Passive Laser Power Stabilization via an Optical Spring

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Metrology experiments can be limited by the noise produced by the laser involved via small fluctuations in the laser's power or frequency. Typically, active power stabilization schemes consisting of an in-loop sensor and a feedback control loop are employed. Those schemes are fundamentally limited by shot noise coupling at the in-loop sensor. In this letter we propose to use the optical spring effect to passively stabilize the classical power fluctuations of a laser beam. In a proof of principle experiment, we show that the relative power noise of the laser is stabilized from approximately 2×10^{-5} Hz$^{-1/2}$ to a minimum value of 1.6×10^{-7} Hz$^{-1/2}$, corresponding to the power noise reduction by a factor of 125. The bandwidth at which stabilization occurs ranges from 400 Hz to 100 kHz. The work reported in this letter further paves the way for high power laser stability techniques which could be implemented in optomechanical experiments and in gravitational wave detectors. © 2022 Optical Society of America

1. INTRODUCTION

Laser power stabilization is important for many modern experiments, since power fluctuations can limit their sensitivity[1, 2]. Currently, interferometric gravitational wave detectors require the most stringent power stability levels, where a relative power noise (RPN) of roughly 2×10^{-9} Hz$^{-1/2}$ is required at 10 Hz by the Advanced LIGO detectors [3]. A third generation of gravitational wave detectors is currently being planned, which will most likely require even higher power stability. So far strict requirements at low frequencies were mostly achieved using active power stabilization schemes, where an in-loop photodetector is used in conjunction with a feedback control loop. Those schemes are usually limited by noise sources coupling in the in-loop detector, and often require a large power detection which can exceed the power threshold of the in-loop sensors [4]. Re-

cently, an alternative technique was demonstrated in which the full beam power of the laser and its fluctuations are sensed via a Michelson interferometer with a movable mirror [5]. In this Letter, we propose to use a movable mirror in a Fabry-Perot cavity with a strong optical spring. The technique demonstrated in this paper is passive, and thus does not require a power sensor. We show here that this technique can provide large suppressions of classical power fluctuations such as to produce a beam in transmission of the cavity which is shot noise limited. In [5], a transfer and a sensing beam are used with a single movable cantilever mirror, in order to demonstrate active power stabilization from 1 Hz to 10 kHz. Here, we instead use a Fabry-Perot cavity with a strong optical spring to passively stabilize the power fluctuations transmitted by the cavity. Unlike other power stabilization techniques employing optical cavities [6], the experiment proposed here provides power noise reduction below the cavity pole. This is an advantage, since it dispenses the use of long and high finesse cavities for stabilization at low frequencies.

We first start with a cavity comprised of a movable end mirror pumped with a laser (see Figure 1). By detuning the cavity away from its resonance, an optical spring [7] is formed, whose dynamic response reduces power fluctuations in transmission of the cavity. To show that an optical spring can passively stabilize the power fluctuations of a laser we first start with the equation of motion of the movable mirror:

$$m\ddot{x} = F_{rad} + F_{res} + F_{ext}, \qquad (1)$$

where F_{rad} is the force on the mirror due to radiation pressure, F_{res} is the restorative spring force, F_{ext} is any external force, such as thermal noise, and m is the mass of the mirror. Eq 1 in the frequency domain then becomes:

$$-m\Omega^2 \tilde{x} = \frac{2\tilde{P}_{\text{circ}}}{c} - k\tilde{x} + \tilde{F}_{ext}. \qquad (2)$$

where c is the speed of light, k is the spring constant corresponding to the restorative mechanical spring force of the movable mirror, and P_{circ} is the circulating power in the cavity.

The power fluctuations (ΔP_{circ}) inside the cavity are dependent on the motion of the movable mirror and on the change of the maximally circulating power (ΔP_{max}) at resonance in the

cavity, which in turn depends on the fluctuations of the injected laser power. These fluctuations can be written as:

$$\Delta P_{\text{circ}} = \frac{dP_{\text{circ}}}{dx}\Delta x + \frac{dP_{\text{circ}}}{dP_{\text{max}}}\Delta P_{\text{max}}. \quad (3)$$

The circulating power in the cavity can be written in terms of the detuning (δ, in units of linewidth) and the maximum power circulating in the cavity (P_{max}) as [7]:

$$P_{\text{circ}}(\delta) = \frac{P_{\text{max}}}{1 + \delta^2}. \quad (4)$$

Additionally, the position of the movable mirror can be written in terms of the detuning as [7]:

$$x = \frac{\delta \lambda A}{8\pi}, \quad (5)$$

where λ is the wavelength of light and A, the total losses of the mirrors, including their transmissivity, in which $A = 0$ is two perfectly reflective mirrors. Given the equations for power in terms of detuning (Eq 4) and position in terms of detuning (Eq 5), it is useful to rewrite Eq 3 as:

$$\Delta P_{\text{circ}} = \frac{dP_{\text{circ}}}{d\delta}\frac{d\delta}{dx}\Delta x + \frac{dP_{\text{circ}}}{dP_{\text{max}}}\Delta P_{\text{max}}. \quad (6)$$

Plugging in respective derivatives into Eq 6 yields:

$$\Delta P_{\text{circ}} = -\frac{16\pi P_{\text{max}}\delta}{\lambda A(1 + \delta^2)^2}\Delta x + \frac{\Delta P_{\text{max}}}{1 + \delta^2}. \quad (7)$$

Solving Eq 2 for Δx to use in Eq 7 yields:

$$\Delta P_{\text{circ}} = \frac{K_{os}\Delta P_{\text{circ}}}{m(\Omega^2 - \Omega_{\text{fund}}^2)} + \frac{\Delta P_{\text{max}}}{1 + \delta^2} + \frac{cK_{os}\Delta F_{ext}}{2m(\Omega^2 - \Omega_{\text{fund}}^2)}, \quad (8)$$

where we have dropped the tilde notation and made the substitution for the optical spring constant K_{os}: [8]

$$K_{os} = \frac{32\pi P_{max}\delta}{\lambda Ac(1 + \delta^2)^2}. \quad (9)$$

Note that this equation for the optical spring constant assumes that the response of the cavity is sufficiently slow such that it may be regarded as instantaneous. Solving Eq 8 for the intracavity power fluctuations yields,

$$\Delta P_{\text{circ}} = \frac{\Delta P_{\text{max}}}{1 + \delta^2}\left(\frac{\Omega^2 - \Omega_{\text{fund}}^2}{\Omega^2 - \Omega_{os}^2 - \Omega_{\text{fund}}^2}\right) \\ - \frac{c}{2}\Delta F_{ext}\left(\frac{\Omega_{os}^2}{\Omega^2 - \Omega_{os}^2 - \Omega_{\text{fund}}^2}\right), \quad (10)$$

where Ω_{fund} is the resonance frequency of the fundamental mode of the cantilever mirror and Ω_{os} is the optical spring frequency, where both are defined as:

$$K_{os} = m\Omega_{os}^2 \quad \text{and} \quad k = m\Omega_{\text{fund}}^2. \quad (11)$$

An independent calculation of Eq 10 can be found in [9].

For frequencies much smaller than the cavity pole, the connection between power fluctuations at the cavity input (ΔP_{in}) and the intracavity power fluctuations on resonance is $\Delta P_{\text{max}} = PB * \Delta P_{\text{in}}$, where PB is the power buildup of the cavity. Hence, as seen from Eq 10, for frequencies much less than the optical spring resonance frequency ($\Omega \ll \Omega_{os}$) and much greater than the fundamental mode of the cantilever mirror ($\Omega \gg \Omega_{\text{fund}}$), the
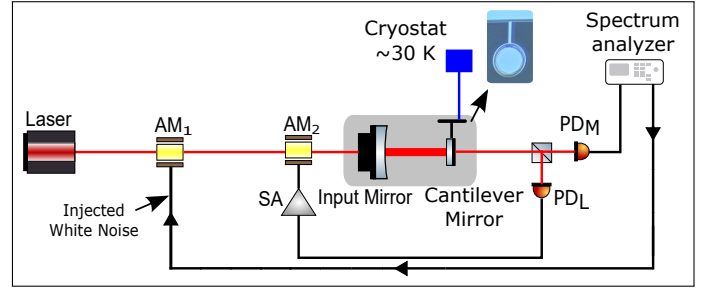

Fig. 1. Simplified experimental setup of the passive laser power stabilization scheme via an optical spring. PD_L is used solely stabilize the optical spring above 100 kHz and not for active power stabilization.

power fluctuations in the cavity are reduced compared to the injected field's power fluctuations by a factor of $(\frac{\Omega}{\Omega_{os}})^2$. We can also see that the external force, F_{ext}, imprints power fluctuations on the laser beam and will limit the stability achievable by this scheme. We have chosen an entirely classical derivation due to the fact that quantum radiation pressure effects are evaded when measuring the amplitude quadrature in transmission of the cavity [10].

The schematic of the experiment used to demonstrate this power fluctuation reduction is shown in Figure 1. It consists of an optomechanical cavity kept at cryogenic temperatures (~ 30 K). The cavity is pumped with a 1064 nm Nd:YAG nonplanar ring oscillator (NPRO) laser and is housed in a vacuum chamber kept at 10^{-8} torr. The movable mirror used in this setup is a cantilever mirror [7] with a mass of 50 nanograms, a fundamental frequency of 876 Hz, and a quality factor of around 25400. We note here that this quality factor is much larger than in previous experiments (16000) [10, 11] due to the reduced temperature and pressure of the cryogenically cooled cavity. The input mirror in the cavity is a 0.5 inch diameter, rigidly mounted mirror with a radius of curvature of 1 cm. The cavity is just under 1 cm long, with a pole greater than 100kHz. Inside the vacuum chamber there is a vibration isolation platform which all the optics are mounted to, reducing seismic vibrations above 100 Hz.

The requirement to keep a system like this stable is a positive optical spring constant and a positive damping coefficient [12, 13] which is not the case for our system. To keep the configuration of the cavity stable we employ a feedback loop actuating on an amplitude modulator (AM$_2$ in Fig 1). This ensures the cavity stays at a constant detuning during a measurement. This feedback loop uses a photodetector (PD$_L$ in Fig 1) in transmission of the cavity as the in-loop sensor and applies relevant correction signals to AM$_2$ only for frequencies close to the optical spring frequency (i.e. above 100 kHz for the red curve in Fig 2). This feedback loop does not provide any power stabilization of the injected beam. Another photodetector in transmission of the cavity is used to monitor the power noise of the transmitted beam, labeled PD$_M$. Additional details describing the experimental setup can be found in [11, 14].

In order to demonstrate the power stabilization we inject white noise into the amplitude modulator before the cavity, AM$_1$, at a voltage yielding a baseline relative power fluctuations of around 2×10^{-5} Hz$^{-1/2}$. This is done to demonstrate that a large noise suppression is possible, since the free running relative power noise of the laser is at roughly 4×10^{-6}Hz$^{-1/2}$ without the noise injection.
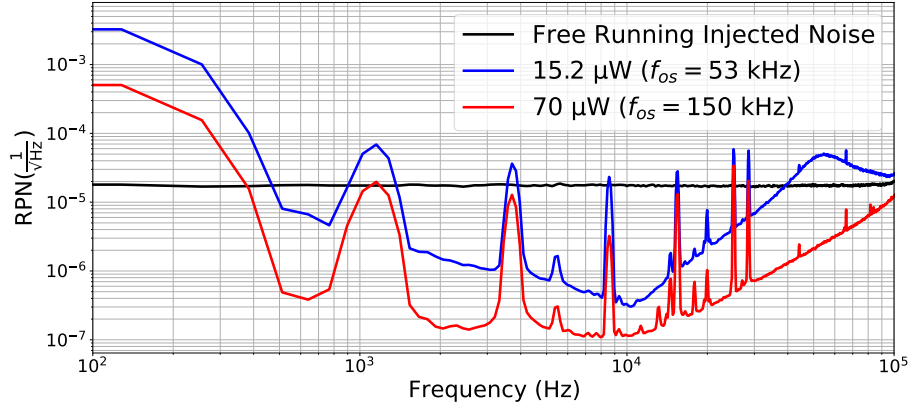
Fig. 2. Amplitude spectral density measurements showing reduction of classic power noise. White noise is added to the free running laser noise to bring the overall noise level up to approximately 2×10^{-5} Hz$^{-1/2}$. The blue curve represents a cavity detuning of 3.6 linewidths and the red curve a detuning 1.5 linewidths. The large features at 3.7 kHz, 15.5 kHz and 28.5 kHz are the coupling from the Yaw, Pitch, and Side-to-Side modes respectively of the cantilever mirror.

2. RESULTS

Figure 2 shows the results of the laser power stabilization for different optical spring strengths. The blue and red curves represent the cavity locked at a detuning corresponding to an optical spring frequency of 53 ± 2 kHz and 150 ± 3 kHz respectively. These optical spring frequencies correspond to a power transmitted by the cavity and incident on PD$_M$ of 15.2 and 70 µW respectively. Additionally, the input power for both these measurements is 6.5mW. The black curve represents the sum of the free running laser noise and the intentionally imprinted white noise, which is measured at a photodetector just after AM$_1$, not pictured in Fig 1. As seen from Fig 2, the injected noise is suppressed by a greater amount with a stronger optical spring, as expected from Eq 10. The maximum suppression of the stronger optical spring measurement occurs at 7900 Hz with a stabilized noise level of 1.6×10^{-7} Hz$^{-1/2}$. This corresponds to the optical spring suppressing the injected noise by a factor of 125. The blue curve measured for a cavity detuning of 3.6 linewidths has a steeper feature below 10 kHz due to being thermal noise limited, whereas the red curve (1.5 linewidths detuned) is mainly shot noise limited. Above 10kHz, both curves are limited by the noise suppression provided by the optical spring.

Figure 3 compares the highest power noise suppression measurement (red curve) with an uncorrelated sum of fundamental limits of this experiment (blue curve) and the power noise suppression by the optical spring. These fundamental limits are comprised of relative shot noise and thermal noise of the cantilever mirror added in quadrature. The total limit for the relative power noise detected by PD$_M$ can be obtained by dividing Eq. 10 by the mean circulating power P$_{circ}$, since, below the cavity pole, the classical RPN transmitted by the cavity (RPN$_M$) should be the same as the classical RPN inside the cavity. Hence, the amplitude spectral density (ASD) of the RPN at PD$_M$ is:

$$
\begin{aligned}
\text{RPN}_M{}^2 = \text{RPN}_{\text{in}}{}^2 & \left(\frac{\Omega^2 - \Omega_{\text{fund}}^2}{\Omega^2 - \Omega_{os}^2 - \Omega_{\text{fund}}^2} \right)^2 \\
& - \left(\frac{c F_{\text{ext,ASD}}}{2 P_{circ}} \right)^2 \left(\frac{\Omega_{os}^2}{\Omega^2 - \Omega_{os}^2 - \Omega_{\text{fund}}^2} \right)^2 + \frac{2hc}{\lambda P_M},
\end{aligned}
$$
(12)

where RPN$_{\text{in}}$ and F$_{\text{ext,ASD}}$ are the ASD of the RPN at the input of the cavity, and of the external force. The first term in Eq 12

represents the power noise suppression by the optical spring. The third term in Eq 12 is the relative shot noise of the measured power on PD$_M$, where h is Planck's constant, and P_M is the mean power transmitted by the cavity and detected on PD$_M$. This relative shot noise level is additionally corroborated by measuring the shot noise of the photodetector experimentally. Both by experimental methods and the relative shot noise term in Eq 12 the average shot noise level was determined to be 7.3×10^{-8} Hz$^{-1/2}$ and is depicted by the brown curve in Fig 3. The next part in the fundamental limits curve is the thermal noise term contribution:

$$
\text{RPN}_{\text{tn}} = \left(\frac{\Omega^2 - \Omega_{\text{fund}}^2}{\Omega^2 - \Omega_{os}^2 - \Omega_{\text{fund}}^2} \right) \frac{c \Omega_{\text{fund}}}{P_{circ}} \sqrt{\frac{k_b T m}{Q \Omega}},
$$
(13)

where Ω_{fund} is the fundamental resonance frequency of the cantilever mirror, Q is the structural quality factor of the movable mirror, T is the temperature, and k_b is the Boltzmann constant. This equation was calculated by substituting F_{ext} in Eq 12 by the thermal noise force considering structural damping [15] and is the minimum relative power noise in transmission of the cavity, limited by thermal noise of the movable mirror. This quantity is a similar result as obtained in [5, 16], but here with a dependence on the intracavity power. This is an advantage for this scheme since a trade off of using a mirror with high spring constant can be made by increasing the intracavity power.

Because the laser beam is not perfectly centered on the movable mirror, we see a coupling of the cantilever modes, pitch (3.7kHz), yaw (15.5kHz), and side-to-side (28.5 kHz), in our measurement. If the beam was perfectly aligned these features would not be observed. In order to account for this in our fundamental limits we use a modeling code that uses the two photon formalism [17] that accounts for the centering of the beam when calculating thermal noise. The result for the thermal noise model in this experiment is shown by the lilac curve in Figure 3. The temperature recorded for these measurements refers to the upper limit of the temperature of the cavity. This is because the cryostat introduces mechanical vibrations strong enough to interfere with the locking capabilities of the cavity. For this reason, the cryostat compressor is turned off and the cavity slowly warms as the measurement is performed. Generally, by the time a measurement is finished the cavity is at 30K.

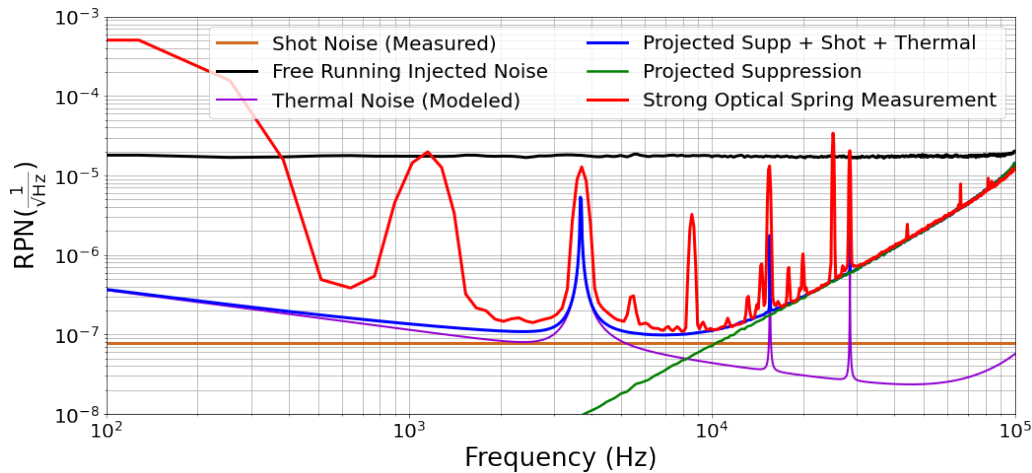The final contribution to the total limit is the residual input

Fig. 3. $70\,\mu$W power noise spectrum post cavity measurement with its associated fundamental noise limitations, which includes relative shot noise, the calculated (modeled) RPN, and projected suppression of the cavity. The circulating power corresponding to $70\,\mu$W of transmitted power is 250 mW.

noise limited by the suppression by the optical spring. This is calculated by taking the injected noise and multiplying by the suppression term (first term) in Eq 12. For the parameters of the stronger optical spring measurement, the optical spring has the potential to provide a power noise suppression of approximately 3×10^4 at 10 Hz. This factor is quite large and thermal or quantum noise typically limit the performance at low frequency.

Given these parameters, we find the total noise budget agrees with the measured spectrum for most frequencies. At low frequencies, the experiment is limited by seismic noise, hence the additional noise in the measurement with respect to the blue curve in Figure 3. In theory, it is possible to lock a Fabry-Perot cavity like this one without the use of feedback, instead using a large, positively detuned carrier beam and a small, negatively detuned sub-carrier beam [12, 13]. This has been tested to show the stability of the double optical spring effect, but not yet on its ability to stabilize the power of the laser. In this regime however, it would be possible to lock the cavity and have the power stabilized without the use of feedback anywhere.

In order to achieve a lower RPN and a larger power in transmission of the cavity, the input power, and therefore the circulating power, needs to be increased. This reduces the contribution from thermal noise and shot noise, as shown in Eq. 12. In this experiment, the circulating power of 250 mW was limited by the damage threshold of the micro-oscillators. However, the suspended mirrors described in [18], as well as [19], utilize parameters that could provide a total RPN close to the needs of current gravitational wave detectors. With a 5-mg suspended mirror having a resonance on the order of 10 Hz, Q of 1×10^5, an intracavity power of 200 W, and a temperature of 20K, we find that the RPN for such a system would be $1.4 \times 10^{-9}\mathrm{Hz}^{-1/2}$ at 10 Hz and $4.4 \times 10^{-10}\mathrm{Hz}^{-1/2}$ at 100 Hz. These parameters are realizable and would give the RPN levels necessary for aLIGO.

Disclosures

The authors declare no conflicts of interest.

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