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Robust non-Markovianity in ultracold gases

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Abstract
We study the effect of thermal fluctuations on a probe qubit interacting with a Bose–Einstein condensed (BEC) reservoir. The zero-temperature case was studied in our previous work (Haikka *et al* 2011 *Phys. Rev.* A **84** 031602), where we proposed a method for probing the effects of dimensionality and scattering length of a BEC based on its behavior as an environment. In this paper, we show that the sensitivity of the probe qubit is remarkably robust against thermal noise. We give an intuitive explanation for the thermal resilience, showing that it is due to the unique choice of the probe qubit architecture of our model.

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(Some figures may appear in color only in the online journal)

1. Introduction

Over the last three decades, quantum computing has been the holy grail of quantum information sciences [1]. Lately, there has been a notable shift of focus from studies of circuit-based quantum computers, where a long computation involving many qubits is broken down into elementary one- and two-qubit quantum gates, to studies of quantum simulators, where a physical system is modeled using another physical realization of the original Hamiltonian [2]. One celebrated example of the latter is the simulation of a Bose–Hubbard model in optically trapped ultracold gases. Mapping between the Bose–Hubbard Hamiltonian and the Hamiltonian describing ultracold atoms in an optical lattice was proposed in 1998 by D Jaksch *et al* [3] and realized experimentally a few years later by the group of I Bloch [4]. Since this milestone there has been an explosion in studies of the systems that can be simulated with ultracold quantum gases [5, 6].

Quantum simulations have been considered in the context of open quantum systems with proposals of simulating the spin-boson model using, for example, a quantum dot coupled to a Luttinger liquid [7] or a more general Bose–Einstein condensed (BEC) reservoir [8]. Both cases realize the independent-boson Hamiltonian with an Ohmic-like spectrum of the reservoir [9]. Another proposal in this direction was presented in [10], where an impurity atom in a double-well potential is immersed in a BEC reservoir, forming a spin-boson model with a reservoir spectral function that can be tuned from sub-Ohmic to Ohmic to super-Ohmic. With a super-Ohmic spectrum the spin-boson model can acquire non-Markovian properties [11], thus simulating a prototype of a non-Markovian open quantum system model.

Non-Markovian systems have been the subject of intensive studies over the last few years, boosted by the recent introductions of several non-Markovianity measures that define and quantify the amount of non-Markovianity in a quantum process [12–14]. Fundamental interest in non-Markovian processes stems from the fact that Markovian dynamics is typically only an approximation, which is no longer valid when considering shorter time scales and/or stronger system–environment couplings. Furthermore, there have been proposals for using non-Markovianity as a resource in the context of quantum metrology [15] and quantum key distribution [16], to name a couple of examples.

Spin systems coupled to ultracold gases are important not only for quantum simulations, but also because they can be used to probe ultracold gases: the way a spin decoheres under the action of an ultracold gas may depend crucially on certain properties of the gas. Hence, it is possible to recover information about the large and generally inaccessible environment by looking at the spin alone. Indeed, the afore-mentioned independent boson models can be used to probe the Luttinger liquid parameter [7] and the density fluctuations of the BEC [8]. In [11], we demonstrated that the non-Markovian properties of the impurity atom in a double-well potential give us indications of the effective dimensionality of the BEC reservoir. In this work, we further consider this model, taking a step toward a more realistic scenario by considering the effect of thermal fluctuations on the sensitivity of the probe qubit. We demonstrate that the double-well qubit is remarkably robust against thermal noise and is therefore a good candidate for probing ultracold gases.

2. The model

We consider a qubit model based on a single atomic impurity trapped in a double-well potential, where the pseudo-spin states are represented by the presence of the impurity atom in the left or the right well of the double-well potential. The qubit is immersed in a thermally equilibrated BEC reservoir. The Hamiltonian of the total closed system is ($\hbar = 1$)

$$\hat{H} = \hat{H}_A + \hat{H}_B + \hat{H}_{AB}, \tag{1}$$

where

$$\hat{H}_A = \int \mathrm{d}\mathbf{x}\, \hat{\Psi}^\dagger(\mathbf{x}) \left[\frac{\mathbf{p}_A^2}{2m_A} + V_A(\mathbf{x}) \right] \hat{\Psi}(\mathbf{x}) \tag{2}$$

is the Hamiltonian of the impurity atom with $\hat{\Psi}(\mathbf{x})$ the impurity field operator and $V_A(\mathbf{x})$ the double-well potential formed by an optical lattice,

$$\hat{H}_B = \int \mathrm{d}\mathbf{x}\, \hat{\Phi}^\dagger(\mathbf{x}) \left[\frac{\mathbf{p}_B^2}{2m_B} + V_B(\mathbf{x}) + \frac{g_B}{2} \hat{\Phi}^\dagger(\mathbf{x})\hat{\Phi}(\mathbf{x}) \right] \hat{\Phi}(\mathbf{x}) \tag{3}$$

is the Hamiltonian for the BEC with $\hat{\Phi}(\mathbf{x})$ the condensate field operator, $V_B(\mathbf{x})$ the harmonic trapping potential and $g_B = 4\pi\hbar^2 a_B/m_B$ the boson–boson coupling constant, and finally

$$\hat{H}_{AB} = g_{AB} \int \mathrm{d}\mathbf{x}\, \hat{\Psi}^\dagger(\mathbf{x})\hat{\Phi}^\dagger(\mathbf{x})\hat{\Phi}(\mathbf{x})\hat{\Psi}(\mathbf{x}) \tag{4}$$

is the interaction Hamiltonian with $g_{AB} = 2\pi\hbar^2 a_{AB}/m_{AB}$ the coupling between the impurity atom and the condensate gas. Masses of the impurity atom and the background bosons are $m_{A/B}$, $m_{AB} = (m_A + m_B)/(m_A m_B)$ is their reduced mass and $a_{B/AB}$ are the s-wave scattering lengths for the boson–boson and impurity–boson collisions, respectively.

We would like to stress that the boson–boson scattering length can be manipulated by Feshbach resonances, providing a controllable environment of interacting bosons. The significance of this is twofold: firstly, many typical models of open qubit systems assume a non-interacting bosonic environment and it is fundamentally interesting to study interacting models. Secondly, the ability to have experimentally feasible and precise control over the environment is vital for reservoir engineering.

We assume that the BEC is trapped in such shallow potential that it may be considered to be homogeneous, while the double-well trap for the impurity atom is so deep that tunneling from one well to the other is suppressed. The condensate is treated in the Bogoliubov approximation, assuming weak to moderate boson–boson coupling. After imposing these assumptions on Hamiltonians (2)–(4) the qubit dynamics can be derived without any further approximation. The result is purely dephasing dynamics of the qubit with constant populations and off-diagonal elements of the qubit density matrix decaying as

$$|\rho_{01}(t)| = \mathrm{e}^{-\Gamma(t)} |\rho_{01}(0)|. \tag{5}$$

The decoherence factor is

$$\Gamma(t) = 8g_{AB}^2 n_0 \sum_{\mathbf{k}} (|u_k| - |v_k|)^2 \,\mathrm{e}^{-k^2\tau^2/2} \times \frac{\sin^2(E_k t/2\hbar)}{E_k^2} \coth\left(\frac{\beta E_k}{2}\right) \sin^2(\mathbf{k}\cdot\mathbf{L}), \tag{6}$$

where n_0 is the condensate density, $|u_k|$ and $|v_k|$ are the kth Bogoliubov mode amplitudes with energy $E_k = \sqrt{2\epsilon_k n_0 g_B + \epsilon_k^2}$, free modes have energy $\epsilon_k = \hbar^2 k^2/(2m_B)$, τ is the width of the impurity wavefunction, assumed Gaussian, in each well of the double well and $\beta = 1/k_\mathrm{B}T$. The spatial separation between the two wells is $\mathbf{L}$. For a detailed derivation of the decoherence factor, see [10, 11].

3. Non-Markovianity measure

With the decoherence factor at hand one has a full description of the qubit dynamics. We proved in [11] that in this case non-Markovianity is directly connected to the negativity of the decay rate $\gamma(t) = \mathrm{d}\Gamma(t)/\mathrm{d}t$. In this section, the connection is briefly reviewed and extended to the case of thermal reservoirs. We focus on the approach of [12], which defines Markovianity to be a property of a dynamical map $\rho(0) \mapsto \rho(t) = \Phi(t,0)\rho(0)$ that monotonically decreases the distinguishability $D[\rho_1, \rho_2] = \frac{1}{2}|\rho_1 - \rho_2|$ of any two system states $\rho_{1,2}(t)$. Non-Markovianity is then the ability of a dynamical map to temporarily increase the distinguishability of two states. The temporal change in the distinguishability $\sigma = \mathrm{d}D[\rho_1, \rho_2, t]/\mathrm{d}t$ can be associated with information flowing from the system to its environment ($\sigma < 0$) or back to the system ($\sigma > 0$). The amount of non-Markovianity in a quantum process is given by the cumulant of the positive information flux, $\mathcal{N} = \max_{\rho_1,\rho_2} \int_{\sigma<0} \mathrm{d}s\, \sigma(s)$, with a maximization done over all possible pairs of states to find the largest amount of information that the system can recover from the environment.

The maximization required in the calculation of the non-Markovianity measure is generally difficult. In the case of pure qubit dephasing, however, it has been proven that the optimizing pair is formed by two antipodal states in the equator of the Bloch sphere and in this case the measure can be recovered analytically. One easily finds that information flows back to the qubit from the environment iff the decay rate is negative. Moreover, in the model considered in this paper there is at most a single interval of time $a < t < b$ such that $\gamma(t) < 0$ and thus we introduce a normalized version of the non-Markovianity measure, which measures the amount of recovered information against the amount that was lost to the environment in the time interval $0 < t < a$. Summarizing, the measure we use in this work to study the non-Markovian properties of a qubit dephasing in a BEC environment is

$$\mathcal{N} = \frac{\mathrm{e}^{-\Gamma(b)} - \mathrm{e}^{-\Gamma(a)}}{\mathrm{e}^{-\Gamma(0)} - \mathrm{e}^{-\Gamma(a)}}, \quad \gamma(t) = \frac{\mathrm{d}\Gamma(t)}{\mathrm{d}t} < 0 \Longleftrightarrow t \in [a, b]. \tag{7}$$

In [11], we studied the changes in the non-Markovianity measure induced by different effective dimensions of the reservoir and for a range of different values of the scattering length of the environment, assuming a zero-T environment. We found the existence of a dimension-dependent critical scattering length such that when $0 \leqslant a_B < a_{\mathrm{crit}}$ the dynamics of the qubit is Markovian and when $a_B > a_{\mathrm{crit}}$ it is non-Markovian. The dependence on the effective dimension of the BEC is such that $a_{\mathrm{crit,3D}} < a_{\mathrm{crit,2D}} < a_{\mathrm{crit,1D}}$; that is, the higher the dimension, the smaller the critical scattering length. Hence, one has at hand a model where the Markovian-to-non-Markovian crossover can be controlled either by changing the scattering length of the background bosons or by lowering the effective dimension of the BEC. Conversely, one may deduce these properties of the environment by looking at the qubit alone, without directly measuring the BEC. In this work, we proceed to consider the effect of thermal fluctuations on this result. Thermal fluctuations can, in principle, wash out non-Markovian effects and thus compromise the sensitivity of the probe qubit. Fortunately, we find the double-well qubit model to have a remarkable robustness against thermal effects, as shown in the next section.

4. Results

Figure 1 shows the decoherence factor $\Gamma(t)$ and decay rate $\gamma(t)$ for a three-dimensional (3D) and a one-dimensional (1D) ^{87}Rb condensate with a range of temperatures $T = 0$–200 nK and $T = 0$–20 nK, respectively. We take the same parameters as in [11] and a fixed value $a_B = a_{\mathrm{Rb}}$ for the scattering length. In the 1D case, the negative part of the decay rate, indicating the existence of non-Markovian effects, decreases in size with increasing temperature until it reaches a critical temperature of about $T = 6.5$ nK, where it vanishes completely. When this happens, the qubit dynamics is Markovian. In the 3D case, the negative part of the decay rate splits into two lobes: the low-temperature lobe, enclosed by the line corresponding to the zero-T decay rate, and the high-temperature lobe, indicated by the high-temperature decay rate. The transition between the two lobes, indicated by the decay rate for $T = 50$ nK, corresponds to the transition from the low-temperature regime to the high-temperature regime. We next demonstrate

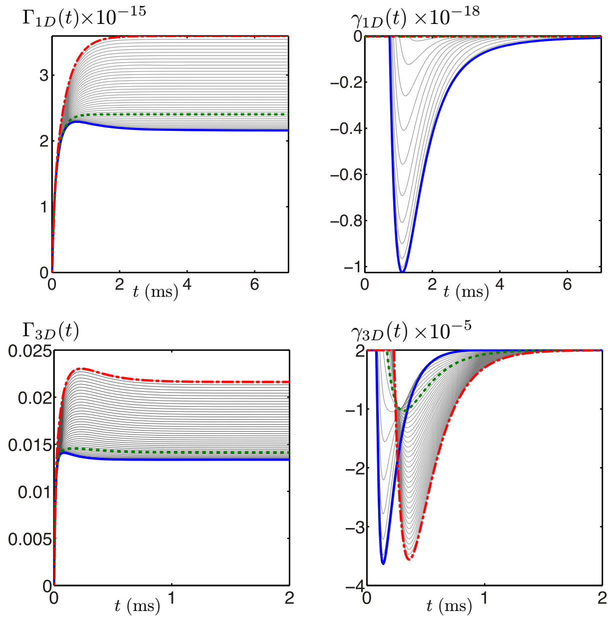

Figure 1. Top: the decoherence factor, $\Gamma_{\mathrm{1D}}(t)$, and the decay rate, $\gamma_{\mathrm{1D}}(t)$, for a 1D environment for temperatures ranging between 0 K (blue solid line) and 20 nK (red dotted line). The $T = 6.5$ nK line, shown in dashed green, shows the transition from the non-Markovian low-temperature limit and the Markovian high-temperature limit. Bottom: the decoherence factor, $\Gamma_{\mathrm{3D}}(t)$, and decay rate, $\gamma_{\mathrm{3D}}(t)$, for a 3D environment for temperatures ranging between 0 K (blue solid line) and 200 nK (red dotted line). The $T = 50$ nK line, shown in dashed green, shows the intermediate stage between the low- and high-temperature limits.

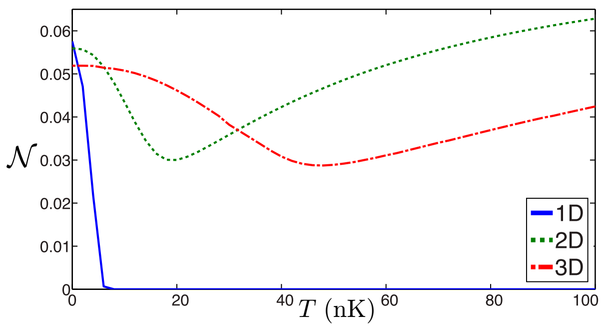

Figure 2. Non-Markovianity measure $\mathcal{N}$ as a function of temperature for quasi-1D (blue solid), quasi-2D (green dashed) and 3D (red dotted) environments. The scattering length is fixed at $a_B = a_{\mathrm{Rb}}$.

that the transition from low to high temperatures is clearly visible also in the non-Markovianity measure.

The temperature dependence of the non-Markovianity measure $\mathcal{N}$ is shown in figure 2 for all the three effective dimensions. In the case of a quasi-1D condensate, the system is non-Markovian only for very low temperatures $T \sim 1$ nK, and for a higher temperature thermal fluctuations wash out the memory effects in the qubit dynamics. When the qubit is embedded in a quasi-2D or a 3D condensate, the dynamics is more robust against thermal effects. In these cases, the non-Markovianity measure is almost constant for low temperatures when $\coth(\beta E_k/2) \approx 1$. When the temperature is increased, the value of the non-Markovianity measure

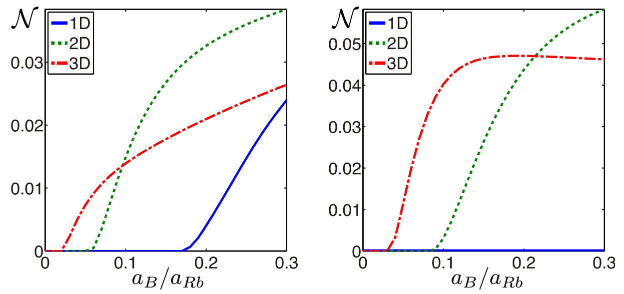

Figure 3. Non-Markovianity measure $\mathcal{N}$ as a function of relative scattering length a_B/a_{Rb} for quasi-1D, quasi-2D and 3D environments with temperatures $T = 0.5$ and 100 nK.

decreases as the system moves toward the high-temperature regime, whereas in the high-T regime the measure increases in value again. The minima in figure 2 correspond to the decay rate moving from the low-T lobe to the high-T lobe (see figure 1). In the high-T regime, $\coth(\beta E_k/2) \approx (\beta E_k/2)^{-1}$, and temperature acts as a coefficient for the decoherence factor of equation (6). Consequently, the whole dynamics is amplified, also leading to higher values of the non-Markovianity measure.

Finally, figure 3 shows the non-Markovianity measure $\mathcal{N}$ as a function of the manipulated scattering length of a Bose–Einstein condensate for temperatures $T = 0.5$ and 100 nK. In both cases, we reproduce the main result of [11], namely that the qubit system has a transition from Markovian to non-Markovian dynamics with increasing scattering length, and that the critical scattering length depends on the effective dimensionality of the BEC: $a_{\mathrm{crit,3D}} < a_{\mathrm{crit,2D}} < a_{\mathrm{crit,1D}}$. For high temperatures the quasi-1D environment is unable to return information back to the system, leading to purely Markovian dynamics. However, since the quasi-2D and the 3D environments still induce non-Markovian dynamics, information obtained on the effective dimensionality of the environment by looking at the qubit dynamics is the same. Thus, we found our main result: thermal effects do not compromise the ability of the probe qubit to detect information about the effective dimensionality and the scattering length of the environment.

5. Discussion and conclusions

The remarkable sensitivity of the probe qubit we propose in this paper derives from its general robustness against thermal effects. This, in turn, is due to the very specific qubit architecture we choose. The deep double-well potential, in which the impurity atom is trapped imposes limitations on the contribution of certain Bogoliubov modes to the qubit dynamics. The decoherence function $\Gamma(t)$ is defined as an integral over all modes k; however, in this model there are two cut-off momenta: $1/\tau$ relates to the size of each harmonic potential in the double well and $1/L$ characterizes the distance between the minima of the wells, and only excitations corresponding to $1/L < k < 1/\tau$ contribute to the dynamics. For high temperatures the temperature-dependent term $\coth(\beta E_k/2)$ diverges at $E_k = 0$, that is, when $k = 0$. However, the lower cut-off frequency excludes the diverging terms and thus prevents non-Markovianity being washed out in the higher-temperature regimes. This is a feature specifically due to the spatial nature of the double-well qubit, rendering this model well suited for realistic temperatures.

In summary, the double-well probe qubit model provides an ideal system for observing non-Markovianity in an atomic system, and for exploiting the Markovian-to-non-Markovian crossover to probe the BEC environment. In addition to involving systems that are straightforwardly combined experimentally, it is an example of a system showing a Markovian-to-non-Markovian crossover in accessible parameter ranges. We have shown here that in addition to the above advantages, this system is also robust to temperature effects, with the measured quantity maintaining its size for temperatures up to and, in some cases, beyond those necessary for experimental realization of these systems.

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