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On the structural optimization in presence of base isolating devices

F. Giambanco, S. Benfratello, L. Palizzolo and P. Tabbuso

Department of Civil, Environmental, Aerospace, Material Engineering (DICAM)
University of Palermo, Italy
Viale delle Scienze - 90128 Palermo
e-mail: francesco.giambanco@unipa.it, salvatore.benfratello@unipa.it,
luigi.palizzolo@unipa.it, pietro.tabbuso@unipa.it

1. Abstract
The minimum volume design of plane frames constituted by elastic perfectly plastic material and subjected to appropriate combinations of fixed, cyclic and dynamic loads is studied. The influence on the design, in terms of cost (volume) and behavioural features, of seismic protecting devices is particularly focused. The considered protecting device is a lead rubber bearing base isolation system. Two optimal design problem formulations are proposed for the structure with or without the protecting device, both based on the so-called statical approach. The minimum volume frame is reached accounting for three different resistance limits: the purely elastic limit, the (elastic) shakedown limit and the instantaneous collapse limit. The adopted load combinations are alternatively characterized by the presence of only fixed loads, of amplified fixed loads and quasi-static perfect cyclic loads due to the wind action, of suitably reduced fixed loads and dynamic actions due to the earthquake. The linear elastic effects of the dynamic actions are studied by utilizing a modal technique. Reference is made to the most recent Italian code related to the structural analysis and design. The solution of the optimization problem is reached by using a suitable subroutine available into the optimization toolbox of MATLAB® appropriate to the proposed formulations. A flexural frame is studied with and without the relevant seismic protecting device in order to study the influence on the design of such a base isolation system. The related minimum volume structures are obtained assuming the stiffness and the damping feature of the base isolation system as variables within assigned suitable ranges. The Bree diagrams of the obtained optimal designs are also determined in order to characterize and compare their structural and safety behaviour.
2. Keywords: structural optimization, seismic loadings, isolating devices.

3. Introduction
As known, except for wind and snow loads, the actions potentially acting upon a civil or an industrial construction are substantially related to the structural masses by a proportional law. The latter depend on the structure geometry and on the chosen material weight, while the former just depend on the geometry of the construction; furthermore, both depend on the particular geographical site where the building must be realized. It is worth noticing that the actions related to the gravitational loads are usually much more influent on the design of the relevant structure with respect to wind and/or snow actions. Therefore, it would be advisable to realize structures with small dimension elements (characterized by a low weight) but able to guarantee the structure safety in terms of resistance, with suitably chosen safety factors, and provided with sufficient stiffness so that the relevant construction could be used in an optimal way and without any awkward breaks. The above described aim substantially represents the natural objective of the structural optimization which is usually devoted to the search of the minimum weight structure, i.e. among the infinite feasible structures which satisfy the assigned requirements, usually imposed in terms of resistance and/or stiffness, find the structure characterized by the minimum volume. Furthermore, it must be remarked that, as known, the presence of too heavy masses induces its more dangerous effects during the earthquake, so the suitable mass reduction and/or the utilization of appropriate devices able to reduce these undesired seismic effects on the structure represent a fundamental task in order to guarantee the structure safety and the complete usability of the construction. Focusing the attention to the utilization of special devices in order to reduce the earthquake effects, two are the main ways available for the designer. The first one is that of stiffening the structure by introducing suitably disposed cross bracing elements; following this approach the structure floor drifts reduce and consequently the stresses on the beams and pillars reduce, so that, often, the cross braced structures can be designed in such a way that beams and pillars remain elastic while just the cross bracing elements can plasticize and dissipate the prescribed amount of plastic energy (see, e.g. [1]). The second one is that of reducing the amount of seismic energy coming out from the ground to the overhanging structure; this approach is very effective and mainly consists in inserting suitable devices (base isolation systems) between the soil foundations and the structure. The main feature of the base isolation systems is that to increase the first natural period of the whole structure-base isolation system in such a way to make the structure less sensitive to seismic actions. This effect can be obtained by means of different approaches, alternatively adopting a passive control, an active control or a semi-active control. These devices must possess suitable mechanical characteristics such that to increase the first natural period of the isolated structure through a decoupling of the structure motion from the one of the soil. The differences among the above referenced approaches lie in the fact whether the mechanical features of the device can change, depending on the load history, or not. Clearly, passive control devices are such that their characteristics do not change depending on those of the seismic action, while the active control ones are able to do that. In this framework a very large amount of studies are available in literature (see, e.g., [2-4]). To the author's knowledge, the base isolation system based on passive devices is, at present, one of the most efficient and economic technique, able to dissipate most of the input energy even preventing damage to the structure. Generally speaking, the isolator should dissipate energy at frequencies dynamically interacting with the structure and transmit only energy acting in a frequency range that poorly excites the structures. Recent approaches devoted to the design of passive devices take into account for the randomness of the seismic actions (see, e.g., [5]). These approaches have been acknowledged by the greater part of the actual structural codes related to the analysis and design of civil and industrial constructions and in particular by the Italian one

[6], which is referred to in the paper.
Principal aim of the present paper is the proposition of appropriate multicriterion optimal design formulations for elastic perfectly plastic steel frames accounting for the presence of some seismic protection devices. In this study some recent researches of the same authors (see, e.g., [7-9]) are extended, proposing new formulations of the minimum volume design of elastic plastic frames by including in the relevant formulations the dynamic response of the structure obtained by a modal technique, by considering more complete load combinations, by considering the mechanical features of the base isolation system among the design variables and effecting several suitably chosen numerical applications useful to better understand the sensitivity of the structural response. Therefore, the minimum volume design of steel frames with and without suitably defined protecting devices and subjected to the action of static as well as dynamic loadings is studied.
The frame model is thought as constituted by linear elastic beam elements, rigid nodes and rigid perfectly plastic hinges. The base isolation system is modeled as a passive control one(lead rubber bearing base isolation system) whose mechanical features have to be optimized. The relevant optimal design problem is formulated as a minimum volume problem and the so-called statical approach is utilized.
The minimum volume structure is determined under suitable constraints on the design variables including the features of the protecting device as well as accounting for three different resistance limits: it is required that the optimal structure behaves in a purely elastic manner, satisfies the elastic shakedown limit and prevents the instantaneous collapse limit, considering acting for each different limit condition a suitably chosen load combination and imposing for each different condition suitably chosen load amplifiers. The adopted load combinations are characterized by the presence of fixed loads, of quasi-statical perfect cyclic (wind) actions and dynamic (seismic) loads. The linear elastic effects of the dynamic actions are studied by utilizing a modal technique.
The proposed treatment is explicitly referred to the most recent Italian code related to the structural analysis and design, but the concepts exploited in the paper may be easily extended to the other codes of other countries; therefore, the minimum volume design is developed at first for the flexural frame as the search for the optimal structure with simultaneous constraints on the purely elastic behaviour (related to basic gravitational load conditions), on the elastic shakedown behaviour (related to serviceability conditions), on the instantaneous collapse (under the combination of amplified fixed loads and cyclic actions due to the wind effect), and on the instantaneous collapse (under the combination of reduced fixed loads and seismic actions). Subsequently, the minimum volume design is developed for the base isolated frame as the search for the optimal structure with simultaneous constraints on the purely elastic behaviour (related to basic gravitational load conditions), on the elastic shakedown behaviour (related to the combination of reduced fixed loads and seismic actions), on the instantaneous collapse (under the combination of amplified fixed loads and cyclic actions due to the wind effect).
The solution of the previously described problems is computationally reached by using a suitable subroutine available into the optimization toolbox of MATLAB® appropriate to the proposed formulations. The Bree diagrams of the optimal structures are also determined in order to characterize the structural behaviour. The comparison among all the obtained results allows us to deduce several useful information regarding the cost and the response sensitivity of the different optimal structures.

4. Fundamentals and structural model

As described in the foregoing section, principal aim of the present paper is the formulation of two appropriate multicriterion minimum volume design problems for elastic perfectly plastic frame structures subjected to fixed, quasi-static (cyclic) and seismic loadings properly combined together and each suitably amplified by appropriate selected parameters. The first formulation is mainly devoted to flexural frames without any protecting device and the relevant optimal design is reached as the minimum volume structure which behaves elastically under the assigned fixed loads, does not violate the elastic shakedown limit in serviceability (seismic) conditions and prevents instantaneous collapse under ultimate load conditions related to high seismic loads or to wind actions, both combined with appropriate fixed loads of given intensity. Whereas, the second formulation is devoted to base isolated flexural frames and the relevant optimal design is reached as the minimum volume structure which behaves elastically under the assigned fixed loads, is safe against plastic shakedown and/or ratchetting under ultimate load conditions related to high seismic loads combined with suitably reduced fixed loads and prevents instantaneous collapse under ultimate load conditions related to wind actions combined with suitably amplified fixed loads. The above mentioned reduced and/or amplified fixed load intensities which define the described load combinations utilized for the searching of the optimal design are provided by the already referenced Italian code.
In order to appropriately describe the cited formulations, some fundamentals must be introduced mainly regarding the definition of some appropriate model both for the frame structure and for the acting loads.
As known, the classical formulation of the static linear elastic analysis problem for frames constituted by n_b beam type elements, described by the Navier kinematical model, and n_N standard nodes (each characterized by three degrees of freedom) is given as follows:

$$\boldsymbol{d} = \boldsymbol{C}\boldsymbol{u} \tag{1a}$$

$$\boldsymbol{Q} = \boldsymbol{D}\boldsymbol{d} + \boldsymbol{Q}^{*} \tag{1b}$$

$$\tilde{\boldsymbol{C}}\boldsymbol{Q} = \boldsymbol{F} \tag{1c}$$

where $\boldsymbol{d}$ is the element nodal displacement vector of dimension $6 \cdot n_b$, $\boldsymbol{C}$ is the compatibility matrix with order $6 \cdot n_b \times 3 \cdot n_N$, $\boldsymbol{u}$ is the frame nodal displacement vector of dimension $3 \cdot n_N$, $\boldsymbol{Q}$ is the generalized stress vector evaluated at the extremes of the elements with order $6 \cdot n_b$, $\boldsymbol{D}$ is the frame internal (square block diagonal) stiffness matrix with order $6 \cdot n_b \times 6 \cdot n_b$, $\boldsymbol{Q}^*$ is the perfectly clamped element generalized stress vector with the same dimension of $\boldsymbol{Q}$, and $\boldsymbol{F}$ is the frame nodal force vector with the same dimension of $\boldsymbol{u}$. The solution to problem (1) is given by:

$$\boldsymbol{u} = \boldsymbol{K}^{-1}\boldsymbol{F}^{*} \tag{2a}$$

$$\boldsymbol{Q} = \boldsymbol{DCu} + \boldsymbol{Q}^{*} = \boldsymbol{DCK}^{-1}\boldsymbol{F}^{*} + \boldsymbol{Q}^{*} \tag{2b}$$

in terms of displacements and generalized stresses, respectively, with $\boldsymbol{K} = \tilde{\boldsymbol{C}}\boldsymbol{DC}$ frame external square stiffness matrix of order $3 \cdot n_N \times 3 \cdot n_N$, and $\boldsymbol{F}^{*} = \boldsymbol{F} - \tilde{\boldsymbol{C}}\boldsymbol{Q}^{*}$ is the equivalent (in terms of structure node displacements) nodal force vector, where the over tilde means the transpose of the relevant quantity.

According with the guidelines of the greater part of international codes, in particular with the Italian one, the design of the relevant structure must be performed taking into account a fixed action, mainly related with the gravitational loads, a quasi statical (cyclic) load related to the wind effect, and a dynamic perfect cyclic load related to seismic actions; the mentioned loads must be suitably combined adopting for each single action a suitably defined amplifier. In the present context even the load related to the wind is modeled as a perfect cyclic load; actually, in any case a generic cyclic load can be described through the superposition of a fixed and a perfect cyclic load.

Making reference to the seismic actions, let us consider the relevant frame as a shear plane frame just subjected to an horizontal ground acceleration $a_g(t)$, modeled as a Multi-Degree-Of-Freedom (MDOF) structure, such that the total number of degrees of freedom is equal to the number of floors n_f.

As usual, the dynamic equilibrium equations can be written in the following form:

$$\boldsymbol{M}\,\ddot{\boldsymbol{s}}(t) + \boldsymbol{A}\,\dot{\boldsymbol{s}}(t) + \boldsymbol{K}_s\,\boldsymbol{s}(t) = \boldsymbol{f}(t) \tag{3}$$

being $\boldsymbol{f}(t) = -\boldsymbol{M\tau}\,a_g(t)$, where $\boldsymbol{\tau}$ is the influence vector with dimension n_f; $\boldsymbol{s}$ represents the displacement vector related to the structure dynamic degrees of freedom and equations (3) has to be solved according to the initial conditions that hereinafter are assumed to be $\boldsymbol{s}(0) = \boldsymbol{0}$, $\dot{\boldsymbol{s}}(0) = \boldsymbol{0}$.

In equation (3) $\boldsymbol{M}$ and $\boldsymbol{A}$ are the mass and damping matrices (with dimensions $n_f \times n_f$), $\boldsymbol{K}_s = \tilde{\boldsymbol{E}}\boldsymbol{KE}$ is the dynamic stiffness matrix of order $n_f \times n_f$ related just to the horizontal floor displacements, being $\boldsymbol{E}$ an appropriate condensation compatibility operator which applied to $\boldsymbol{s}$ provides the frame nodal displacement $\boldsymbol{u}$ $(\boldsymbol{u} = \boldsymbol{Es})$. All the matrices in equation (3) are assumed to be positive ones. Furthermore, $\dot{\boldsymbol{s}}(t)$ and $\ddot{\boldsymbol{s}}(t)$ are the velocity and the acceleration vectors of the system with dimension n_f, respectively, and the over dot means time derivative of the relevant quantity.

As it is known, the dynamic characteristics of the structural behaviour are identified in terms of natural frequencies as well as damping coefficients. In this framework, as usual, the following coordinate transformation is adopted:

$$\boldsymbol{s}(t) = \boldsymbol{\Phi z}(t) \tag{4}$$

being $\boldsymbol{z}(t)$ the modal displacement vector with dimension n_f and $\boldsymbol{\Phi}$ the so-called modal matrix of order $n_f \times n_f$, normalized with respect to the mass matrix and whose columns are the eigenvectors of the undamped structure, given by the solution to the following eigenproblem:

$$\boldsymbol{K}_s^{-1}\boldsymbol{M}\,\boldsymbol{\Phi} = \boldsymbol{\Phi}\,\boldsymbol{\Omega}^{-2} \tag{5a}$$

$$\tilde{\boldsymbol{\Phi}}\,\boldsymbol{M}\,\boldsymbol{\Phi} = \boldsymbol{I}_{n_f} \tag{5b}$$

$$\tilde{\boldsymbol{\Phi}}\,\boldsymbol{K}_s\,\boldsymbol{\Phi} = \boldsymbol{\Omega}^{2} \tag{5c}$$

In equations (5a,c), besides the already known symbols, $\boldsymbol{I}_{n_f}$ represents the $n_f \times n_f$ identity matrix while $\boldsymbol{\Omega}^2$ is a diagonal matrix of order $n_f \times n_f$ listing the square of the natural frequencies of the structure.

Once the modal matrix $\boldsymbol{\Phi}$ has been determined, the structure can be defined as a classically-damped one if $\tilde{\boldsymbol{\Phi}}\boldsymbol{A\Phi} = \boldsymbol{\Xi}$ is a diagonal matrix of order $n_f \times n_f$ whose typical non zero element Ξ_{jj} is equal to $2\zeta_j\,\omega_j$, being ω_j and ζ_j the j^{th} natural frequency and the j^{th} damping coefficient, respectively.

According to the referenced Italian code, the study related to the frame structure without any base isolation system is performed taking into account all structural modes and assuming a constant damping coefficient equal to 0.05.

Making reference to the elastic response spectrum $S_e(T)$ defined in the relevant code and once the natural frequencies and the modal matrix are known, the displacement vector due to the j^{th} mode can be determined as follows:

$$\boldsymbol{s}_j = \boldsymbol{\Phi}_j\,\frac{\boldsymbol{\Phi}_j^T\boldsymbol{M\tau}\,S_e(T_j)}{\omega_j^2} \tag{6}$$

According to the above referred guidelines the displacements $\boldsymbol{s}$ and the generalized stresses $\boldsymbol{Q}$ can be combined in a full quadratic way following the equation:

$$E_\ell = \sqrt{\sum\nolimits_k \sum\nolimits_j \rho_{jk}\,E_{j\ell}\,E_{k\ell}} \tag{7}$$

being E_ℓ the ℓ^{th} component of the combined effect of the relevant quantity, $E_{j\ell}, E_{k\ell}$ the ℓ^{th} component of the effect due to j^{th} and k^{th} modes, respectively, and ρ_{ij} the correlation coefficients between j^{th} and k^{th} modes expressed by the equation:

$$\rho_{jk} = \frac{8\zeta^2 \beta_{jk}^{3/2}}{\left(1+\beta_{jk}\right)\left[\left(1-\beta_{jk}\right)^2 + 4\zeta_j \zeta_k \beta_{jk}\right]} \tag{8}$$

in which $\beta_{jk} = T_k / T_j$, being T_j, T_k the periods of the j^{th} and k^{th} mode.

In the present framework related to the optimal design of earthquake resistant elastic plastic frame structures, the utilization of a base isolation system can be a very important tool which has been deeply investigated in the recent past. When a passive isolation technique is adopted the relevant equations of dynamic equilibrium are the same as reported in (3) (plus the appropriate initial conditions) but the mass and the stiffness matrices assume the following new form:

$$\hat{\boldsymbol{M}} = \begin{bmatrix} m_{iso} & & & & \\ & m_1 & & & \\ & & m_2 & & \\ & & & . & \\ & & & & m_{n_f} \end{bmatrix}, \quad \hat{\boldsymbol{K}}_s = \begin{bmatrix} k_{iso}^{tot} + k_1 & -k_1 & & & \\ -k_1 & k_1 + k_2 & -k_2 & & \\ & -k_2 & . & & \\ & & & . & -k_{n_f} \\ & & & -k_{n_f} & k_{n_f} \end{bmatrix} \tag{9a,b}$$

being m_{iso} the mass and k_{iso}^{tot} the total stiffness coefficients of the base isolated level and m_j, k_j, $\left(j = 1,2,...,n_f\right)$ the mass and the stiffness coefficients of the j^{th} storey of the main structure. For the aim of the present paper m_{iso} represents a known value of the problem, while $\boldsymbol{k}_{iso}$ must be considered as a variable vector with element appertaining to a suitably assigned range. In the same way, in the case of presence of base isolation device even the damping coefficient of the relevant mode must be considered as variable within a suitably assigned range and, consequently, equation (8) must be written as follows:

$$\rho_{jk} = \frac{8\zeta_j \zeta_k \beta_{jk}^{3/2}}{\left(1+\beta_{jk}\right)\left[\left(1-\beta_{jk}\right)^2 + 4\zeta_j \zeta_k \beta_{jk}\right]} \tag{8'}$$

for all the combinations which involve the mode related to the base isolation system.

Always according with the guidelines of the referenced Italian code, the seismic loads have to be evaluate for two different conditions: the serviceability conditions related to a limit condition of full usability of the building and the exceptional ones in which the structure finds itself in a condition of impending collapse. Clearly, the intensity of seismic actions is very different between the above referenced conditions and it strictly depends on the up-crossing probability of selected intensity levels during the lifetime of the structure.

Therefore, for the aim of the present paper and taking into account the Italian code, we now assume that the actions are represented by four appropriate combinations of the above referred loads each of which related to different suitably chosen limit conditions. The first combination is characterized by the only presence of the full fixed loads $\boldsymbol{F}_0^*$; the second combination is defined as the superimposition of appropriate reduced fixed loads $\boldsymbol{F}_{0e}^*$ (actually, it is widely accepted the hypothesis that the probability of the presence of the full fixed loads during the seismic event is definitely low) and appropriately low seismic actions related to the response spectrum S_e^S (serviceability conditions), function of a suitably selected up-crossing probability in the lifetime of the structure; the third combination is characterized by the superimposition of suitably amplified fixed loads $\boldsymbol{F}_{0w}^*$ and perfect cyclic load related to the wind actions $\boldsymbol{F}_{ciw}$ (in this case the amplifiers of the fixed loads represents suitably chosen safety factors against the impending collapse); the last combination is characterized by the superimposition of the above described reduced fixed loads $\boldsymbol{F}_{0e}^*$ and seismic actions related to the response spectrum S_e^I (ultimate conditions), function of a different suitably selected up-crossing probability in the lifetime of the structure.

In the above defined combinations, $\boldsymbol{F}_{0e}^*$ and $\boldsymbol{F}_{0w}^*$ are special combinations of gravitational loads as prescribed by the referenced code, S_e^S and S_e^I are the response spectra related to serviceability and instantaneous collapse conditions, respectively, while the reference mechanical cyclic loads related to the wind action are defined as two opposite and independent load conditions $\boldsymbol{F}_{ciw}$, ($i = 1,2$), such that $\boldsymbol{F}_{c1w} = \boldsymbol{F}_{cw}$ and $\boldsymbol{F}_{c2w} = -\boldsymbol{F}_{cw}$; therefore, $\boldsymbol{F}_{ciw}$ is modeled as a perfect cyclic load.

Clearly, since the design problem under investigation is a minimum volume search one, the structural geometry is not known a priori and, therefore, let the typical v^{th} element geometry be fully described by the m components of the vector $\boldsymbol{t}_v \left(v = 1,2,...,n_b\right)$ so that $\tilde{\boldsymbol{t}} = \left[\tilde{\boldsymbol{t}}_1, \tilde{\boldsymbol{t}}_2, ..., \tilde{\boldsymbol{t}}_v, ..., \tilde{\boldsymbol{t}}_{n_b}\right]$ represents the $n_b \times m$ supervector collecting all the design variables.

5. Optimal design problem formulation

Let us consider now an elastic perfectly plastic frame structure, as above described, without any protecting device and, according to the Italian code and to the above described loading model, let it be subjected to fixed mechanical loads, quasi static perfect cyclic loads (wind effect) and perfect cyclic dynamic (seismic) loads. Furthermore, let us impose that for the first load combination as above described (full fixed loads $\boldsymbol{F}_0^*$) it behaves in a purely elastic manner; for the second one, i.e. in serviceability conditions (appropriately reduced fixed loads $\boldsymbol{F}_{0e}^*$ and low seismic actions related to the response spectrum S_e^S), it respects the elastic shakedown limit; while for the remaining two load combinations (alternatively, suitably amplified fixed loads $\boldsymbol{F}_{0w}^*$ and perfect cyclic load related to the wind actions $\boldsymbol{F}_{ciw}$, and reduced fixed loads $\boldsymbol{F}_{0e}^*$ and seismic actions related to the response spectrum S_e^I) it is able to prevent the instantaneous collapse.

As a consequence, the multicriterion (minimum volume) design problem formulation, where suitable constraints are imposed on the purely elastic behaviour, on the elastic shakedown behaviour and on the instantaneous collapse, can be written as follows:

$$\min_{\left(\boldsymbol{t},\boldsymbol{u}_0,\boldsymbol{u}_{0e},\boldsymbol{u}_{0w},\boldsymbol{u}_{cw},\boldsymbol{s}_{jce}^S,\boldsymbol{s}_{jce}^I,\boldsymbol{u}_{jce}^S,\boldsymbol{u}_{jce}^I,\boldsymbol{Y}_0^S,\boldsymbol{Y}_{0iw}^I,\boldsymbol{Y}_{0ie}^I\right)} V \tag{10a}$$

subjected to:

$$\bar{\boldsymbol{t}}_{min} \geq \boldsymbol{t} \geq \bar{\boldsymbol{t}}_{max} \tag{10b}$$

$$\boldsymbol{H}\,\boldsymbol{t} - \bar{\boldsymbol{h}} \geq \boldsymbol{0} \tag{10c}$$

$$\boldsymbol{Q}_0 = \boldsymbol{D}\boldsymbol{C}\boldsymbol{u}_0 + \boldsymbol{Q}_0^*\,, \quad \boldsymbol{K}\boldsymbol{u}_0 - \boldsymbol{F}_0^* = \boldsymbol{0} \tag{10d}$$

$$\boldsymbol{Q}_{0e} = \boldsymbol{D}\boldsymbol{C}\boldsymbol{u}_{0e} + \boldsymbol{Q}_{0e}^*\,, \quad \boldsymbol{K}\boldsymbol{u}_{0e} - \boldsymbol{F}_{0e}^* = \boldsymbol{0} \tag{10e}$$

$$\boldsymbol{Q}_{0w} = \boldsymbol{D}\boldsymbol{C}\boldsymbol{u}_{0w} + \boldsymbol{Q}_{0w}^*\,, \quad \boldsymbol{K}\boldsymbol{u}_{0w} - \boldsymbol{F}_{0w}^* = \boldsymbol{0} \tag{10f}$$

$$\boldsymbol{Q}_{cw} = \boldsymbol{D}\boldsymbol{C}\boldsymbol{u}_{cw}\,, \quad \boldsymbol{K}\boldsymbol{u}_{cw} - \boldsymbol{F}_{cw} = \boldsymbol{0} \tag{10g}$$

$$\boldsymbol{s}_{jce}^S = \boldsymbol{\Phi}_j \frac{\tilde{\boldsymbol{\Phi}}_j \boldsymbol{M}\boldsymbol{\tau}\, S_e^S\left(T_j\right)}{\omega_j^2}\,, \quad \boldsymbol{Q}_{jce}^S = \boldsymbol{D}\boldsymbol{C}\boldsymbol{u}_{jce}^S\,, \quad Q_{ce\ell}^S = \sqrt{\sum_j \sum_k \rho_{kj}\, Q_{kce\ell}^S Q_{jce\ell}^S} \tag{10h}$$

$$\boldsymbol{s}_{jce}^I = \boldsymbol{\Phi}_j \frac{\tilde{\boldsymbol{\Phi}}_j \boldsymbol{M}\boldsymbol{\tau}\, S_e^I\left(T_j\right)}{\omega_j^2}\,, \quad \boldsymbol{Q}_{jce}^I = \boldsymbol{D}\boldsymbol{C}\boldsymbol{u}_{jce}^I\,, \quad Q_{ce\ell}^I = \sqrt{\sum_j \sum_k \rho_{kj}\, Q_{kce\ell}^I Q_{jce\ell}^I} \tag{10i}$$

$$\boldsymbol{\varphi}^E \equiv \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\boldsymbol{Q}_0 - \boldsymbol{R} \leq \boldsymbol{0}\,, \tag{10j}$$

$$\boldsymbol{\varphi}_{ie}^S \equiv \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\boldsymbol{Q}_{0e} + \left(-1\right)^i \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\boldsymbol{Q}_{ce}^S - \boldsymbol{S}\boldsymbol{Y}_0^S - \boldsymbol{R} \leq \boldsymbol{0}\,, \quad \boldsymbol{Y}_0^S \geq \boldsymbol{0} \tag{10k}$$

$$\boldsymbol{\varphi}_{iw}^I \equiv \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\boldsymbol{Q}_{0w} + \left(-1\right)^i \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\boldsymbol{Q}_{cw} - \boldsymbol{S}\boldsymbol{Y}_{0iw}^I - \boldsymbol{R} \leq \boldsymbol{0}\,, \quad \boldsymbol{Y}_{0iw}^I \geq \boldsymbol{0} \tag{10ℓ}$$

$$\boldsymbol{\varphi}_{ie}^I \equiv \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\boldsymbol{Q}_{0e} + \left(-1\right)^i \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\boldsymbol{Q}_{ce}^I - \boldsymbol{S}\boldsymbol{Y}_{0ie}^I - \boldsymbol{R} \leq \boldsymbol{0}\,, \quad \boldsymbol{Y}_{0ie}^I \geq \boldsymbol{0} \tag{10m}$$

where equations (10k,ℓ,m) hold for $i = 1,2$ and $\ell = 1,2,.....,6 \cdot n_b$.

In equations (10b,c) $\boldsymbol{t}$ is the design variable vector, whit $\bar{\boldsymbol{t}}_{min}$ and $\bar{\boldsymbol{t}}_{max}$ representing the vectors collecting the suitably chosen imposed limit values of the admissible range for $\boldsymbol{t}$, and $\boldsymbol{H}$ is the technological constraint matrix, with $\bar{\boldsymbol{h}}$ representing a suitably chosen technological vector.

In equations (10d-i) $\boldsymbol{u}_0$ and $\boldsymbol{Q}_0$, $\boldsymbol{u}_{0e}$ and $\boldsymbol{Q}_{0e}$, $\boldsymbol{u}_{0w}$ and $\boldsymbol{Q}_{0w}$, $\boldsymbol{u}_{cw}$ and $\boldsymbol{Q}_{cw}$, $\boldsymbol{u}_{jce}^S = \boldsymbol{K}^{-1}\tilde{\boldsymbol{E}}\boldsymbol{K}_s\boldsymbol{s}_{jce}^S$ and $\boldsymbol{Q}_{jce}^S$, $\boldsymbol{u}_{jce}^I = \boldsymbol{K}^{-1}\tilde{\boldsymbol{E}}\boldsymbol{K}_s\boldsymbol{s}_{jce}^I$ and $\boldsymbol{Q}_{jce}^I$ are the purely elastic response to the assigned full fixed loads, to the appropriately reduced fixed loads to join with seismic actions, to the appropriately amplified fixed loads to join with wind actions, to the mechanical cyclic loads (wind), to the low dynamic load related to the j^{th} structural mode, to the full dynamic load related to the j^{th} structural mode, respectively, evaluated in terms of structure node displacements and element node generalized stresses.

Finally, in equations (10j,k,ℓ,m) $\boldsymbol{\varphi}^E$, $\boldsymbol{\varphi}_{ie}^S$, $\boldsymbol{\varphi}_{iw}^I$ and $\boldsymbol{\varphi}_{ie}^I$ are the plastic potential vectors related to the purely elastic limit (apex *E*), to the elastic shakedown limit (apex *S*) and to the instantaneous collapse limit (apex *I*), respectively, and $\boldsymbol{Y}_0^S$, $\boldsymbol{Y}_{0iw}^I$ and $\boldsymbol{Y}_{0ie}^I$ are the fictitious plastic activation intensity vectors related to the elastic shakedown limit (apex *S*) and to the impending instantaneous collapse(apex *I*), respectively. In addition, $\tilde{\boldsymbol{N}}$ is the matrix of the external normals to the discrete elastic domain boundary, $\tilde{\boldsymbol{G}}_p$ is an appropriate equilibrium matrix which, applied to element nodal generalized stresses, provides the generalized stresses acting upon the plastic nodes of the elements, $\boldsymbol{Q}_{ce}^S$ and $\boldsymbol{Q}_{ce}^I$ the combined generalized stress vectors related to low and full seismic actions, $-\boldsymbol{S} = \tilde{\boldsymbol{N}}\left(\boldsymbol{D}\boldsymbol{C}\tilde{\boldsymbol{G}}_p\boldsymbol{K}^{-1}\boldsymbol{G}_p\tilde{\boldsymbol{C}}\tilde{\boldsymbol{D}} - \boldsymbol{D}\right)\boldsymbol{N}$ is a time independent symmetric structural matrix which transforms the plastic activation intensities into the plastic potentials and $\boldsymbol{R}$ is the relevant plastic resistance vector.

Problem (10) can be easily specialized to the case of plane frames protected by a suitably disposed base isolation system, as previously described. In such a case, yet according to the already referenced Italian code, the load combinations to be considered as well as the limit criteria to be imposed are different. Actually, the code prescribes that the optimal isolated structure maintains an elastic shakedown behaviour even when subjected to high seismic loads. As a consequence, we now assume that the actions are represented just by three appropriate combinations of the loads defined at the previous section each of which related to different suitably chosen limit conditions. The first combination is characterized by the only presence of the full fixed loads $\boldsymbol{F}_0^*$; the second combination is defined as the superimposition of appropriate reduced fixed loads $\boldsymbol{F}_{0e}^*$ and seismic actions related to the response spectrum S_e^I; the third combination is characterized by the superimposition of suitably amplified fixed loads $\boldsymbol{F}_{0w}^*$ and perfect cyclic load related to the wind actions $\boldsymbol{F}_{ciw}$. Furthermore, let us impose that for the first load combination the structure behaves in a purely elastic manner; for the second one, it respects the elastic shakedown limit; while for the third combination it is able to prevent the instantaneous collapse.

Therefore, the multicriterion (minimum volume) design problem formulation for the base isolated structure, where the features of the protecting device are introduced as further variables of the problem, can be written as follows:

$$\min_{\left(\boldsymbol{t},\boldsymbol{k}_{iso},\zeta_{iso},\hat{\boldsymbol{u}}_0,\hat{\boldsymbol{u}}_{0e},\hat{\boldsymbol{u}}_{0w},\hat{\boldsymbol{u}}_{cw},\hat{\boldsymbol{s}}_{jce}^I,\hat{\boldsymbol{u}}_{jce}^I,\boldsymbol{Y}_0^S,\boldsymbol{Y}_{0iw}^I\right)} V \tag{11a}$$

subjected to:

$$\overline{\boldsymbol{t}}_{min} \geq \boldsymbol{t} \geq \overline{\boldsymbol{t}}_{max} \tag{11b}$$

$$\overline{\boldsymbol{k}}_{min} \geq \boldsymbol{k}_{iso} \geq \overline{\boldsymbol{k}}_{max} \tag{11c}$$

$$\overline{\zeta}_{min} \geq \overline{\zeta}_{iso} \geq \overline{\zeta}_{max} \tag{11d}$$

$$\boldsymbol{H}\,\boldsymbol{t} - \overline{\boldsymbol{h}} \geq \boldsymbol{0} \tag{11e}$$

$$\boldsymbol{Q}_0 = \boldsymbol{D}\hat{\boldsymbol{C}}\hat{\boldsymbol{u}}_0 + \boldsymbol{Q}_0^*, \quad \hat{\boldsymbol{K}}\hat{\boldsymbol{u}}_0 - \hat{\boldsymbol{F}}_0^* = \boldsymbol{0} \tag{11f}$$

$$\boldsymbol{Q}_{0e} = \boldsymbol{D}\hat{\boldsymbol{C}}\hat{\boldsymbol{u}}_{0e} + \boldsymbol{Q}_{0e}^*, \quad \hat{\boldsymbol{K}}\hat{\boldsymbol{u}}_{0e} - \hat{\boldsymbol{F}}_{0e}^* = \boldsymbol{0} \tag{11g}$$

$$\boldsymbol{Q}_{0w} = \boldsymbol{D}\hat{\boldsymbol{C}}\hat{\boldsymbol{u}}_{0w} + \boldsymbol{Q}_{0w}^*, \quad \hat{\boldsymbol{K}}\hat{\boldsymbol{u}}_{0w} - \hat{\boldsymbol{F}}_{0w}^* = \boldsymbol{0} \tag{11h}$$

$$\boldsymbol{Q}_{cw} = \boldsymbol{D}\hat{\boldsymbol{C}}\hat{\boldsymbol{u}}_{cw}, \quad \hat{\boldsymbol{K}}\hat{\boldsymbol{u}}_{cw} - \hat{\boldsymbol{F}}_{cw} = \boldsymbol{0} \tag{11i}$$

$$\hat{\boldsymbol{s}}_{jce}^I = \hat{\boldsymbol{\Phi}}_j \frac{\tilde{\hat{\boldsymbol{\Phi}}}_j \hat{\boldsymbol{M}}\hat{\boldsymbol{\tau}}\,\hat{S}_e^I\left(T_j\right)}{\hat{\omega}_j^2}, \quad \boldsymbol{Q}_{jce}^I = \boldsymbol{D}\hat{\boldsymbol{C}}\hat{\boldsymbol{u}}_{jce}^I, \quad Q_{ce\ell}^I = \sqrt{\sum_j \sum_k \hat{\rho}_{kj}\, Q_{kce\ell}^I Q_{jce\ell}^I} \tag{11j}$$

$$\boldsymbol{\varphi}_{iso} \equiv \eta_{iso}\boldsymbol{Q}_{iso} - \boldsymbol{R}_{iso} \leq \boldsymbol{0}, \tag{11k}$$

$$\boldsymbol{\varphi}^E \equiv \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\boldsymbol{Q}_0 - \boldsymbol{R} \leq \boldsymbol{0}, \tag{11ℓ}$$

$$\boldsymbol{\varphi}_{ie}^S \equiv \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\boldsymbol{Q}_{0e} + \left(-1\right)^i \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\boldsymbol{Q}_{ce}^I - \hat{\boldsymbol{S}}\boldsymbol{Y}_0^S - \boldsymbol{R} \leq \boldsymbol{0}, \quad \boldsymbol{Y}_0^S \geq \boldsymbol{0} \tag{11m}$$

$$\boldsymbol{\varphi}_{iw}^I \equiv \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\boldsymbol{Q}_{0w} + \left(-1\right)^i \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\boldsymbol{Q}_{cw} - \hat{\boldsymbol{S}}\boldsymbol{Y}_{0iw}^I - \boldsymbol{R} \leq \boldsymbol{0}, \quad \boldsymbol{Y}_{0iw}^I \geq \boldsymbol{0} \tag{11n}$$

where behind the already known symbols, $\boldsymbol{k}_{iso}$ is the introduced variable vector containing the unknown stiffness of the n_{iso} base isolators, being $\overline{\boldsymbol{k}}_{min}$ and $\overline{\boldsymbol{k}}_{max}$ the vectors collecting the suitably chosen imposed limit values of the admissible range for $\boldsymbol{k}_{iso}$, and ζ_{iso} is the introduced variable related to the damping coefficient of the base isolation system, being $\overline{\zeta}_{min}$ and $\overline{\zeta}_{max}$ the suitably chosen imposed limit values of the admissible range for ζ_{iso}, $\boldsymbol{Q}_{iso}$ is the vector collecting the shear force acting on the base isolators, $\eta_{iso} \geq 1$ is a suitably chosen integer (safety factor) and $\boldsymbol{R}_{iso}$ is the relevant resistance vector, while $\boldsymbol{\varphi}_{iso}$ is the plastic potential vector related to the resistance limit of the base isolation system. It is worth noticing that the chosen typology for the base isolation system allows us to define the base isolator resistance as linear function of the relevant stiffness through a coefficient provided by the manufacturer and depending on the maximum admissible horizontal displacement, i.e. $\boldsymbol{R}_{iso} = \alpha\boldsymbol{k}_{iso}$. Finally, the symbol $\left(\hat{\bullet}\right)$ characterizes all the quantities already present in problem (10) the dimensions of which expand directly depending on the imposed base isolated floor.

6. Applications

In this section the optimal designs of elastic perfectly plastic steel frames have been numerically obtained making reference to the different structures considered and to the related formulations proposed into the previous sections. In particular, a multicriterion design (simultaneously according to purely elastic, elastic shakedown and instantaneous collapse limit criteria) has been determined for two steel frames constituted by four floors: a classical flexural frame and the same frame but provided by a base isolation system. The solutions of the optimization problems (10) and (11) are reached by using a suitable subroutine available into the optimization toolbox of MATLAB® appropriate to the proposed formulations (*fmincon*).

The non isolated flexural frame under examination is plotted in Fig. 1a. It is constituted by rectangular box cross section elements (Fig. 1b) with $b = 300$ mm and $h = 600$ mm, and the thickness is assumed as a constant features for each element. The element thicknesses are assumed as design variables and collected in the vector $\boldsymbol{t}$. Furthermore, $L_1 = 600$ cm, $L_2 = 400$ cm, $H = 400$ cm, Young modulus $E = 21\,\mathrm{MN/cm^2}$ and yield stress $\sigma_y = 23.5\,\mathrm{kN/cm^2}$ has been assumed. Two rigid perfectly plastic hinges are located at the extremes of all elements, considered to be purely elastic, and an additional hinge is located in the middle point of the longer beams (Fig.1c). The interaction between bending moment M and axial force N has been taken into account. In Fig. 1d the dimensionless rigid plastic domain of the typical rigid perfectly plastic hinge is plotted in the plane N/N_y , M/M_y , being N_y and M_y the yield generalized stress corresponding to N and M , respectively.

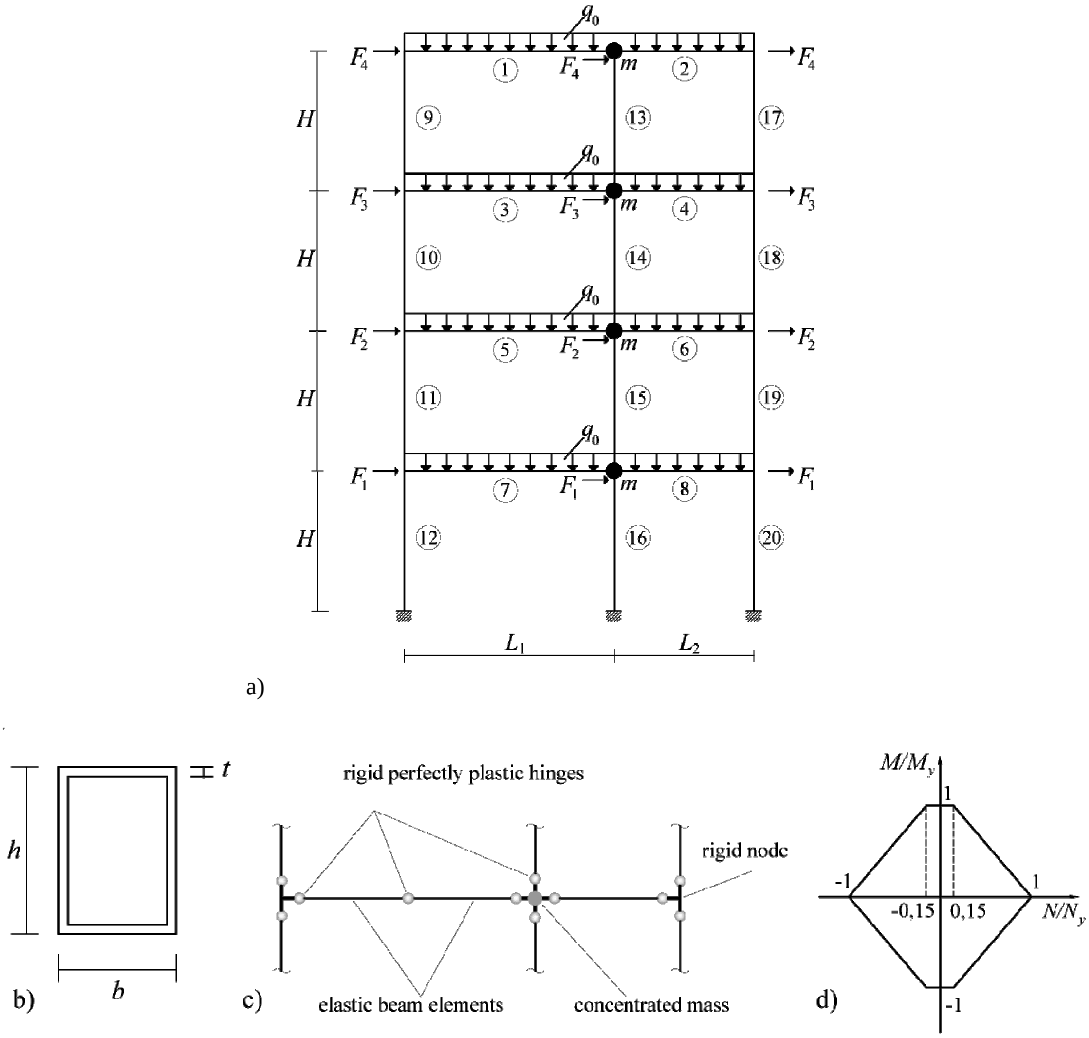


Figure 1: Non isolated flexural frame: a) geometry and load condition; b) typical box cross section; c) structural scheme; d) rigid plastic domain of the typical hinge.

The structure is subjected to a fixed uniformly distributed vertical load on the beams, $q_0 = 70$ kN/m , to perfect cyclic concentrated horizontal loads applied on all the nodes (wind effect) described by the vector $\tilde{\boldsymbol{F}}_{cw} = |35.07 \quad 41.06 \quad 46.45 \quad 25.22|$ (kN), where the typical component F_{cwj} is the resultant force at the j^{th} floor, and to seismic actions. Vector $\boldsymbol{F}_{cw}$ is computed referring to the Italian code for a building in Palermo, assuming a class type B, a category type III and an impact surface for the typical floor equal to 28 m^2. It is worth noticing that the typical load F_j $(j=1,2,3,4)$ represented in Fig. 1a is deduced as $F_j = F_{cwj}/3$. Furthermore, we assume that the seismic masses are equal for each floor, $m = 57.08 \text{ kN} \cdot \text{sec}^2/\text{m}$, and located in the intermediate node at each floor, (Fig. 1a).
The selected response spectra for serviceability conditions (up-crossing probability in the lifetime 81%) and for instantaneous collapse (up-crossing probability in the lifetime 5%) are those corresponding to Palermo, with a soil type B, life time 100 years and class IV.
The optimal multicriterion design has been computed solving problem (10), assuming $F^*_{0ej}/F^*_{0j} = 0.8$ and $F^*_{0wj}/F^*_{0j} = 1.25$, with F^*_{0ej} , F^*_{0wj} and F^*_{0j} the j^{th} components of the relevant vectors.
The results obtained for the plane frame plotted in Fig. 1a have been determined by solving problem (10) and they are reported in terms of thicknesses in Table 1. The optimal reached volume has been $V = 1.71$ m^3 .

Table 1. Optimal thicknesses (mm) of the optimal flexural frame.

El.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
s	4.64	2.62	8.11	12.46	3.72	28.56	2.76	3.83	4.65	4.48
El.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
s	2.60	4.90	7.38	13.23	20.33	27.11	2.75	9.61	18.12	22.49

In order to investigate the features of the obtained design the relevant Bree diagrams reported in Fig. 2a,b have been determined. In particular, in Fig. 2a the Bree diagram describing the response of the structure to the combination of fixed and seismic loads is plotted, indicating with ξ_{0e} and ξ_{ce} the multipliers of the fixed and seismic actions, respectively, while in Fig. 2b the Bree diagram describing the response of the structure to the combination of fixed and wind loads is plotted, indicating with ξ_0 and ξ_c the multipliers of the fixed and wind actions, respectively.

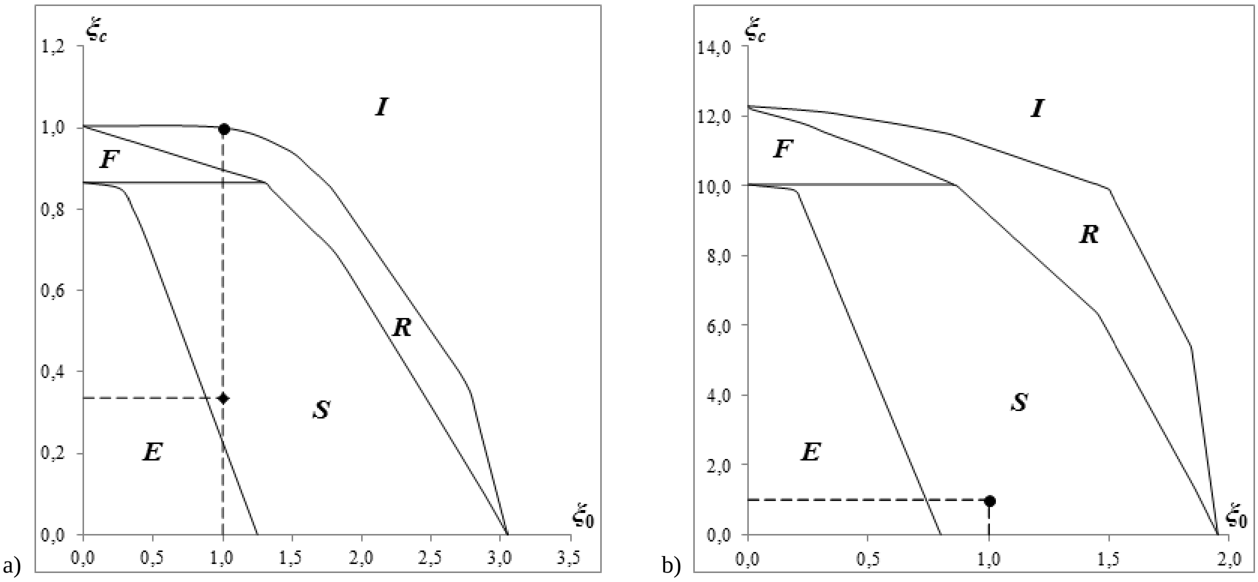


Figure 2: Bree diagrams of the non isolated flexural frame: a) fixed and seismic loads, b) fixed and wind actions.

The second frame under examination is plotted in Fig. 3, where a suitably disposed base isolation system is considered. The isolated structure is constituted by the same rectangular cross box section elements adopted for the non isolated frame (Fig. 1b) and yet the thicknesses, constant for each element, are assumed as design variables and collected in the vector $\boldsymbol{t}$. The geometrical and mechanical features as well as the loading model of the base isolated frame in Fig 3 are the same as the ones defined for the flexural frame plotted in Fig. 1.

It is worth noticing that the further degree of freedom introduced at the base isolated level imposes to consider three additional wind actions $F_{iso} = 5.84\ \text{kN}$ on the three ground level nodes. Among the different passive device systems available on sale, attention has been focused on the lead rubber bearings which differ from the elastomeric isolators by the presence of a central lead core. The reason of such a selection is mainly due to the high dissipated energy characterizing such a devices together with the property of a simple bi-linear force-displacements constitutive law. The mechanical characteristics of the base isolation system are defined in terms of mass, assuming even for the base isolation level $m_{iso} = m = 57.08\ \text{kN} \cdot \text{sec}^2/\text{m}$, while the stiffness, $\boldsymbol{k}_{iso}$, and damping ratio, ζ_{iso}, of the base isolation system are assumed as variables. The assigned values $\bar{k}_{min,j} = 1.55\ \text{kN/mm}$, $\bar{k}_{max,j} = 4.55\ \text{kN/mm}$ $(j = 1,2,3)$, and $\bar{\zeta}_{min} = 20\%$, $\bar{\zeta}_{max} = 30\%$ for the base isolation behaviour ranges have been deduced as typical ones for isolation system on sale for the relevant case. Furthermore, the maximum horizontal displacement of the base isolation level $u_{hiso} = 150\ \text{mm}$ and the safety factor for the base isolating devices $\eta_{iso} = 1.30$ have been assumed.

The results obtained for the plane frame plotted in Fig. 2 have been determined by solving problem (11) and they are reported in terms of thicknesses in Table 2.

For the simple case studied the optimal structure has been characterized by values of isolator stiffness substantially identical $k_{iso,j} = 1.85\ \text{kN/mm}$ $(j = 1,2,3)$, while the optimal damping coefficient for the base isolation system has been found $\zeta_{iso} = 22\%$. The optimal reached volume has been $V = 0.617\ \text{m}^3$.

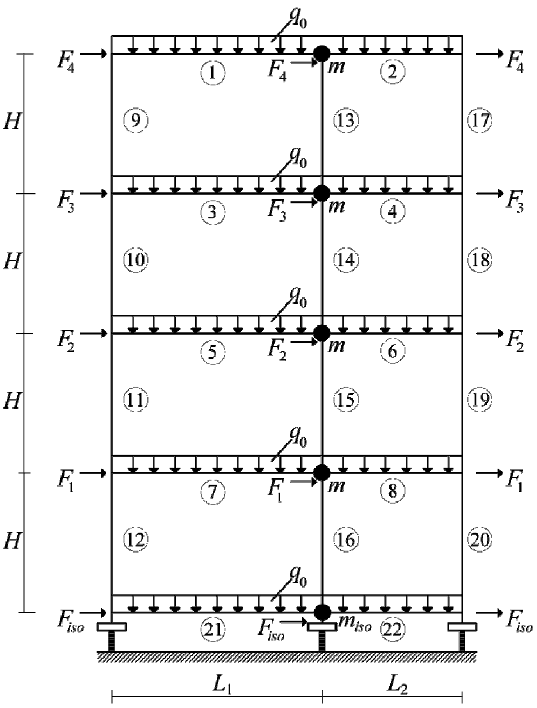


Figure 3: Flexural steel frame equipped with a base isolation system.

Table 2. Optimal thicknesses (mm) of the optimal base isolated flexural frame.

El.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
s	2.49	1.55	2.49	2.28	2.50	5.36	2.52	8.00	1.80	2.03	2.65
El.	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
s	3.52	2.04	3.99	6.39	8.61	1.00	2.18	4.80	6.11	2.65	5.27

As usual, the features of the obtained design can be deduced by the analysis of the relevant Bree diagrams reported in Fig. 4a,b. In particular, in Fig. 4a the Bree diagram describing the response of the structure to the combination of fixed and seismic loads is plotted, where again ξ_{0e} and ξ_{ce} are the multipliers of the fixed and seismic actions, respectively, while in Fig. 4b the Bree diagram describing the response of the structure to the combination of fixed and wind loads is plotted, where again ξ_0 and ξ_c are the multipliers of the fixed and wind actions, respectively.

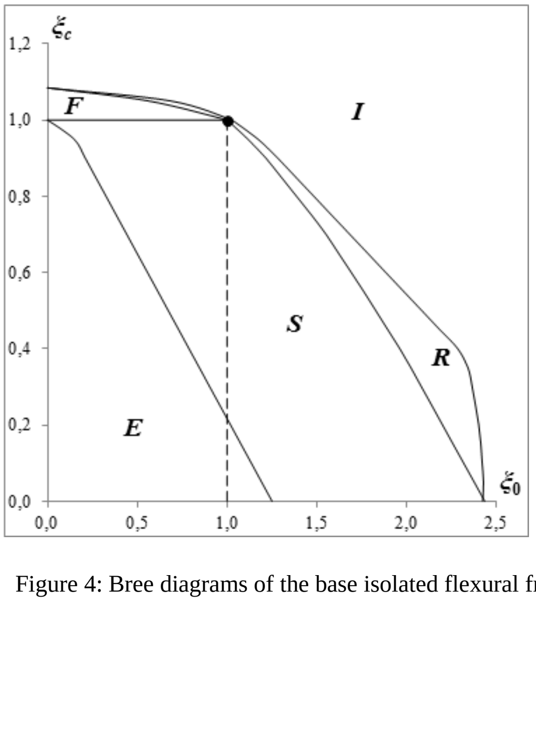
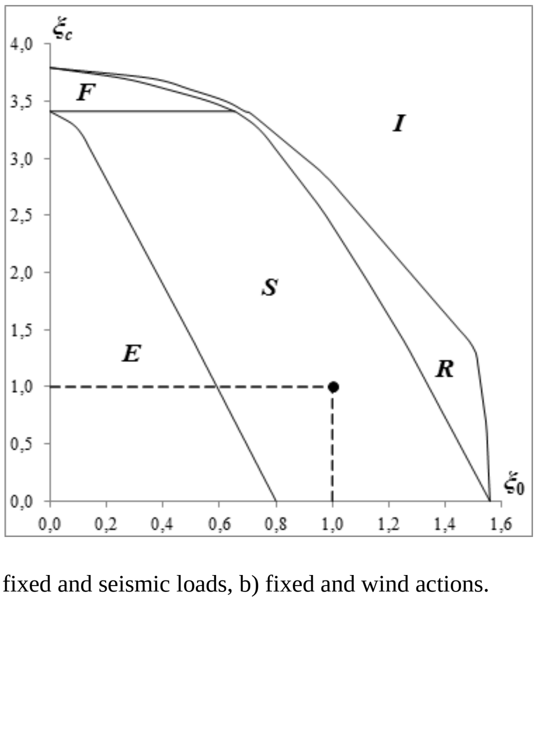
a)  b) 

Figure 4: Bree diagrams of the base isolated flexural frame: a) fixed and seismic loads, b) fixed and wind actions.

The analysis of the obtained results allows us to make some useful remarks. First of all, as it was easy to expect, the structural volume of the optimal base isolated frame turned out to be noticeably lower than the analogous volume of the optimal frame without protecting device, with a percentage decrease of 64%. Such an occurrence, even if the cost related to the realization of the protecting devices must be considered as well as the cost of its maintenance must be suffered, however guarantees a great economy. Yet, it must be consider that, due to the imposed elastic structural behaviour of the isolated frame even for high seismic load conditions, it is not expected the need of structural recovering works after the seismic event. Furthermore, the stiffness of the base isolation system trends to be as low as possible always accounting for its limit resistance. The examination of the obtained Bree diagrams shows that both the optimal frames are substantially insensitive to the wind actions while they find themselves in a condition of impending collapse for high level seismic actions. Such an occurrence was expected, and imposed, for the non isolated frame, while it represents a bad condition for the base isolated structure; actually, the imposed elastic shakedown behaviour for ultimate seismic loads does not produce a sufficiently safe behaviour against the prescribed loads. In order to limit this undesired behaviour different approaches can be utilized (see, e.g. [9]), but they lies outside of the specific interest of the present paper.

7. Conclusions

The present paper has been devoted to develop a suitable approach for taking into account in the optimal design of elastic perfectly plastic frames subjected to different load conditions defined as suitable combinations of fixed loads, perfectly cyclic (wind) loads and seismic actions, as well as the presence of base isolating devices. This problem is very important in the framework of structural analysis and design since the base isolating system, especially the passive one, is widely adopted when it is required and/or convenient to reduce the energy amount transmitted from the ground to the overhanging structure. The chance of adopting base isolating devices has been introduced in the most recent international codes related to structural analysis and design and in particular in the Italian one which has been referred to all along the paper. In order to fulfill the requirements of the code it is necessary to perform a multicriterion design problem able to take into account all the prescribed load combinations. Therefore, in the present paper, the optimal design problem has been formulated, on the grounds on a statical approach, as the search for the minimum volume structure and three different resistance limits have been simultaneously considered: the purely elastic limit, the elastic shakedown limit and the instantaneous collapse limit. The actions which the structure can suffer are defined as four different combinations as follows: a basic load combination defined taking into account just the vertical (gravitational) actions; a serviceability combination characterized by the simultaneous presence of suitably reduced fixed loads and (low) seismic loads (related to an 81% up-crossing probability in the structure lifetime); two ultimate limit load combinations characterized alternatively by the presence of suitably amplified fixed loads and perfect cyclic (wind) actions, or by the presence of suitably reduced fixed loads and (high) seismic loads (related to a 5% up-crossing probability in the structure lifetime).

Two different formulations of the minimum volume design have been proposed: the first one is devoted to the optimal design of flexural frames without any protection device imposing constraints on the purely elastic behaviour for the basic load combination, on the elastic shakedown behaviour related to the serviceability combination and on the instantaneous collapse related to suitably alternative combinations of fixed and perfectly cyclic or seismic actions; the second one is devoted to the optimal design of base isolated flexural frames with constraints on the purely elastic behaviour for the basic load combination, on the elastic shakedown behaviour for the combination of suitably reduced fixed loads and high seismic actions, and on the instantaneous collapse just for the combination of suitably amplified fixed loads and perfectly cyclic (wind) actions.

The effected numerical applications are related to a four plane steel frame. The obtained optimal structures have been compared in order to interpret the safety and behavioural structure features. Obviously, it has been deduced that the optimal volume of the structure equipped with the described protecting devices is definitely smaller than the one related to the unprotected structure. In particular, the volume percentage saving resulted in 64% for the base isolated structure. Moreover, the optimal structure equipped with the considered protecting device is characterized by a more safe behaviour; actually it exhibits an elastic shakedown behaviour even suffering the action of high seismic loadings, even if, as already remarked, the elastic shakedown limit practically coincides with the instantaneous collapse one. Anyway, no steady-state plastic deformations must be expected for the base isolated frame and, consequently, no further cost must be considered related to the maintenance of the structure nor problems related to its loss of functionality.

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