and, for these reasons, we decided to shape the W by following a Guided Inquiry approach, with the main objective of facilitating the students' understanding about the identification of appropriate selection methods, reading a distribution graph and identification of relevant quantities.

Students' learning was assessed by comparing the answers to a specifically designed questionnaire before and after the W. Moreover, a qualitative analysis of videos registering students' experimental activities and discussions was also performed. In fact, many research papers [14-16] show that a detailed analysis of the phrases or utterances used by students when carrying out an activity involving human-to-human interaction, can provide evidence of the cognitive style(s) used when tackling a given issue or problem.

The administered questionnaire was aimed at investigating the student knowledge with respect to the distribution concept, and to obtain information for the gauging of the interventions. The questionnaire implementation was carried out by following two main steps:

1. The two questions were generated and, then, reviewed during the meeting with the experienced faculty professors. They report experiments and/or simple situations and are organized by following a thematic ordering. We chose an open-answer questionnaire format, as it guarantees the advantage to highlight the spontaneous responses, avoiding the bias that

may result from suggesting responses to individuals, typical of closed-answer items. However, we were aware that the analysis of the outcomes from open-ended questions usually needs an extensive coding of the answers and, for this reason, we performed this action firstly during the questionnaire validation phase (see point II), and, secondly, with the answers provided by the students selected for this study.

2. The second step concerned the questionnaire validation process aimed at evaluating the appropriateness, meaningfulness, and usefulness of the specific items. The researchers first tried to search for all possible answers that a hypothetical student could supply when facing the problematic situations described in each item. This analysis was conducted independently of the observation with our student sample (hence the term 'a-priori'), in order to provide a reference point for the subsequent study of the observation ('a-posteriori') data. According to Brousseau [17], and other researchers [18,19] a search for possible student answering strategies can be very useful to highlight weak points in the questions, and modify them before administering the questionnaire. The answer lists, provided independently by the three researchers, were compared and discussed in a form of content validation [20], and a revised version of the questionnaire and of the answer list were developed. Successively, the questionnaire was administered to a sample of five engineering students not participating to the W, in order to test our pilot validation on learners having received the same traditional physics instruction of our research sample. A focus group was conducted with these students in order to clarify the meaning of some unclear answers.

At the end of the workshop the same questionnaire was re-administered (post-test), in order to verify the instruction effectiveness, but the students were informed of the post test administration only immediately before it was given them.

The analysis of students' answers was separately performed by the researchers and was based on the careful reading of student answers within a framework provided by domain-specific expertise.

Results of the pedagogical experimentation

Qualitative analysis

The workshop design explicitly requested students to work in small groups, discussing the results within the group itself and/or with the teacher (usually after each measurement or analysis stage). A class discussion was the typical conclusion of each workshop stage.

Below we report some aspects of the learning paths that emerged, mostly during the group discussions, in order to make explicit the students' learning difficulties and progresses. To do this, we report some transcripts of discussions[1] between students performing a task in the same group, or between students and the instructor, or general questions raised by the students.

A learning difficulty, identified at the beginning of the W, was the understanding of the model of electron gas that describes electrons as a gas similar to the ideal gas studied in thermodynamics.

Students had already studied the thermionic emission, but they had never thought about the characteristics of the emission rate. A few students, in fact, were aware that electrons can be

[1]Interview excerpts are not always literally translated from Italian into English. We tried to convey the sense of the originals, rather than reporting the exact terms and expressions used by students. Only the typical expressions we identified as relevant for the analysis are directly translated.

emitted at different speeds, but the majority thought of a single output speed. A large number showed difficulties in giving a meaning to the concept of "velocity distribution of emitted electrons". In fact, the majority represented the electrons emitted all at the same speed by using a constant distribution. Moreover, some believed that electrons could have velocities distributed according to a specific law, but only when they are within the metal and that they lose this property once out of the metal.

The understanding of the methodology used to select the electrons with velocities in a given range was a fundamental aspect of the learning path. Several students easily identified the voltage or the electric field applied between the two electrodes as the responsible for the selection of the electrons on the basis of their kinetic energy.

A typical student idea was:

"*The presence of such a retarding voltage has a filtering effect, in the sense that the higher the voltage, the lower the anodic current, and therefore a smaller amount of electrons will reach the anode. This retarding effect is due to the presence of an electric field between anode and cathode that opposes to the electron motion.*"

Many students proposed arguments based on the principle of mechanical energy conservation, identifying a value of voltage above which there is no current, but did not identify how it is possible to arrive at this value of zero current.

For example:

"…the electric field between anode and cathode is opposite to the motion of emitted electrons and ...in other words it supplies a way to discriminate electrons with more kinetic energy from those having less energy. By remembering the formula of kinetic energy ($E_k = ½mv^2$), we can conclude that the electrons arriving to the anode are those with a greater velocity.

By synthesizing, the method of retarding potential was easily understood by the students: they showed to understand that applying a negative potential with respect to the emitter allows us to make a selection of electrons on the base of their speed. Moreover, they were able to deduce from the current-voltage exponential graph that the electrons were not emitted all with the same speed, even if they were not yet able to determine the shape of the distribution.

A typical argumentation was:

"*To measure the velocity distribution we apply a field ...: increasing this field, if all electrons had the same speed we should note a zero current, beyond a certain field strength. Actually, since the electrons have different speeds, a number of electrons is still able to reach the anode*."

The analytical procedure describing the relationship between the voltage-current characteristic and the velocity distribution of the electrons was a critical aspect of the whole learning path. No student was able to identify it.

It is well known [8] that for high values of filament temperature the agreement between the experimental data and the theoretical curves is not good. Some students developed some argumentations on this issue. For example, some reported as a motivation the high space charge density which leads to a high interaction between the electrons, thus violating the hypothesis of weak interaction of the particles each other. For example:

"When the filament voltage increases, the temperature of the cathode also increases and thus the electron emission is greater. This makes the space charge density no longer negligible, and this implies that the particles begin to interact with each other."

Quantitative analysis: pre/post questionnaire comparison

The pre/post-instruction questionnaire was structured in 2 items. Below, we report the items, our categorization and the results obtained from the pre/post-instruction comparison.

Item n. 1 describes (in a simplified form) a historic experimental apparatus (the Zartman apparatus [21]), which has been used to determine the speed distribution of bismuth gas molecules emitted in a high temperature oven. These molecules, which are emitted at different speeds, can leave a mark on a drum rotating at a constant velocity, on which the shape of the distribution is thus determined.

Figure 1 represents the categorization of students' answers and their distributions in pre/post tests along following categories of students' answers:

Level 0: Not answer or only partial answers.
Level 1: Student describes the apparatus but does not give any information about the method of selection.
Level 2: Correctly identifies the method of selection.

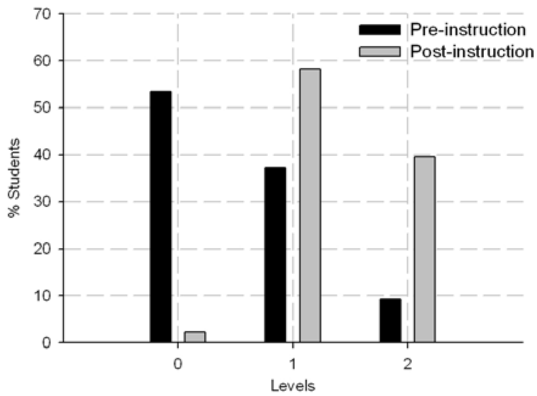

Figure 1. Categories of students' answers in Item 1 and their percentages in each category.
$\chi^2 = 30.19;\ p = 2.8 \cdot 10^{-7}$

By comparing pre-test and post-tests results, we can confidently say that a considerable improvement in the post-instruction answers was obtained. Although the proposed apparatus was never described during the activity, after instruction all the students were able to at least describe it correctly. Furthermore, 40% of them managed to identify the selection method, clearly explaining it in their answers. The post-instruction answers show that the identification of the selection method in the description of the apparatus had attained a considerable improvement, compared with the pre-instruction.

In Item 2 students have to compare three different distributions at three different temperatures. Figure 2 represents the categorization of students' answers and their distributions in pre/post tests along following categories of students' answers:

Level 0: Student does not identify any information about the distribution.
Level 1: Student identifies some information but not the variance or the connection with temperature.
Level 2: Student identifies the variance as a relevant characteristic of distributions and compares them correctly, but does not know the connection with temperature.
Level 3: Student identifies the variances, compares correctly them and knows the connection with temperature.

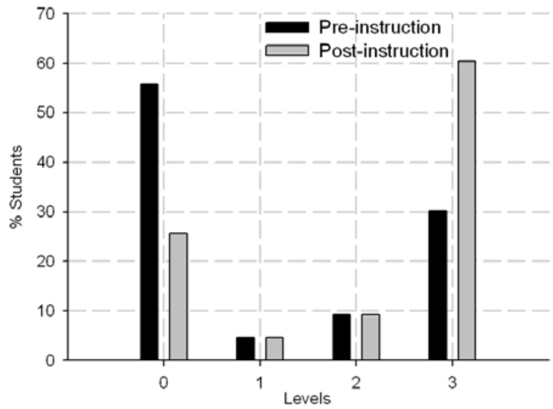

Figure 2. Categories of students' answers in Item 2 and their percentages in each category.
$\chi^2 = 9.16$; $p = 0.03$

As shown in Figure 2, the improvement in post-instruction answers with respect to the pre-instruction ones is considerable. This is highlighted by both the decreasing of percentage of students who did not manage to identify an answer, and the considerable percentage (60%) of students that managed to correctly identify the variance as a relevant quantity, to relating it to temperature. The correct answers were also well explained and complete.

Findings

In this paper we describe a 20 hours' workshop for undergraduate engineering students of University of Palermo that uses an Inquiry Approach aimed at searching answers to a problematic question that involves the understanding of the statistical distribution concept in the field of statistical mechanics.

The majority of students showed a considerable interest with respect to both the subject and the way it was dealt with. Almost all were easily involved by the laboratory measurements, mainly for the character of the approach; they were guided only by the research questions and were free to design appropriate experimental set ups.

Some preliminary results of the students' learning, mainly focused on the comparison of pre and post instruction questionnaire answers and on a qualitative analysis of students ideas discussed during the workshop activities, are reported. These show that students easily understood the use of the method of retarding potential to build distributions. All students showed the awareness of the need for accurate experimental designs and models able to describe the experimental data. Moreover, although a high percentage of students initially showed difficulties especially in the construction of distributions and in identifying the quantity characterizing the distributions, their active construction of distributions, aimed at explaining their experimental data, stimulated their understanding of general concepts.

The analysis of the videos recording the students' behaviours during the group work investigations is in progress. It highlights how students became confident with the investigation methods and the dynamics of their learning. The main point that such analysis is pointing out is how the students during the activities became more and more comfortable with the main characteristics of scientific inquiry and their relationships.

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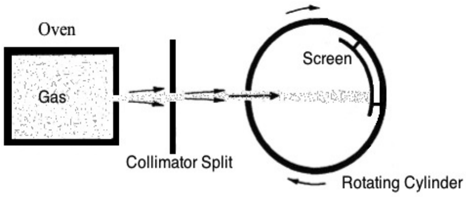
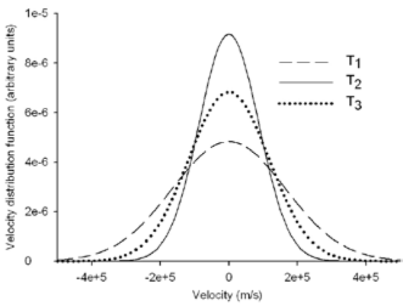
Appendix: Questionnaire

1) A historical apparatus for the study of velocity distribution of atoms/molecules is the following: an oven emits atoms/molecules of bismuth, which are collimated on a drum (as a screen) that can rotate at a constant velocity. The surface of the drum is covered with a material that emits light when struck by atoms/molecules of bismuth. In this way the atoms/molecules leave a persistent trace on the drum and can therefore be detected.
Explain why the drum is maintained rotating.



2) Three Maxwellian velocity distributions at three different temperatures are reported. Analyse the plots and state the relation between the temperatures, giving reasons for your answer.



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