

Variational measures in the theory of integration

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We will present here some results concerning the variational measures associated to a real valued function, or, in a more general setting, to a vector valued function. Roughly speaking, given a function Φ defined on an interval $[a, b]$ of the real line it is possible to construct, using suitable families of intervals, a measure μ_Φ which carries information about Φ . If Φ is a real valued function, then the σ -finiteness of the measure μ_Φ implies the a.e. differentiability of Φ , while the absolute continuity of the measure μ_Φ characterizes the functions Φ which are Henstock-Kurzweil primitives. The situation becomes more complicated if we consider functions taking values in an infinite dimensional Banach space. If the Banach space has the Radon-Nikodým property, then it is possible to obtain properties similar to those of the real case. But it is surprising that by means of the variational measures it is possible to characterize the Banach space having the Radon-Nikodým property.

References

- [1] B. Bongiorno, L. Di Piazza and V. Skvortsov, *A new full descriptive characterization of Denjoy-Perron integral*, Real Analysis Exchange, **21** (1995/96), 256–263.
- [2] B. Bongiorno, L. Di Piazza and K. Musiał, *A characterization of the Radon-Nikodým property by finitely additive interval functions*, Illinois Journal of Mathematics. Volume 53, Number 1 (2009), 87-99.
- [3] D. Bongiorno, *Stepanoff's theorem in separable Banach spaces*, Comment. Math. Univ. Carolinae, **39** (1998), 323–335.
- [4] L. Di Piazza, *Variational measures in the theory of the integration in R^m* , Czechos. Math. Jour. 51(126) (2001), no. 1, 95–110.
- [5] V. Marraffa, *A descriptive characterization of the variational Henstock integral*, Proceedings of the International Mathematics Conference (Manila, 1998), Matimýás Mat. **22** (1999), no. 2, 73–84.