Testing for public debt sustainability using a Time-Scale Decomposition analysis

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Abstract

In this paper we estimate the response of primary surplus to debt ratios to GDP (following Bohn, 1998) to test for debt sustainability within the 12 core EMU countries using a factor model. The analysis is split into two stages. In the first stage we retrieve the cyclical components of primary surplus and debt ratios for each EMU country by using a wavelet decomposition of primary surplus to debt ratios to GDP, based on the Maximal Overlapping Discrete Wavelet Transform. In the second stage, we use Full Information Maximum Likelihood for a factor decomposition of the cross covariance matrix of the wavelet coefficients of primary deficit and debt to GDP ratios at different scales (each associated with a given frequency range). This enable us to gain insight of the standardized reaction (over the cycle) of the primary surplus to debt.

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1. Introduction

The recent turmoil affecting Eurozone sovereign debt has been characterized by raising concerns by financial markets about the ability of countries such as Portugal, Ireland, Italy, Greece and Spain to service their debts. In this paper, we follow the suggestions of Bohn (1998) to assess debt sustainability within the 12 core EMU countries, by estimating the response of primary surplus to lagged debt.

Our contribution to the literature on the estimation of a fiscal reaction function is twofold. First, wee need to account for cyclical effects on fiscal covariates which can, in the short run, undermine fiscal solvency. The approaches adopted in the Stability and Growth Pact, signed in 1997, and in the new Fiscal Compact, signed in December 2011, recognize that cyclical influences on fiscal series cancel out over the business cycle, hence EU Member States have to aim at balancing their budgets over the business cycle. Thereforey, in this paper we study whether the fiscal reaction of primary surplus to lagged debt holds on average over the business cycle. While previous studies have used, for that purpose, estimated regressors as proxies for temporary government spending and for real GDP (subject to measurement errors) among the explanatory variables entering the fiscal reaction function, we avoid the use of such proxies since they could bias the estimates.

Second, previous studies focus on the estimation of the unstandardized response of surplus to debt (which is sensitive to the size of fiscal shocks), we concentrate on the standardized response.

For that purpose, we apply the variance, (cross) covariance decomposition of fiscal time series for those scales corresponding to frequency ranges covering business cycle horizon.

The analysis is split in two stages. In the first stage we use of the Maximal Overlapping Discrete Wavelet Transform to obtain the series of the wavelet coefficients series of primary surplus and lagged debt ratios for each EMU country. In a second stage, we exploit the findings of Witcher (2000) which show that the (cross) covariance of two time series at a given scale can be computed by using the corresponding time series wavelet coefficients. In particular, we apply Full Information Maximum Likelihood, FIML, to implement a factor decomposition of the wavelet coefficients corresponding to the first two scales: the first is associated with a cycle of two to four years and the second one associated with a cycle of four to eight years.

The structure of the paper is as follows. Section 2 describes the literature review on government solvency. The empirical methodology is described in Section 3; Section 4 discusses the empirical evidence and Section 5 concludes. A detailed description of wavelet analysis and of the scale by scale variance-covariance decomposition is presented in the Appendix.

2. Literature Review on government intertemporal solvency

The government intertemporal budget constraint can be stated as the requirement that the current stock of public debt has to be equal to the present discounted value of future primary budget surpluses (e.g. revenues minus non-interest outlays):

$$B_t = \sum_{n=0}^{\infty} E_t\left(u_{t,n} S_{t+n}\right) \tag{1}$$

where B_t is the public debt at the start of a period, $u_{t,n}$ is the period t stochastic discount factor used for discounting state-contingent primary surpluses (primary meaning: excluding interest) S in period $t+n$. Following Bohn (1998), given that both $u_{t,n}$ and the primary surplus S_{t+n} are stochastic, one can write:

$$E_t\left(u_{t,n} S_{t+n}\right) = E_t\left(u_{t,n}\right) * E_t\left(S_{t+n}\right) + \mathrm{cov}_t\left(u_{t,n}, S_{t+n}\right) \tag{2}$$

Therefore, the current stock of public debt, for which intertemporal solvency holds, can be written as:

$$B_t = \sum_{n=0}^{\infty} E_t(u_{t,n}) E_t(S_{t+n}) + \sum_{n=0}^{\infty} \text{cov}_t(u_{t,n}, S_{t+n}) \tag{3}$$

As long as surpluses co-vary positively with u_{tn}, hence with systematic risk included in the stochastic discount factor, the intertemporal solvency constraint can be consistent with negative $E_t(S_{t+n})$, that is with primary surpluses that are negative on average. Ad hoc sustainability test ignore the covariance term in equation (3) and, as shown by Trehan and Walsh (1991), they rely on exploring the unit-root and co-integration features of fiscal data. Model based sustainability test takes into account the last addend in (3), recognizing that the stochastic discount factor can be interpreted as investors' marginal rate of substitution between periods t and $t+n$. Consequently, model based sustainability tests acknowledge that the discount factor applied to S_{t+n} is not the interest rate on public debt, and, it has to be consistent with the general equilibrium conditions that link the government and the private sector. Given the intertemporal solvency condition in (3) can also be stated in terms of primary surplus and debt scaled by GDP, a sufficient condition to test model based sustainability (see Bohn, 1998) consists in estimating the following policy reaction function for the primary surplus:

$$s_t = \alpha + \rho b_{t-1} + \mu_t + \varepsilon_t \tag{4}$$

where the lower case letters are for the fiscal variables scaled by GDP and μ_t controls for temporary determinants of the primary deficit (due to output fluctuations over the business cycle or wars). Debt sustainability occurs if ρ is positive, e.g. when there is an increase in primary surplus in response to rising debt, once we control for temporary fluctuations in government spending and GDP, included in μ_t. The empirical findings of Bohn (1998 and 2008) for the United States and of Mendoza and Ostry (2008) for a large panel of industrial and emerging countries, show that fiscal data are consistent with model based sustainability test. More recently, the study of Fincke and

Geiner (2011) supports model based debt sustainability for PIIGS countries, with the only exception being Greece.

3. Empirical methodology

The methodological contribution to previous studies on model based sustainability (see Bohn, 1998 and 2008; Mendoza and Ostry, 2008; Ghosh et al., 2011, Fincke and Geiner, 2011) is twofold.

First, contrary to the aforementioned studies, our modeling approach does not require the use of estimated regressors entering the term μ_t, in eq. (4). Estimated proxies of cyclical output and government spending might be subject to measurement errors and this can bias the fiscal reaction function parameter estimates. Second, contrary to the previous studies, our focus is on the unstandardized response of primary surplus which is not sensitive to size of the shock hitting the fiscal variables.

3.1 MODWT and factor analysis

Our modeling approach is split in two stages. In the first stage, we apply the Maximal Overlapping Discrete Wavelet Transform, MODWT (see Percival and Walden, 2000; Whitcher, 2000) to obtain a decomposition of each time series into different scales, *j*, each associated to a given frequency range, localized in time (see Appendix for more details). In particular, our focus is on the first two scales. More specifically, since we use yearly data, the time series component at the first scale capture the dynamics of a time series over a short run horizon ranging between two and four years, whereas the time series component at the second scale capture the dynamics of a time series over a medium run horizon ranging between four and eight years.[1] Overall, the focus on the first two scales corresponds to an horizon up to eight-years, that is, a business cycle horizon.

[1] The j-th scale entails a cycle period less than 2^{j+1} years.

Therefore, in the first stage of the analysis, the MODWT filter is used to obtain the wavelet coefficients $\bar{W}^{s}_{j,t}, \bar{W}^{b}_{j,t}$ of surplus and debt (ratios to GDP) for each country. According to Whitcher, (2000), the wavelet cross covariance for the two fractionally integrated time series *s* and *b* (with the orders of integration d_1 and d_2, respectively) for scale λ_j and lag τ is defined as $\gamma_{\tau,s,b}(\lambda_j)$ and it is given by:

$$\frac{1}{N_j} \sum_{l=L_j-1}^{N-\tau-1} \bar{W}^{s}_{j,t} \bar{W}^{b}_{j,t} \qquad (5)$$

where $N_j = N - L_j + 1$ and $L_j = (2^j-1)(L-1)$; *L* stands for the filter length. Given the raw time series data available for each country range from 17 to 42, we prefer to use short filters such as Haar or a Daubechies filter of length $L = 4$, due to the limited number of time series observations. This choice is motivated by the requirement, when selecting the wavelet coefficients to be included in setting up the log-likelihood function, of avoiding trimming too many initial observations for the wavelet coefficients affected by the boundary. Trimming is the price to pay when using a relatively long filter which, on the other hand, guarantees to rely on standard asymptotics when drawing inference (see Whitcher, 2000)[2].

In the second stage of the analysis, once we have used a wavelet decomposition of each time series of length *T* up to scale two, we employ Full Information Maximum Likelihood to the wavelet coefficients differing across scales and across 12 EMU core countries to get a factor a decomposition of the cross covariance matrix (at lag 1 and for the first two scales) of the surplus and debt GDP ratios. Tthe Gaussian log-likelihood we maximize is given by:

[2] In presence of time series having a unit root, condition $L > 2d$ necessary to rely on the central limit theorem (hence to make standard asymptotics inference), would suggest the use of a filter with length bigger than two. Panel unit root test for debt-ratios (see Antonini et al., 2011) would point at a value of *d*, the fractional integration order parameter, equal to unity.

$$\sum_{i=1}^{12}\sum_{j=1}^{2}\sum_{t=1}^{T_i} L\left(\bar{W}^{i}_{jt}; A\Omega^{i}_{j}A'\right) \tag{6}$$

where $L(.)$ is the Gaussian log-density at time t and for scale j and corresponding to country i. The observables entering the log-density are given by $\bar{W}^{i}_{jt}$, the bi-dimensional vector of wavelets coefficients of s_t and b_{t-1} at time t and scale j, and for country i. The unknown coefficients enter A and Ω_{ji} matrices. More specifically, for each scale j, the country specific unknown coefficients are in the diagonal covariance matrix of structural form shocks, Ω^{i}_{j} (for i =1,2), that is:

$$\Omega^{i}_{1} = \begin{bmatrix} (\sigma^{1}_{si})^2 & 0 \\ 0 & (\sigma^{1}_{bi})^2 \end{bmatrix}; \Omega^{i}_{2} = \begin{bmatrix} (\sigma^{2}_{si})^2 & 0 \\ 0 & (\sigma^{2}_{bi})^2 \end{bmatrix} \tag{7}$$

The constant factor loading matrix A is:

$$A = \begin{bmatrix} 1 & \beta \\ 0 & 1 \end{bmatrix} \tag{8}$$

The coefficient β is the parameter measuring the average response (over the cycle) of the primary surplus (scaled by GDP) to a one standard deviation shock to lagged debt.

The specification of the structural form coefficients in Ω^{i}_{j} and A, underlying the reduced form cross covariances at different scales, is equivalent to the following factor model fitted to an unbalanced panel of the 12 EMU countries:

$$
\begin{aligned}
s^i_{jt} &= \beta\sigma^i_{bj}\eta^i_{jt} + \sigma^i_{sj}\nu^i_{jt} \\
b^i_{jt-1} &= \sigma^i_{bj}\eta^i_{jt}
\end{aligned}
\quad ; j = 1,2 \qquad (9')
$$

where s^i_{jt} and b^i_{jt} are the primary surplus and public debt (ratios to GDP) observations at time t, for country i and at scale j. The shock ν^i_{jt} hits only the time series s whereas η^i_{jt} is a common shock to both s and b. The size (measured by the standard deviation) of the shocks ν^i_{jt} and η^i_{jt} are given the parameters σ^i_{bj} and σ^i_{bj} varying across scales and countries. The coefficient β is the standardized primary surplus response to lagged debt. Our focus is on the first two scales which are able to capture a period ranging between two and eight years (which is typically the time horizon corresponding to business cycle frequencies). Since the parameter β is constant across scales and countries, then this coefficient is meant to capture the average response, over the cycle, of primary surplus to debt within the EMU area.

We also consider a second version of the factor model, by letting the the parameters σ_{si} and σ_{bi} varies only across scales:

$$
\begin{aligned}
s^i_{jt} &= \beta\sigma_{bj}\eta^i_{jt} + \sigma_{sj}\nu^i_{jt} \\
b^i_{jt-1} &= \sigma_{bj}\eta^i_{jt}
\end{aligned}
\quad ; j = 1,2 \qquad (9'')
$$

3.2 Test for over-identifying restrictions

Given the use of wavelet coefficients of two variables observed across two scales and across twelve countries, we have a total of 72 moment conditions (6 per country). If the focus is the estimation of the factor model given by (9'), then the total number of unknowns is equal to 49; if the focus is the estimation of the factor model given by (9''), then the total number of unknowns is equal to 5. Therefore in case of model (9') we have 23 over-identifying restrictions, and in case of model (9'') we have 67 over-identifying restrictions. A likelihood ratio test is implemented by comparing the (maximized) log-likelihood function given by either eq. (9') or by eq. (9'') with the (maximized) log-likelihood function for the exactly identified model given by:

$$\sum_{i=1}^{12} \sum_{j=1}^{2} \sum_{t=1}^{T_i} L\left(\bar{W}^{i}_{jt}; \Sigma^{i}_{j} \right) \tag{10}$$

where Σ^{i}_{j} is the sample covariance matrix for the time series s_t and b_{t-1} corresponding to scale *j* and to country *i*.

4. Empirical evidence

The annual data source (ending in 2011) for the General Government consolidated gross debt ratio to GDP and for the primary surplus ratio to GDP is AMECO. The countries under investigation are Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands Portugal and Spain, whose sample starts in 1976, 1970, 1975, 1978, 1991, 1988, 1985, 1980, 1990, 1975, 1977, 1995, respectively. All data are in hundreds. In Table 1 are reported the descriptive stastics.

From Table 2 we can observe that shocks to debt ratio (for both scales) are bigger (in terms of standard deviations) than shocks to surplus ratios. The estimated standardized response of primary surplus ratio to debt ratio according to wavelet decomposition based on the LA(4) and Haar filters are equal to 0.0213 and 0.0254 respectively. However, a Likelihood ratio test rejects the 67 over-identifying restriction characterizing the "pooled" factor model specification given by equation

(9''). We now turn our focus on the estimation results of equation (9') accounting for country specific standard deviation of shocks hitting the two fiscal series.

From Table 3 we can observe that the largest size of shock hitting the public debt to GDP ratio over a period ranging from two to four years (e.g. the one associated with the first scale when using annual data) are for Greece, Portugal and Italy, since the point estimates of the first scale standard deviations (when using the LA(4) filter) are equal to 8.57%, 5.31%, and 4.38%, respectively. The same ranking is observed when using the Haar filter, although we can observe higher values (equal to 10.92%, 6.85%, 5.29%) for the point estimates of the debt ratios standard deviation.
Moreover, from Table 3 we can observe that, if we use the LA(4) filter, then the largest size of shock hitting the public debt to GDP ratio over a period ranging from four to eight years (e.g. the one associated with the second scale when using annual data) is the one for Finland and Greece: the point estimates of the standard deviation of *b*, are 3.59% and 3.23%, respectively. The point estimates of the standard deviation of the debt ratio, based on the Haar filter wavelet coefficients, are higher than the corresponding ones based on the LA(4) filter, and they rank first Greece and France. Luxembourg is the country with the smallest size of shock hitting public debt for both scales.
Furthermore, from Table 3, we can observe that the size of the shock hitting debt ratios is bigger than the one for primary surplus ratios. In particular, if the focus is on the first scale, then Germany, Finland, and Netherlands show the largest standard deviation of *s*, with values ranging between equal to 0.95% and 1.30%. The same countries exhibit the largest standard deviation of primary surplus ratios when the focus is on the LA(4) filter used to obtain the second scale component of *s*.

Inspection of Table 2 and Table 5 gives evidence of a positive standardized positive response (over the cycle) of primary surplus to lagged debt. This evidence is robust to the factor model specification and the filter chosen.

Finally, we can observe (see Table 6) that the maximized log-likelihood functions of the structural and reduced form model are very close to each other only when we employ the LA(4) filter to obtain the wavelet coefficient at different scales. This finding is confirmed by a likelihood ratio test for the 23 over-identifying restrictions which are accepted only when we use a LA(4) filter with length greater than the one corresponding to the Haar filter.

If we focus on the LA(4) filter, the estimated response of primary surplus to debt (see Table 4) would set to 0.024 the value of the interest rate-growth rate differential, *r-g* , capable to stabilize the debt/GDP ratio. More specifically, the condition for a constant debt-to-GDP ratio (over the cycle) requires that βb_{t-1}, which is the estimated size of (cyclically adjusted) primary surplus (according to eq. 4), should be equal to total interest payments, $(r-g)b_{t-1}$. The coefficient *r-g* is the interest rate growth differential. Taking into account the uncertainty surrounding the point estimates of β, measured by the FIML parameter standard error, equal to 0.010 (if we focus on the LA(4) filter, see Table 4), then we should set the interest rate adjust for growth to 4.24% (which is obtained by summing the point estimate of β to two standard deviations of this coefficient) for the worst case scenario. These estimates should help to set the nominal rate of return on a Eurobond, once growth projections are taken into account[3]. The higher are growth projections (within EMU), the higher can be the "affordable" nominal rate of return on a Eurobond, that is a return rate which does not let the public debt ratio to GDP grow unbounded.

[3] Since the second half of 2011, as a possible device to cope with the rising concern of long term public debt sustainability for the peripheral EMU countries, it has been suggested to pool part of eurozone countries' debt. This could, for instance, be achieved, by issuing long term bond with joint guarantee by the whole set of EMU countries

5. Conclusions

In this paper, we assess public debt sustainability within the 12 core EMU countries employing a factor model fitted to an unbalanced panel. Following the suggestions of Bohn (1998), we estimate a fiscal reaction function, and, in particular, the response of primary surplus to lagged debt. Our contribution to previous studies on model based sustainability along the lines of Bohn (1998) is twofold. First, we control for cyclical factors affecting the dynamics of primary surplus (and of public debt) ratio without using proxies of temporary government spending and of real GDP (which might be subject to measurement errors). More specifically, we use a factor decomposition of the cross covariance matrix (at lag one) of the cyclical components of primary surplus and of public debt ratios to GDP. Moreover, the cyclical components are obtained without resorting to an autoregressive dynamics model specification (which might be subject to lag length misspecification). For this purpose, we employ the Maximal Overlapping Discrete Wavelet Transform. As shown by Whichter et al. (2000), the cross covariance matrix of the raw data at a given scale (we focus only on the first two which are associated to a period ranging between two and eight years, covering a business cycle horizon) is obtained by using the sample cross covariance of the wavelet coefficients for that scale. Second, contrary to previous studies focusing on the unstandardized response of surplus to debt (which is sensitive to size of shocks hitting the fiscal series), we apply a factor decomposition of the cross-covariances at the first two scales to get the standardized response (over the cycle). We employ Full Information Maximum Likelihood to estimate and make inference on the parameters of interest.

According to a Likelihood ratio test the data support an over-identified factor model based on wavelet coefficients obtained through an LA(4) filter. The estimated fiscal reaction coefficient (over the cycle) equal 0.0224 suggest that the 12 Eurozone countries as whole are on a sustainable public debt path. Finally, the estimated coefficient measuring the response of primary surplus to debt (and its standard error) can be used to set the nominal rate of return on Eurobond (once growth projections are taken into account).

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Table 1: Descriptive statistics

	AUS	BEL	FIN	FRA	GER	GRE	IRE	ITA	LUX	NED	PORT	SPAIN
Sample Mean												
Debt	50.46	97.19	28.48	47.36	61.37	69.38	66.84	87.85	9.85	60.19	51.15	39.20
Surplus	0.32	1.58	3.89	-0.67	0.30	-0.74	0.69	0.58	2.26	1.31	-1.13	-0.13
Sample Std Deviation												
	AUS	BEL	FIN	FRA	GER	GRE	IRE	ITA	LUX	NED	PORT	SPAIN
Debt	18.13	25.95	17.92	19.39	11.26	41.09	26.81	26.76	4.43	12.98	19.26	18.89
Surplus	1.30	3.40	3.44	1.52	2.07	3.83	7.28	3.21	2.14	2.06	2.49	4.06
Minimum Value												
	AUS	BEL	FIN	FRA	GER	GRE	IRE	ITA	LUX	NED	PORT	SPAIN
Debt	16.66	54.28	6.15	19.82	39.54	15.74	24.71	37.25	4.06	39.55	13.49	11.81
Surplus	-1.99	-7.38	-3.82	-5.14	-5.98	-10.42	-28.03	-5.76	-0.93	-3.58	-7.32	-9.41
Maximum Value												
	AUS	BEL	FIN	FRA	GER	GRE	IRE	ITA	LUX	NED	PORT	SPAIN
Debt	72.15	134.07	57.63	85.84	83.04	165.34	111.70	121.25	20.26	78.48	107.76	68.47
Surplus	3.32	6.84	9.74	1.36	4.34	4.37	6.72	6.51	6.44	5.62	3.46	4.01

Table 2: Estimation results of equation (9’’)

	Filter LA(4)	**Filter Haar**
first scale σ_s	0.0079 (0.0003)	0.0086 (0.0003)
first scale σ_b	0.0227 (0.0106)	0.0482 (0.0018)
second scale σ_s	0.0091 (0.0004)	0.0121 (0.0004)
second scale σ_b	0.0386 (0.0015)	0.0623 (0.0024)
B	0.0213 (0.0009)	0.0254 (0.0072)
Log-Likelihood over-identified model	3050.59	3056.38
Log-Likelihood exactly identified model	3238.8966 3328.9924	3328.9924
p-value Likelihood ratio test	0.0000	0.0000

Note: Standard errors in parenthesis. The Likelihood ratio test under the null is a $\chi^2(67)$.

Table 3: Estimation results of equation (9'); volatilities first scale

		Filter LA(4)		Filter Haar	
Austria	σ_s	0.0059	(0.0007)	0.0063	(0.0007)
	σ_b	0.0304	(0.0038)	0.0363	(0.0044)
Belgium	σ_s	0.0074	(0.0008)	0.0074	(0.0008)
	σ_b	0.0238	(0.0027)	0.0287	(0.0032)
Finland	σ_s	0.0095	(0.0011)	0.0095	(0.0011)
	σ_b	0.0289	(0.0035)	0.0345	(0.0041)
France	σ_s	0.0043	(0.0005)	0.0043	(0.0005)
	σ_b	0.0409	(0.0052)	0.0512	(0.0064)
Germany	σ_s	0.0131	(0.0022)	0.0130	(0.0021)
	σ_b	0.0379	(0.0065)	0.0452	(0.0073)
Greece	σ_s	0.0074	(0.0011)	0.0077	(0.0011)
	σ_b	0.0857	(0.0135)	0.1092	(0.0164)
Ireland	σ_s	0.0074	(0.0010)	0.0119	(0.0016)
	σ_b	0.0245	(0.0036)	0.0296	(0.0041)
Italy	σ_s	0.0055	(0.0007)	0.0061	(0.0008)
	σ_b	0.0438	(0.0058)	0.0529	(0.0068)
Luxemb	σ_s	0.0080	(0.0013)	0.0097	(0.0015)
	σ_b	0.0118	(0.0019)	0.0147	(0.0023)
Netherlands	σ_s	0.0097	(0.0012)	0.0095	(0.0011)
	σ_b	0.0183	(0.0022)	0.0218	(0.0026)
Portugal	σ_s	0.0080	(0.0010)	0.0090	(0.0011)
	σ_b	0.0531	(0.0067)	0.0685	(0.0084)
Spain	σ_s	0.0069	(0.0013)	0.0070	(0.0012)
	σ_b	0.0134	(0.0026)	0.0155	(0.0028)

Note: Standard errors in parenthesis.

Table 4: Estimation results of equation (9’’); volatilities second scale

		Filter LA(4)		Filter Haar	
Austria	σ_s	0.0070	(0.0009)	0.0068	(0.0008)
	σ_b	0.0133	(0.0018)	0.0472	(0.0059)
Belgium	σ_s	0.0062	(0.0007)	0.0063	(0.0007)
	σ_b	0.0208	(0.0026)	0.0384	(0.0044)
Finland	σ_s	0.0147	(0.0020)	0.0169	(0.0020)
	σ_b	0.0359	(0.0048)	0.0548	(0.0067)
France	σ_s	0.0056	(0.0008)	0.0080	(0.0010)
	σ_b	0.0137	(0.0019)	0.0678	(0.0087)
Germany	σ_s	0.0125	(0.0026)	0.0114	(0.0019)
	σ_b	0.0158	(0.0033)	0.0655	(0.0112)
Greece	σ_s	0.0064	(0.0012)	0.0077	(0.0012)
	σ_b	0.0323	(0.0061)	0.1320	(0.0208)
Ireland	σ_s	0.0114	(0.0019)	0.0273	(0.0040)
	σ_b	0.0257	(0.0044)	0.0474	(0.0070)
Italy	σ_s	0.0050	(0.0007)	0.0064	(0.0008)
	σ_b	0.0220	(0.0033)	0.0728	(0.0097)
Luxemb	σ_s	0.0119	(0.0024)	0.0119	(0.0019)
	σ_b	0.0041	(0.0008)	0.0223	(0.0037)
Netherlands	σ_s	0.0104	(0.0014)	0.0112	(0.0013)
	σ_b	0.0143	(0.0019)	0.0300	(0.0037)
Portugal	σ_s	0.0064	(0.0009)	0.0084	(0.0010)
	σ_b	0.0179	(0.0025)	0.0789	(0.0100)
Spain	σ_s	0.0075	(0.0020)	0.0109	(0.0021)
	σ_b	0.0066	(0.0017)	0.0249	(0.0048)

Note: Standard errors in parenthesis.

Table 5: Estimation results of equation (9’’); Fiscal reaction coefficient

LA(4)	0.0224	0.0100
Haar	0.0250	0.0061

Note: Standard errors in parenthesis.

Table 6: Estimation results of equation (6''); Log-Likelihood function and Test for over-identify restrictions

LA(4)	Log-Likelihood exactly identified model	3238.8966
	Log-Likelihood over-identified model	3225.0765
	Likelihood Ratio p-value	0.2302
Haar	Log-Likelihood exactly identified model	3328.9924
	Log-Likelihood over-identified model	3302.9699
	Likelihood Ratio p-value	0.0004

The Likelihood ratio test under the null is a $\chi^2(23)$.

Appendix

Frequency domain approaches provide an insightful representation of econometric data by decomposing it into sinusoidal components at various frequencies, which have intensities that vary across the frequency spectrum. The shortcoming of Fourier analysis is related to the assumption of intensities constant through time. This feature makes Fourier methods ineffective in analysing signals containing local irregularities, such as spikes or discontinuities. Wavelets can be a particular useful tool when the signal is localized in time as well as frequency. Discontinuities in signals can be described in terms of very short (compressed) basis functions with a high-frequency content, whereas a fine analysis at low frequencies can be achieved using highly dilated (stretched) basis functions. In other words, the wavelet is contracted or dilated to change the scale at which one looks at a signal. The wavelet is then shifted or translated in time to correspond to different part of the signal. The procedure is called multiresolution analysis. In particular, in case of a dyadic multiresolution analysis, the dilated and translated family of wavelets functions can be defined as[4]:

$$\psi_{j,k}(t) = 2^{-j/2}\psi(2^{-j}t - k); j,k \in I \qquad \text{(A1)}$$

Where j and k are the integer parameters governing the scale resolution (i.e. 2^{-j}) and translation in time, respectively.
All the wavelet basis functions, $\psi_{j,k}$, are self-similar, namely, they differ only by translation and change of scale from one another. These functions result from a *mother* wavelet, $\psi(t)$, which is any oscillating function with unit energy, i.e.:

$$\int_{-\infty}^{+\infty} \psi(t)dt = 0$$
$$\int_{-\infty}^{+\infty} |\psi(t)|^2 dt = 1 \qquad \text{(A2)}$$

The object of a wavelet analysis is to associate an amplitude coefficient to each of the wavelet. The task is accomplished by the Discrete Wavelet Transform which is implemented via the pyramid algorithm of Mallat (1987). If certain conditions are satisfied, these coefficients completely characterize the signal which is resolved in terms of a coarse approximation and the sum of fine details:

$$x(t) = \sum_k v_{J,k}\phi_{J,k}(t) + \sum_j \sum_k w_{j,k}\psi_{j,k} \qquad \text{(A3)}$$

Here J is the highest possible level of decomposition; $\phi_{J,k}$ is the set of translated orthogonal scaling functions spanning the lower frequency range $[0, \pi/2^{(J)})$. Therefore, the first term

[4] Given a time series with T observations, conventional dyadic multiresolution analysis applies to a succession of frequency intervals in the form of $(\pi/2^{(j)}, \pi/2^{(j-1)})$, with the decomposition level j running from 1 to J. The bandwidths are halved (downsampled by 2) repeatedly descending from high to low frequencies. By the j^{th} round, there will be j wavelet bands and one accompanying scaling function band . At the decomposition level j, one obtains a set of $T/2^j$ mutually orthogonal wavelets functions given by equation (A.1), separated from each other by 2^j points.

$\sum_k v_{J,k}\phi_{J,k}(t)$ in (A3) is the coarse approximation of the signal, and the second term $\sum_j\sum_k w_{j,k}\psi_{j,k}$ in A(3) is the sum of fine details.
The scaling and wavelet coefficients $v_{j,k}$ and $w_{j,k}$ are the following projections of *x(t)* on the bases $\phi_{j,k}$ and $\psi_{j,k}$ respectively:

$$v_{j,k} = \int x(t)\phi_{j,k}(t)dt \qquad \text{(A4)}$$

$$w_{j,k} = \int x(t)\psi_{j,k}(t)dt \qquad \text{(A5)}$$

The signal can then be written as a set of orthogonal components at resolutions 1 to J:

$$x(t) = S_J + D_J + D_{J-1} + \ldots\ldots + D_1 \qquad \text{(A6)}$$

An important feature of a wavelet analysis consists in the fact that it is an energy-preserving transform; as a consequence, the variance of the signal is perfectly captured by the variance of the wavelet coefficients, *w*. In other words, the overall variance of the data can be expressed as a sum of the variances within the frequency bands, which may be indexed by j:

$$\sigma^2 = \sum_{j=1}^{\infty}\sigma_j^2 \qquad \text{(A7)}$$

where σ_j^2 is the contribution of the variability at scale 2^{-j} to the overall variability of the process:

$$\sigma_j^2 = \frac{1}{2^j}Var(w_{j,t}) \qquad \text{(A8)}$$

Similarly, as shown by Whitcher (1998) and by Whitcher et al. (2000), the wavelet covariance decomposes the covariance between two stochastic processes on a scale-by-scale basis. For a bivariate stochastic process $X_t = (x_{1,t}, x_{2,t})$, there will be:

$$\sum_{j=1}^{\infty}Cov_x(j) = Cov(x_{1,t}, x_{2,t}) \qquad \text{(A9)}$$

where

$$Cov_x(j) = \frac{1}{2^j}Cov(w_{1,j,t}, w_{2,j,t}) \qquad \text{(A10)}$$

A disadvantage of the conventional dyadic wavelet analysis is the restriction on the sample size T which has to be a power of 2. A further problem lies in the fact that the DWT depends upon a nonsymmetric filter that is liable to induce a phase lag in the processed data. These difficulties can be circumvented by the Maximum Overlapping Discrete Wavelet Transform (MODWT), which represents an attempt to generate a transform that is not sensitive to the choice of the starting point for the data series. In order to avoid such sensitivity, the filtered output at each stage of the pyramid algorithm is not subjected to downsampling. As a consequence, the number of coefficients generated at the j-th stage of the decomposition are in number equal to the sample size, T, instead that equal to $T/2^j$. An important feature of the MODWT is that, besides handling any sample size, the detail and smooth coefficients of the multiresolution analysis are associated with linear phase filters. The consequence is that it is possible to align the features of the original time series with those of the multiresolution analysis.

The DWT, as well as its variants, the Partial DWT and the MODWT, makes use of circular filtering. he series under investigation is treated as if it is a portion of a periodic sequence with period N. In other words, the transform considers x_{N-1}, x_{N-2}.... as useful surrogates for the unobserved x_{-1} , x_{-2} …. . This can be a questionable assumption for some time series. The effects of this assumption, and solutions to the problems created, are fully explored in Percival and Walden (2000). A problem with the periodic extension can occur when there is a large discontinuity between the end of one replication of the sample and the beginning of the next. In such cases the coefficients produced by the transform result remarkably high and the reconstructed details are affected. To reduce this problem the data should be suitably de-trended.