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An Improved Solver for the M/EEG Forward Problem

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***Abstract*—Noninvasive investigation of the brain activity via electroencephalography (EEG) and magnetoencephalography (MEG) involves a typical inverse problem whose solution process requires an accurate and fast forward solver. We propose the Method of Fundamental Solutions (MFS) as a truly meshfree alternative to the Boundary Element Method (BEM) for solving the M/EEG forward problem. The solution of the forward problem is obtained, via the Method of Particular Solutions (MPS), by numerically solving a set of coupled boundary value problems for the 3D Laplace equation. Numerical accuracy and computational load are investigated for spherical geometries and comparisons with a state-of-the-art BEM solver shows that the proposed method is competitive.**

***Index Terms*—EEG, MEG, method of fundamental solutions, meshfree methods.**

I. The M/EEG Forward Problem

On the basis of the quasi-stationary approximation of the Maxwell's equations, the EEG forward problem for a piecewise-constant conductivity head model can be formulated as a set of coupled Boundary Value Problems (BVPs) for the 3D Laplace operator. In particular, the governing PDE in a certain region is a Poisson equation if a neural source is located in that region, or a Laplace equation otherwise. Also the MEG forward problem involves the evaluation of the electric potential at the interfaces between different media [1]. So far, traditional mesh-based methods have been used to address the M/EEG forward problem [2]. Among these methods, the Boundary Element Method (BEM) has become the method of choice because of its efficiency with respect to the Finite Elements Method, and it is currently implemented in widely used software packages for M/EEG source analysis. However, the BEM requires high quality meshing of the domain boundaries, which is not a trivial task, involves costly numerical integration and could potentially introduce mesh-related artifacts in the reconstructed neural activation patterns.

II. Methodology

To remedy the drawbacks of the BEM, in this paper we propose the application of the Method of Fundamental Solutions (MFS) [3] via the Method of Particular Solutions (MPS) for solving the M/EEG forward problem. The MFS approximates the solution of the given BVP by a linear combination of fundamental solutions of the governing homogeneous PDE. The coefficients of the combination are determined by enforcing it to satisfy the boundary conditions. In order to avoid the singularities of the fundamental solutions, the centers of the expansion need to be placed on a fictitious boundary outside the physical domain. The proposed method is truly meshfree, since normals to boundaries and pairwise distances between points are the only geometric quantities that are needed. Moreover, no numerical integration has to be performed, the method has the potential for spectral accuracy and its implementation is straightforward.

III. Numerical Results

We have compared the proposed method with the state-of-the-art implementation of the BEM in solving the EEG forward problem for a unitary dipolar current source in a multi-layered sphere representing the head, with realistic geometric proportions and electric conductivities. Figure 1 shows that

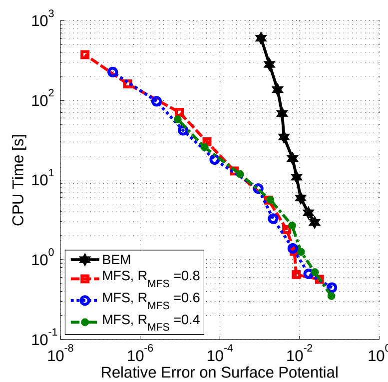

Fig. 1. Dipole in a three-layered sphere – Cost per Accuracy

the proposed method, implemented for different ratios R_{MFS} between the number of collocation points and the number of centers, is extremely competitive when compared to BEM from an accuracy vs. CPU time standpoint. This advantage is important in the perspective of the iterative solution of the inverse problem and it becomes more significant as the desired accuracy increases.

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