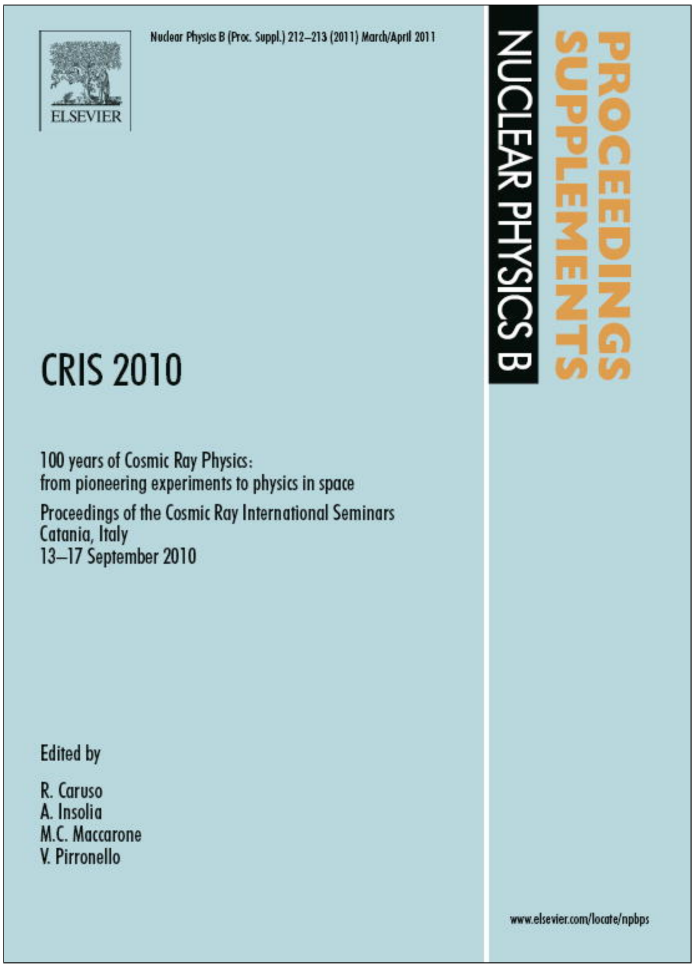
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A discrimination technique for extensive air showers based on multiscale, lacunarity and neural network analysis

Antonio Pagliaro[a b *], G. D'Alí Staiti[c b], F. D'Anna[a]

[a]Istituto di Astrofisica Spaziale e Fisica Cosmica di Palermo - Istituto Nazionale di Astrofisica, Via Ugo La Malfa 153, 90146 Palermo, Italy

[b]Istituto Nazionale di Fisica Nucleare, Sezione di Catania, Viale A. Doria 6, 95125 Catania, Italy

[c]Università degli Studi di Palermo, Dipartimento di Fisica e Tecnologie Relative, Viale delle Scienze, Edificio 18, 90128 Palermo, Italy

We present a new method for the identification of extensive air showers initiated by different primaries. The method uses the multiscale concept and is based on the analysis of multifractal behaviour and lacunarity of secondary particle distributions together with a properly designed and trained artificial neural network. In the present work the method is discussed and applied to a set of fully simulated vertical showers, in the experimental framework of ARGO-YBJ, to obtain hadron to gamma primary separation. We show that the presented approach gives very good results, leading, in the 1–10 TeV energy range, to a clear improvement of the discrimination power with respect to the existing figures for extended shower detectors.

1. Introduction

The study of cosmic ray composition is crucial, mainly in connection to the energy spectrum, for the quest of primary particles origin and sources nature (see e.g. [1]). Separating hadron-initiated showers from gamma-initiated ones is the first step, the next one being the estimate of the relative abundance of the different nuclei.

The aim of this paper is to introduce a separation technique, on an event by event basis, based on the different topology of gamma and hadron-initiated showers, both as far as the space and time distribution in the shower front are concerned. A detailed picture of the shower is therefore mandatory for the method to be applied. The main idea is to exploit at best the differences in the multifractal behaviour of the space distribution and the differences in the arrival time distribution for showers initiated by different progenitors.

Fractals and multifractals are used to model hierarchical, inhomogeneous structures in several areas of astrophysics and multifractal analysis has been used for the determination of the mass composition of cosmic rays [2], as well as a separation technique based on multifractal behaviour [3]. For what concerns extensive air showers, the multifractal analysis is motivated by the self-similar nature of the interactions.

Our technique uses a multifractal-wavelet and lacunarity based analysis similar to the one used by Rastegaarzadeh and Samimi [3]. We improve Rastegaarzadeh and Samimi method by introducing a lacunarity parameter on time arrival of the showers and an artificial neural network that allows better separation. The concept of lacunarity was originally developed to describe a property of fractals [4]. It quantifies the geometric arrangement of gaps in solid objects (lacunae). It can be extended to the description of distribution of data sets including, but not restricted to, those with fractal and multifractal distributions. In landscape ecology lacunarity was recently adopted to describe gaps in habitat cover-

*Contact e.mail: pagliaro@ifc.inaf.it

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age, providing a new way to quantify landscape texture [5]. In our method, lacunarity is used to describe gaps in time arrivals of showers. It is assessed by calculating a type of variance-to-mean ratio of the presence of a particle, repeated over a range of time scale see [6].

The method applies at best in the experimental framework where a detailed picture of the shower front is given. We have been testing it using the ARGO-YBJ detector [7]. ARGO-YBJ is a compact array of RPC with a sampling area of about 100×110 m^2, consisting of a central carpet of about 5600 m^2 with 92% coverage, surrounded by a guard ring with coarse sampling. Thus, in the central carpet surface a fully detailed map of the shower front can be worked out, with a space resolution for multiple hit counting given by 7×56 cm^2 (induction strip size) and a time resolution of the order of 1 ns in a slightly larger unit (OR of 8 adjacent strip covering a surface of 62×56 cm^2).

Our plan: in section 2 we introduce wavelet and (multi)fractal analysis; in section 3 we define lacunarity; section 4 is devoted to simulated showers; in section 5 we describe our method for spatial separation; in section 6 we describe our method for time separation; in section 7 we introduce the artificial neural network. Section 8 is devoted to the result of a test run and section 9 to concluding remarks.

2. Wavelet methods and fractal analysis

The wavelet transform is, by definition, the decomposition of a function on a basis obtained by translation and dilation of a particular function localized in both physical and frequency space.

A wavelet analysis of a density field associates each point with a real number which represents the smoothed local density contrast at a given scale (see for example Pagliaro and Becciani [8], Gambera et al [9]).

Fractal analysis, on the other hand, is concerned with the measurement of the local smoothness of the signals. The basic idea is that, most often, the significant information in a signal does not reside in its amplitude but in the local variations of its regularity (e.g.[4]).

A deterministic fractal is defined using the concept of self-similarity: given a bounded set A in a Euclidian n-space, the set A is said to be self-similar when A is the union of N distinct (non overlapping) copies of itself, each of them scaled down by a ratio r. The fractal dimension D is related to the number N and the ratio r: $D = logN/log(1/r)$ and plays a central role. It is a measure of how the members are distributed in space. Intuitively, the larger the fractal dimension, the rougher the texture. Self-similar multifractals are geometrical objects invariant by dilation. However, multifractality is not characterized by a single fractal dimension, but by a function.

The large numbers of interactions in showers from the same progenitor are self-similar. For this reason showers may be characterized by their fractal dimension. Moreover, the distribution of the shower particles near the core has a self-similar character different from those far away. Different physics is involved in production of different secondary particles. For these reasons, extensive air showers have a multifractal behaviour and more than a fractal dimension.

In our method, for a distribution of particles on a plane, the number of particles inside a radius R is computed. If a scaling law of the form $N(R) \propto R^D$ holds and D is a single non-integer value, the distribution has a fractal distribution with dimension D. Multifractal behaviour can be revealed by studying the scaling laws for secondary particles at different core distances.

Wavelet transforms have been used as a natural tool to investigate the self-similar properties of fractal objects at different spatial locations and length scales.

Holschneider [10] has shown that if a function f has scaling law with exponent D around x_0: $f(R(x_0, \lambda\epsilon)) \sim \lambda^D f(R(x_0, \epsilon))$ then the wavelet transform $W(s,t) = \int g(x; s,t) f(x) dx$, where s is the single dilation parameter (scale) and t is the translational parameter, has the same scaling exponent, if the wavelet g has a zero average and decays fast at infinity. Thus the local scaling behaviour is represented by $W(s,t) \sim s^{D(t)}$.

Therefore for any distribution function f the slope of the plot of $\log W(s,t)$ versus $\log s$ will give the fractal dimension of the distribution around point t for the range of the scale s.

3. Lacunarity

Lacunarity is a counterpart to the fractal dimension that describes the texture of a fractal (Mandelbrot [4], Lin and Yang [11], Gefen, Meir and Aharony [12]). The properties and characteristics of a fractal set are not completely determined by its fractal dimension D (it is easy to construct a family of fractals that share the same D but differ sharply). These sets have different texture, more specifically, different lacunarity. Lacunarity is a notion distinct and independent from D; it is not related with the topology of the fractal and needs more than one numerical variable to be fully determined. Lacunarity is strongly related with the size distribution of the holes on the fractal and with its deviation from translational invariance.

As stated by Gefen, Meir and Aharony [12], lacunarity is the deviation of a fractal from translational invariance and can be extended to the description of spatial distribution of real data sets, including those with multifractal distributions. Lacunarity is defined as $\Lambda = E(\frac{M}{E(M)} - 1)^2$ where M is the mass of the fractal set (defined as the total number of points in the image) and $E(M)$ is the expected value of the mass computed for the fractal. This measures the discrepancy between the actual mass and the expected value. Lacunarity is small when texture is fine and large when texture is coarse. The mass of the fractal set is related to the length by $M(L) = kL^D$.

A number of algorithms have been proposed to measure lacunarity. We adopt the intuitively clear and computationally simple gliding box method of Allain and Cloitre [13].

We decided to use lacunarity to quantify heterogeneity in time arrivals of showers, so adding a third dimension to our spatial analysis. We assess time pattern lacunarity by gridding each time arrival array into squares as in McIntyre and Wiens [6]. Squares that contains a particle were denoted 1, empty squares were denoted 0; lacunarity is then computed following the gliding box protocol[13].

4. Simulated showers

To test our separation technique we analysed simulated showers initiated both by gamma-rays and protons. The showers were simulated by means of the CORSIKA code with QGSJET model [14], and the ARGO-G code [15],[16] to simulate the pattern of the shower front hit as detected in the ARGO-YBJ detector. The ARGO-G detector simulation code, a tool developed within the ARGO-YBJ collaboration and based on the GEANT3 package [17], gives as an output the space distribution of the particles hitting the detectors modules, taking into account the space resolution and the overall detectors efficiency, as well as the arrival time distribution of particles. A detailed description of the ARGO-YBJ detector performances is beyond our scope. A total number of 1000 events were chosen for the analysis. The showers have zenith angles $0 < \theta < 10$ and primary energies:

- $8 < E < 10 \cdot 10^{12} eV$ for the proton generated ones;
- $4 < E < 5 \cdot 10^{12} eV$ for the gamma generated ones.

Differences in energies ensure that the numbers of hits on the analysis carpet are comparable. The energy spectrum was generated according to power laws with spectral index -2.7 for protons and -2.5 for gammas. The spatial distribution of the secondary particles at ARGO-YBJ altitude (4300m a.s.l.) is used for the analysis. The time analysis is performed on the shower from time zero (first detection on the carpet of a secondary particle) to time 2000 ns. The output of the simulated showers includes the effect of the detector response as far as space and time distribution on the carpet is concerned.

5. Spatial separation

Our spatial separation technique uses the differences in wavelet based multifractal behaviours of showers of different progenitors. These are: the mean and the standard deviation of individual Gaussian fits to the distributions of multifractal dimensions.

For each shower, the dependence with respect to the distance from the shower core of the fractal dimension is almost linear [3]. This has a consequence: we need to specify the mean and the standard deviation just for two different mean distances from the core. So we choose to analyze two regions: a circle of fixed radius r_0 centred at the shower core (inner ring) and a ring with fixed inner and outer radii r_i and r_o (outer ring). In our analysis we choose the values: $r_0 = 7$ m, $r_i = 9$ m, $r_o = 12$ m. The radii have been chosen such that on average the two rings contain about an equal number of hits.

Our shower core is computed on the wavelet transform of the spatial map on the maximum scale ($s = 2^5$). On that scale, in this analysis, the shower core is found as the coordinates $xcore$ and $ycore$ of the maximum value of the wavelet coefficients.

The multifractal behaviour of each individual shower is then specified by four quantities: the mean of the Gaussian fit in the inner and the outer ring (μ_I, μ_O), the standard deviation in the inner and the outer ring (σ_I, σ_O).

The wavelet transform is a linear operator that can be written as:

$$W(s,t) = \langle f|\psi\rangle = s^{-1/2}\int_{-\infty}^{+\infty} f(x)\psi^*\left(\frac{x-t}{s}\right)dx$$

where $s(>0)$ is the scale on which the analysis is performed, $t \in \Re$ is the spatial translation parameter and ψ is the the Grossmann-Morlet (1984, 1987) analyzing wavelet function.$\psi_{(1,0)}(x)$ is called mother wavelet. It generates the other wavelet function $\psi_{(s,t)}(x), s > 1$.

We choose a mother wavelet similar to the Mexican hat in order to use the *à trous* algorithm in the following.

The set of scales are powers of two: $s = 2^r$ and the first scale always corresponds to the size of 1 pixel. The scale s may be considered as the resolution. In other words, if we perform a calculation on a scale s_0, we expect the wavelet transform to be sensitive to structures with typical size of about s_0 and to find out those structures. We choose to investigate on the scales 2 to 32 that on the ARGO-YBJ carpet correspond to physical sizes ≈ 1.2 m and ≈ 20 m, respectively.

Our result is a set of matrices of wavelet coefficients, one matrix for each scale investigated. An example is shown in Fig. 1

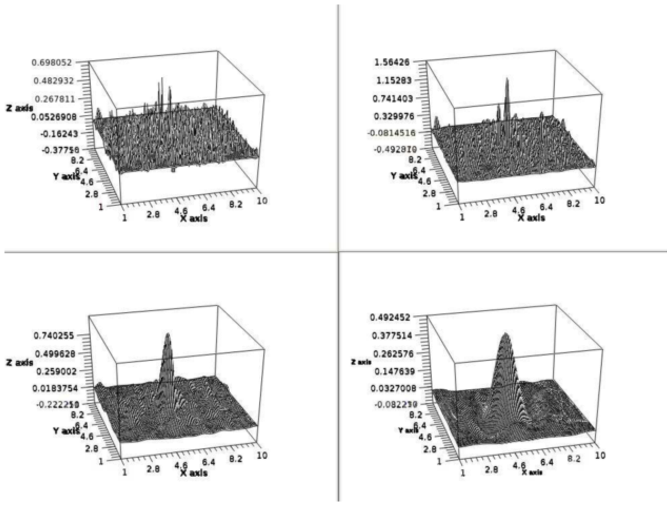

Figure 1. Matrices of wavelet coefficients for a single photon event. One matrix for each scale investigated. Scales shown are: top left: 4, top right: 8, bottom left: 16, bottom right: 32. Our shower core is computed on the wavelet transform of the spatial map at the maximum scale

As seen in Sect.2, if a function f has scaling law with exponent D around x_0: $f(R(x_0,\lambda\epsilon)) \sim \lambda^D f(R(x_0,\epsilon))$ then the wavelet transform $W(s,t) = \int g(x;s,t)f(x)dx$ has the same scaling exponent. Thus:

- The local scaling behaviour is represented by $W(s,t) \sim s^{D(t)}$.
- For any distribution function f, the slope of the plot of $\log W(s,t)$ versus $\log s$ will give the fractal dimension of the distribution around point t for the range of the scale s.

We compute a $log(W)/log(s)$ matrix for each scale s on the two regions selected (inner and outer ring).

$$D = log(W)/log(s)$$

is the fractal dimension and has a Gaussian distribution. If μ_I, μ_O are the average values of D in the region and σ_I, σ_O its standard deviations we compute average values of $\mu_I, \mu_O, \sigma_I, \sigma_O$ on all the scales and obtain our parameters: $\overline{\mu}_I, \overline{\mu}_O, \overline{\sigma}_I, \overline{\sigma}_O$. All these quantities have a nearly monotonic dependence on the mass number of the progenitor. However, they show large fluctuations, prohibiting de facto mass separation.

So, to get a high-resolution separation technique, we need at least one more parameter and then define and train an artificial neural network.

6. Time separation

Our separation technique on time arrival of the shower use the lacunarity technique in the same two regions as the spatial separation: the inner and outer ring. First, we need to compute our time array as $T = T_{max} - T_{min}$ where T_{min} is the time arrival of the first secondary particle on the carpet and is set to 0 and T_{max} is the time arrival of the last secondary particle we include in our analysis. Maximum value of T is 2000 ns. Then we need to define the time scale on which we compute lacunarity. This is a crucial parameter. We call it t_{lac} and to compute $T = T_{max} - T_{min}$

A box of length t_{lac} is placed at the origin of the sets. The number of occupied sites within the box (box mass k) is then determined. The box is moved one space along the set and the mass is computed again. This process is repeated over the entire set, producing a frequency distribution of the box masses $n(k, t_{lac})$. This frequency distribution is converted into a probability distribution $Q(k, t_{lac})$ by dividing by the total number of boxes $N(t_{lac})$ of size t_{lac}.

$$Q(k, t_{lac}) = n(k, t_{lac})/N(t_{lac})$$

The first and second moments of the distribution are computed, Z_1 and Z_2.

$$Z_1(t_{lac}) = \sum_k k \cdot Q(k, t_{lac})$$

$$Z_2(t_{lac}) = \sum_k k^2 \cdot Q(k, t_{lac})$$

The lacunarity is now defined as:

$$\Lambda(t_{lac}) = Z_2/Z_1^2$$

Lacunarity is computed both in the inner and the outer ring (Λ_I, Λ_O). By trial and error, we find that 5 ns is a good choice for the t_{lac} parameter.

7. Artificial Neural Network

We assume therefore that the mass of the progenitor can be estimated with the use of an artificial neural network of the six variables. The neural network is a standard three layer perceptron with only one output neuron (1=hadron, 0=gamma). The input layer consist of six neurons each of them reading one of the parameters:

- Average of the fractal dimensions on the spatial scales in the inner and outer region:

$$\overline{\mu}_I, \overline{\mu}_O$$

- Average of the standard deviation of the fractal dimensions on the spatial scales in the inner and outer region:

$$\overline{\sigma}_I, \overline{\sigma}_O$$

- Lacunarity of the time arrivals arrays in the inner and outer region:

$$\Lambda_I, \Lambda_O$$

The neural network we choose is of the feed forward type and it is made of three perceptron layers. The input is made of six neurons, the hidden layer is made of four neurons, while the output vector is defined in a one dimensional space and it is trained to be 0 for gamma initiated events and 1 for hadronic ones.

Network was implemented and optimized by using the Stuttgart Neural Network Simulator Tool (SNNS) [18]. SNNS is a simulator for neural networks developed at the Institute for Parallel and Distributed High Performance Systems at the University of Stuttgart. The network was trained by using one thousand events from indipendent samples.

8. Results for a test run

In Fig. 2 we present the results of applying our method to 1000 simulated showers of two different primaries (gamma and hadron). As is seen in the histogram, the identification is achieved with a very high resolution and almost no overlap is present between showers from different primaries. Almost 90% of the hadron showers have neural network output neuron > 0.5 and almost 90% of the gamma showers have neural network output neuron < 0.5.

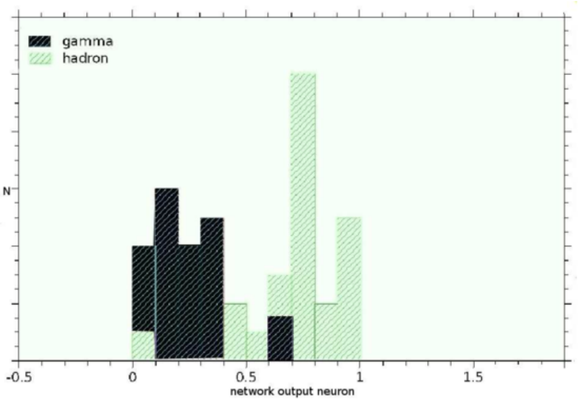

Figure 2. Gamma-hadron separation for our test run with 1000 events (500 gammas and 500 hadrons)

The most important parameter in gamma/hadron discrimination is the Q (quality) factor. The Q factor is defined as

$$Q = \frac{\varepsilon_\gamma}{\sqrt{1-\varepsilon_h}}$$

where

- ε_γ is the fraction of showers induced by photons correctly identified by the discrimination criterion;
- ε_h is the fraction of showers induced by protons correctly identified by the discrimination criterion;

so that $1-\varepsilon_h$ is the background contamination (fraction of events induced by protons and erroneously identified as gamma induced ones). In our test run a value of $Q \approx 2.47$ has been reached, which is among the largest obtained in the field.

9. Summary and concluding remarks

A new technique for separating extensive air showers initiated by different progenitors, based on multifractal and lacunarity analysis, has been developed.

It is well known that fractals are sets of points that possess the property of being invariant by dilation. When a fractal set is self-similar a central role is played by a quantity called fractal dimension, a measure of how the members are distributed in space. Self-similar multifractals are also geometrical objects invariant by dilation. The large numbers of interactions in showers from the same progenitor are self-similar. For this reason showers may be characterized by their fractal dimension.

Since, for each shower, the dependence with respect to the distance from the shower core of the fractal dimension is almost linear [3], we need to specify the mean and the standard deviation of the fractal dimension just for two different distances from the core. So we choose to analyze two regions: a circle of fixed radius r_0 centred at the shower core (inner ring) and a ring with fixed inner and outer radii r_i and r_o (outer ring).

On the other hand, lacunarity is a counterpart to the fractal dimension that describes the texture of a fractal. The properties and characteristics of a fractal set are not completely determined by its fractal dimension. These sets may have different texture, more specifically: different lacunarity. Lacunarity is not related with the topology of the fractal and is strongly related with the size distribution of the holes on the fractal and with its deviation from translational invariance. In our approach, lacunarity is used to describe gaps in time arrivals of showers. We choose to compute lacunarity on time arrival structure of the showers in order to exploit a third dimension in our analysis. The regions are the same as for multifractal-wavelet analysis (inner ring and outer ring).

Finally, the multifractal behaviour and lacunarity time structure of each shower is represented by six variables: average of the fractal dimensions on the spatial scales in the inner and outer ring, average of the standard deviation of the fractal dimensions on the spatial scales in the inner and outer ring, lacunarity of the time arrivals arrays in the inner and outer ring.

Due to large shower to shower fluctuations, the differences in any single one of these variables have a very poor separation power [19]. So, on these six quantities, a neural network analysis has been performed. The neural network we choose is a standard three layer perceptron with only one output neuron of the feed forward type. The output vector is defined in a one dimensional space and it is trained to be 0 for gamma initiated events and 1 for hadronic ones.

Network were implemented and optimized by using the Stuttgart Neural Network Simulator Tool, a simulator for neural networks developed at the Institute for Parallel and Distributed High Performance Systems at the University of Stuttgart, and trained by using several thousand events from indipendent samples.

It is well known that the most important parameter in gamma/hadron discrimination is the Q (quality) factor, defined as $Q = \frac{\varepsilon_\gamma}{\sqrt{1-\varepsilon_h}}$ where ε_γ is the fraction of showers induced by photons correctly identified by the discrimination criterion and ε_h is the fraction of showers induced by protons correctly identified by the discrimination criterion, so that $1 - \varepsilon_h$ is the background contamination (fraction of events induced by protons and erroneously identified as gamma induced ones).

Our approach gives very good results, leading to a clear improvement of the discrimination power with respect to the existing figures for extended shower detectors: the value $Q \approx 2.47$ obtained in our test run is among the largest obtained in the field. The technique shows up to be very promising and its application may have important astrophysical prospects in the experimental environment of extensive air shower study.

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