Estimation of Sparse Generalized Linear Models: the dglars package

Luigi Augugliaro and Angelo M. Mineo

Abstract dglars is a public available R package that implements the method proposed in Augugliaro, Mineo and Wit (2013) developed to study the sparse structure of a generalized linear model. This method, called dgLARS, is based on a differential geometrical extension of the least angle regression method (LARS). The core of the dglars package consists of two algorithms implemented in Fortran 90 to efficiently compute the solution curve; specifically a predictor-corrector algorithm and a cyclic coordinate descent algorithm.

Key words: generalized linear models, dgLARS, predictor-corrector algorithm, cyclic coordinate descent algorithm, sparse models, variable selection

1 Introduction

Nowadays, high-dimensional data sets, namely data sets where the number of predictors, say p, is larger than the sample size N, are becoming more and more common. Modern statistical methods developed to study this kind of data sets are usually based on the idea to use a penalty function to estimate a solution curve embedded in the parameter space and then to find the point that represents the best compromise between sparsity and predictive behaviour of the model. Recent statistical literature has a great number of contributions devoted to this problem: some important examples are the L_1 penalty function [8], the SCAD method [5] and the MC+ penalty function [9], among others.

Luigi Augugliaro

University of Palermo, Viale delle Scienze Ed. 13 e-mail: luigi.augugliaro@unipa.it Angelo M. Mineo

University of Palermo, Viale delle Scienze Ed. 13 e-mail: angelo.mineo@unipa.it

Differently from the methods cited above, in [3] the authors propose a new approach based on the differential geometrical representation of a GLM. The derived method, that does not require an explicit penalty function, has been called differential geometric LARS (dgLARS) method because it is defined generalizing the geometrical ideas on which the least angle regression (LARS), proposed in [4], is based. Using the differential geometric characterization of the classical signed Rao score test statistic, dgLARS gains important theoretical properties that are not shared by other methods. From a computational point of view, the dgLARS method consists essentially in the computation of the implicitly defined solution curve. In [3] this problem is satisfactorily solved by using a predictor-corrector (PC) algorithm. In [2] is proposed a much more efficient cyclic coordinate descend (ccd) algorithm to fit the dgLARS solution curve when we work with an high-dimensional data set.

In this paper we present the dglars package that implements both the algorithms to compute the solution curve implicitly defined by dgLARS. The object returned by these functions is a S3 class object, for which specific methods and functions have been implemented. The package dglars is available under general public licence (GPL-2) from the Comprehensive R Archive Network at http://CRAN.R-project.org/package=dglars.

2 The dglars package

The dglars package is an R [6] package containing a collection of tools related to the dgLARS method. In the following of this section we describe the main functions available with this package. For a complete description of the all functions implemented in the dglars package the reader is referred to the manual of the package.

The function dglars() is the main function of the proposed package. It can be called with the following arguments

```
dglars(X, y, family = c("binomial", "poisson"), control = list())
```

where X is the design matrix of dimension $n \times p$, y is the n-dimensional response vector and family is the error distribution used in the model. Finally control is a named list of control parameters. For a complete description of this list, the interested readed is referred to the corresponding help page.

To gain more insight on how to use the dglars() function we consider a simulated data set. We simulate a data set from a logistic regression model with sample size equal to 100 and p = 5 predictors. We also assume that only the first 2 predictors influence the response variable. The used R code is

```
R> n <- 100; p <- 4; s <- 2; X <- matrix(rnorm(n * p), n, p)
R> bs <- rep(1, s); Xs <- X[, 1 : s]
R> eta <- drop(1 + drop(Xs %*% bs))</pre>
```

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The dglars R package

R> mu <- binomial()\$linkinv(eta); y <- rbinom(n, 1, mu)
R> out_dglasso_pc <- dglars(X = X, y = y, family = "binomial")</pre>

dglars() returns a S3 class object

R> class(out_dglasso_pc)
[1] "dglars"

As shown in the following R code, the method function print.dglars() can be used to print out the basic information contained in a dglars object.

```
R> out_dglasso_pc
```

```
Call: dglars(X = X, y = y, family = "binomial")
```

Sequence	g	Dev	%Dev	df
+x1	3.67566	134.6	0.00000	1
	3.06853	130.5	0.03080	2
+x2	3.04937	130.3	0.03171	2
	0.21800	109.0	0.19047	3
+x4	0.20859	109.0	0.19055	3
	0.05396	108.8	0.19188	4
+x3	0.03199	108.8	0.19194	4
	0.00010	108.8	0.19198	5

```
Algorithm pc ( method = dgLASSO ) with exit = 0
```

The column Sequence shows that the dgLARS method first finds the true predictors and then includes the other false predictors. The column g reports the value of the parameter γ used in the dgLARS method to select the trade-off between sparsity of the estimated model and prediction behavior [3]. To be more specific, at the starting step the predictor x1 makes the smallest angle with the tangent residual vector and then is included in the active set. The predictor x2 is included in the active set at $\gamma^{(2)} = 3.04937$, this means that $\hat{\beta}_2(\gamma^{(2)})$ is equal to zero and then the number of non-zero estimated coefficients is equal to 2. The predictor x4 is included at $\gamma^{(3)} = 0.20859$ and so on.

More information about the estimated sequence of models can be obtained using the method function symmary.dglars(). The output printed out by summary.dglars() is divided in three different sections. The first section completes the basic information printed out by print.default() showing the sequence of the Akaike Information Criterion (AIC) [1] and the sequence of the Bayesian Information Criterion (BIC) [7]. The ranking of the estimated models obtained by these two criteria are also showed and the corresponding best model is identified by an arrow. The last two sections show the estimated coefficients of the two best models. For the sake of brevity, the following R code shows only the first section printed out by summary.dglars():

```
R> summary(out_dglasso_pc)
Call: dglars(X = X, y = y, family = "binomial")
                                                Rank.AIC Rank.BIC
                                                                       BIC
                            Complexity
                                           AIC
 Sequence
                       Dev
                  g
            3.67566
                     134.6
                                        136.6
                                                    8
                                                                 6
                                                                    139.2
       +x1
                                      1
            3.06853
                     130.5
                                      2
                                         134.5
                                                    7
                                                                 8
                                                                    139.7
                                        134.3
                                                    6
       +x2
            3.04937
                     130.3
                                      2
                                                                 7
                                                                    139.5
                     109.0
                                      3
                                        115.0
                                                    2
                                                                 2
                                                                    122.8
            0.21800
       +x4
            0.20859
                     109.0
                                      3
                                        115.0
                                                    1 <-
                                                              -> 1
                                                                    122.8
            0.05396
                     108.8
                                      4
                                        116.8
                                                    4
                                                                 4 127.2
       +x3
            0.03199
                     108.8
                                      4
                                        116.8
                                                    3
                                                                 3 127.2
            0.00010
                                      5 118.8
                                                    5
                                                                   131.8
                     108.8
                                                                 5
```

3 Conclusion

In this paper we have described the R package dglars. This package implements the differential geometric extension of the LARS method proposed in [3] and called dgLARS. The use of this package is shown by means of a simulated data set. The output of the functions are presented in a way that is easy to interpret for people familiar with standard lm, glm or gam output.

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