Prospective elementary teachers' perceptions of the processes of modeling: A case study

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In this paper we discuss a study on the approaches to modeling of students of the 4-year elementary school teacher program at the University of Palermo, Italy. The answers to a specially designed questionnaire are analyzed on the basis of an *a priori* analysis made using a general scheme of reference on the epistemology of mathematics and physics. The study is performed by using quantitative data analysis methods, i.e. factorial analysis of the correspondences and implicative analysis. A qualitative analysis of key words and terms used by students during interviews is also used to examine some aspects that emerged from the quantitative analysis. The students have been classified on the basis of their different epistemological approaches to knowledge construction, and implications between different conceptual strategies used to answer the questionnaire have been highlighted. The study's conclusions are consistent with previous research, but the use of quantitative data analysis allowed us to classify the students into three "profiles" related to different epistemological approaches to knowledge construction, and to show the implications of the different conceptual strategies used to answer the questionnaire, giving an estimation of the classification or implication "strength." Some hints on how a course for elementary school physics and mathematics education can be planned to orient the future teachers to the construction of models of explanation are reported.

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I. INTRODUCTION

In recent years the education research community has shown great interest in the problems that can arise when doing activities based on scientific and mathematical modeling in schools.

Different definitions of modeling in education have been put forward. When we refer to the "modeling" of a situation or phenomenon we mean the process of (i) finding the variables which may be relevant in the description of the phenomenon itself, (ii) giving a verbal and schematic description of the phenomenon, (iii) determining the existing relationships between the variables, (iv) representing these relationships through equations and/or rules that give the model a predictive value [1]. According to Niss [2], models can be used in science and mathematics teaching in order to help students analyze and assess a given situation, and to consolidate the analytical skills acquired during learning. The specific, scientific contexts in which models are constructed and discussed are also important.

Many researchers have recommended shifting the focus of science education from traditional subject matter to more general themes and developing students' skills,

Published by the American Physical Society under the terms of the Creative Commons Attribution 3.0 License. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. giving more relevance to modeling activities in the teaching and learning of scientific disciplines [3–5]. Moreover, it has been argued that it would be useful to rethink the nature of the mathematical problem-solving experiences students are provided with at school, in terms of contents, approaches to learning, and ways of assessing learning. Lesh *et al.* [6] have proposed doing this by means of mathematical modeling. This approach has traditionally been reserved for secondary schools [7], but recent research has indicated that it can also be used at elementary school level [8].

On the other hand, as Viennot *et al.* have pointed out [9], it is important to take the teacher role into account. Just as students cannot be considered passive receivers of what they are taught, teachers are not simply passive transmitters of teaching innovation defined by research. More or less explicit disregard of critical details of a proposed teaching sequence and more or less evident dislike for planned strategies are likely to deeply influence student's learning. Moreover, apart from their knowledge and technical ability to implement the suggested approaches in the classroom, the teacher will probably look at them more or less favorably depending on their personal lines of thought, or beliefs, about knowledge construction.

Prawat [10] argues that teachers' beliefs pose an obstacle to educational reform "because of their adherence to outmoded forms of instruction that emphasize factual and procedural knowledge at the expense of deeper levels of understanding" (p. 354). The applications of teachers'

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personal beliefs and cognitive styles to modeling have also been an important theme in research in mathematics teaching, as can be seen from specific studies [11] and from studies originating from many national and international conferences on this subject, for example, ICME (International Congresses on Mathematical Education) working groups and the related publications on modeling in different countries' curricula and in teacher training courses [12].

Kagan [13] refers to beliefs as a "particularly provocative form of personal knowledge" and argues that teachers' professional knowledge can be influenced by beliefs. According to Kagan, this knowledge improves as teachers' experiences in classrooms, and with colleagues working in the same environment, grow. A highly personalized pedagogy is thus formed, which actually restricts the teacher's perception, judgment, and behavior. The teacher must, therefore, not be left alone either while training or when in a teaching post, in order to try to orient their beliefs about knowledge construction towards modeling and innovative use of technological tools. Particularly important among these tools are real time laboratory systems and computer simulation and modeling environments, which are well recognized by educational research as useful for improving the modeling experience.

An appropriate analysis of the teachers' existing modeling abilities and personal lines of thought about modeling is needed within the framework of teacher training courses. This is particularly relevant in university courses for elementary school teacher training, where, at least in Italy, science and scientific approaches are not always considered the most fundamental subjects for an effective teacher training program.

In this paper, we discuss some of the results of a research study on the conceptions and beliefs about knowledge construction of student teachers (STs) attending the 4-year program for elementary teacher training at the University of Palermo and their approaches to modeling activities. Students attending the program mainly come from secondary schools not specializing in mathematics and science. As a consequence, many of them do not have these disciplines as their "first choices." Their attitude towards science is remarkably

similar to what Palmer reported [14]. He found that little or no success in the STs' own experience in science influences their attitudes and self-confidence towards science and science teaching. This is also highlighted by the results of the admission tests they underwent to access the program.

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The main hypothesis of our work is that elementary school student teachers implicitly have their own beliefs about the construction of scientific knowledge and the understanding of reality, which then become explicit when engaged in modeling activities and processes. These beliefs are the result of their past experience as students and the implicit behavior of their past school teachers. Experiences in the social context in which they live and work can also play a significant role.

II. EPISTEMOLOGICAL APPROACHES TO LEARNING MATHEMATICS AND PHYSICS: A REFERENCE FRAMEWORK

Much has been written about different epistemological approaches to learning mathematics and physics, with particular relevance to modeling abilities. In our research we concentrate only on the most well-known approaches, as they are easily identifiable in future teacher behavior.

- (1) The behaviorist approach [15–17] sees knowledge construction as a response to concrete external stimuli coming from the real life world. Learning is achieved best when the learner can confront real, concrete situations. One uses descriptions of reality mainly based on concrete data coming from the environment. Memory is often used "on the spot" and out of context.
- (2) Cognitive psychology claims that knowledge construction is a process that mainly involves the use of memory (i.e., recalling of cognitive resources), motivation, and thinking. Learning is a personal mental process which depends on the learner's processing capacity, the amount of effort spent in the learning process, the depth of the processing [18,19], and the learner's existing knowledge structure [20]. Description or interpretation of new situations is based on previous learning experience and on contextualized use of memory and other cognitive resources.
- (3) Constructivist theory claims that knowledge construction is the product of observation, processing, interpretation, and personalization of information into personal knowledge structures [21,22]. The learner interprets the world and builds their knowledge by making analogies with their previous models of knowledge and making abstract references to objects and ideas coming from experience. Learning is often promoted by peer to peer work and by contextualization for immediate application in order to acquire personal meaning.

¹The program for elementary teacher training has its origins in a previous program, named "Pedagogical Sciences," traditionally made up of very intense theoretical study of general teaching practice and psychology, which mainly dealt with humanistic education. For the past ten years workshop activities for the teaching of mathematics and science in elementary schools have also been included in the program's curriculum. The general approach of these courses was rather traditional, with lectures and problem-solving activities. At the end of the course, student teachers were also requested to produce a written teaching sequence for elementary schools on specific topics related to mathematics and science which was then discussed during an oral examination.

TABLE I. Standard approaches to knowledge, with reference to modeling.

Behaviorist Cognitivist Constructivist

The learner adapts their behavior in order to respond to given learning stimuli from the external world, i.e., the true reality. A model is seen as a repetition of facts, or a reproduction of objects that really exist, or a scale reproduction of reality, aimed at completely describing what is observed.

The learner activates their cognitive resources (memory, past learning experience, etc.) to make sense of the surrounding reality. The model is therefore a mental construction aimed at making sense of reality by comparing it with the learner resources. The learner is able to recall the relevant variables from a previously studied phenomenon and to find useful relationships between them but is not always able to explain the reasons for their answers.

The learner learns by designing and constructing abstract "objects," concepts, descriptions, and interpretations of natural phenomena, in different ways, including peer to peer interaction. The model is a mental construction built by analogy to situations not necessarily related to the phenomenological world and can be applied to it, if needed.

However, when the behaviorist, cognitivist, and constructivist schools of thought are closely analyzed, many overlaps in their ideas and principles become apparent. According to Ertmer and Newby [23], teaching or learning of mathematic or scientific disciplines can include principles from all the three schools of thought and the three approaches can be used as a system of classification, or "taxonomy," for learning. Behaviorist strategies can be used to teach the "what" (facts), cognitive strategies can be applied to teach the "how" (processes and principles), and, finally, constructivist strategies can be used to teach the "why" (higher level thinking that promotes personal meaning and situated and contextual learning).

For example, let us consider the traditional approach followed by many physics or mathematics textbooks in their "exercise" sections, which can be considered to be based on a behaviorist line, although some aspects of cognitivism can be found too. In fact, the more or less implicit idea suggested in textbooks is to have a general look at all the data and variables contained in the exercise and to limit the solution strategy to these, excluding strategies that do not make use of all the data that the problem suggests. Although it provides an immediate solution to many textbook problems, this strategy is very often insufficient for efficient long-term learning. Many students that get accustomed to it often find it difficult to face new, more abstract problems, where more data than strictly necessary is given or when data is given as symbols and not as concrete numerical values to be applied immediately. In this case a wider approach, based on constructivist strategies, is needed.

Table I summarizes the three approaches to knowledge construction described, with particular reference to modeling processes.

III. RESEARCH

A. Aims and research questions

This study is based on quantitative and qualitative research methods. It involves the analysis of data obtained

from an open-ended questionnaire and the analysis of interviews. Data from the questionnaire have been analyzed by means of implicative statistics methods [24] (see the following section). The main objective is not a final testing of the hypothesis resulting from previous studies (see, for example, [13,25]) but further development of it and the search for hints and suggestions for improving teacher training courses for elementary school teachers.

The main research questions involved in this study are the following.

- What are the main epistemological approaches to mathematics and physics learning put into action by a sample of student teachers that followed a traditional high school curriculum?
- Do our primary school student teachers begin mathematics and physics courses already able to correctly connect mathematical models to real situations?

B. Research methodology and sample

An open-ended questionnaire was administered to 78 STs in the third year of the program for elementary school teacher training in the academic year 2009-2010, before the beginning of the courses of physics and mathematics education.² The participants in the survey were enrolled on a voluntary basis and explicitly agreed to contribute to our study on the cognitive styles of elementary school student teachers.

²The physics education course was made up of 60 hours of lectures and workshop activities. The mathematics education course involved similar activities, oriented to mathematics, and was 40 hours long. The two courses were taught by two of the authors of this paper (who have long experience in physics and mathematics education research) and were closely integrated with respect to the general teaching approach and to the theoretical framework. The third author, a physics education researcher, was not involved in the courses. He acted as an independent observer during the administration of the questionnaire and also conducted the interviews with the STs, making clear that these were in no way connected to course evaluation.

The questionnaire is made up of two parts: a sixquestion section on the processes of modeling and a fourquestion section on the connections between models and real situations, i.e., how personal ideas on modeling are put into action when related to concrete situations. Some of the questions (mainly the ones in the first part) are inspired by other questionnaires on the processes of modeling, which we thought relevant for our survey [26,27].

The analysis of the questionnaire was conducted in three separate stages. In the first one, an *a priori* analysis of the possible answers to the questions was performed. According to Brousseau [28], this analysis allows researchers to highlight the answering strategies expected from students facing a problematic situation and the potential alternative responses that may appear. The analysis is conducted independently of the observation (hence the term *a priori*), in order to provide a reference point for the subsequent study of the "postobservations"—in our case the approaches to modeling highlighted by STs in answers to the questions.

The *a priori* analysis was independently performed by the three researchers, and then a consensus was negotiated to obtain the final, shared version of the analysis.

In the second stage, actual ST's answers were independently analyzed by the researchers, by taking into account the strategies that we found during the *a priori* analysis. As a first result we found that some of the hypothesized answering strategies were not used by the STs and, by contrast, some unforeseen strategies were put into action. In line with previous research [27,29,30] these strategies were *a posteriori* added to the *a priori* answering strategies, in order to obtain a global list useful to better classify ST behavior. This list is shown below and reports the questionnaire's 10 questions, in bold. Each question is followed by the set of possible strategies we hypothesized STs would put into action when answering the question, and the unforeseen strategies, in italics.

During the analysis each researcher used the list to draw up a table containing the strategies actually used by each ST to answer the questions. The inter-rater reliability of the analysis was very high. Discordances between researcher tables were only found when a ST answer was classified by one or more researchers not considering just one of the a priori or a posteriori strategies, but identifying in it elements of two or more strategies. In a few cases discordances were due to different researcher interpretations of STs' statements. This happened 8 times when comparing tables of researchers 1 and 2, 5 times for researchers 2 and 3, and 4 times for researchers 1 and 3. Hence we obtained percentages of accordance of about 99% between the analysis tables of each researcher couple. The differences between the three tables were compared and discussed by the researchers to reach a consensus on a common table to use for the study.

Questions and their respective answering strategies

1. Models are very common in science and mathematics, but what actually is a model in physics?

1A A faithful or reduced scale reproduction of a real object.

1B An operative procedure to follow in order to simplify and describe phenomena from the natural world.

1C A reproduction of a real object, not necessarily on a reduced scale, aimed at helping us to interact with it and/or describe it.

1D A stylized or simplified reproduction of a real object, aimed at helping us to interact with it and/or describe it.

1E A mental representation of a real object or phenomenon, which accounts more or less accurately for its mechanisms of functioning.

1F A real or abstract object that behaves like another real object, but does not necessarily look like it.

1G A physical model is a mental formalization of real phenomena.

2. And what is a model in mathematics?

2A A picture of a geometrical shape, maintaining fixed proportions between its elements.

2B It is a method to faithfully describe reality.

2C It is a quantitative but essential reproduction of a phenomenon.

2D A mathematical model is a symbolic or quantitative representation of a situation or phenomenon.

2E A mathematical model is a guideline or a formula, aimed at resolving a problem.

2F A mathematical model is a simplified representation of a system, whose basic elements (variables, sources and contexts) are connected by relationships (a set of rules).

2G A mathematical model is a reference for the construction of a line of reasoning or the demonstration of a hypothesis.

2H It is a description of a situation or phenomenon that is useful for predicting the evolution of the situation or phenomenon itself.

2I It is an abstract construction that allows different quantitative representations of the same object to be built.

3. Are the models creations of human thought or do they already exist in nature?

3A They are creations of human thought based on preexisting "natural models".

3B Models really exist and are simple, real life situations.

3C Models already exist in nature and humans try to understand them, sometimes only imperfectly.

3D Models are simply creations of the human mind, like mathematical formulas.

3E Many models are creations of the human mind and are what we call "theories".

- 3F Models are creations of human thought; their creation comes from continuous interaction with the "real" external world.
- 3G Models are creations of human thought, and their purpose is to predict and make sense of natural phenomena.

4. What are the main characteristics of a model? Give at least one example.

- 4A A model has to start from some hypotheses of the real world that have to be verified.
- 4B It must be able to account for all the features of the real object it represents.
- 4C The model must simply be a description of the reality.
- 4D It must be fixed and immutable. It is the modeler that chooses a model suitable for the real situation.
- 4E It must be able to account for the features of the real that are of practical interest.
- 4F It must highlight the variables that are relevant for the description or explanation of the phenomenon and their relationships.
- 4G It must be expressed in mathematical language and/ or accepted by a scientific community.
- 4H It can be qualitative, semi quantitative or quantitative.
- 4I It must allow what we observe about different phenomena or situations to be generalized.
- 4J It must be useful for analyzing and making predictions about the behavior of a more or less complex system.

5. Can all natural phenomena be described or explained by a model? Carefully explain your answer.

- 5A Yes. A natural phenomenon can always be described by a physical model, as physics is the natural world, with all its laws.
- 5B No. There are phenomena that cannot be described or explained with a model and/or that cannot be defined in terms of precise physical quantities.
- 5C Not always. Even the ablest modeler will not be able to reproduce particularly complex systems (for example human behavior).
- 5D No. Some phenomena still have not been explained, but they will be in the future.
- 5E Yes. It just depends on the modeler's ability to carefully reproduce the features of interest.
- 5F Yes and no. In fact the way nature works is not completely known to man, so further study is necessary to explain all phenomena.
- 5G Yes. If the modeler is able to find all the relevant variables that characterize the phenomenon.

6. Is a mathematical formula always a way to express a real situation? Carefully explain your answer.

6A No, as mathematics is an abstract construction and does not always represent reality.

- 6B Yes, but only if it quantitatively describes the entire real situation.
- 6C No, because reality is so complex that it cannot always be expressed by a mathematical formula.
- 6D No, because not all phenomena can be described mathematically or quantitatively.
- 6E Yes, because mathematics is the language the human brain uses to quantitatively describe or explain a real situation.
- 6F No, as a real phenomenon can have characteristics that cannot easily be expressed in mathematical language.
- 6G Yes, because a mathematical law is always verifiable, starting from well-defined hypotheses.
- 6H Yes, but it is necessary to carefully choose the mathematical variables needed to express the real situation.

7. Can the mathematical formula y = ax be used to calculate the circumference of a circle? Carefully explain your answer.

7A No, as in the formula for circumference calculation the radius and the circumference are present, and not the variables x and y.

7B No, because the constant a does not have the correct value, i.e. 2π .

7C No, because y = ax is a direct proportionality, i.e., a straight line, while the circumference is a curve.

7D No, because the formula y = ax is an algebraic one, while the circumference calculation is a geometric task.

7E No, because y = ax is not the correct mathematical relationship between x and y.

7F Yes, because the circumference is directly proportional to the radius, as y is with respect to x in the formula y = ax.

8. An object is free falling. Report the variables that you think are relevant for the description of the phenomenon and verbally describe the relation that you think exists between these variables. Carefully explain your answer.

- 8A The speed of the object depends on certain parameters, like the object's weight, its shape or the forces acting on the object.
- 8B The relevant variables are space and time. They are linearly dependent.
- 8C The relevant variables are space and time. Space is proportional to the square root of time.
- 8D The relevant variables are the acceleration caused by gravity and/or the starting height and/or the mass and/or the force of gravity.
- 8E In order to describe the phenomenon we must determine all the forces acting on the object and then use Newton's 2nd law.
- 8F The relevant variables are time, space, velocity and acceleration—an explanation is given, but the relationships between the variables are not completely or clearly expressed.

8G The relevant variables are time, space, velocity and acceleration. Space is proportional to the squared time and/ or velocity is proportional to time—examples are clearly given.

9. Write the mathematical formula that represents the relation you found in the previous question. Carefully explain your answer.

9A Verbal explanation based on concrete situations, but no formula reported.

9B Graphic representation of non significant variables that come from real experience.

9C Use of incorrect formulas, like s = vt and/or F = Ma.

9D $s = \frac{1}{2}at^2$ —no explanation.

9E $s = \frac{1}{2}at^2$ and v = at—no explanation.

9F v-t and/or s-t formulas or graphs are reported and correctly commented on or applied.

10. Consider the free falling object in the previous two questions. How would you modify the model to take into account other elements that can influence the motion of the object, like the medium in which the motion takes place?

10A The motion of the object can be influenced by environmental conditions, like wind or temperature.

10B The motion can be influenced by a collision with another object.

10C Friction with air can influence the motion of an object.

10D Friction with air can influence the motion of an object, so density may be a relevant variable.

10E If we want to improve the model, we should take into account one or more forces opposite to motion, for example, friction with air, which increases with the velocity of the object, with its surface area, etc.

In the last stage of the questionnaire analysis, we drew up a table that identifies three "profiles" containing the answering strategies that can be considered typical of each epistemological approach reported in Sec. II. Each profile defines the "ideal model" of a ST answering all the questions by always adopting that given epistemological approach. These profiles, shown in Table II, have been used for the quantitative analysis of the research data, which is further explained in Sec. IV.

In order to better clarify the meaning of the three profiles, the answers from the *a priori* or *a posteriori* analysis, which each of the three ideal STs would give to the questions according to our analysis criteria, are shown in the Appendix.

After the questionnaire had been administered, interviews were conducted with a selected group of 15 STs, in order to get relevant information on the STs' behavior and to widen the analysis of their cognitive styles by highlighting points of interest or unusual elements in

TABLE II. Ideal profiles of STs and the related answering strategies for the 10-item questionnaire.

Behaviorist	Cognitivist	Constructivist	
1A, 1B	1C, 1D	1E, 1F, 1G	
2A, 2B, 2C,	2D, 2E, 2F, 2G	2H, 2I	
3A, 3B, 3C,	3D, 3E	3F, 3G	
4A, 4B, 4C, 4D	4E, 4F, 4G, 4H	4I, 4J	
5A, 5B, 5C, 5D	5E, 5F	5G	
6A, 6B, 6C, 6D	6E, 6F, 6G	6H	
7A, 7B	7C, 7D, 7E	7F	
8A	8B, 8C, 8D, 8E	8F, 8G	
9A, 9B	9C, 9D, 9E	9F	
10A, 10B	10C, 10D	10E	

the questionnaire answers. The interview protocol was predesigned by all three researchers, but the interviews were conducted by one of the researchers face to face with the ST. In some cases, questions not in the interview protocol were asked in order to better clarify specific situations that emerged during the discussion. Interviews were audio recorded and then analyzed together by the three researchers, on the basis of a search for key words, and specific aspects of the STs' answers that provided evidence of the cognitive style(s) used, in relation to our research questions [31,32].

IV. DATA ANALYSIS

A. General overview of the quantitative method used

"If a question is more complex than one that follows it, then every pupil who succeeds in the first one should also succeed in the second one" [24]. Every teacher knows that there are exceptions to this situation, whatever the degree of complexity of the question. The evaluation and the structuring of implicative relationships between variables (in our case, teaching situations, learning strategies, etc.) are the general problems at the origin of the development of statistical implicative analysis (SIA) [33,34]. These problems, which have also drawn attention from psychologists interested in ability tests [35], have become the subject of significant renewed interest during the past decade.

SIA provides a complete framework to evaluate the relevance of relationships between variables and to structure them in order to get correlations at different granularity levels. The underlying objective is to highlight the emerging properties of the whole system, which cannot be deduced by simply breaking it up into parts [36]. All these properties, which emerge from complex interactions, contribute to the interpretation of the nature of the system as a whole. A more complete description of SIA is given in [24].

In this work we use two well-known SIA indexes, the *similarity* and the *implication*, aimed at getting fine detail

about the properties of our sample system (a group of 78 STs and 72 answering strategies), on the basis of our research hypothesis and in relation to the research questions.

Let us consider two generic STs *i* and *j*. Lerman's similarity index [37,38] classifies STs according to hierarchical clustering [39,40] and allows us to calculate similarities in ST behavior (i.e., similar answering strategies). It is defined as follows:

$$s(i,j) = \frac{n_{i \wedge j} - \frac{n_i n_j}{n}}{\sqrt{\frac{n_i n_j}{n}}} \quad \forall \ n_j \neq n,$$

where n_i and n_j are the number of answering strategies put into action by i and j, respectively, n is the total number of answering strategies (72, in our case), and $n_{i \wedge j}$ is the number of common answering strategies used by i and j.

For fixed values of n_i and n_j the greater $n_{i \wedge j}$ is (i.e., the more i and j are "similar"), the more s(i,j) is positive. When i and j put completely different strategies into action $(n_{i \wedge j} = 0)$ the similarity index assumes negative values.

If we take into account two generic answering strategies, a and b, we can define the implication index, $q(a, \bar{b})$:

$$q(a,\bar{b}) = \frac{n_{a\wedge\bar{b}} - \frac{n_a n_{\bar{b}}}{n}}{\sqrt{\frac{n_a n_{\bar{b}}}{n}}} \quad \forall \ n_{\bar{b}} \neq 0,$$

where n_a is the number of STs that put strategy a into action, $n_{\bar{b}}$ is the number of STs not putting strategy b into action (i.e., using all possible strategies except b), n is the total number of STs (78, in our case), and $n_{a \wedge \bar{b}}$ is the number of STs both using strategy a and not using strategy b.

These two indexes are better described in [24], where a full theoretical discussion of their origin and meaning is given. In order to clarify the use we made of them, we remark that s(i, j) is mainly used to reveal whether there is a grouping of ST behaviors and if it is possible to identify clusters of behaviors with respect to their similarity to the epistemological approaches to knowledge discussed in Sec. II. This analysis is done by considering the three "ideal profiles" of STs defined in Table II. The use of ideal profiles of individuals participating in a survey or research is common in many research papers [24,29,30] and the results of previous research validate this method both theoretically and experimentally.

On the other hand, the implication index, $q(a, \bar{b})$, allows us to find relationships (or implications) between strategies activated in each answer and to study their coherence in the proposed framework of epistemological approaches to knowledge described in Sec. II. The possibility offered by $q(a, \bar{b})$ to get fine detail about the implications of the strategies used allowed us to better specify the results

TABLE III. Data matrix for C.H.I.C. analysis. The 78 STs are shown as s1, s2, ..., s78. The three ideal ST profiles are shown as behav. IST, cogn. IST, and const. IST, respectively, and the 72 answering strategies are represented by 1A, 1B, ..., 10E.

Student Teacher	Strategy				
	1A	1B		10D	10E
s1					
s2					
s3					
s78					
behav. IST					
cogn. IST					
const. IST					

obtained by the use of s(i, j) and also to obtain the data necessary to answer our second research question.

B. Sample data analysis

In order to analyze the data we used C.H.I.C. (classification hiérarchique implicative et cohésitive) software [41–43]. It enables the calculation of associations (implicative and similarity) from a set of data and the construction of dendrograms in the form of "implicative graphs" and "similarity trees," for an easy comparison of the results. The software also provides a level of significance for each index value. In fact, an implication between two strategies is identified on the basis of the percentage of STs making use of both the first and the second answering strategies. The similarity between two STs is also expressed by a percentage indicating the similarity level, i.e., the confidence assigned by C.H.I.C. to the similarity relationship between them.³

The matrix that we build in order to use C.H.I.C. has the form of the one shown in Table III

For example, if student teacher s1 used strategies 1B and 10D in his answers, the s1 row in Table III will contain the binary digit 1 in the related cells, while all other cells will contain 0. C.H.I.C. works on this table to determine implications between ST strategies and uses the transposed one for similarity analysis. The last three rows represent the ideal ST models described in Sec. II. They contain 1 and 0 according to the profiles defined in Table II.

More information about the software and its use in the framework of SIA can be found in Ref. [44].

³For each answer to a question, C.H.I.C. completely identifies a student with one of the three ideal profiles if the student used at least one of the answering strategies in Table II for that question. That is, if a student used strategy 6E and/or 6F and/or 6G, he is classified as 100% similar (for question 6) to the "cognitivist" profile.

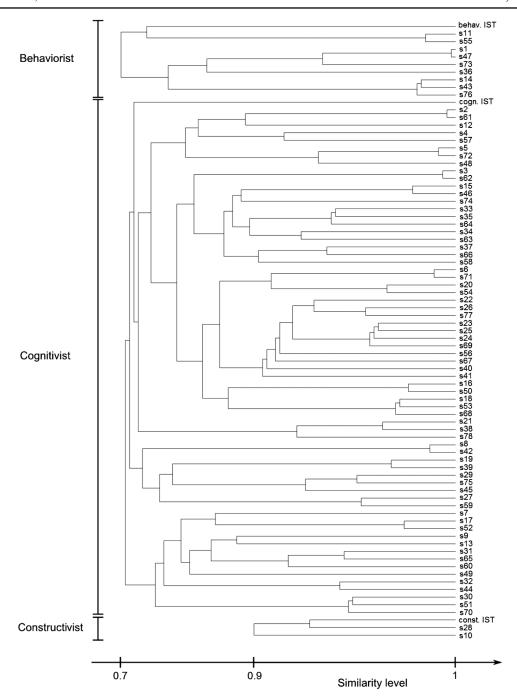


FIG. 1. Similarity tree for the study variables (STs answering to the test and the three ideal profiles identified on the basis of the *a priori* or *a posteriori* analysis). The horizontal axis shows the similarity level between variables, but it is not represented to scale.

V. ANALYSIS OF RESULTS AND DISCUSSION

A. Quantitative analysis

Figure 1 shows the similarity tree obtained from our data. Each ST is represented by s_i (where i goes from 1 to 78) in the right-hand column of the graph. The three ideal profiles: behav. IST, cogn. IST, and const. IST (representing the behaviorist, cognitivist, and constructivist approaches to knowledge, respectively) are considered as ideal STs and placed in the same column. The tree shows

relationships and similarities between the general answering strategies used by STs, and also allows the similarity between each ST and the ideal ST profiles to be studied.

The horizontal axis shows the similarity level⁴ between STs. For example, similarity between s_{11} and s_{55} is weaker

⁴Similarity trees are represented by C.H.I.C. without respecting a common scale factor in the similarity values reported in the horizontal axis. This is a known issue of the software and it is going to be fixed in future versions.

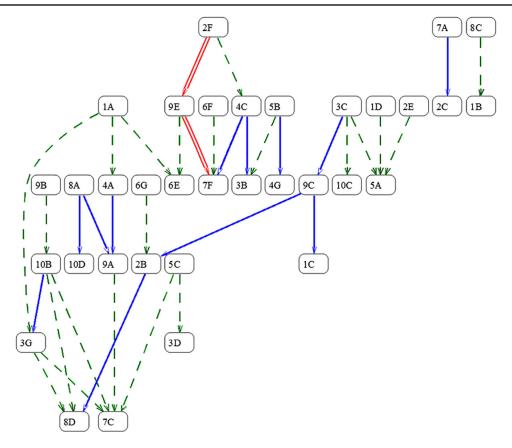


FIG. 2 (color online). Implicative graph for answers to the test. Rectangles represent STs' strategies as reported in the *a priori* or *a posteriori* analysis in Sec. III. Red double-line arrows indicate a 99% incidence of implication between two strategies, blue solid arrows indicate a 95% incidence, and green dashed arrows indicate a 90% incidence of implication. Implications are to be read only between neighboring pairs of strategies.

than similarity between s_1 and s_{47} as the link between the first two variables corresponds to a lower similarity level than the link between s_1 and s_{47} . Note also that these four variables are linked to the behaviorist ideal profile, although they have a lower link strength.

Figure 1 shows that the STs are grouped in several similarity clusters, at different levels of strength of the link. With respect to STs' similarity to the ideal profiles, three "macroclusters" are evident. The vast majority of elements in our sample exhibit a cognitivist answering strategy overall, with a 74% confidence level, and nine show behavioristlike attitudes, with a 70% confidence level. Only two STs show answering strategies that can be considered constructivist, with an 89% confidence level.

The three macroclusters shown in Fig. 1 are disjointed at the confidence levels shown. We will see that the implicative analysis, discussed below, demonstrates that in some cases STs classified in one of these macroclusters use answering strategies that can be classified as typical of one or both the other two ideal profiles.

Figure 2 shows the implicative graph we obtained by means of C.H.I.C. ST answering strategies are connected to each other by means of arrows. For the sake of simplicity, in Fig. 2 we chose to represent only the answering

strategies that imply another one with a significance level of 99% (red double lines), 95% (blue solid lines), and 90% (green dashed lines). We should point out that in C.H.I.C. graphs implications are to be read only between pairs of strategies. For example, the implication chain 2F-9E-7F is therefore to be read by considering that 99% of all STs using answering strategy 2F (10 in our survey data) also use strategy 9E and 99% of all STs showing strategy 9E (18) also put into action strategy 7F.

We will now discuss some of the implications, by considering the higher percentages of implications, but also taking into account the number of STs involved.⁵

Implication between strategies 2F and 9E (revealed in all the 10 STs using strategy 2F) demonstrates a close link between two cognitivist strategies, i.e., the recognition of the formal structure of the model and the ability to report mathematical formulas to summarize a phenomenon (not supported, however, by an explanation of the reasons for the answer).

The 99% implication between 9E and 7F seems to show that 18 STs, although incapable of explaining the reasons

 $^{^5\}mbox{We}$ will not discuss implications involving less than 10 STs here.

for their (correct) writing of mathematical formulas to summarize a phenomenon, appear to be able to put into action a constructivistlike answering strategy in question 7. The 95% significant implication between strategies 10B (behaviorist) and 3G (constructivist), revealed in 12 STs, seems to again demonstrate a correlation between the use of low-level and higher-level strategies. On the other hand, we must also consider that question 3 is one of the questions aimed at exploring the general, theoretical aspects of the process of modeling, while question 10 is an applicative one, aimed at testing the actual capabilities to expand a model to represent new situations. Therefore, implication 10B-3G actually reveals 12 STs that are not able to generalize a model by applying it to wider situations and whose constructivist-type idea of the nature of a model, expressed in 3G, is probably to be considered developed only at a declarative level.

The 90% implication between strategy 3G and a cognitivist one (7C), shown by 17 STs, and the 90% one between 3G and the cognitivist 8D (again true for 17 STs), go along the same line. We can, therefore, hypothesize that the previously considered implications can be ascribed to the use of resources from previous instruction experience, i.e., "memories" from mathematics and physics courses attended by the STs in previous years (all our STs attended a 5 year course of physics and mathematics at secondary school, although limited to a few hours per week).

Implications 1A-4A and 8A-9A, true for 10 and 11 STs, respectively, give evidence of the persistence of behaviorist strategies applied by STs in answers to questions on the nature and characteristics of models and when trying to give a verbal and then a formal description of a real physical situation. A persistence of behaviorist strategies can also be found in the interesting implication 5B-3B (true for 14 STs), where the idea of the existence of phenomena that cannot be explained by a model is related to the idea of a model as something that really exists in nature and identified as simple, real life situations.

The 90% implication between answering strategies 9A and 7C (in 13 STs) makes evident a link between a behaviorist approach to the formalization of a situation and a cognitivistlike use of formulas. Although it is evidence of the correct recognition of y = ax as a direct proportionality, the use of strategy 7C reveals an imperfect transfer between algebraic and geometrical representations, and can, therefore, be an obstacle to procedures based on mathematic modeling.

Other implications worth noting are the ones between 3C and 5A and those between 2E and 5A (both in 10 STs). These implications highlight that the idea of a model as something that exists in nature, which humans can only imperfectly understand, or as a formula aimed at solving a problem, is linked to the direct identification of physics with the natural world.

B. Qualitative analysis

In this section we quote some excerpts from STs' interviews. As stated in Sec. III, interviews are aimed at deepening the analysis of the cognitive styles of STs by highlighting points of interest or controversial behavior in the questionnaire answers. Interviews were analyzed on the basis of a search for key words and specific aspects of the STs' answers that could give evidence of the cognitive style(s) used to answer our research questions [31,32].

We start by quoting parts of discussions between a researcher and three of the STs that reveal implications between strategies 9E and 7F in the analysis of the questionnaire answers, as shown in Fig. 2. ST answers are shown together, but they were obtained in single interviewer-interviewee face to face setups:

Researcher. In your answer to item 9 you formalized the equations between space, velocity and time in the free fall of a body by using the equations $s = \frac{1}{2}at^2$ and v = at, but you did not give any reason for writing these particular relationships between s, v and t. Can you now explain your answer?

Elena. I know a falling body performs an accelerated motion, so the correct equations should be the ones I reported in my answer to question 9.

Daniela. Time, space and velocity are the significant variables to describe the free fall of a body. The s-t and v-t graphs are quadratic and linear, respectively.

Francesca. Galilei showed that all falling bodies have the same constant acceleration, so I wrote that $s = \frac{1}{2}gt^2$ and v = gt, where g is gravity.

The three STs try to justify their use of the uniformly accelerated motion equations by using specific terms or concepts like "accelerated motion," "graphs are quadratic or linear," "all falling bodies have constant acceleration," but are not able to explain their use of a quadratic expression for s(t) and a linear one for v(t) by linking the meaning of a uniformly accelerated motion to these variables. It is worth noting that all of them answered question 8 by naming space, velocity, and time (strategy 8F), without being able to verbally express the relationships between them, and they confirm their imperfect understanding in their answer to this researcher question as well. The

⁷An implication between 8F and 9E is present in the implicative graph of our data, but only at a significance level of 75%. For this reason, it is not shown in Fig. 2.

⁶Interview excerpts are not always literally translated into English from Italian. We tried to convey the sense of the originals, rather than reporting the exact terms and expressions used by STs. Only the key words and typical expressions we identified as relevant for the analysis are directly translated.

cognitive style shown here by Elena, Daniela, and Francesca seems to be driven by a mere recall of concepts studied before, without any clear evidence of an understanding of the reasons leading to the mathematical expressions of s(t) and v(t).

Researcher. What does it actually mean that a variable is proportional to another one? Try to explain with reference to the answer you gave to question 7.

Elena. It means that when one variable increases the other also increases. This is certainly true for the circumference and the radius of a circle.

Daniela. If I am not wrong, proportionality should be expressed by a linear graph. I remember solving a math problem at school, where we were asked to plot the graph of circumference vs radius in a circle. It was a line, so I am sure that y = ax can also be used in the case of a circle.

Francesca. A variable is proportional to another when it varies together with it. In a circle the circumference varies with the radius, as it does for variables y and x.

The answering strategies put into action here by Elena, Daniela, and Francesca again seem to be linked more to the recalling of previously studied facts than to a real understanding of the concept of proportionality. The association of the linear dependence relationship to the definitions "when one variable increases the other also increases" and "the variables vary together with" is very common in STs who went to secondary schools where mathematics was not a main subject.

Researcher. Can you give another example of the application of the formula y = ax to a situation you can find in real life or that you have previously studied?

Elena. I am not sure ... maybe the falling of a body can be described by this formula, but I have difficulty in relating y and x to variables useful for the study of motion.

Daniela. I think that the relationship between the force of gravity and the mass of a body can be expressed as y = ax. In physics books the force is written as F = mg, an equation that seems to me similar to a linear relationship between F and m.

Francesca. I remember that the velocity of a body can be defined as space over time. So, if I solve the equation, space is proportional to time, exactly as y is proportional to x.

Here Daniela and Francesca seem to deal with the question with some success, but a close inspection of their ideas reveals the use of terms or phrases, like "I remember that...," "in physics books it is written..." that can again be due to recalling previous instruction more than a real ability to apply an abstract idea to a concrete situation.

The analysis of these interview answers seems to show that the use made by Elena, Daniela, and Francesca of strategy 7F is more due to a cognitivist line of thought than to the application of constructivist abilities. In the similarity tree shown in Fig. 1 the three STs are coded as s19, s69, and s77 and are collectively classified as cognitivist.

We continue by discussing some of the answers of two STs, which highlight an implication between 10B and 3G.

Researcher. You wrote that models are creations of human thought. Can you explain this idea better?

Valentina. Yes, it is a way the human brain has to describe, explain and predict what it really exists in nature. It is a specific experimental result or argument that has been validated by a scientific community.

Antonella. A model is something given by a line of reasoning, so it comes from the human mind but it is then applied to concrete situations, like an experiment or a formula. A model is the starting point to solve or demonstrate something observed.

Here we have a confirmation of the fact that both Valentina and Antonella have ideas about models that can be considered part of the line of thought typical of a constructivist, at least at a declarative level. The use of words or concepts like "explain" and "predict" or "validated by a scientific community" and "starting point to demonstrate" lead us to suppose that Valentina and Antonella think of scientific knowledge as a construction based on the interpretation of experimental results and the sharing of ideas in a community.

Researcher. What happens if you have a model to explain some phenomena that works well, and you suddenly make some observations of a phenomenon that is apparently similar to the first ones, but it is not well explained by your model?

Valentina. Well, I think that if the model is working it should explain all similar phenomena. I think that the scientific community should have taken into account all similar situations before building an accepted model.

Antonella. If the model does not explain similar situations the mathematic formula describing it is probably wrong. Maybe it is necessary to find another one, with

different variables, that can be fitted to the new phenomenon better...

Here, Valentina is showing that she has a somehow "rigid" idea about what science is. She seems convinced that once a model is "working" it has to be immutable, as it is the result of "the scientific community's" work. Antonella, on the other hand, thinks that in these situations an error in the "formula" can be detected, and different "variables" can be found to make changes to the model. Here the use of the concept "find another formula," together with "fitted to the new phenomenon better" leads us suppose that Antonella is showing a mixed but constructivist—dominant line of thinking.

Researcher. Let's consider the situation described in item 10 of the questionnaire. We asked you to try and modify the free falling body model in order to take into account the influence of elements other than gravity that affect motion. You cited a collision with other objects as one of the possibilities, but do you think there may be other variables that can be considered and measured during the fall of a body? If yes, how can you choose these variables?

Valentina. You said "other variables" to consider I thought that in the study of the fall of an object the correct formula was $s = \frac{1}{2}at^2$. I have to think about this more carefully ... 8

Antonella. Yes, I think that one of the ways in which a falling object can modify its motion is by colliding with another body. I don't think other elements can influence its motion . . .

Here the researcher goes on a concrete situation and asks for an existing model to be extended in order to take new experimental evidence into account. Both Valentina and Antonella are now puzzled and seem not to be able to build any extension to the free falling body model.

Researcher. And what about air? Do you think it influences the motion of a falling object? If yes, is it possible to take it into account in some way and modify the original model?

Valentina. Yes! Now you mention it I remember that when I am in a car and I put my arm out of the window it is pushed backwards by the wind, that is by air . . . So

air must affect the motion of an object. But I don't know how . . .

Antonella. Mmm ... every medium in which a body moves has different characteristics, like density ... so I think we could try to use these characteristics to modify the model ... [no concrete answer is then given to the researcher's question].

The researcher tries to help Valentina and Antonella. A possible element that can influence the motion of a falling object, other than gravity, is mentioned (air). Valentina immediately recalls from her memory an experimental situation that confirms the relevance of air as an element that can influence the motion of bodies, but she is not able to find a new variable, related to air, that can be inserted into the free falling body model to take into account the new situation. Antonella detects a "characteristic" of air that can, in her opinion, influence motion. She shows that she has the idea of the "modification" of a model as a consequence of observing new facts, but she is not able to concretize her intuition with an extended model for the fall of a body in air.

Valentina and Antonella seem to confirm that simply being able to correctly give definitions of the nature and uses of models is not sufficient to say that a correct understanding of the idea of modeling has been gained. They need to work on modeling processes to improve their understanding, and, in particular, they need to learn how to generalize a model by choosing supplementary variables relevant for describing new situations and inserting them into the model. A deeper understanding of the use of mathematical concepts and tools is also needed here.

We will now consider some parts of the interviews with three STs that highlighted implications between strategies 8A and 9A, two behaviorist strategies.

Researcher. You wrote in your answer to item 8 that the speed of a free falling body can depend on the body's weight, on its shape, or on the forces acting on it. Can you explain how, and why, you chose them?

Francesca. When a body falls, only its weight is important. Once the weight is found, one can use Newton's laws to calculate the body's acceleration, another important variable for motion.

Maurizio. We must take into account all the relevant parameters, like forces, acting on the body. Then, when the time runs the space traveled by the body increases according to the forces, as can easily be seen in real situations.

Oriana. Mass, force and gravity are ... the variables that influence motion. I chose these as I can see from my experience that they are the relevant ones.

 $^{^8}$ Valentina is actually one of the students that answered question 9 by using strategy 9D, i.e., by recalling only the s(t) relationship between space and time in uniformly accelerated motion, without any attempt to explain why this formula is correct and from where it originates.

All the answers of the three STs confirm that their lines of reasoning can be defined as behaviorist. The use of concepts like "in falling only weight is important," "velocity of the body increases ..., as can easily be seen in real situations," "I can see from my experience that they are the relevant ones" suggest that the basic ideas behind Francesca's, Maurizio's, and Oriana's reasoning is mainly an "on the spot" comparison between the proposed situation and their own perception of the phenomenon from the real world.

Researcher. Yes, but are you sure that the entities you named are actually variables or parameters? What does a variable have to do in order to be so defined?

Francesca. Well, a variable is something important for the description of the phenomenon ...

Maurizio. A variable should be used in a formula in order to obtain useful results and information about the body's motion.

Oriana. Mass, force and gravity are variables because they influence the body's motion.

Here the researcher is trying to understand if the use made of the term "variable" by STs is based on a real understanding of what a variable is. Francesca draws attention to the idea that a variable should "describe" a situation. Maurizio goes further, and seems to show an understanding that a variable must be related to another one in a "formula" to "obtain useful results about motion." Oriana still repeats her somewhat vague idea of the relevance of a variable because it "influences" the motion of a body.

Researcher. In mathematics, what do you think the difference is between a variable and a parameter?

Francesca. A parameter should define something that remains fixed, while a variable is something relevant for the description of a real situation.

Maurizio. Judging from the names, I would guess that a variable varies and a parameter maybe not In this actual case, the forces are probably parameters and time is a variable, as it runs.

Oriana. I am not sure... I think that both should describe a real situation but I don't know what the real difference is ...

The use made here of terms or concepts like "a parameter defines something fixed," "variables describe real situations," "a variable varies," "time is a variable as it runs" seems to show that Francesca, Maurizio, and Oriana

have memories of mathematical or physical concepts from past studies. On the other hand, these resources are vaguely recalled by both Francesca and Oriana. Only Maurizio seems to be able to construct a logical line that starts from a semantic analysis of the word "variable" as something "varying" to understand that time can better be considered a variable in relation to a (constant) force acting on a body.

Researcher. In your answers to item 9 you did not write any formula representing the motion of the free falling body. Can you now use the variables and parameters we found before in order to write the formula?

Francesca. I am not able to write a formula ... I always had problems with mathematics I can only say that the weight is directed downwards.

Maurizio. I find it difficult to translate my ideas into a math formula. Surely time is a variable that increases, and also space, and ... yes, velocity is a variable that increases with time. In fact it can easily be seen that the more a body takes to fall, the higher is its velocity when it hits the ground.

Oriana. Mass and gravity should probably be multiplied to obtain a correct expression for the free body falling. I wrote this in the answer to item 9 but I am not really able to say why.

Here the researcher wants to see if the STs can find a relationship between the variables they have chosen as relevant for the description of the phenomenon and write it in a formalized way. Francesca immediately says she has difficulties with mathematics and reports the concrete result that the weight is directed downwards. Maurizio understands that velocity could also be defined as a variable, as it also varies. As proof of this, he recalls the concrete idea that "the more a body takes to fall, the higher its velocity is when it arrives to the ground." Oriana simply states that mass and gravity are to be multiplied to get correct information about the falling of a body (maybe a memory of previous instruction about the calculation of the force of gravity?).

Francesca, Maurizio, and Oriana clearly show a persistently behaviorist line of thought, although in some cases aspects of a cognitivist one can also be identified. This result is not surprising, as it may be due to previous experience of mathematics and physics teaching.

VI. CONCLUSIONS AND IMPLICATIONS FOR TEACHING

The similarity tree analysis discussed in Sec. Vallows us to say that the vast majority of our elementary school student teachers made use of answering strategies that can be generally defined as cognitivist, with only a small group showing clear behaviorist attitudes and only two showing a well-defined constructivist approach. The implication graph and the qualitative analysis of interviews help us to refine this result, by giving more detail about relationships and implications between the answering strategies used by STs.

The personal views revealed by our analysis of the beliefs and approaches to modeling of our sample may be the result of the typical way in which STs have been taught mathematics and science in the past. In fact, very often, concepts relevant for the understanding of scientific questions are introduced in Italian schools by following a traditional teaching approach, without any advantage for the understanding of the implicit scientific content concerning real life phenomena. Such traditional approach is usually based only on the transmission of contents and integrated with sometimes meaningless workshop activities. In fact, these are very often performed directly by teachers with the sole purpose of contextualizing ideas already taught as they are presented in traditional textbooks and passively accepted by students. These types of teaching methods tend to stimulate rigid mnemonic attitudes in more passive students, fostering a behavioristlike approach to new situations that are presented to them. More active students are at best motivated to build links bridging the concepts they have studied with real life contexts in an attempt to give them meaning, thus making use of a typical cognitivistlike approach, which is not wrong in itself but not always sufficient for a meaningful approach to scientific knowledge.

However, it is well known that an effective scientific education needs to be supported by activities deeply rooted in a constructivistlike approach [45–47], capable of helping students to observe and make sense of suitably designed experiences related to everyday life phenomena. In fact, as research into science education has largely shown, the more learning environments are able to stimulate student interests related to their own everyday lives, the more effective they are. As a consequence, recommendations about school curricula and teaching are today more and more oriented towards integrated community-based tasks and activities for learners arising from real life problems [5,48].

Our results are consistent with data from the literature [49–51]. In particular, we find that STs' beliefs can be eclectic, and sometimes contradictory. Many STs hold more than one view about knowledge construction, with particular reference to strategies that are inefficient for correctly connecting mathematical modeling to real situations. This is an ability that can be considered an important part of the construction of elementary school STs' own science understanding [52]. Our findings allow us to go in depth with respect to the STs' epistemological approaches to knowledge. They highlight a significant presence of behaviorist ideas, even in student teachers that generally

adopt cognitivist strategies. Moreover, our data evidence a ST general approach to knowledge too grounded on a rigid use of cognitive resources, mainly coming from memories of past instruction and not based on a solid understanding of modeling strategies. When we compare qualitative and quantitative findings we also notice that in some cases constructivist strategies are used by STs, although often only at a declarative level. In these cases such use is not supported by a suitable application of constructivist strategies to the analysis of the real situations presented.

Through the calculation of *similarity* and *implication* indexes our analysis methods give meaning to the relationships found between the study variables, focusing on the most relevant ones, identified on the basis of their strength. Pictures are supplied of significant typical ST behaviors and specific strategies used by STs in answering the questionnaire items. The use of interview analysis based on the search for key words in ST statements makes stronger and deepens the results obtained with the quantitative analysis.

Our results about elementary school student teachers' perception of the process of modeling are limited by the context of the Italian school system. However, they show a strong similarity with results obtained in different school systems and can give hints on how STs' personal views and beliefs about modeling can be used and redirected to plan a teacher training program that can influence teacher's perception, judgment, and behavior with respect to science and science teaching.

In line with these considerations, the two courses of mathematics and physics education held by us have been restructured, taking into account the need to carry out an educational reconstruction of the scientific content to be taught [53–55], aimed at orienting the course towards the construction of models of explanation. In doing this we have attempted to bring about a paradigmatic change of direction, aimed at shifting student teacher attention from a traditional "concept to context" learning approach to a more effective "context to concept" one [56,57].

The key point for the courses is in the structuring of learning environments in which different topics are presented as real life, problematic situations and questions to be solved. The aim of this is to orient STs towards contents through pathways of investigation and discovery related to tangible context. Moreover, as pointed out by Nilsson and van Driel [58], in a teacher training program STs should be encouraged to modify their approach to content knowledge and teaching methods. This can be done if their views and beliefs are taken into account and developed, in order to orient STs towards a meaningful construction of scientific knowledge for teaching, and to allow them to see themselves as learners of both science or mathematics subjects and science or mathematics teaching.

A complete report of the new approach with a detailed analysis of student activities, answers to questionnaires, interviews, and videos of STs performing experimental and modeling tasks and giving their "simulated" lessons in a peer to peer context is in preparation and will be presented in a forthcoming paper. Preliminary qualitative results show that searching for models in real life contexts stimulates student teachers' investigative skills (searching for relevant variables, formulating hypotheses, etc.) much more than when STs are only requested to look for already formalized concepts or equations to be applied in more or less abstract contexts. The great majority of STs showed to be able to think about and build kinds of "mechanism of functioning" of the situations they have analyzed, so to supply meaning to the relationships between the involved variables. An educational reconstruction of scientific content based on laboratory and modeling activities seems, then, to support in STs the building and development of constructivistlike strategies, the ones that should continue to play a relevant role in student teachers' future careers as elementary school teachers.

APPENDIX

Answers, drawn from the *a priori* and *a posteriori* analysis, that each of the three "ideal student teachers" would give to the questionnaire items, according to our analysis criterion.

1. Behaviorist

Models in physics are a faithful or reduced scale reproduction of a real object.

Models in physics are an operative procedure to follow in order to simplify and describe phenomena from natural world.

Models in mathematics are a picture of a geometrical shape, maintaining fixed proportions between its elements.

In mathematics a model is a method to faithfully describe reality.

In mathematics a model is a quantitative, but essential, reproduction of a phenomenon.

Models are creations of human thought based on preexisting "natural models."

Models really exist and are simple, real life situations.

Models already exist in nature and humans try to understand them, sometimes only imperfectly.

A model has to start from some hypotheses of the real world that have to be verified.

A model must be able to account for all the features of the real object it represents.

A model must be a simple description of the reality.

A model must be fixed and immutable. It is the modeler that chooses a model suitable to the real situation.

A natural phenomenon can always be described by a physical model, as physics is the natural world, with all its laws.

There are phenomena that cannot be described or explained with a model and/or that cannot be defined in terms of precise physical quantities.

A natural phenomenon cannot always be described or explained by a physical model. Even the ablest modeler will not be able to reproduce particularly complex systems (for example, human behavior).

Some phenomena still have not been explained, but they will be in the future.

A mathematical formula cannot always express a real situation, as mathematics is an abstract construction and does not always represent reality.

A mathematical formula can always express a real situation, but only if it quantitatively describes the entire real situation

A mathematical formula cannot always express a real situation, because reality is so complex that it cannot always be expressed by a mathematical formula.

A mathematical formula cannot always express a real situation, because not all phenomena can be described mathematically or quantitatively.

y = ax cannot be used to calculate the circumference of a circle, as in the formula for circumference calculation the radius and the circumference are present, and not the variables x and y.

y = ax cannot be used to calculate the circumference of a circle, because the constant a does not have the correct value, i.e., 2π .

The speed of a free falling object depends on certain parameters, like the object's weight, its shape, or the forces acting on the object.

For the free fall a verbal explanation, based on concrete situations, is given, but no formula is reported.

For the free fall a graphic representation of nonsignificant variables, that come from real experience, is reported.

The motion of a free falling object can be influenced by environmental conditions, like wind or temperature.

The motion of a free falling object can be influenced by a collision with another object.

2. Cognitivist

A physical model is a reproduction of a real object not necessarily on a reduced scale, aimed at helping us to interact with it and/or describe it.

A physical model is a stylized or simplified reproduction of a real object, aimed at helping us to interact with it and/ or describe it.

A mathematical model is a symbolic or quantitative representation of a situation or phenomenon.

A mathematical model is a guideline or a formula, aimed at resolving a problem.

A mathematical model is a simplified representation of a system, whose basic elements (variables, sources, and contexts) are connected by relationships (a set of rules).

A mathematic model is a reference for the construction of a line of reasoning or the demonstration of a hypothesis.

Models are simply creations of the human mind, like mathematical formulas.

Many models are creations of the human mind and are what we call "theories."

A model must be able to account for the features of the real object that are of practical interest.

A model must highlight the variables that are relevant for the description or explanation of the phenomenon and their relationships.

A model must be expressed in mathematical language and/or accepted by a scientific community.

A model can be qualitative, semiquantitative, or quantitative.

A natural phenomenon can always be described or explained by a physical model, it just depends on the modeler's ability to carefully reproduce the features of interest.

Sometimes a natural phenomenon can be described or explained by a physical model and others not. In fact, the way nature works is not completely known to man, so further study is necessary to explain all phenomena.

A mathematical formula can always express a real situation, because mathematics is the language the human brain uses to quantitatively describe or explain a real situation.

A mathematical formula cannot always express a real situation, as a real phenomenon can have characteristics that cannot easily be expressed in mathematical language.

A mathematical formula can always express a real situation, because a mathematical law is always verifiable starting from well-defined hypotheses.

y = ax cannot be used to calculate the circumference of a circle, because y = ax is a direct proportionality, i.e., a straight line, while the circumference is a curve.

y = ax cannot be used to calculate the circumference of a circle, because the formula y = ax is an algebraic one, while the circumference calculation is a geometric task

y = ax cannot be used to calculate the circumference of a circle, because y = ax is not the correct mathematical relationship between x and y.

The variables describing a free falling object are space and time. They are linearly dependent.

The variables describing a free falling object are space and time. Space is proportional to the square of time.

The variables describing a free falling object are the acceleration caused by gravity and/or the starting height and/or the mass and/or the force of gravity.

In order to describe the phenomenon we must determine all the forces acting on the object and then use Newton's 2nd law.

The formula linking the variables relevant to describe a free falling object is s = vt and/or F = ma.

The formula linking the variables relevant to describe a free falling object is $s = \frac{1}{2}at^2$ —no explanation.

The mathematical formulas that describe the motion of a free falling object are $s = \frac{1}{2}at^2/v = at$ —no explanation.

Friction with air can influence the motion of a free falling object.

Friction with air can influence the motion of a free falling object, so density may be a relevant variable.

3. Constructivist

A physical model is a mental representation of a real object or phenomenon, which accounts more or less accurately for its mechanisms of functioning.

A physical model is a real or abstract object that behaves like another real object, but does not necessarily look like it.

A physical model is a mental formalization of real phenomena

A mathematical model is a description of a situation or phenomenon that is useful for predicting the evolution of the situation or phenomenon itself.

A mathematical model is an abstract construction that allows different quantitative representations of the same object to be built.

Models are creations of human thought; their creation comes from the continuous interaction with the "real" external world.

Models are creations of human thought, and their purpose is to predict and make sense of natural phenomena.

A model must allow what we observe about different phenomena or situations to be generalized.

A model must be useful for analyzing and making predictions about the behavior of a more or less complex system.

A natural phenomenon can always be described or explained by a physical model if the modeler is able to find all the relevant variables that characterize the phenomenon.

A mathematical formula can always express a real situation, but it is necessary to carefully choose the mathematical variables needed to express the real situation.

y = ax can be used to calculate the circumference of a circle, because the circumference is directly proportional to the radius, as y is with respect to x in the formula.

The variables relevant to describe the free falling object motion are time, space, and velocity—an explanation is given but the relationships between the variables are not completely or clearly expressed.

The variables relevant to describe the free falling object motion are time, space, and velocity. Space is proportional to the squared time and/or velocity is proportional to time—clear examples are given.

v-t and/or s-t graphs are reported and correctly commented on or applied.

If we want to improve the free falling object model, we should take into account one or more forces opposite to motion. For example, friction with air, which increases with the velocity of the object, with its surface, etc.

- [1] J. K. Gilbert, C. Boultere, and M. Rutherford, Models in explanations, Part 1: Horses for courses?, Int. J. Sci. Educ. **20**, 83 (1998).
- [2] M. Niss, Issues and problems of research on the teaching and learning of applications and modeling, in *Modeling and Mathematics Education: ICTMA 9—Applications in Science and Technology*, edited by J. F. Matos, W. Blum, S. K. Houston and P. Carreira (Horwood, Chichester, England, 2001), p. 72.
- [3] J. Clement, Model based learning as a key research area for science education, Int. J. Sci. Educ. **22**, 1041 (2000).
- [4] National Research Council, National Science Education Standards (National Academy Press, Washington, DC, 1996).
- [5] M. Rocard, P. Csermely, D. Jorde, D. Lenzen, H. Walberg-Henriksson, and V. Hemmo, *Science Education NOW: A Renewed Pedagogy for the Future of Europe* (Office for Official Publications of the European Communities, Luxembourg, 2007).
- [6] R. Lesh, K. Cramer, H. Doerr, T. Post, and J. Zawojewski, Model development sequences, in *Beyond Constructivism:* A Models and Modeling Perspective on Mathematics Teaching, Learning, and Problem Solving, edited by R. A. Lesh and H. Doerr (Lawrence Erlbaum, Mahwah, NJ, 2003), p. 35.
- [7] P. L. Galbraith, W. Blum, G. Hooker, and I. D. Huntley, Mathematical Modeling: Teaching and Assessment in a Technology-Rich World (Horwood, West Sussex, 1998).
- [8] H. Doerr and L. D. English, A modeling perspective on students' mathematical reasoning about data, J. Res. Math. Educ. 34, 110 (2003).
- [9] L. Viennot, F. Chauvet, P. Colin, G. Rebmann, Designing strategies and tools for teacher training: The role of critical details, examples in optics, Sci. Educ. **89**, 13 (2005).
- [10] R. Prawat, Teachers' beliefs about teaching and learning: A constructivist perspective, Am. J. Educ. 100, 354 (1992).
- [11] W. Blum, ICMI Study 14. Applications and modeling in mathematics education, Educ. Stud. Math. 51, 149 (2002).
- [12] An International Review of Applications in School Mathematics—The Elusive El Dorado, edited by H. Burkhardt (Information Reference Center (ERIC/IRC), The Ohio State University, Columbus, OH, 1983).
- [13] D. Kagan, Implications of research on teacher belief, Educ. Psychol. 27, 65 (1992).
- [14] D. H. Palmer, Factors contributing to exchange amongst preservice elementary teachers, Sci. Educ. **86**, 122 (2001).
- [15] E. L. Thorndike, Educational Psychology: The Psychology of Learning (Teachers College Press, New York, 1913).
- [16] I.P. Pavlov, *Conditioned Reflexes* (Clarendon, London, 1927).
- [17] B. F. Skinner, *About Behaviorism* (Knopf, New York, 1974).
- [18] F. I. M. Craik and R. S. Lockhart, Levels of processing: A framework for memory research, J. Verb. Learn. Verb. Behav. 11, 671 (1972).
- [19] F. I. M. Craik and E. Tulving, Depth of processing and the retention of words in episodic memory, J. Exp. Psychol. 104, 268 (1975).

- [20] D. P. Ausubel, *Educational Psychology: A Cognitive View* (Holt, Rinehart and Winston, New York, 1974).
- [21] P. A. Cooper, Paradigm shifts in designing instruction: From behaviorism to cognitivism to constructivism, Educ. Technol. **33**, 12 (1993).
- [22] B. G. Wilson, Reflections on constructivism and instructional design, in *Instructional Development Paradigms*, edited by C. R. Dills and A. J. Romiszowski (Educational Technology Publications, Englewood Cliffs, NJ, 1997), p. 63.
- [23] P. A. Ertmer and T. J. Newby, Behaviorism, cognitivism, constructivism: Comparing critical features from an instructional design perspective, Perform. Improv. Q. 6, 50 (1993).
- [24] R. Gras, F. Suzuki, F. Guillet, and F. Spagnolo, Statistical Implicative Analysis: Theory and Applications (Springer, New York, 2008).
- [25] P. Pintrich, Implication of psychological research on student learning and college teaching for teacher education, in *Handbook of Research on Teacher Education*, edited by W. Houston (Macmillan, New York, 1990), p. 826.
- [26] N. G. Lederman, F. Abd-El-Khalick, R. L. Bell, and R. Schwartz, Views of nature of science questionnaire: Toward valid and meaningful assessment of learner's conceptions of nature of science, J. Res. Sci. Teach. 39, 497 (2002).
- [27] C. Fazio and F. Spagnolo, Conceptions on modelling processes in Italian high school prospective mathematics and physics teachers, S. Afr. J. Educ. 28, 469 (2008).
- [28] G. Brousseau, Theory of Didactical Situations in Mathematics, edited by M. Cooper, N. Balacheff, R. Sutherland, and V. Warfield (Kluwer Academic, Dordrecht, 1997).
- [29] F. Spagnolo, Insegnare le Matematiche nella Scuola Secondaria (La Nuova Italia, Firenze, 1998).
- [30] F. Spagnolo, La modélisation dans la recherche en didactiques des mathématiques: Les obstacles épistémologiques, Rech. Didact. Math. 26, 337 (2006).
- [31] H. P. Ginsburg, N. E. Kossan, R. Schwartz, and D. Swanson, Protocol methods in research on mathematical thinking, in *The Development of Mathematical Thinking*, edited by H. P. Ginsburg (Academic, New York 1983), p. 7.
- [32] R. P. Hunting, Clinical interview methods in mathematics education research and practice, J. Math. Behav. 16, 145 (1997).
- [33] R. Gras, Contribution à l'étude expérimentale et à l'analyse de certaines acquisitions cognitives et de certains objectifs en didactique des mathématiques, Ph.D. thesis, Université de Rennes, 1979, p. 1.
- [34] R. Gras, Les fondements de l'analyse statistique implicative, Quad. Ric. Didattica 9, 187 (2000) [http://dipmat .math.unipa.it/~grim/quaderno9.htm].
- [35] J. M. Bernard and S. Poitrenaud, L'analyse implicative bayesienne d'un questionnaire binaire: Quasi-implications et treillis de galois simplifié. Mathématiques, Informatique et Sciences Humaines 147, 25 (1999).
- [36] L. Seve, *Emergence, Complexité et Dialectique* (Odile Jacob, Paris, 2005).
- [37] I.C. Lerman, Classification et Analyse Ordinale Des Données (Dunod, Paris 1981).

- [38] I. C. Lerman, R. Gras, and H. Rostam, Elaboration et évaluation d'un indice d'implication pour des données binaires I, Math. Sci. Hum. 74, 5 (1981); I. C. Lerman, R. Gras, and H. Rostam, Elaboration et évaluation d'un indice d'implication pour des données binaires 2, Math. Sci. Hum. 75, 5 (1981).
- [39] A. Fernández and Sergio Gómez, Solving non-uniqueness in agglomerative hierarchical clustering using multidendrograms, J. Classif. 25, 43 (2008).
- [40] A. D. Gordon, *Classification* (Chapman & Hall, London, 1999), 2nd ed.
- [41] R. Couturier, R. Gras, and F. Guillet, Reducing the number of variables using implicative analysis, in *Classification*, *Clustering*, and *Data Mining Applications: Proceedings of the Meeting of the International Federation of Classification Societies (IFCS 2004)* (Springer Verlag, Berlin, 2004), p. 277.
- [42] R. Couturier, Un système de recommandation basé sur l'A.S.I., in *Proceedings of Troisieme Rencontre International de l'Analyse Statistique Implicative*, edited by R. Gras, F. Spagnolo, and J. David [Quad. Ric. Didattica 15, Suppl. 2, 157 (2005)] [http://math.unipa.it/~grim/asi/suppl_quad_15_2.htm].
- [43] A. Markos, G. Menexes, and I. Papadimitriou, The C.H.I.C. analysis software v1.0., in *Classification as a Tool for Research: Proceedings of the 11th International Federation of Classification Conference*, edited by H. Loracek-Junge and C. Weihs (Springer, Berlin, 2010), p. 409.
- [44] Association de Recherche en Didactique des Mathématiques (A.R.D.M.), http://ardm.eu/contenu/logiciel-d-analyse-de-donnees-chic.
- [45] N. Nersessian, Constructive modeling in doing and learning physics, in *Thinking Physics for Teaching*, edited by C. Bernardini, C. Tarsitani, and M. Vicentini (Plenum, New York, 1995), p. 77.
- [46] M. Watts and Z. Jofily, Towards critical constructivistic teaching, Int. J. Sci. Educ. 20, 173 (1998).
- [47] E. von Glasersfeld, Questions and answers about radical constructivism, in *The Practice of Constructivism in Science Education*, edited by K. Tobin (Lawrence Erlbaum, Hillsdale, NJ, 1994), p. 18.

- [48] M. C. Linn, P. Bell, and E. A. Davis, Specific design principles: Elaborating the scaffolded knowledge integration framework, in *Internet Environments for Science Education*, edited by M. C. Linn, E. A. Davis, and P. Bell (Lawrence Erlbaum, New York, 2004), p. 315.
- [49] P. Klein, Preservice teachers' beliefs about learning and knowledge, Alberta J. Educ. Res. **42**, 361 (1996).
- [50] V. Collinson, Staff development through inquiry: Opening a pandora's box of teachers beliefs, in Proceedings of the Annual Meeting of the Association of Teacher Educators, St. Louis, MO, 1996, Eric Document Reproduction Service No. ED. 393 842.
- [51] J. Calderhead, Teachers: Beliefs and knowledge, in Handbook of Educational Psychology, edited by D. Berliner and R. Calife (Simon and Schuster— Macmillian, New York, 1996), p. 709.
- [52] J. Koch, in *Elementary Science Teacher Education*, edited by K. Appleton (Lawrence Erlbaum Associates, New Jersey, 2006), p. 91.
- [53] R. Duit, H. Gropengie βer, and U. Kattmann, Toward science education research that is relevant for improving practice: The model of educational reconstruction, in *Developing Standard in Research on Science Education*, edited by H.E. Fisher (Taylor and Francis, London, England, 2005), p. 1.
- [54] C. Fazio, I. Guastella, R. M. Sperandeo-Mineo, and G. Tarantino, Modeling mechanical wave propagation: Guidelines and experimentation of a teaching-learning sequence, Int. J. Sci. Educ. 30, 1491 (2008).
- [55] R. M. Sperandeo-Mineo, C. Fazio, and G. Tarantino, Pedagogical content knowledge development and preservice physics teacher education: A case study, Res. Sci. Educ. **36**, 235 (2006).
- [56] H. Lye, M. Fry, and C. Hart, What does it mean to teach physics "in context"?, Aust. Sci. Teach. J. 48, 16 (2001).
- [57] E. Whitelegg and M. Parry, Real-life contexts for learning physics: Meanings, issues and practice, Phys. Educ. 34, 68 (1999).
- [58] P. Nilsson and J. van Driel, How will we understand what we teach?—Primary student teachers' perceptions of their development of knowledge and attitudes towards physics, Res. Sci. Educ. 41, 541 (2011).