Non linear analysis of 3D elastoplastic framed structures

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Studies on the post-elastic behavior of materials and structures have permitted the updating of national and international codes with particular reference to the design of structures in seismic zones. More severe conditions are also imposed for those regions declared seismic and zones considered non-seismic in the past are today included into these categories. Most of buildings are therefore inadequate with respect to the prescriptions of actual codes. In the worst case some structures have not been designed to absorb horizontal actions. The evaluation of the vulnerability of existing structures to seismic loads is therefore of extreme importance and can be done by performing nonlinear finite element analyses. In literature, with respect to framed structures, three different finite element models are utilized to describe the elastoplastic behavior of a beam/column element [1]: lumped models, distributed nonlinearity models, fiber models. Lumped models consider the constitutive nonlinearity concentrated at a section level of a frame element, usually employing nonlinear springs at the ends of beam/column elements [2]. Distributed nonlinearity models average the nonlinearity over a finite element by considering the possibility to form plastic hinges at different evaluation points of the element and calculating weighted integrals of the section responses [3]. Fiber models subdivide a section with a large number of finite elements and nonlinearity is related to the stress-strain relationship of a single finite element [4].
The present work concentrates on the framework of lumped models. A new three-dimensional two-node Euler-Bernoulli beam/column finite element is proposed with the aim to run nonlinear analyses on 3D RC framed structures. With respect to the existing lumped models where plastic hinges can develop only at the two ends of an element, the proposed model is based on the possible formation of a maximum of three hinges, in any way positioned inside the finite element. This choice reduces the computational costs associated with the employment of remeshing procedures when a hinge doesn't form at the ends of an element.
The basic concept is related to the possibility to split the kinematical behavior of a frame element in two different parts when a new hinge is formed at a certain point of the element. In other words, an entire element can be thought as the connection of two or more sub-elements in each one of them a displacement field can be defined and linked to the others by beams of an Heaviside function:

$$w(x) = w_1(x) H(a-x) + w_2(x) H(x-a) \tag{1}$$

where $w(x)$, $w_1(x)$, and $w_2(x)$ are the displacement fields of the entire element, the first sub-element and of second sub-elements respectively, $H(\bullet)$ is the Heaviside function of the enclosed quantity, a identifies the position of the plastic hinge.
The nonlinear behavior of the hinge is defined in the framework of a thermo-dynamically consistent elastoplastic theory. An elastic-perfect plastic constitutive behavior of the hinge is considered.
State equations and flow rules are derived from a Helmholtz free potential energy [5,6]. A polyhedric activation domain is defined on the internal normal force and bending moments reference system to control the onset and evolution of plasticity at the hinge. For a reinforced concrete cross-section with generic dimensions and position and quantity of steel bars, the

polyhedric domain is obtained considering a finite number of points, related to the most important states of stress acting on the section [7].

The hinge is assumed to have a plastic behavior only with respect to the section curvatures, while an elastic behavior is considered in the axial direction. A non associative procedure is followed: on the base of the internal axial force registered at a certain section of the element, the polyhedric domain is cut by a plane at a constant value of axial force to obtain a 2D activation domain depending on the bending moments only.

The elastoplastic frame element has been introduced in a finite element analysis program to run nonlinear simulations on 2D and 3D framed structures. To this end state equations and flow rules have been rewritten in a discrete manner. A classical elastic predictor phase is followed by a plastic corrector phase in the case of activation of the inelastic phenomenon. The corrector phase is based on the evaluation of correct bending moments by employing the closest point method, which permits to satisfy the following loading-unloading conditions:

$$\phi_{pi} \le 0; \quad \dot{\lambda}_{pi} \ge 0; \quad \phi_{pi}\dot{\lambda}_{pi} = 0 \tag{2}$$

where ϕ_{pi} is the generic plane which determines part of the limit surface of the elastic domain and $\dot{\lambda}_{pi}$ is the lagrangian multiplier associated.

The formation of one or more hinges inside a finite element modifies the distribution of stresses inside that element and its stiffness matrix. As a consequence, the global stiffness matrix is continuously modified at each plastic load step.

Two Newton-Raphson iterative loops have been implemented. The first one is within a single time step and permits to reach the convergence inside a single element when plasticity is activated and a hinge formed. The second one permits to reach the convergence of the overall structure as in a standard finite element procedure. The iterative procedure is stopped when the structure becomes labile and the first collapse mechanism is formed.

Numerical examples are finally reported to validate the efficiency of the proposed model. The effectiveness of the model is obtained by comparing the results with those available in literature.

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