On the optimal design of base isolation devices

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SUMMARY. The paper deals with the optimal design of a base isolation system for a given structure subjected to seismic loads. In particular, an appropriate minimum displacement seismic protection device optimal design formulation is proposed for an assigned elastic perfectly plastic steel frame constrained to behave in conditions of elastic shakedown. The chosen base isolation device is constituted by elastomeric isolators. Suitable combinations of fixed and seismic loads are considered. According to the unrestricted shakedown theory, the seismic input is given as any load history appertaining to a suitably defined seismic load admissibility domain. The relevant dynamic structural response is obtained by means of a modal analysis making reference to the non-classically damped structural model. Some numerical applications conclude the paper.

1 INTRODUCTION

In last decades an ever increasing attention has been paid to seismic actions causing the worst effects either on civil or manufacturing or infrastructure structures. Therefore, the safeguard of such structures is the first goal to be achieved in the structural engineering framework. Two main objectives belong to this framework: the first one is to avoid the partial or global collapse with the corresponding human, social and economic outcomes; the second one, mainly devoted to high cardinal structures (such as hospitals, schools and so forth), requires a minimal structural efficiency both during and after the earthquake. In order to achieve the last objective it is required to design the structure in such a way that its response under the expected seismic event guarantees its usability. For elastic plastic structures the latter requirement can be obtained if the structural design imposes an elastic shakedown behaviour under the expected seismic actions. Clearly, a new structure can be easily designed to possess such behaviour, but usually resulting in an over-dimensioning with respect to the serviceability loading conditions. From the other hand the designer will face higher difficulties in upgrading an existing structure. In the latter case the more convenient strategy seems to be the adopting of appropriate seismic protection devices. Two main strategies are available: the first one is that of stiffening the structure by introducing suitably disposed cross bracing elements; the second one is that of reducing the amount of seismic energy coming out from the ground to the overhanging structure. In the first strategy the structure floor drifts are reduced as well as the stresses on the beams and pillars (see, e.g. [1]). The second strategy is regarded as very effective and mainly consists in inserting suitable devices (base isolation systems) between the soil foundations and the structure able to increase the first natural period of the isolated system making the structure less sensitive to seismic actions.

This effect can be obtained alternatively adopting a passive control, an active control or a semi-active control. In passive control devices the mechanical characteristics do not change depending on those of the seismic action, while in active control ones it is possible (see, e.g., [2-4]). To author’s knowledge, the base isolation system based on passive devices is one of the most efficient and economic technique. Recent approaches devoted to the design of passive devices take into account for the randomness of the seismic actions (see, e.g., [5]).

The optimal design of a base isolation system can be formulated in different ways [3-6]. As an example, the isolating device can be designed searching for the minimum drift of a chosen structure floor within an admissible range for the protecting device stiffness, or searching for the minimum base isolation system displacement according to fixed maximum structural drifts. Aim of the present paper is the formulation of an appropriate minimum displacement protection device design problem for an assigned structure constrained to elastically shakedown.

In the present case, the seismic loads are unknown; further, the shakedown theory is related to the structural analysis under cyclic or repeated loads belonging to an admissible domain. To this aim in the paper reference is made to the so-called unrestricted shakedown theory [7]. According to such theory an appropriate seismic load domain is generated through the definition of a suitable number of dynamic basic load conditions. The relevant dynamic structural response is obtained by means of a modal analysis making reference to the non-classically damped structural model being the relevant structure provided by a base isolation system. Some examples related to plane steel frames conclude the paper.

2 STRUCTURE AND LOADING MODEL DEFINITIONS

Let us consider the plane frame plotted in Fig. 1a, constituted by Navier’s beam type elements provided with a base isolation system constituted by viscoelastic devices disposed under each pillar. The purely elastic behaviour of each isolation device is described by the relation

$$F_{x,j}^{iso} = k_j^{iso} u_{x,j}^{iso}, \quad j = 1,2,...,n_{iso}, \tag{1}$$

with $F_{x,j}^{iso}$, k_j^{iso}, $u_{x,j}^{iso}$ horizontal force, shear stiffness, horizontal displacement related to the j-th device, n_{iso} being the number of the relevant isolation devices. Therefore, the described devices totally prevent vertical displacements and rotations of the constrained cross section elements and they result elastically flexible with regard to the horizontal displacements (Fig. 1b).

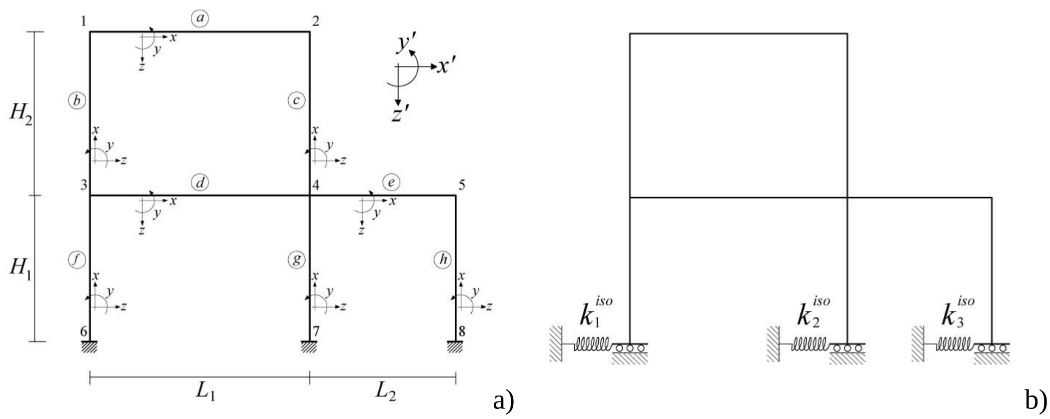

Figure 1: a) plane frame provided with a base isolation system;
b) assumed elastic model for the base isolation devices.

If no dynamic actions are present, the classical formulation of the static linear elastic analysis problem for plane frames constituted by n_b beam type elements, n_N standard nodes (with three degrees of freedom) and n_{iso} elastically flexible external nodes, is given as follows:

$$\boldsymbol{d} = \boldsymbol{C}\boldsymbol{u}, \qquad \boldsymbol{Q} = \boldsymbol{D}\boldsymbol{d} + \boldsymbol{Q}^*, \qquad \tilde{\boldsymbol{C}}\boldsymbol{Q} = \boldsymbol{F} \tag{2a,b,c}$$

where (Fig. 2) $\boldsymbol{d}$, $\boldsymbol{Q}$ and $\boldsymbol{Q}^*$ are the displacement, generalized stress and perfectly clamped generalized stress vectors of the beam element extremes of dimension $6 \cdot n_b$, respectively, $\boldsymbol{D}$ is the frame internal (square block diagonal) stiffness matrix with order $6 \cdot n_b$.

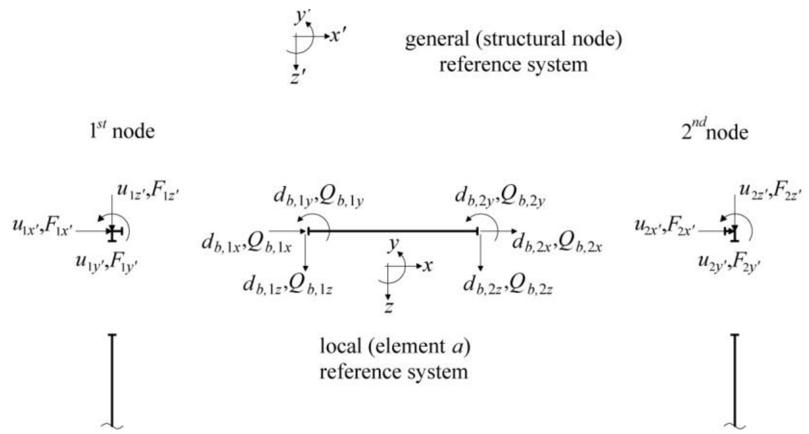

Figure 2: Plane frame: displacement and force vector components and reference systems.

Furthermore, $\tilde{\boldsymbol{u}} = \left| \tilde{\boldsymbol{u}}^{iso} \quad \tilde{\boldsymbol{u}}_N \right|$ and $\tilde{\boldsymbol{F}} = \left| \tilde{\boldsymbol{F}}^{iso} \quad \tilde{\boldsymbol{F}}_N \right|$ are frame nodal displacement and nodal force vectors of dimension $n_{iso} + 3 \cdot n_N$; $\boldsymbol{C}$ is the compatibility matrix with order $6 \cdot n_b \times \left(n_{iso} + 3 \cdot n_N \right)$ and its transpose $\tilde{\boldsymbol{C}}$ is the related equilibrium matrix. The solution to problem (2) is given by:

$$\boldsymbol{u} = \hat{\boldsymbol{K}}^{-1}\boldsymbol{F}^*, \qquad \boldsymbol{Q} = \boldsymbol{D}\boldsymbol{C}\boldsymbol{u} + \boldsymbol{Q}^* = \boldsymbol{D}\boldsymbol{C}\hat{\boldsymbol{K}}^{-1}\boldsymbol{F}^* + \boldsymbol{Q}^* \tag{3a,b}$$

in terms of structure node displacements and element generalized stresses, respectively, with $\hat{\boldsymbol{K}}$ frame external square stiffness matrix of order $\left(n_{iso} + 3 \cdot n_N \right)$ obtained by $\boldsymbol{K} = \tilde{\boldsymbol{C}}\boldsymbol{D}\boldsymbol{C}$ with $\hat{K}_{jj} = K_{jj} + k_j^{iso}$ for $j = 1,2,...,n_{iso}$, and $\boldsymbol{F}^* = \boldsymbol{F} - \tilde{\boldsymbol{C}}\boldsymbol{Q}^*$ equivalent nodal force vector.

Making reference to seismic actions, let us consider the relevant frame provided by viscoelastic isolation devices, just subjected to an horizontal ground acceleration $a_g\left(t\right)$. The model to be used for the elastic dynamic analysis can be deduced by the frame model already utilized in eqs. (2-3).

With this aim, at first, let us reorder the elements of vector $\boldsymbol{u}$, i.e.

$$\boldsymbol{u}_d = \boldsymbol{E}_1\boldsymbol{u} = \left| \tilde{\boldsymbol{u}}_t \quad \tilde{\boldsymbol{u}}_r \right|^T \tag{4}$$

with $\boldsymbol{E}_1$ appropriate reordering matrix, $\boldsymbol{u}_t$ horizontal displacement components and $\boldsymbol{u}_r$ remaining displacement components, where the apex T denotes the transpose of the relevant quantity.

Analogously, matrix $\hat{\boldsymbol{K}}$ must be reordered:

$$\boldsymbol{K}_d = \boldsymbol{E}_1 \hat{\boldsymbol{K}} \tilde{\boldsymbol{E}}_1 = \begin{vmatrix} \boldsymbol{K}_{tt} & \boldsymbol{K}_{tr} \\ \boldsymbol{K}_{rt} & \boldsymbol{K}_{rr} \end{vmatrix} \tag{5}$$

with trivial meanings of the utilized symbols and being $\tilde{\boldsymbol{E}}_1 \boldsymbol{E}_1 = \boldsymbol{I}$, with $\boldsymbol{I}$ identity matrix. Furthermore, in order to describe the classical frame model, the equality of the horizontal displacements at the same floor must be imposed, i.e.

$$\boldsymbol{u}_t = \boldsymbol{E}_2 \boldsymbol{s} \tag{6}$$

where $\boldsymbol{E}_2$ is an appropriate condensation matrix of order $\left(n_{iso} + n_N\right) \times n_f$, with n_f number of floors (including the base isolation floor), and $\boldsymbol{s} = \begin{vmatrix} s_b & \tilde{\boldsymbol{s}}_s \end{vmatrix}^T$ is the (horizontal) displacement vector related to the frame floors (dynamic degrees of freedom), with s_b base isolation displacement and $\boldsymbol{s}_s$ structural floor displacement vector with respect to the base isolation level.

Finally, it is usual to model the isolated structure as the superimposition of a classical $\left(n_f - 1\right)$ floors clamped frame over the base isolation level as represented in Fig. 3. On the ground of such representation the dynamic equilibrium equations can be written in the following form:

$$\begin{vmatrix} m_{tot} & \tilde{\boldsymbol{\tau}}_s \boldsymbol{M}_s & \tilde{\boldsymbol{0}} \\ \boldsymbol{M}_s \boldsymbol{\tau}_s & \boldsymbol{M}_s & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{vmatrix} \begin{vmatrix} \ddot{s}_b(t) \\ \ddot{\boldsymbol{s}}_s(t) \\ \ddot{\boldsymbol{u}}_r(t) \end{vmatrix} + \begin{vmatrix} a_b & \tilde{\boldsymbol{0}} & \tilde{\boldsymbol{0}} \\ \boldsymbol{0} & \boldsymbol{A}_s & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{vmatrix} \begin{vmatrix} \dot{s}_b(t) \\ \dot{\boldsymbol{s}}_s(t) \\ \dot{\boldsymbol{u}}_r(t) \end{vmatrix} + \begin{vmatrix} k_b & \tilde{\boldsymbol{0}} & \tilde{\boldsymbol{0}} \\ \boldsymbol{0} & \boldsymbol{K}_s & \boldsymbol{K}_{sr} \\ \boldsymbol{0} & \boldsymbol{K}_{rs} & \boldsymbol{K}_{rr} \end{vmatrix} \begin{vmatrix} s_b(t) \\ \boldsymbol{s}_s(t) \\ \boldsymbol{u}_r(t) \end{vmatrix} = - \begin{vmatrix} m_{tot} \\ \boldsymbol{M}_s \boldsymbol{\tau}_s \\ \boldsymbol{0} \end{vmatrix} a_g(t) \tag{7}$$

or, explicitly:

$$m_{tot} \ddot{s}_b + \tilde{\boldsymbol{\tau}}_s \boldsymbol{M}_s \ddot{\boldsymbol{s}}_s + a_b \dot{s}_b + k_b s_b = -m_{tot} a_g \tag{8a}$$

$$\boldsymbol{M}_s \boldsymbol{\tau}_s \ddot{s}_b + \boldsymbol{M}_s \ddot{\boldsymbol{s}}_s + \boldsymbol{A}_s \dot{\boldsymbol{s}}_s + \boldsymbol{K}_s \boldsymbol{s}_s + \boldsymbol{K}_{sr} \boldsymbol{u}_r = -\boldsymbol{M}_s \boldsymbol{\tau}_s a_g \tag{8b}$$

$$\boldsymbol{K}_{rs} \boldsymbol{s}_s + \boldsymbol{K}_{rr} \boldsymbol{u}_r = \boldsymbol{0} \tag{8c}$$

where: $m_{tot} = m_b + \sum_{i=1}^{\left(n_f - 1\right)} m_i$ is the total mass of the isolated structure, m_b being the mass of the base isolation level; $\boldsymbol{M}_s$ is the mass matrix of the clamped frame; $\boldsymbol{\tau}_s$ is the influence vector of the over frame; a_b is the damping coefficient related to the base isolation device; $k_b = \sum_{j=1}^{n_{iso}} k_j^{iso}$ is the total stiffness of the base isolation devices; $\boldsymbol{A}_s$ is the damping matrix related to the clamped frame. The following relations hold:

$$K_{s(i,j)} = \left(\tilde{\boldsymbol{E}}_2 \boldsymbol{K}_{tt} \boldsymbol{E}_2\right)_{(1+i,1+j)}, \; K_{sr(i,j)} = \left(\tilde{\boldsymbol{E}}_2 \boldsymbol{K}_{tr}\right)_{(1+i,j)}, \; \boldsymbol{K}_{rs} = \tilde{\boldsymbol{K}}_{sr} \qquad \text{(9a,b,c)}$$

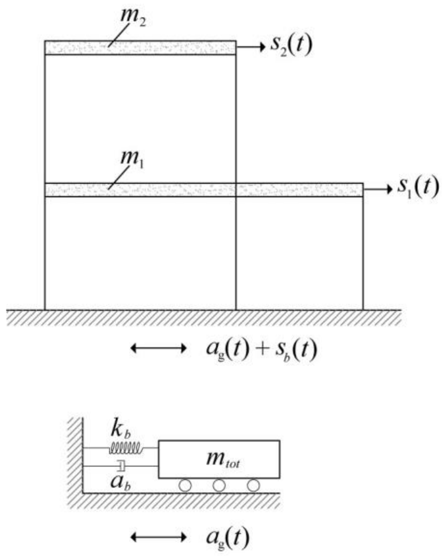

Figure 3: Isolated frame structural model.

Finally, $\dot{s}_b(t)$, $\dot{\boldsymbol{s}}_s(t)$ and $\ddot{s}_b(t)$, $\ddot{\boldsymbol{s}}_s(t)$ represent the velocity and the acceleration vectors of the base isolation system and of the over frame structure, respectively, the over dot meaning time derivative of the relevant quantity. From (8c) one obtains

$$\boldsymbol{u}_r = -\boldsymbol{K}_{rr}^{-1} \boldsymbol{K}_{rs} \boldsymbol{s}_s \qquad (10)$$

and eqs. (8a,b) can be rewritten in the following compact form:

$$\begin{vmatrix} m_{tot} & \tilde{\boldsymbol{\tau}}_s \boldsymbol{M}_s \\ \boldsymbol{M}_s \boldsymbol{\tau}_s & \boldsymbol{M}_s \end{vmatrix} \begin{vmatrix} \ddot{s}_b(t) \\ \ddot{\boldsymbol{s}}_s(t) \end{vmatrix} + \begin{vmatrix} a_b & \tilde{\mathbf{0}} \\ \mathbf{0} & \boldsymbol{A}_s \end{vmatrix} \begin{vmatrix} \dot{s}_b(t) \\ \dot{\boldsymbol{s}}_s(t) \end{vmatrix} + \begin{vmatrix} k_b & \tilde{\mathbf{0}} \\ \mathbf{0} & \left(\boldsymbol{K}_s - \boldsymbol{K}_{sr} \boldsymbol{K}_{rr}^{-1} \boldsymbol{K}_{rs}\right) \end{vmatrix} \begin{vmatrix} s_b(t) \\ \boldsymbol{s}_s(t) \end{vmatrix} = -\begin{vmatrix} m_{tot} \\ \boldsymbol{M}_s \boldsymbol{\tau}_s \end{vmatrix} a_g(t) \qquad (11)$$

It is worth noting that the base isolation system damping coefficient a_b can be computed once assigned the relevant isolation system damping coefficient ζ_b and once evaluated its stiffness:

$$\omega_b = \sqrt{\frac{k_b}{m_{tot}}}; \qquad a_b = 2 m_{tot} \omega_b \zeta_b \qquad \text{(12a,b)}$$

with ω_b natural frequency related to the base isolation system.

Furthermore, it must be observed that the mass, damping and stiffness matrices in equation (11) do not satisfy the Caughey-O'Kelly condition [8] namely, the relevant system is not a classically damped one. As a consequence, the elastodynamic analysis can be effected as synthetically described in the following.

As known, eq. (11) can be reformulated in the following way:

$$\begin{vmatrix} \bar{\boldsymbol{A}} & \bar{\boldsymbol{M}} \\ \bar{\boldsymbol{M}} & \boldsymbol{0} \end{vmatrix} \begin{vmatrix} \dot{\boldsymbol{s}}(t) \\ \ddot{\boldsymbol{s}}(t) \end{vmatrix} + \begin{vmatrix} \bar{\boldsymbol{K}} & \boldsymbol{0} \\ \boldsymbol{0} & -\bar{\boldsymbol{M}} \end{vmatrix} \begin{vmatrix} \boldsymbol{s}(t) \\ \dot{\boldsymbol{s}}(t) \end{vmatrix} = \begin{vmatrix} \boldsymbol{f}(t) \\ \boldsymbol{0} \end{vmatrix} \tag{13}$$

where

$$\bar{\boldsymbol{A}} = \begin{vmatrix} a_b & \tilde{\boldsymbol{0}} \\ \boldsymbol{0} & \boldsymbol{A}_s \end{vmatrix},\ \bar{\boldsymbol{M}} = \begin{vmatrix} m_{tot} & \tilde{\boldsymbol{\tau}}_s \boldsymbol{M}_s \\ \boldsymbol{M}_s \boldsymbol{\tau}_s & \boldsymbol{M}_s \end{vmatrix},\ \bar{\boldsymbol{K}} = \begin{vmatrix} k_b & \tilde{\boldsymbol{0}} \\ \boldsymbol{0} & \left(\boldsymbol{K}_s - \boldsymbol{K}_{sr}\boldsymbol{K}_{rr}^{-1}\boldsymbol{K}_{rs}\right) \end{vmatrix},\ \boldsymbol{f}(t) = -\begin{vmatrix} m_{tot} \\ \boldsymbol{M}_s \boldsymbol{\tau}_s \end{vmatrix} a_g \tag{14a,b,c,d}$$

The solution of the system (13) together with the corresponding initial conditions provides the structural response in terms of floor horizontal displacements and allows to determine (see, e.g. [9]) the natural frequencies and the damping ratios related to the non-classically damped system. Once these last are known the complete frame node displacement vector and the related element generalized stress vector due to the dynamic actions can be determined.

In the present context, the interest is focused in the determination of the characteristic of the isolation device which guarantees the shakedown of the structure. Since the real seismic load history is not known, reference must be made to a suitably defined admissible load domain. The definition of such a domain is made referring to the unrestricted dynamic shakedown theory [7]. Following this theory the seismic acceleration $a_g(t)$ is expressed as the superposition of a discrete set of single-frequency wave components $\bar{\psi}_{ij}(\tau)$:

$$a_g(t) = \sum_{i=1}^{n_f}\sum_{j=1}^{4} \xi_{ij}\,\bar{\psi}_{ij}(\tau),\quad \left(0 \le \tau \le T\right), \qquad \xi_{ij} \ge 0,\quad \sum_{i=1}^{n_f}\sum_{j=1}^{4} \xi_{ij} = 1 \tag{15a,b}$$

being T the duration of the seismic action, ξ_{ij} some arbitrary coefficients required to satisfy the admissibility conditions (15b) [10] and

$$\bar{\psi}_{ij}(\tau) = \left[1 - 2\cdot\mathrm{int}\left(\frac{j-1}{2}\right)\right] c_0 E(\tau)\left[(-1)^j \cos\omega_i\tau + \sin\omega_i\tau\right] \qquad j = 1,2,3,4 \tag{16}$$

where c_0 is a parameter related to the maximum power of the seismic input and $E(\tau)$ a suitable defined envelope function [7]. In equation (16) the intensity of the i-th single-frequency wave component is related to the power spectral density, here modeled by the well-known Kanai-Tajimi filter, of the considered earthquake corresponding to i-th structural natural mode [7].

Finally, an appropriate elastic plastic model for the structure is adopted as shown in Fig. 4a. In particular, beams and columns are considered as purely elastic elements and at their extremes are located rigid perfectly plastic hinges where the mechanical resistance limit is verified. The domain which describes the rigid perfectly plastic behaviour of the cited hinges can be represented just in terms of bending moments or it can take into account also the influence of the axial forces (as known, especially for steel frame structures constituted by quite slender elements, it is usual to neglect the shear force influence). In the first case, adopted in the present context and certainly reliable during the initial phase of structure dimensioning, the hinge dimensionless domain is constituted by a segment with extremes +1 and −1 (Fig. 4b), being M_y the full plastic bending moment of the relevant cross section.

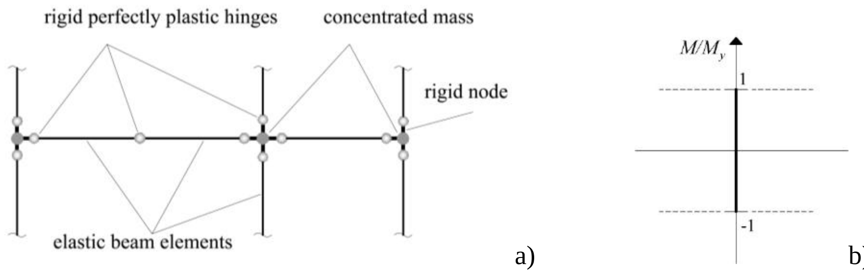

Figure 4: a) elastic plastic structural scheme; b) rigid plastic domain of the typical hinge.

3 OPTIMAL DESIGN PROBLEM FORMULATION

Making reference to the isolated elastic perfectly plastic frame structure as above described, according to the assumed loading model, let it be subjected to fixed mechanical loads and seismic loads. The minimum displacement base isolation system design problem formulation, with constraints on the elastic shakedown, can be written as follows:

$$\min_{\left(s_b, k_b, \boldsymbol{u}_0, \boldsymbol{s}_i, \boldsymbol{Y}_0^S\right)} s_b \tag{17a}$$

subjected to:

$$\hat{\boldsymbol{K}}\boldsymbol{u}_0 = \boldsymbol{F}_0^* \tag{17b}$$

$$\boldsymbol{Q}_0 = \boldsymbol{DCu}_0 + \boldsymbol{Q}_0^* \tag{17c}$$

$$\begin{vmatrix} \bar{\boldsymbol{A}} & \bar{\boldsymbol{M}} \\ \bar{\boldsymbol{M}} & \boldsymbol{0} \end{vmatrix} \begin{vmatrix} \dot{\boldsymbol{s}}_k(t) \\ \ddot{\boldsymbol{s}}_k(t) \end{vmatrix} + \begin{vmatrix} \bar{\boldsymbol{K}} & \boldsymbol{0} \\ \boldsymbol{0} & -\bar{\boldsymbol{M}} \end{vmatrix} \begin{vmatrix} \boldsymbol{s}_k(t) \\ \dot{\boldsymbol{s}}_k(t) \end{vmatrix} = \begin{vmatrix} \boldsymbol{f}_k(t) \\ \boldsymbol{0} \end{vmatrix} \qquad \forall k \in I(m) \tag{17d}$$

$$\boldsymbol{s}_k(t) = \left| s_{bk}(t) \quad \tilde{\boldsymbol{s}}_{sk}(t) \right|^T \qquad \forall k \in I(m) \tag{17e}$$

$$\boldsymbol{u}_{tk}(t) = \boldsymbol{E}_2 \boldsymbol{s}_k(t) \qquad \forall k \in I(m) \tag{17f}$$

$$\boldsymbol{u}_{rk}(t) = -\boldsymbol{K}_{rr}^{-1}\boldsymbol{K}_{rs}\boldsymbol{s}_{sk}(t) \qquad \forall k \in I(m) \tag{17g}$$

$$\boldsymbol{u}_{dk}(t) = \left| \tilde{\boldsymbol{u}}_{tk}(t) \quad \tilde{\boldsymbol{u}}_{rk}(t) \right|^T \qquad \forall k \in I(m) \tag{17h}$$

$$\boldsymbol{u}_k^S(t) = \tilde{\boldsymbol{E}}_1 \boldsymbol{u}_{dk}(t) \qquad \forall k \in I(m) \tag{17i}$$

$$\boldsymbol{Q}_k^S(t) = \boldsymbol{DCu}_k^S(t) \qquad \forall k \in I(m) \tag{17j}$$

$$s_b = \max_{k \in I(m)} \max_{0 \le t \le T} s_{bk}(t) \tag{17k}$$

$$\boldsymbol{Q}_+^S = \max_{k \in I(m)} \max_{0 \le t \le T} \boldsymbol{Q}_k^S(t) \tag{17ℓ}$$

$$\boldsymbol{Q}_-^S = \min_{k \in I(m)} \min_{0 \le t \le T} \boldsymbol{Q}_k^S(t) \tag{17m}$$

$$\boldsymbol{\varphi}_{+}^{S} = \xi_0^S \tilde{\boldsymbol{G}}_p \boldsymbol{Q}_0 + \xi^S \tilde{\boldsymbol{G}}_p \boldsymbol{Q}_{+}^S + \boldsymbol{S}\boldsymbol{Y}_0^S - \boldsymbol{R} \le \boldsymbol{0} \tag{17n}$$

$$\boldsymbol{\varphi}_{-}^{S} = -\xi_0^S \tilde{\boldsymbol{G}}_p \boldsymbol{Q}_0 - \xi^S \tilde{\boldsymbol{G}}_p \boldsymbol{Q}_{-}^S - \boldsymbol{S}\boldsymbol{Y}_0^S - \boldsymbol{R} \le \boldsymbol{0} \tag{17o}$$

$$\boldsymbol{Y}_0^S \ge \boldsymbol{0} \tag{17p}$$

where, besides the already defined symbols, $\boldsymbol{u}_k^S(t)$ and $\boldsymbol{Q}_k^S(t)$, $k \in I(m)$ with m number of basic load conditions, are the purely elastic response to the k-th seismic action, $\boldsymbol{\varphi}_{+}^{S}$, $\boldsymbol{\varphi}_{-}^{S}$ are the plastic potential vectors related to the elastic shakedown limit (apex *S*), $\tilde{\boldsymbol{G}}_p$ is an appropriate equilibrium matrix which applied to element nodal generalized stresses provides the bending moments acting upon the plastic nodes of the elements, $\xi_0^S \le 1$ and $\xi^S \ge 1$ are scalar load multipliers suitable to define the chosen load combination, $-\boldsymbol{S} = \boldsymbol{D}\boldsymbol{C}\tilde{\boldsymbol{G}}_p \hat{\boldsymbol{K}}^{-1} \boldsymbol{G}_p \tilde{\boldsymbol{C}}\tilde{\boldsymbol{D}} - \boldsymbol{D}$ is a time independent symmetric structural matrix which transforms the plastic activation intensities into the plastic potentials, $\boldsymbol{Y}_0^S$ are the fictitious plastic activation intensity vectors related to the elastic shakedown limit and $\boldsymbol{R}$ is the relevant plastic resistance vector. The problem is solved by searching for the minimum base isolation system displacement within the admissible domain for base isolation stiffness, i.e. the domain which characterize the safe shakedown behaviour for the structure.

4 NUMERICAL APPLICATIONS

The minimum displacement design of the base isolation device for the steel frame in Fig. 5 has been obtained referring to the previously proposed formulation. The design problem (17) has been solved utilizing a suitable MATLAB direct search subroutine. The frame is constituted by square box cross section elements with $\ell = 250\,\text{mm}$ and constant thicknesses as listed in Table 1. Furthermore, the following geometrical and material characteristics have been assumed: $L_1 = 7.00\,\text{m}$, $L_2 = 4.00\,\text{m}$, $H = 4.00\,\text{m}$, Young modulus $E = 210\,\text{GPa}$, yield stress $\sigma_y = 235\,\text{MPa}$. The rigid perfectly plastic hinges are located at the extremes of all elements and an additional hinge is located in the middle point of the longer beams.

Table 1. Thicknesses (mm) of the frame.

El.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
t	16	16	16	16	19	16	19	24	19	34	19	40	16	16	16	16
El.	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**
t	16	16	16	19	23	29	36	40	16	16	16	16	18	30	16	16

The structure is subjected to a fixed uniformly distributed vertical load on the beams, $q_0 = 40\,\text{kN/m}$ and to seismic actions. The seismic masses are equal at each floor: $m = 35.88\,\text{kN s}^2/\text{m}$. The equivalent damping coefficient of the base isolation system has been assigned $\zeta_b = 0.10$. In the case under examination the ground acceleration a_g has been characterized by the following Kanai-Tajimi parameters: $\zeta_g = 0.65$, $\omega_g = 19\,\text{rad/s}$ and $S_0 = 0.0050$. The adopted load combinations are defined by an assigned fixed load multipliers

$\xi_0^S = 0.8$ and to an imposed minimum seismic load multiplier $\xi^S = 1$.

The optimal base isolation displacement has been found $s_b = 103\,\text{mm}$ related to a base isolation stiffness $k_b = 0.81\,\text{kN/mm}$ and to a shakedown load multiplier $\xi^S = 4.88$. It is worth noting (Fig. 6a,b) that the minimum value of the seismic load multiplier is reached for $k_b = 3.78\,\text{kN/mm}$ and $s_b = 121\,\text{mm}$, but the searched displacement decreases on decreasing the base isolation stiffness till its minimum with a great safety margin with respect to the shakedown.

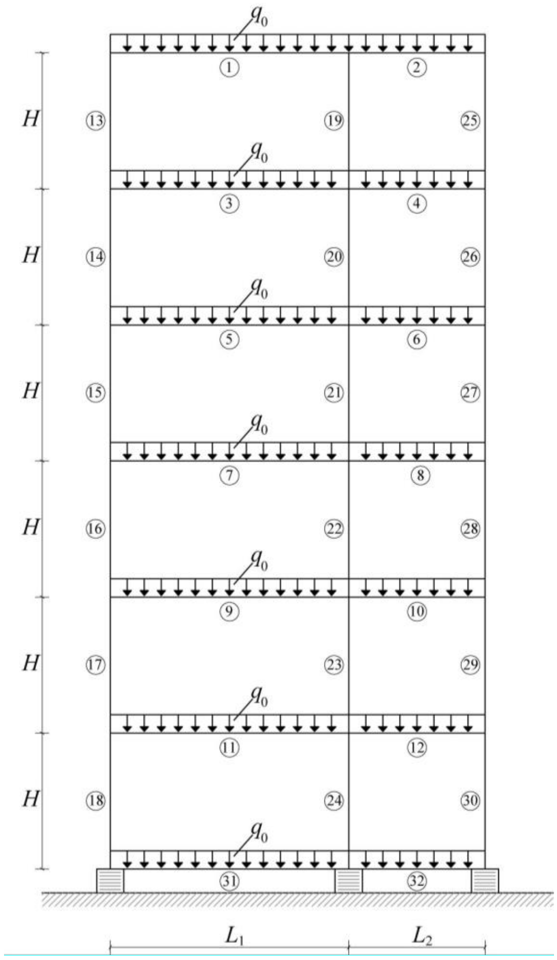

Figure 5 – Frame under examination.

1 CONCLUSIONS

The present paper has been devoted to the optimal design of a base isolation system for a given structure subjected to seismic loads. In particular, an appropriate minimum displacement seismic protection device optimal design formulation is proposed for an assigned elastic perfectly plastic steel frame constrained to behave in conditions of elastic shakedown. The overhanging structure has been assumed as a plane steel frame subjected to a suitable combinations of fixed and seismic loads and the selected isolation system is an elastomeric isolator.

The main problem to be solved when facing the proposed design is that, in the case of real seismic actions, the load history is not known but a suitably defined admissible load domain is required in order to perform the shakedown behaviour design. In order to achieve this aim, in the paper reference has been made to the unrestricted dynamic shakedown theory.

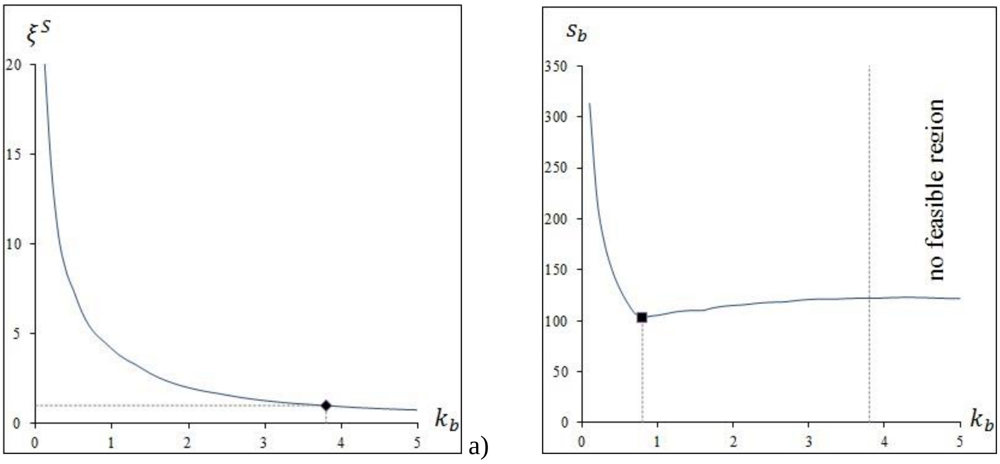

Figure 6: a) shakedown multiplier as function of the base isolation stiffness (♦ $\xi^S = 1$);
b) base isolation displacement as function of the base isolation stiffness (■ $s_b = 103\,\text{mm}$).

The dynamic structural response is obtained by means of a modal analysis making reference to the non-classically damped structural model. Some numerical applications conclude the paper

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