Heterogeneous structures studied by interphase elasto-damaging model.

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SUMMARY. For all structures that are constituted by heterogeneous materials, the meso-modelling approach is the most rigorous since it analyzes such structures as an assembly of distinct elements connected by joints, the latter commonly simulated by apposite interface models. In particular, the zero-thickness interface (ZTI) models are extensively used in those cases where the joint thickness is small if compared to the other dimensions of the heterogeneosu material. In ZTI models the constitutive laws relate the contact tractions to the displacement discontinuities at the interface, but in many cases the joint response depends also on internal stresses and strains within the bulk material. In this sense the interphase model represents an enhancement of the ZTI because is able to introduce the effect of internal stresses into the analysis. Particular attention is spent to the definition of a damage model in order to describe the propagation of a fracture inside the interphase element. The damage model is developed in a thermodinamically context for plane stress applications.

1 INTRODUCTION

The mechanical response of all those structures that are constituted by heterogeneous materials is dependent by their microstructure and by all the static and kinematic phenomena occurring in each constituent and at their joints. Material degradation due to nucleation, growth and coalescence of microvoids and microcracks is usually accompanied by plastic deformations that cause strain softening and induced anisotropy.
The mesoscopic approach is by now the most diffused technique to study this kind of structures, because it overcomes the problems associated with the strong simplifications that have to be introduced, for example, when the macroscopic approach is applied. In particular, with the mesoscopic approach all the material constituents are modelled individually and their interactions are regulated by using appropriate devices able to reproduce the inelastic phenomena that usually occur at the physical interfaces. In literature, these mechanical devices are generally called contact elements, normally distinguished between link elements, thin layer elements and zero-thickness interface elements. Among them, in the last decades interface elements have been applied in several engineering applications due to their simple formulation and to their easiness to be implemented in finite element codes [1]-[6].

The interface constitutive laws are expressed in terms of contact tractions and displacement discontinuities which are considered as generalized joints strains. In order to model the nonlinear behaviour caused by plastic deformations and damage evolution the constitutive laws of the interface elements are formulated making use of concepts borrowed by theory of plasticity and continuum damage.

However, in many cases the structural response depends also on internal stresses and strains within the joint. It is sufficient to think to the fracture that appears in the middle of masonry blocks caused by the horizontal tangential contact stresses between the mortar and the block when the masonry assembly is subjected to a pure compressive load. These tangential stresses cannot be captured by the classical ZTI model. Therefore, the usual assumption used in zero-thickness interface elements, where the response is governed by contact stress components may require a correction by introducing the effect of the internal stresses into the analysis. This enhancement of the interface element is known as interphase element, for the first time proposed by Giambanco and Mróz [7].

The interphase element has been formulated by authors as a new contact element and introduced in a scientific oriented finite element code. Patch tests have been carried out in elasticity to investigate the numerical performance and convergence of the element. All the results are shown in the paper written by Giambanco el al. [8]. In particular, in that paper is shown how strategies such as the Reduced Selective Integration or the Enhanced Assumed Strain methods are necessary to avoid shear locking effects of the model.

In this work the same interphase element is improved by introducing an isotropic damage model in order to describe the nonlinear response due to the evolution of fractures inside the interphase. The basic relations of the interphase model are reported in Section 2 for seek of completeness. In Section 3 the damage model adopted in this work is shown while Section 4 is dedicated to numerical applications in order to show the effectiveness of the proposed model.

2 THE INTERPHASE MODEL.

Let us consider, in the Euclidean space $\Re^3$ referred to the orthonormal frame $(O,\, \mathbf{i_1},\, \mathbf{i_2},\, \mathbf{i_3})$, a structure formed by two adherents Ω^+, Ω^- connected by a third material Ω in contact with the two bodies by means of the two physical interfaces Σ^+ and Σ^- respectively, as in Fig. 1.

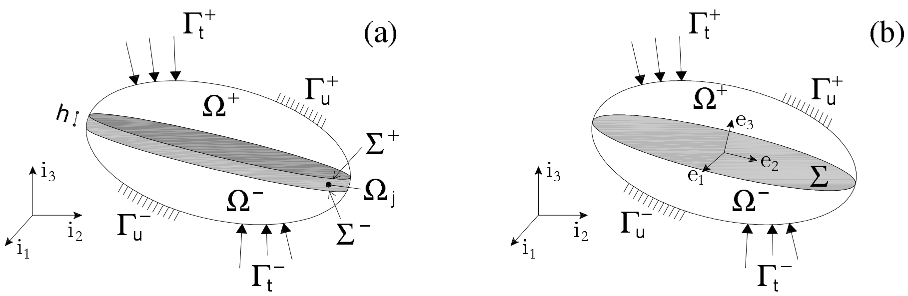

Figure 1: (a) Mechanical scheme of a third body iterposed between two adherents; (b) Interphase mechanical scheme

It is assumed that the thickness h of the joint is small if compared with the characteristic dimensions of the bonded assembly.

The boundary of the two adherents is divided in the two parts $\Gamma_u^\pm$ and $\Gamma_t^\pm$, where kinematic and loading conditions are specified respectively.

The joint interacts with the two adherents through the following traction components:

$$\mathbf{t}^\pm = t_1^\pm \mathbf{e}_1 + t_2^\pm \mathbf{e}_2 + t_3^\pm \mathbf{e}_3 \tag{1}$$

which can be considered as the external surface loads for the joint.

In Eq. 1 $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ are the unit vectors of the local reference system, with $\mathbf{e}_3$ oriented along the normal to the middle surface Σ and directed towards the adherent Ω^+.

The joint can be regarded as an interphase model. It is assumed that the fibers inside the interphase and directed along $\mathbf{e}_3$ are maintained rectilinear during the deformation process. In view of this hypothesis the interphase dieplacement field $\mathbf{u}$ can be easily obtained from the displacement $\mathbf{u}^+$ and $\mathbf{u}^-$ of the interfaces Σ^+ and Σ^-, thus

$$\mathbf{u}(x_1,\, x_2,\, x_3) \;= \left(\frac{1}{2}+\frac{x_3}{h}\right)\, \mathbf{u}^+\,(x_1,\, x_2) + \left(\frac{1}{2}-\frac{x_3}{h}\right)\, \mathbf{u}^-\,(x_1,\, x_2) \tag{2}$$

with x_1, x_2 and x_3 the Cartesian coordinates in the orthonormal frame $(O,\, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$.

Since the thickness of the joint is generally small if compared to the characteristic dimensions of the adherents, we can assume the strain state ε uniform along the $\mathbf{e}_3$ direction and given by:

$$\varepsilon(x_1,\, x_2) \;= \frac{1}{h} \int_{-h/2}^{h/2} \nabla^s \mathbf{u}\,(x_1,\, x_2,\, x_3)\; dx_3. \tag{3}$$

Substituting the expression (2) we have:

$$\varepsilon(x_1,\, x_2) \;= \frac{1}{2h}\,([\mathbf{u}] \otimes \mathbf{I}_3 + \mathbf{I}_3 \otimes [\mathbf{u}]) + \frac{1}{2}\nabla^s \left(\mathbf{u}^+ + \mathbf{u}^-\right) \tag{4}$$

where $[\mathbf{u}] = \mathbf{u}^+ - \mathbf{u}^-$, $\mathbf{I}_3 = \{\delta_{i3}\}$ and ∇^s is the symmetric gradient operator defined as $\nabla^s = \frac{1}{2}\left(\nabla + \nabla^T\right)$.

Let us note that in the interphase model the joint curvatures generated by displacement field (2) and the related flexural effect are neglected.

Equilibrium equations are derived by applying the principle of virtual displacements (PVD) that asserts that the external work produced by the contact tractions equals the internal work developed in the joint. By applying the divergence theorem and assuming that $\Sigma = \Sigma^+ = \Sigma^-$, the PVD leads to the following local equilibrium relation of the interphase model:

$$\mathbf{t}^+ - \sigma \cdot \mathbf{I}_3 + \frac{h}{2} div\sigma = \mathbf{0}; \quad \mathbf{t}^- + \sigma \cdot \mathbf{I}_3 + \frac{h}{2} div\sigma = \mathbf{0} \quad on\ \Sigma, \tag{5}$$

$$\mathbf{m} \cdot \sigma = \mathbf{0} \quad on\ \Gamma. \tag{6}$$

3 A SIMPLIFIED ISOTROPIC DAMAGE MODEL.

Respect to the ZTI model, the interphase presents the innovative aspect of decomposing the stress state in an external and internal part. The external stress state, or contact tractions, are responsible for the loss of adhesion at the joint-units interfaces while the internal stress state dominates the progressive damage of the bulk material.

The present work is a first attempt to describe the nonlinear material response of the cohesive joints making use of the interphase model. The attention is focused on the damage of the bulk material, thus the nonlinear behaviour of the joint is caused by the evolution of microcraks and microvoids occurring in the material interposed between the adherents and a perfect adhesion is considered at the physical interfaces.

The joint material is a quasi-brittle material with different tensile and compressive strengths. The constitutive model adopted is a simplified isotropic damage model where the global damage parameter ω is a weighted combination of the damage variables in tension ω_+ and compression ω_-, thus

$$\omega = \alpha_+ \omega_+ + \alpha_- \omega_- \tag{7}$$

where the weighting coefficients α_+ and α_- are defined as functions of the principal values of the stress tensor as follows:

$$\alpha_+ = \frac{\sum \langle \sigma_p \rangle}{\sum |\sigma_p|} \qquad ; \qquad \alpha_- = \frac{\sum \langle -\sigma_p \rangle}{\sum |\sigma_p|} \tag{8}$$

being $\langle \sigma_p \rangle$ and $\langle -\sigma_p \rangle$ the positive and negative parts of the principal stress tensor respectively and $\sum |\sigma_p|$ the sum of the absolute values of the principal stresses.

The damage constitutive model is similar to those proposed by Tao and Phillips [9] and Voyiadjis and Taqieddin [10].

Under uniaxial loading damage of the bulk material is governed by the corresponding damage parameter. In presence of biaxial stress state, both tensile and compressive damage parameters evolve and their contribution to the global damage is in proportion to the values of the weighting coefficients. The damage parameter ω can assume values in the range $0 \leq \omega \leq 1$, with boundaries having the meaning of a pristine ($\omega = 0$) and a fully damaged ($\omega = 1$) material respectively.

Following a thermodynamical approach, the Helmholtz free energy can be defined as:

$$\Psi\left(\varepsilon, \xi_d^+, \xi_d^-, \omega_+, \omega_-\right) = \frac{1}{2}\left(1-\omega\right)\varepsilon^T \mathbf{E} \varepsilon + \Psi_d^+ + \Psi_d^- \tag{9}$$

where $\mathbf{E}$ is the material undamaged elastic tensor, ε is the strain vector, Ψ_d^+ and Ψ_d^- two convex inelastic potentials accounting for the evolution of damage activation domains as a consequence of tensile and compressive damage mechanisms, respectively. In particular, Ψ_d^+ and Ψ_d^- are written as:

$$\Psi_d^\pm = -h_d^\pm \left[\xi_d^\pm + \ln\left(1 - \xi_d^\pm\right)\right] \tag{10}$$

with ξ_d^+ and ξ_d^- two internal variables used to describe the damage evolution; h_d^+ and h_d^- are material constants.

By imposing the Clausius-Duhem inequality, the derivative of Eq. 9 with respect to all the kinematic variables, leads to the correspondent state equations:

$$\sigma = \frac{\partial \Psi}{\partial \varepsilon} = \left(1-\omega\right)\mathbf{E}\varepsilon; \quad \varsigma^\pm = \frac{\partial \Psi}{\partial \omega_\pm} = \frac{\alpha_\pm}{2}\varepsilon^T \mathbf{E}\varepsilon; \quad \chi_d^\pm = \frac{\partial \Psi}{\partial \xi_d^\pm} = h_d^\pm \frac{\xi_d^\pm}{1-\xi_d^\pm} \tag{11}$$

ς^+ and ς^- are the thermodynamic forces associated to the damage variables ω_+ and ω_-.

The capability of the model to describe the different behaviour of the material during tensile or compressive stresses comes from the definition of two different yield functions, written as:

$$\phi_d^\pm = \varsigma^\pm - \varsigma_0^\pm - \chi_d^\pm \tag{12}$$

where ς_0^+ and ς_0^- are the initial damage thresholds which govern the onset of damage in tension and compression, respectively. The two parameters χ_d^+ and χ_d^-, instead, define the evolution laws of the damage domains by changing the threshold levels.

Flow rules are derived looking for the maximum value of dissipation with respect to the static variables by means of the Lagrangian method. The Lagrangian is defined as:

$$\underset{\varsigma^{\pm},\chi_d^{\pm}}{L} = \varsigma^{+}\dot{\omega}_{+} + \varsigma^{-}\dot{\omega}_{-} - \chi_d^{+}\dot{\xi}_d^{+} - \chi_d^{-}\dot{\xi}_d^{-} - \dot{\lambda}_d^{+}\phi_d^{+} - \dot{\lambda}_d^{-}\phi_d^{-} \tag{13}$$

where the dot symbolizes the time derivative of the corresponding variable and λ_d^{+} and λ_d^{-} are lagrangian multipliers.

λ_d^{+}, λ_d^{-}, ϕ_d^{+} and ϕ_d^{-} have to respect the loading/unloading complementarity conditions:

$$\phi_d^{\pm} \leq 0; \quad \dot{\lambda}_d^{\pm} \geq 0; \quad \phi_d^{\pm}\dot{\lambda}_d^{\pm} = 0. \tag{14}$$

Finally the following flow rules are deduced:

$$\dot{\lambda}_d^{\pm} = \dot{\xi}_d^{\pm} = \dot{\omega}_{\pm} \tag{15}$$

A typical uniaxial response of the model on a single Gauss point for a cyclic loading is shown in Fig. 2. It is clear the change of the elastic modulus during unloadings and the different maximum stresses in tension and compression. It is also visible the effect of crack closing when stresses change their sign.

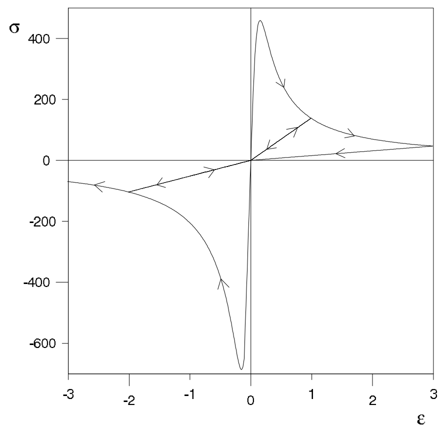

Figure 2: Typical uniaxial response of the model for a cyclic loading.

4 NUMERICAL APPLICATIONS.

The model presented in Sections 2 and 3 has been detailed for 2-D applications in order to assess the numerical performance of the interphase element. The interphase model has been implemented in a scientific oriented finite element code as a new finite element. All numerical applications were carried out under the hypothesis of plane stress state.

In this particular case the elastic matrix $\mathbf{E}$ is written as

$$\mathbf{E} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{16}$$

with E and ν the Young's elastic modulus and Poisson's coefficient respectively.

The principal stress directions coincide with the principal strain directions so that the parameters α_+ and α_- can be simply obtained starting from the knowledge of the principal strains. In particular:

$$\alpha_+ = \frac{\langle \varepsilon_1 + \nu\varepsilon_2 \rangle + \langle \varepsilon_2 + \nu\varepsilon_1 \rangle}{|\varepsilon_1 + \nu\varepsilon_2| + |\varepsilon_2 + \nu\varepsilon_1|}; \quad \alpha_- = \frac{\langle -(\varepsilon_1 + \nu\varepsilon_2) \rangle + \langle -(\varepsilon_2 + \nu\varepsilon_1) \rangle}{|\varepsilon_1 + \nu\varepsilon_2| + |\varepsilon_2 + \nu\varepsilon_1|}. \tag{17}$$

A trial prediction/damage correction procedure is followed at each step of the simulations.

4.1 *Uniaxial compression of a brick-mortar-brick system.*

The example regardes the problem of the uniaxial compression of two masonry blocks joined by a mortar thin layer. The geometry of this test is illustrated in Fig. 3. Two different cases have been considered, depending on the different ratios of elastic moduli between blocks and mortar. In the first case the blocks are characterized by an elastic modulus higher than that one of mortar. In particular $E_b = 10E_m$. In the second case the opposite situation $E_m = 30E_b$ has been considered. Simulations were conducted under controlled displacement.

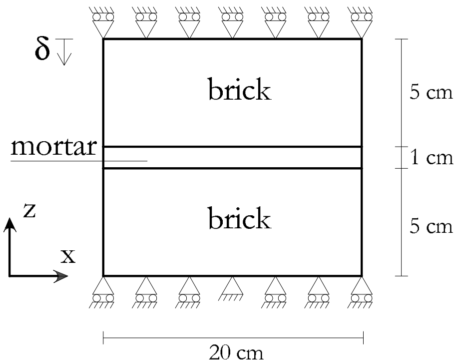

Figure 3: Scheme of the uniaxial compression test of a masonry block.

In the first column of Fig. 4 the results of the first case are shown in terms of global load-displacement curve together with the σ_z, σ_x and ω profiles for the entire mortar layer at certain load steps. The same kind of curves are reported in the second column to have a comparison with the results of the second case.

First of all, it can be noticed how the global response of the assembly is completely different. After an elastic initial response and a small nonlinear branch, the post-peak behaviour is quite different due to the different boundary conditions for the interphase: the figure shows a smooth softening curve that progressively tends to zero for the first case, while a sudden fall is evident for the second case.

In terms of stresses, when the brick is stiffer than mortar the confinement action provided by the blocks leads to compressive stresses in the x-direction. On the other hand, the opposite case happens when mortar is stiffer than bricks. Now the joint provides a sort of confinement action on the blocks with the result that the joint is subjected to tensile stresses in the x-direction.

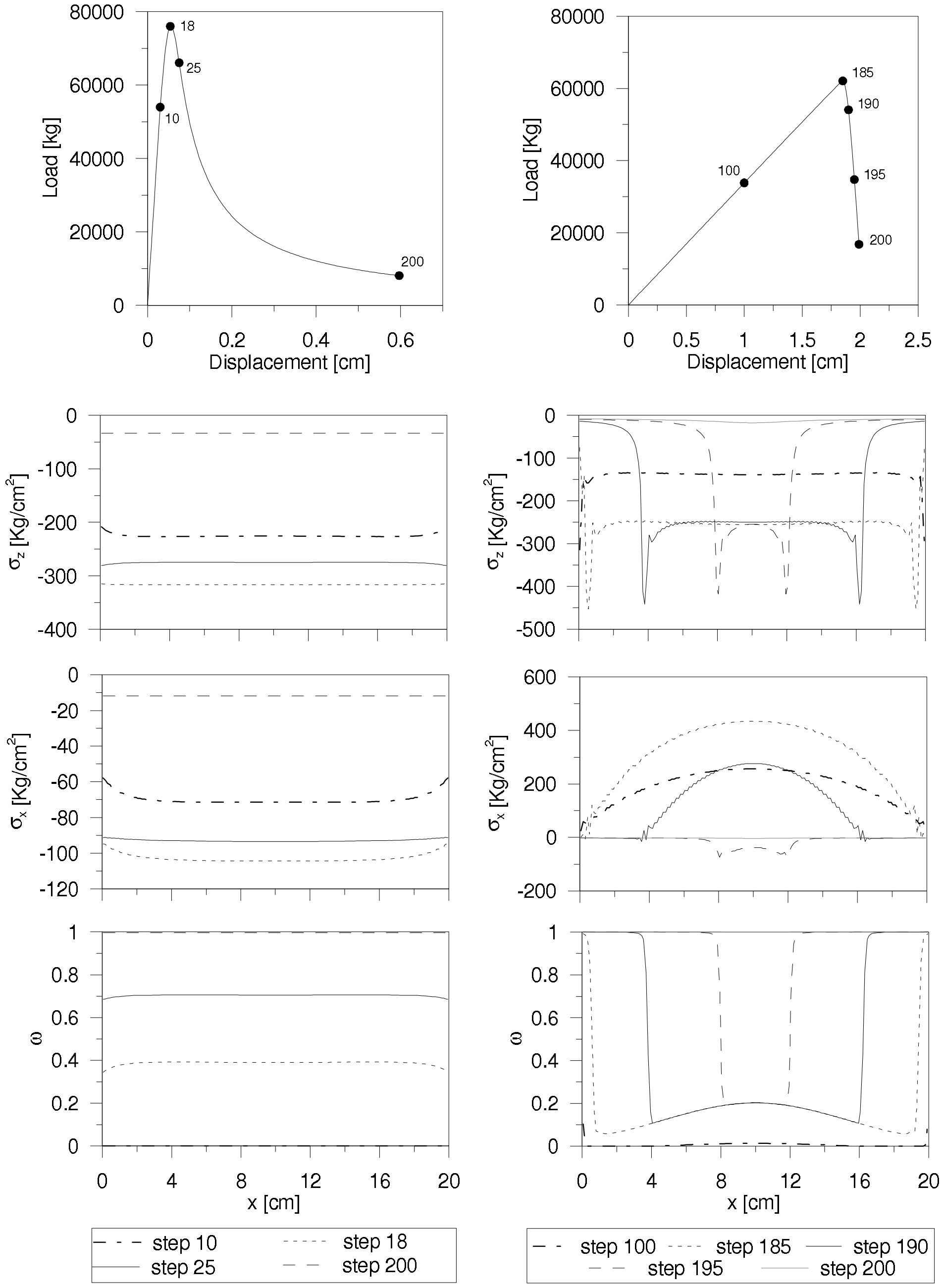

Figure 4: Uniaxial compression test of a masonry block. First coloumn: results for the case $E_b = 10E_m$; second coloumn: results for the case $E_m = 30E_b$.

In particular, the behaviour of the mortar layer when it is confined by two stiffer blocks is similar to a material in edometric conditions, while the case of a mortar layer confined by two softer blocks remembers the solution of Boussinesq for a rigid body on elastic soil, clearly showed by the stress instabilities at the boundaries in the σ_z curves.

Even the evolution of damage is different in both cases. In the first case stresses are almost equal for each Gauss point and damage starts to develop in compression at the end of the elastic branch and progressively evolves up to the value 1 causing a reduction of stresses till a null value in all the joint.

In the second case, instead, at the beginning the stress increases with the load and tensile damage starts to appear in the middle of the joint when tensile strains in the x-direction are able to overcome the threshold of the correspondent activation function. After that, starting from the two external ends of the mortar joint, damage develops also in compression and quickly goes to one annulling the stresses. The crack finally evolves towards the middle until all the layer is fractured.

5 CONCLUSIONS

The present paper deals with the mesomodelling of heterogeneous structures by means of interphase elements, that can be considered as an enhancement of the common interface elements. The nonlinear response of the element has been modelled by introducing a isotropic damage model able to describe the different behaviours under tensile or compressive stress state. Numerical examples are provided to prove the effectiveness of the model to predict structural response and inelastic phenomena. Ongoing and future efforts are devoted to the introduction of a plastic activation function written on the base of contact tractions in order to reproduce plasticity effects by the model.

6 Acknowledgements

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