Quantification of statistical uncertainties in subspace-based operational modal analysis and their applications

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QUANTIFICATION OF STATISTICAL UNCERTAINTIES IN SUBSPACE-BASED OPERATIONAL MODAL ANALYSIS AND THEIR APPLICATIONS

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ABSTRACT

Modal parameters are estimated from vibration data, thus they are inherently afflicted with statistical uncertainties due to the unknown ambient excitation and measurement noise. While the point estimates of the modal parameters can be obtained with several system identification methods, only few of them also provide the associated uncertainties. The quantification of these uncertainties is important for many applications, since they are a means to assess the precision of the estimates, and to evaluate if changes between different datasets are statistically significant or not. As such, they are an added value in modern modal analysis practice and used in applications to e.g., damage detection and localization, reliability analysis, modal tracking and model calibration. For subspace-based system identification, efficient methods for uncertainty quantification have been developed for the last 15 years, yielding reliable estimates of the uncertainties at reasonable computational cost. They cover a wide range of subspace methods and their application areas. In this paper, an overview of the developments is given and the importance of the knowledge of the uncertainties is illustrated.

Keywords: Uncertainty quantification, subspace methods, (operational) modal analysis, monitoring

1. INTRODUCTION

The identification of dynamic system characteristics from vibration measurements is a fundamental task in engineering. Amongst others, subspace-based system identification methods are well-suited for this purpose. They identify the system matrices of a linear time-invariant state-space model that describes the dynamic system behavior [1], from which the modal parameters are retrieved. The estimates based on data are inherently afflicted with statistical uncertainties due to the unknown ambient excitation, measurement noise and limited data length. The quantification of these uncertainties is tied to the deployed identification method. However, the subspace identification methods only produce point estimates but not their related confidence bounds.

The objective for uncertainty quantification is to obtain the modal parameter estimates *and* their confidence bounds from the same dataset. While the statistical properties of estimates from subspace methods have been analyzed in great detail in the automatic control literature in the past, e.g. in [2–4], the expressions therein cannot be directly used for an actual covariance estimation in practical applications, since they require in addition e.g. the estimation of the unknown states and their covariances, which are not computed in the modal parameter estimation. A different approach was proposed in [5], where the covariance of estimated parameters is computed easily from the sample covariances of the underlying output data covariances and their related sensitivities.

The sensitivity-based covariance propagation is a simple and powerful tool for uncertainty quantification, and is theoretically justified by the statistical delta method [6]. It states that a function of an asymptotically Gaussian variable is also asymptotically Gaussian if its sensitivity is non-zero, and gives the respective covariance expression. Since the output covariances that are the basis of any subspace method are asymptotically Gaussian, this strategy can be used for characterizing the statistical distributions and in particular the covariance of the modal parameters and functions thereof.

This paper summarizes some of the developments for uncertainty quantification in the context of subspace methods from the last 15 years. In the first part, theoretical and algorithmic developments are presented regarding the uncertainty quantification of modal parameters for diverse subspace methods, their efficient implementation and the uncertainty quantification of the related modal indicators. In the second part, modal-parameter based methods are presented where the uncertainty information of the underlying modal parameter estimates is integrated, enhancing the originally deterministic methods.

2. BACKGROUND OF SUBSPACE-BASED SYSTEM IDENTIFICATION AND UNCERTAINTY QUANTIFICATION

In this section, the subspace-based system identification method for modal analysis and the associated uncertainty quantification framework are outlined. The schematics of the framework are presented in Fig. 1.

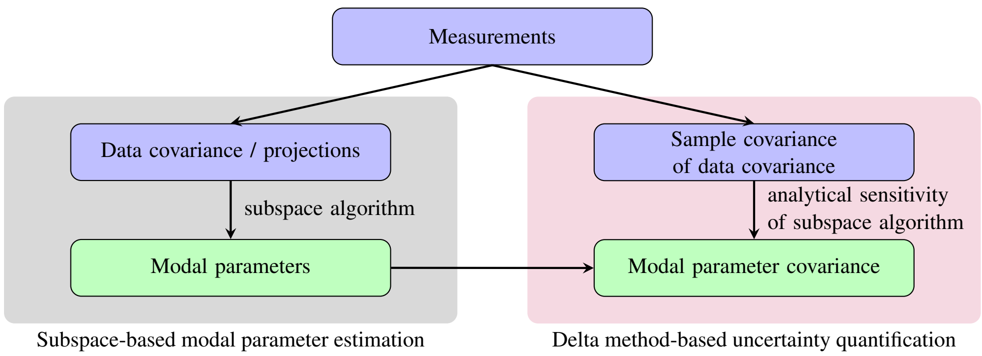


Figure 1: Framework of subspace identification and uncertainty quantification

2.1. Subspace-based system identification

Assume that the vibration behavior of the investigated structure can be modelled by a linear time-invariant system, and that in the simplest case only outputs are measured while inputs (acting forces) are unknown. Then the system dynamics can be described by the discrete-time state space model

$$\left\{ \begin{array}{rcl} x_{k+1} & = & Ax_k + w_k \\ y_k & = & Cx_k + v_k \end{array} \right. , \tag{1}$$

where A is the state transition matrix, C is the output matrix, and k is the integer time step corresponding to the system at time $t = k\Delta t$, where Δt is the sampling rate. Vector $y_k \in \mathbb{R}^r$ contains the measured outputs (such as accelerations, velocities, displacements, strains), and $x_k \in \mathbb{R}^n$ is the state vector. The state noise $w_k \in \mathbb{R}^n$ is related to the unknown ambient excitation, and vector $v_k \in \mathbb{R}^r$ is the output noise. The modal parameters are related to the eigenvalues and eigenvectors (λ_i, ϕ_i), $i = 1, \ldots, n$, of A and to C by

$$\mu_i = \frac{\log(\lambda_i)}{\Delta t}, \quad f_i = \frac{|\mu_i|}{2\pi}, \quad \zeta_i = \frac{-\mathrm{Re}(\mu_i)}{|\mu_i|}, \quad \varphi_i = C\phi_i, \tag{2}$$

where μ_i is an eigenvalue of the corresponding continuous-time system, f_i is the natural frequency, ζ_i is the damping ratio and φ_i is the mode shape at the output coordinates.

To estimate the matrices A and C from the output data y_k of length N, $k = 1, \ldots, N$, and consequently the modal parameters in (2), subspace methods are used. They are based on projecting (or simply multiplying) data Hankel matrices with a future and a past time sample horizon obtaining some matrix $\hat{\mathcal{H}}$, such that a consistent estimate of the system's observability matrix can be obtained from the column space of $\hat{\mathcal{H}}$. The respective data Hankel matrices can be defined as

$$\mathcal{Y}^- = \frac{1}{\sqrt{N}} \begin{bmatrix} y_1 & y_2 & \cdots & y_N \\ y_2 & y_3 & \cdots & y_{N+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_p & y_{p+1} & \cdots & y_{N+p-1} \end{bmatrix}, \quad \mathcal{Y}^+ = \begin{bmatrix} y_{p+1} & y_{p+2} & \cdots & y_{N+p} \\ y_{p+2} & y_{p+3} & \cdots & y_{N+p+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_{2p} & y_{2p+1} & \cdots & y_{N+2p-1} \end{bmatrix},$$

where $\mathcal{Y}^-$ can be built also from a subset of reference sensors or projection channels but not necessarily from all sensors.

There are many subspace methods in the literature [1, 7] with different ways to compute $\hat{\mathcal{H}}$, but from which the modal parameters are retrieved in the same way. For example, the covariance-driven subspace algorithm [8] computes $\hat{\mathcal{H}} = \mathcal{Y}^+ \mathcal{Y}^{-T}$, which corresponds to a matrix containing the output covariances of the data for different time lags. Their theoretical values satisfy the decomposition

$$\mathcal{H} = \mathcal{O}\mathcal{C}, \quad \text{where} \quad \mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{p-1} \end{bmatrix} \tag{3}$$

with the observability and stochastic controllability matrices $\mathcal{O}$ and $\mathcal{C}$. With the singular value decomposition (SVD) truncated at the desired model order

$$\hat{\mathcal{H}} = \begin{bmatrix} U_1 & U_0 \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ 0 & S_0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_0^T \end{bmatrix}, \tag{4}$$

an estimate $\hat{\mathcal{O}} = U_1 S_1^{1/2}$ of the observability matrix is obtained. Then, the output matrix C is estimated from the first block row of the observability matrix, and the state transition matrix is estimated from the shift-invariance property in a least-squares sense as $\hat{A} = (\hat{\mathcal{O}}^\uparrow)^\dagger \hat{\mathcal{O}}^\downarrow$, where $\hat{\mathcal{O}}^\uparrow$ and $\hat{\mathcal{O}}^\downarrow$ are the observability matrix estimate without the last and first block row, respectively. Ultimately, the modal parameters are obtained from the eigenvalues and eigenvectors of $\hat{A}$ and from $\hat{C}$ as in (2).

In engineering practice the modal parameters are obtained at different model orders by successively truncating the SVD in (4) and interpreted in so-called stabilization diagrams with the goal to separate (stable) physical modes from spurious ones. An efficient procedure for the multi-order modal parameter estimation can be achieved by exploiting the structure of the least-squares problem at multiple model orders for estimating $\hat{A}$, as detailed in [9].

2.2. Uncertainty quantification

The delta method is a statistical tool that helps to estimate the covariance of a function of an asymptotically Gaussian variable [6]. It is used to propagate the sample covariance of the data-related covariances that are computed in the first step of the subspace algorithms through all steps of the algorithm down to the modal parameters or functions thereof. Finally, confidence intervals of the computed parameters can be derived.

The first step of the subspace algorithms involves data projections that involve the computation of output covariances. If inputs (e.g. due to artificial excitation) are available in addition to the measured outputs, then this step also involves input/output covariances of the data. These data-related covariances are asymptotically Gaussian, i.e.,

$$\sqrt{N}(\hat{\mathcal{R}} - \mathcal{R}) \rightarrow \mathcal{N}(0, \Sigma_{\mathcal{R}})$$

where $\hat{\mathcal{R}}$ is a vector containing all covariance estimates involved in the chosen subspace method. An estimate $\hat{\Sigma}_{\mathcal{R}}$ of the covariance can be easily evaluated by the sample covariance based on partitions of the available data. The propagation of this covariance to the modal parameter estimates is then based on the delta method, stating that a function $\hat{Y} = f(\hat{\mathcal{R}})$ is also asymptotically Gaussian with

$$\sqrt{N}(\hat{Y} - Y) \rightarrow \mathcal{N}(0, \mathcal{J}_{Y,\mathcal{R}} \Sigma_{\mathcal{R}} \mathcal{J}_{Y,\mathcal{R}}^T).$$

The derivative $\mathcal{J}_{Y,\mathcal{R}}$ of the function with respect to $\mathcal{R}$ is obtained from perturbation theory. For a first-order perturbation it holds $\Delta Y \approx \mathcal{J}_{Y,\mathcal{R}} \Delta\mathcal{R}$. Hence, perturbing the functional relationship between $\mathcal{R}$ and Y analytically and neglecting higher-order terms yields the desired derivative, in particular for cases where the functional relationship is not explicit like for the SVD or eigenvalue decomposition. Subsequently, covariance expressions for the estimates satisfy

$$\hat{\Sigma}_Y \approx \hat{\mathcal{J}}_{Y,\mathcal{R}} \hat{\Sigma}_{\mathcal{R}} \hat{\mathcal{J}}_{Y,\mathcal{R}}^T. \qquad (5)$$

With this principle, the uncertainties of the output covariances from the first step of the subspace method can be propagated step by step through the algorithm down to the modal parameters and related quantities. Finally, confidence intervals of the estimates can be established based on the computed covariance and the fact that the distribution of the estimates can be approximated as Gaussian. For example, if $\hat{Y}$ is the vector containing the modal parameter estimates, then the covariance of each component $\hat{Y}_i$ is related to the respective component $\hat{\sigma}^2_{Y_i}$ on the diagonal of the asymptotic covariance estimate $\hat{\Sigma}_Y$, and the 95% confidence interval is given by

$$(\hat{Y}_i - 2 \cdot \frac{1}{\sqrt{N}} \hat{\sigma}_{Y_i},\ \hat{Y}_i + 2 \cdot \frac{1}{\sqrt{N}} \hat{\sigma}_{Y_i}),$$

i.e. this interval contains the true value Y_i of the estimate $\hat{Y}_i$ with a probability of 95%.

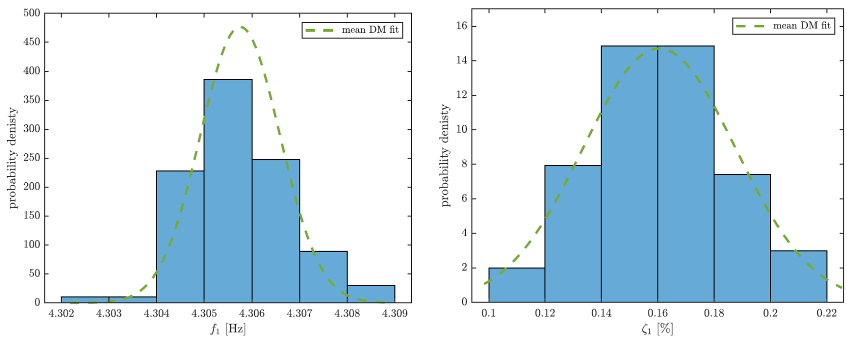


Figure 2: Histograms of modal parameter estimates with the delta method-based Gaussian approximation from [10].

To illustrate the Gaussian character of the modal parameter estimates in a practical setting, histograms of the first frequency and damping ratio are shown in Fig. 2 that are obtained from 100 datasets containing experimental data of a wind turbine blade [10], together with their mean Gaussian approximation provided by the delta method. It can be seen that the Gaussian approximation is appropriate, and that the modal parameter covariance estimated from one dataset (on which the mean fit is based in the figures) describes well the empirical distribution from the histogram.

3. THEORY AND ALGORITHMIC DEVELOPMENTS

3.1. Efficient implementation and uncertainty quantification in stabilization diagrams

The algorithm for uncertainty quantification of modal parameters from covariance-driven subspace identification has been proposed in [5], where the involved analytical sensitivities for the covariance propagation as well as the initial sample covariance are derived. However, in its direct implementation the size of the involved covariance matrices is considerable, which makes it computationally taxing and causing memory problems even for moderately sized problems. To alleviate this problem, a memory efficient and fast computation scheme for this method has been developed in [11] based on a mathematical reformulation of the algorithm, where in particular the structure of the the initial sample covariance estimate is exploited. Moreover, the covariance computation is optimized for multiple model orders in the stabilization diagram in [11], where the fact is exploited that the first columns of the observability matrix estimate at a higher model order are identical to the observability matrix estimate at a lower model order due to the SVD in (4). In this way, redundant operations in the computation of the stabilization diagram uncertainties can be avoided, and the algorithmic speed increased by two orders of magnitude of the maximal model order compared to a direct implementation, resulting in computation times of less than a minute for typical problem sizes.

In Fig. 3, the stabilization diagram from a dataset of the Z24 Bridge is shown, where the estimated standard deviations of the frequencies are shown as horizontal bars. Putting a threshold on the coefficient of variation of the frequencies (standard deviation divided by the frequency) cleans the diagram considerably, as seen in Fig. 3 (right).

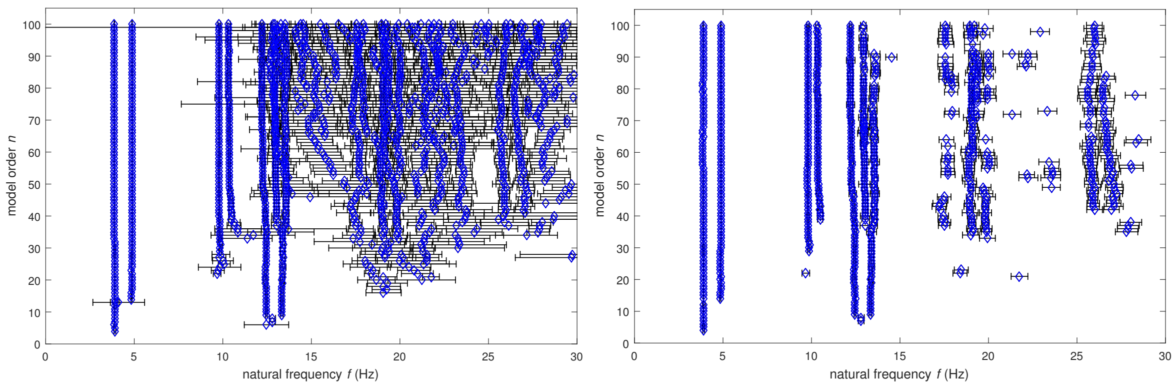


Figure 3: Stabilization diagram of Z24 Bridge without (left) and with threshold on standard deviation of frequencies (right).

The uncertainty information of the modal alignments in the diagram can then be used to obtain global mode estimates [12], and to actually obtain the modal alignments by automated statistical clustering approaches [13].

3.2. Uncertainty quantification for diverse subspace methods

The family of subspace methods is big, including covariance-driven and data-driven methods, methods for output-only (stochastic) system identification, and methods for input/ouput (combined deterministic-stochastic) for the case where some of the inputs are known. The uncertainty quantification strategy for the above-mentioned covariance-driven subspace method has been extended to a wide range of different subspace methods in [14] for the purpose of modal parameter uncertainty quantification.

The main difficulty in the extension to other subspace methods lies in the link between the matrix $\hat{\mathcal{H}}$ and the data-related covariance matrices, in order to perform the first step of uncertainty propagation. The covariance-driven subspace method is the simplest of the subspace methods for uncertainty quantification, since it only depends on one data-related covariance matrix $\mathcal{Y}^+\mathcal{Y}^{-T}$. The sample covariance of this matrix is then propagated throughout the method. In other subspace methods, the construction of the initial 'projection matrix' $\hat{\mathcal{H}}$, from which the observability matrix is obtained, is more complex. For 'data-driven' methods like UPC [1, 8], the projection is not immediately linked to output covariances, but is a matrix that grows with the number of data samples. However, it can be shown that the observability matrix can be equivalently related to $\hat{\mathcal{H}}\hat{\mathcal{H}}^T$ for the purpose of uncertainty propagation, which is again 'covariance-driven'. For the case of UPC where $\hat{\mathcal{H}} = \mathcal{Y}^+\mathcal{Y}^{-T}(\mathcal{Y}^-\mathcal{Y}^{-T})^{-1}\mathcal{Y}^-$, the algorithm depends on two data-related covariance matrices, namely $\mathcal{Y}^+\mathcal{Y}^{-T}$ and $\mathcal{Y}^-\mathcal{Y}^{-T}$. In the presence of known inputs, matrix $\hat{\mathcal{H}}$ depends furthermore on covariance matrices between outputs and inputs, and between inputs. In [14], the related uncertainty quantification schemes are developed for different classes of subspace methods, namely

- Output-only orthogonal projection data-driven algorithm (UPC),
- Input/output covariance-driven algorithm,
- Input/output orthogonal projection data-driven (similar to MOESP),
- Input/output oblique projection data-driven (N4SID).

The developed strategies allow an easy extension also to other subspace methods. While the system matrices A and C and the related modal parameters are obtained from the observability matrix in these methods, there is an alternative way to obtain them based on the state sequences in data-driven subspace methods like UPC, for which the uncertainty quantification scheme has been developed recently in [15].

With the availability of input data, the system matrices B and D of the related input/output state space model can be identified in addition to matrices A and C. With the knowledge of these matrices, the parametric transfer function can be obtained. The associated uncertainties for B and D, and the phase and magnitude of the transfer function have been obtained in [16], showcased in Fig. 4. It can be seen that the computed confidence intervals coincide well with the empirical ones in a Monte Carlo simulation.

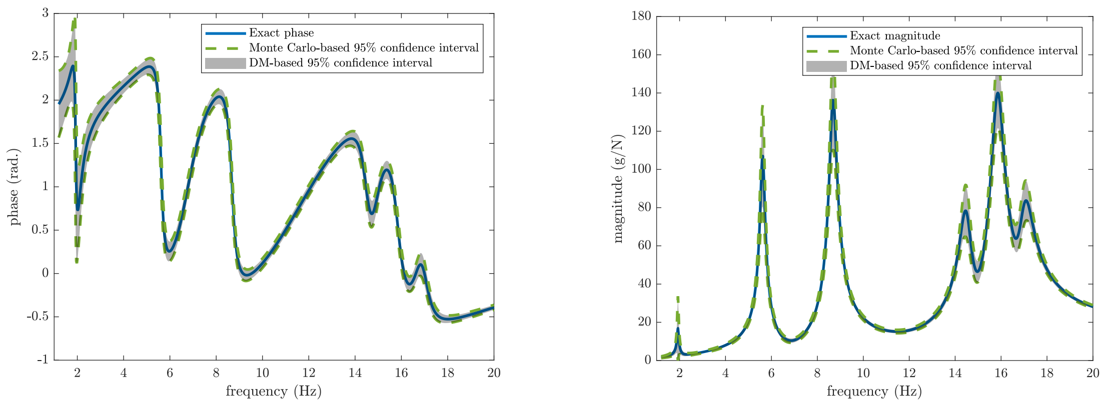


Figure 4: First component of phase and magnitude of $H(z)$ with Monte Carlo and delta method-based confidence intervals from [16].

3.3. Uncertainty quantification of modal indicators

Modal indicators like the Modal Assurance Criterion (MAC) and the Modal Phase Collinearity (MPC) are computed from mode shapes and thus inherit their statistical uncertainties when computed from measurement data. Hence, they will never be exactly one (indicating a perfect mode shape match, or a perfectly real-valued mode shape), but only close to one. Hence, the quantification of the statistical uncertainties is required in order to evaluate if the modal indicators are significantly close to one or not.

The particular difficulty in uncertainty quantification of the modal indicators is their boundedness in the interval $[0, 1]$. When the theoretical value is one, e.g., when the MAC is evaluated for identical mode shapes or when the MPC is evaluated for a real-valued mode shape, then the distribution of the corresponding estimates accumulates near one and is not Gaussian anymore. Hence the previous first-order framework for uncertainty quantification cannot be used anymore. In [17, 18] the distribution properties of the MPC and MAC estimates have been derived, as well as the respective confidence intervals. It turns out that the distributions for theoretical values at the border of the interval (i.e., for MAC or MPC equal to one) can be described by means of the second-order delta method, where the distributions can be approximated by a scaled and shifted χ^2 distribution, whereas in the inside of the interval $[0, 1]$ the distributions are asymptotically Gaussian, as illustrated in Fig. 5.

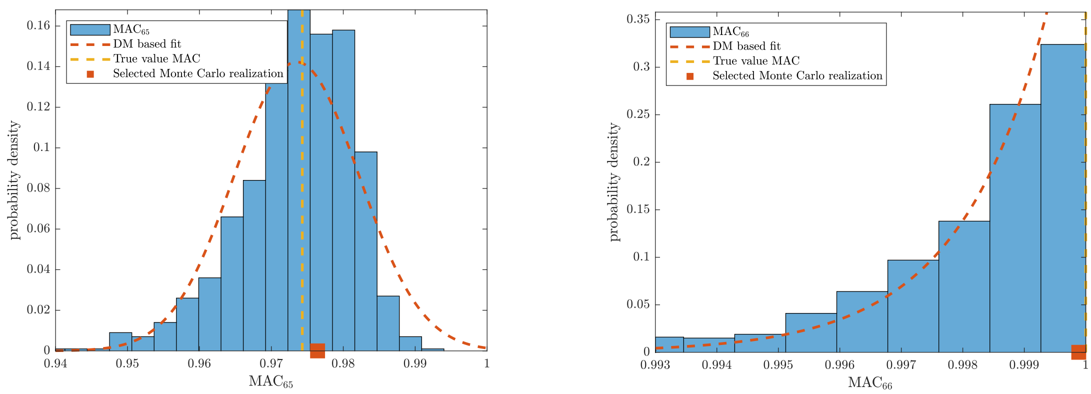


Figure 5: Distribution fits of MAC estimates from delta method, together with histograms from Monte Carlo simulation. Left: MAC of estimates of different mode shapes with Gaussian approximation, right: MAC of estimates of equal mode shape with scaled and shifted χ^2 approximation, from [18].

3.4. Validation

The presented methods for uncertainty quantification of the modal parameters and the modal indicators have not only been extensively validated in simulation studies, but also on experimental data. A first validation of the uncertainties obtained from the covariance-driven subspace methods was reported in [19] using data of a bridge and of a building from different sensor setups. More recently, an extensive validation study on a laboratory test of a large-scale wind turbine blade was carried out in [10] based on 100 experimental data sets, covering all of the presented subspace methods and modal indicators. The results confirm that the delta method is, on average, adequate to characterize the distribution of the considered estimates from all the different methods solely based on the quantities obtained from one data set, validating the use of this statistical framework for uncertainty quantification in practice.

3.5. Some case studies

Modal parameter uncertainties are particularly useful in the monitoring of structures over time. Some large scale case studies have been reported where the modal parameter uncertainty has been evaluated. One example is the S101 Bridge [18, 20], where damages were introduced to the bridge while measurement data was collected continuously during four days. The role of the modal parameter uncertainties was evaluated in [21] for the monitoring the Baixo Sabor arch dam during several years, where it was concluded that the uncertainties are particularly useful for removing outliers in the modal parameter tracking. In [22], the accuracy of the uncertainty estimates during ice-structure interaction was evaluated, and applied to monitoring data of a lighthouse.

4. UNCERTAINTIES IN MODAL PARAMETER-BASED METHODS

Modal parameters and modal indicators are used in many engineering applications. Instead of using their point estimates only, the uncertainties of the estimates are an additional valuable information that can enhance methods. In this section two examples are given where the quantified modal parameter uncertainties are integrated into originally deterministic methods, namely for damage localization and for finite element model updating. Furthermore, some application cases are briefly presented.

4.1. Flexibility evaluation and DLV-based damage localization

The damage locating vector vector approach [23] computes originally a vector in the null space of the change of flexibility matrix that is computed from input/output measurement data from the reference and from the damaged states, and then applies it as a load (at the sensor coordinates) to a finite element model of the structure to compute the stress field. Damage is located where the resulting stress is zero. The approach has been generalized the output-only case [24], where the required load vector is computed from the null space of the transfer matrix difference, which can be evaluated from the modal parameters solely.

Instead of evaluating the resulting stress deterministically to decide if it is zero or not over a structural component, a statistical evaluation is made in [25–27] where the modal parameter uncertainty is propagated to the computed stress field and used for a decision with hypothesis tests. The method has been extended the case of changing temperature in [28], where the temperature influence is removed from the modal parameters. In these works it has been shown that the performance of the damage localization is largely increased when considering the modal parameter uncertainties, as illustrated in Fig. 6. It can be seen that the rate of successful damage localization is always higher when the considering the uncertainties in the statistical tests compared to the deterministic evaluation of the stress estimates.

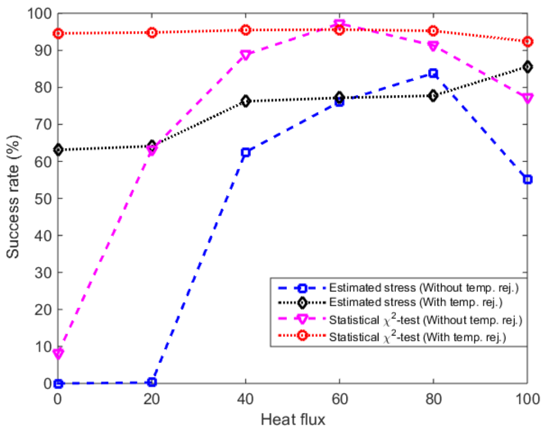


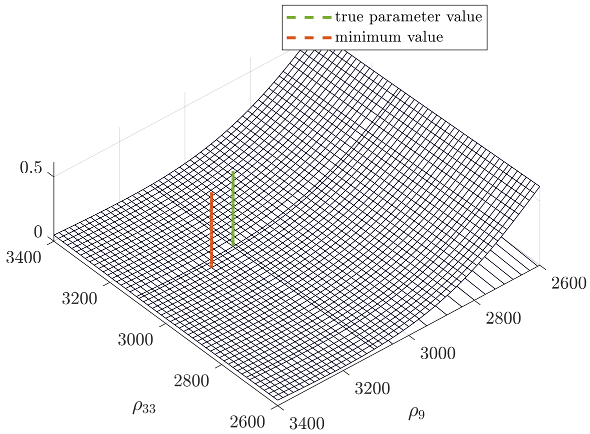
Figure 6: Success rates for DLV-based damage localization with and without considering modal parameter uncertainties for different temperature scenarios, from [28]. Success rates are higher when considering the uncertainties in the evaluation (red vs. black curve, pink vs. blue curve).

In a related work [29], the uncertainty of the modal parameters as well as assumed uncertainties of finite element model parameters have been propagated to the flexibility matrix estimate and associated computed deflections under static loads with the purpose of reliability analysis.

4.2. Finite element model updating

Modal parameters are essential for finite element model updating, and their uncertainties can be valuable for the updating problem. Since the updated model parameters are a function of the modal parameter estimates, the uncertainty of the model parameters due to the uncertainty of the modal parameters can be evaluated by uncertainty propagation. This was carried out in [30], where the propagation is performed through each iteration step of the updating minimization problem in a subspace fitting approach. Furthermore, the uncertainties of the modal parameters can be directly considered in the formulation of the objective function as well as in the stopping criterion for the optimization, as developed in [31]. In this work, model frequencies and mode shapes (via the MAC) are penalized in the objective function when they are outside the confidence bounds of their estimated counterparts, which leads to a steeper objective function. The optimization search can be terminated once the model-based modal parameters are within the confidence bounds of their data-based counterparts, leading to an efficient algorithm for the updating problem.

In Fig. 7, a modal parameter-based objective function is illustrated without and with the consideration of the uncertainties. First of all, it can be seen that the objective function is indeed steeper when considering the uncertainties, which should facilitate the optimization search. Second, it can be seen that the minimum of the objective function (red line) that is computed on the modal parameter estimates is not taken on at the true parameter values (green line) due to the modal parameter uncertainties, but just close to it. This also confirms that convergence of the optimization search to the minimum of the objective function is actually not required, and it is sufficient to stop the search once arrived in the yellow region indicating the values of the objective function where the model-based modal parameters are within the confidence bounds of their estimates.



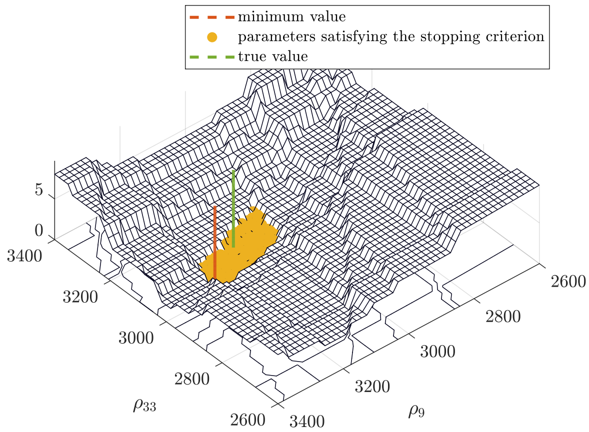


Figure 7: Modal parameter-based objective function to be minimized in the model updating, for two model parameters in the vicinity of their true values. Without (left) and with consideration of modal parameter uncertainties (right), from [31].

5. CONCLUSIONS

The methods for uncertainty quantification in subspace-based operational modal analysis have evolved over the last years, offering computationally efficient tools for a wide range of methods and engineering applications. They also have become part of commercial software [32]. The methods have shown their adequacy in extensive validation studies as well as in diverse case studies on real structures. Future work on the topic should include, e.g., further integration of the quantified uncertainties in methods for damage diagnosis, including in the data normalization step to remove environmental effects, as well as in data-based reliability analysis.

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