A new fitness-based selection operator for genetic algorithms to maintain the equilibrium of selection pressure and population diversity

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Abstract. A genetic algorithm is one of the best optimization techniques for solving complex nature optimization problems. Different selection schemes have been proposed in the literature to address the major weaknesses of GA i.e., premature convergence and low computational efficiency. This article proposed a new selection operator that provides a better trade-off between selection pressure and population diversity while considering the relative importance of each individual. The average accuracy of the proposed operator has been measured by χ^2 goodness of fit test. It has been performed on two different populations to show its consistency. Also, its performance has been evaluated on fourteen benchmark problems while comparing it with competing selection operators. Results show the effective performance in terms of two statistics i.e., less average and standard deviation values. Further, the performance indexes and the GA convergence show that the proposed operator takes better care of selection pressure and population diversity.

Keywords: genetic algorithm; optimization; premature convergence; population diversity; selection operators.

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1. Introduction

During the last five decades, many meta-heuristics approaches have been developed to solve complex nature optimization problems. These include deterministic, stochastic, iterative, population-based approaches, etc. depending upon their division criteria. A stochastic algorithm provides the solution to a problem following the probabilistic rules while a population-based algorithm uses the set of solutions to improve the results. Similarly, a better solution obtained by multiple iterations uses an iterative approach. The evolutionary and the swarm intelligence approaches are classified based on simulation theory with natural phenomena.

Genetic algorithm (GA) is the commonly used method of evolutionary intelligence. Its development stems from Holland's work [8] in 1975 which utilizes the basic principles of Darwin's evolution process. It is a universally used optimization technique that generates a population of possible solutions and returns the better solution based on Darwin's "survival of the fittest" principle. After generating a random population of individuals, it is encoded into a suitable scheme. Then GA iteratively generates a new population of individuals unless it satisfies some predefined criteria like the maximum number of iterations, convergence, etc. The important

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feature of GA is that it searches for the global optima using the population of individuals. GA has vast applications in different fields as highlighted by Goldberg [4].

GA works with the help of three basic operators i.e., selection, crossover, and mutation. Each operator plays a key role in determining the optimal solution. The major issue with GA is that it gets stuck at the local optima i.e., premature convergence. This is due to the loss of population diversity. To tackle this, a lot of work has been done to find a trade-off between selection pressure and population diversity [10, 12]. Therefore, GA provides the best optimal results only if it maintains an equilibrium i.e., explores the new areas of search space (exploration) as well as uses the already detected points (exploitation). This balance is better maintained by using the suitable selection operator or by the adaptation of probabilities associated with recombination operators.

GA iteratively generates a new population from the current generation using a suitable selection scheme. The selection operator defines the procedure to generate a new population. If it maintains the equilibrium i.e., balances the exploration and exploitation, it will provide the best near-optimal results efficiently. Otherwise, it will be a simple random sampling with different results in each iteration. The selection operator is selected according to the problem complexity.

The selection process determines the individuals to be selected for crossover and the number of offspring each individual will participate. The objective of the selection process is to reduce the search space by discarding the worst individuals. Thus, the selection process highly affects the performance of GA. According to Shivaraj et al. [17] using the selection scheme that works well in that problem, the quality of results can be improved. A comparison of the performance of GA has been made using different selection strategies. A detailed analysis of some conventional selection schemes has been done by Goldberg and Deb [3].

This study focuses on the effectiveness of the selection operators on the performance of GA. A new selection operator is proposed to maintain a better trade-off between selection pressure and population diversity. The best individuals, as well as the worst, are selected with some probability to improve the quality of the population as a whole. In this way, it maintains the balance between exploration and exploitation.

The rest of this article is divided into five major sections. The Section 2 discusses the background of selection schemes. The proposed work is presented in the Section 3. In Section 4, criteria for performance evaluation and results are presented. Lastly, the findings of the current study are concluded in Section 5.

2. Traditional selection procedures: theory and methods

The fitness proportional selection (FPS) was the first selection mechanism proposed by Holland[8]. In this scheme, each individual has been assigned a proportion of roulette wheel according to its fitness value in the population. The wheel spun marks the pointer to an individual. A probability p_j is assigned to each individual using the following equation:

$$p_j = \frac{f_j}{\sum_{i=1}^{k} f_i}; \qquad j = 1, 2, 3, ..., k \tag{1}$$

Where f_j is the fitness value of the j^{th} individual in the population and k is the size of population. This scheme assigns more weight to the fittest individual i.e., the individuals with higher fitness value have more chance of being selected as a parent.

The FPS scheme has been widely studied and applied in various fields like menu planning [11], scheduling problems [14, 15], spanning tree [18] etc. The main strengths of FPS have been discussed in the literature such as, it gives a higher probability to the best individuals and the probability and area of selection remain the same throughout the selection process [1].

However, there are some drawbacks also, by replacing the whole population with outstanding individuals, the algorithm will not thoroughly search the possible solutions and will converge to a sub-optimal solution, this is commonly known as premature convergence [6]. Also, transposing the fitness function will change the selection probabilities.

Baker [2] proposed the linear rank selection operator (LRS) as an alternative to the FPS scheme. A probability is assigned to each individual according to their fitness ranking, proportionally to their rank. A sample of individuals is then selected using any sampling procedure such as roulette wheel sampling and stochastic universal sampling. Selection pressure remains constant throughout the process because of uniform scaling. In this way, selection probabilities are assigned regardless of fitness values:

$$p = \frac{1}{k}(\eta^- + (\eta^+ - \eta^-)\frac{i-1}{k-1}); \qquad i = 1, 2, 3, ..., k \tag{2}$$

where η^+ and η^- are the parametric values to control the selection pressure and must satisfy the condition $\eta^- + \eta^+ = 2$. Baker recommended to use $\eta^+ = 1.1$ to maintain the selection pressure. Here η^+/k and η^-/k are the probabilities of best and worst individuals to be sampled respectively. This scheme assigns different rank to all individuals even if they have the same fitness values. It reduces the risk of premature convergence by introducing the population diversity. This scheme has the drawback that due to high population diversity the process slowly converges to the optimal solution.

The tournament selection (TS) has been proposed as an alternative to FPS [3]. In TS, q individuals are randomly selected and compared on the basis of their fitness values. The individual with higher fitness is declared as a winner. In this way, all winners are selected for mating pool. Most commonly used tournament size is 2 i.e., binary tournament scheme (BTS). The tournament size maintains the selection pressure. The higher is the tournament size the more is the selection pressure [13]. The selection probability of the i^th individual in the q tournament size is given as follows:

$$p_i = \frac{1}{k^q}[(i)^q - (i-1)^q] \tag{3}$$

The FPS and the TS allocate probabilities based on fitness ranking. The performance of FPS and BTS is the same in terms of allocation of reproduction probabilities [3]. The allocated probabilities are not assigned by taking into account the magnitude of the difference of fitnesses. This is the reason that the reproduction probabilities differ by the same amount regardless of the difference between the fitness. Moreover, selection pressure is difficult to adjust because it remains constant throughout the selection process. To allow exploration in the process, the selection pressure should be low at the beginning of the search and it should rise towards the end for the convergence of the process [**?**].

Some linear transformations have also been proposed to improve the FPS scheme [6, 16]. The simple is the linear transformation procedure, where each fitness is transformed linearly as:

$$f^{'}(i) = af(i) + b \tag{4}$$

where $f^{'}(i)$ and $f(i)$ are the transformed and the raw fitness of the i^{th} individual respectively and 'a' and 'b' are the two coefficients. Although, this procedure deals with the scaling problem but it fails to adjust the selection pressure. A better selection scheme is always needed to balance the selection pressure and the population diversity. To trade off these two competing criteria is a difficult task. This article mainly contribute by reducing this weakness of the conventional selection schemes. The proposed operator introduces the population diversity while maintaining the selection pressure.

Another selection scheme Split Rank Selection (SRS)has also been proposed [10]. It splits the population into two equal parts after sorting them according to their fitness values. The probability has been assigned according to the following formula:

$$p_i = \begin{cases} \frac{12i}{5K(K+2)}, & i \leq K/2 \\ \\ \frac{28i}{5K(3K+2)}, & i \geq K/2 \end{cases} \tag{5}$$

Further stair-wise selection scheme (SWS) has been introduced by [7]. Its working phenomenon proceeds by dividing the population into five equal parts after sorting them in ascending order. The probabilities are assigned according to the following formula:

$$p_i = \begin{cases} \frac{i}{W(W+5)}, & 1 < i \leq W/5 \\ \\ \frac{4.5i}{W(3W+5)}, & W/5 < i \leq 2W/5 \\ \\ \frac{9i}{W(5W+5)}, & 2W/5 < i \leq 3W/5 \\ \\ \frac{15i}{W(5W+5)}, & 3W/5 < i \leq 4W/5 \\ \\ \frac{20.5i}{W(9W+5)}, & 4W/5 < i \leq W \end{cases} \tag{6}$$

Using the same idea, Split Based Selection (SBS) was proposed by [9]. It splits the population into three categories best fit, average fit, and lower fit. The assigned probabilities are:

$$p_i = \begin{cases} \frac{5i}{k(2k+5)}, & i \leq 2k/5 \\ \\ \frac{1}{k}, & 2k/5 < i \leq 3k/5 \\ \\ \frac{15i}{k(8k+5)}, & i > 3k/5 \end{cases} \tag{7}$$

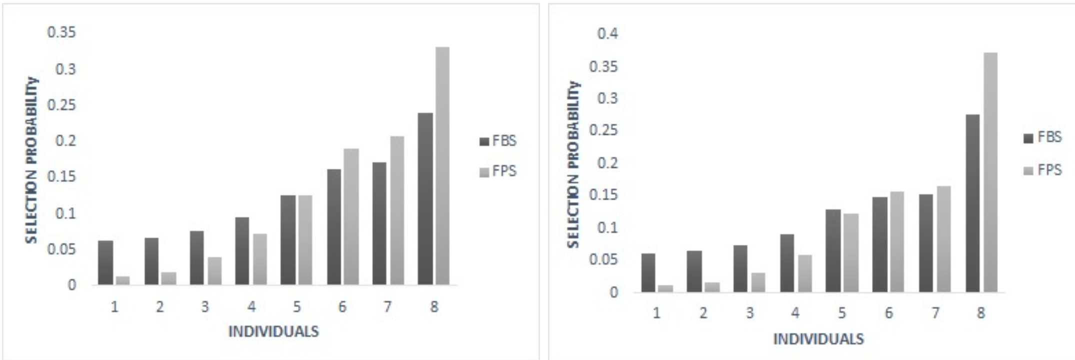
3. Proposed selection operator

3.1. Motivation

The above discussion concludes that some of the conventional selection schemes either overcome the problem of selection pressure or high population diversity. As a result either the fittest parents are selected only for the matting pool or the individuals are selected in such a way that there is no significant difference in the selection probabilities of best and worst individuals. The FPS has a major drawback that when the fitness function is transposed it changes the selection probabilities. As a result, there is not much difference in the selection probabilities of worst and best individuals. None of the techniques discussed above consider the relative importance of each individual.

3.2. Proposed scheme: fitness-based selection (FBS) procedure

The proposed scheme is a better alternative to overcome the drawbacks of the above schemes. Fitness determines the importance of each individual in the population. The FPS, LRS, and TS do not consider the relative importance of each individual. Because of that, these schemes do



(a) *Selection probability of FBS and FPS for pop1*

(b) *Selection probability of FBS and FPS for pop2*

Figure 1: Selection probability of FBS and FPS

not balance the selection pressure and the population diversity. The proposed scheme gives a better trade-off between selection pressure and population diversity because it uses magnitudes of the fitness of each individual.

Let we have 'k' individuals and m_i be the size of i^{th} individual, where $i = 1, 2, 3, ...k$. For i^{th} individual, f_i be the corresponding fitness value. Then

$$M_{f,t} = Median(f_{(1)}, f_{(2)}, f_{(3)}, ..., f_{(k)}),$$

where $f_{(1)}, f_{(2)}, f_{(3)}, ..., f_{(k)}$ be the fitness values arranged in ascending order of magnitude i.e., from worst to best individuals. The expected number of individuals at the next generation will be:

$$M_{i,t+1} = m_{i,t}.n.P_{i,t}$$

where the total number of individuals sum to n. The probability of selection for the i^{th} individual at t^{th} generation is:

$$P_{i,t} = \frac{(f_i + M_{f,t})}{\sum_{j=1}^{k}(m_{j,t}f_j + M_{f,t})}; \qquad i = 1, 2, 3, ..., k \tag{8}$$

where f_i is the fitness value of i^{th} individual and $M_{f,t}$ is the median of the current population at t^{th} generation.

To examine the performance of the proposed operator, a hypothetical population of eight individuals has been taken. The first population is without any outliers in the fitness values. By adding the median fitness values, the probabilities of least fit individuals increases at starting generations, and later on best fit individuals show a high selection probability. It is obvious from Figure 1 that at the starting generations FBS introduces the diversity i.e., it increases the selection probability of worst individuals as compared to FPS. After some generations, it maintains the selection pressure by assigning high probability to the fittest individuals. Following the same pattern, in the second population with outliers in fitness values, FBS excellently maintains the equilibrium of selection pressure and population diversity.

3.3. The Sampling procedure

The process of selecting the individuals for a mating pool is a two-step procedure. In the first step, probabilities are assigned to each individual in the population using any scheme, and individuals are sampled in the second step of selection. The sampled individuals mate to produce the offspring. The roulette wheel sampling scheme has been used. The observed and expected number of individuals for the proposed FBS operator are compared using χ^2 goodness of fit test.

3.4. The χ^2 goodness of fit

The χ^2 has been widely used as a measure of average accuracy [16]. It measures the average difference between the observed and expected number of outcomes. Here it is used to analyze the average difference between the observed and expected number of offspring produced from the proposed FBS operator. Let $C_1, C_2, ..., C_k$ be the k disjoint classes, $\varphi_i = \sum_{i \epsilon c_j} e_i$ be the expected number of outcomes and $O_i = \sum_{i \epsilon c_j} o_i$ be the observed number of offspring. The χ^2 as a measure of average accuracy is given as:

$$\chi^2 = \sum_{i \epsilon c_j} \frac{(O_i - \varphi_i)^2}{\varphi_i}$$

To obtain the results with the required accuracy, the parameters are fixed to be k= 150 as the population size with 10 classes. Tables 1 and 2 show the observed and expected number of offspring produced from FBS and FPS for two different populations: one without outliers and the other with outliers in the fitness values of the population. The purpose is to show the consistency in the average accuracy of FBS as demonstrated in Figs. 2(a) and 2(b). Firstly, a random population is generated and probabilities are assigned using the probability distribution given in Table 1, then the roulette wheel scheme is applied to get the values of O_i, φ_i and χ^2 respectively. The sample mean and variance is obtained by:

$$\hat{e}^{(F,S)} = \frac{1}{m} \sum_{k=1}^{m} \chi^2$$

$$\hat{\sigma^2}^{(F,S)} = \frac{1}{m-1} \sum_{k=1}^{m} (\chi^2 - \hat{e}^{(F,S)})^2$$

While comparing it to the theoretical χ^2 distribution, the mean and variance for 10 classes should be $k-1 = 9 and 2(k-1) = 18$ respectively. The estimated mean (variance) for population 1 and 2 are 9.0449 (8.966) and 19.0650 (18.626) respectively.The estimated values are close to the expected values which confirm the behavior of the sampling scheme corresponding to the probability distribution of FBS.

i	Classes	φ_i	O_i	χ^2	Classes	φ_i	O_i	χ^2
1	1-24	14.768	15.160	0.901	1-45	14.942	14.600	0.774
2	25-45	15.334	15.180	0.825	46-67	15.258	15.507	0.963
3	46-63	15.085	14.727	1.077	68-82	14.759	14.253	0.768
4	64-79	15.305	15.700	0.764	83-95	15.372	16.047	1.072
5	80-93	15.076	15.007	0.944	96-106	14.688	15.547	1.089
6	94-106	15.077	15.000	0.931	107-116	14.691	14.893	1.037
7	107-118	14.846	14.267	0.740	117-125	14.236	14.187	0.702
8	119-130	15.654	16.267	1.361	126-134	15.190	14.640	0.933
9	131-140	14.011	13.767	1.056	135-142	14.944	14.540	0.813
10	141-150	14.844	14.927	0.913	143-150	15.919	15.787	1.166

Table 1: *Classes C_i in Pop1 with expected (φ_i) and observed (O_i) number of individuals for FBS and FPS*

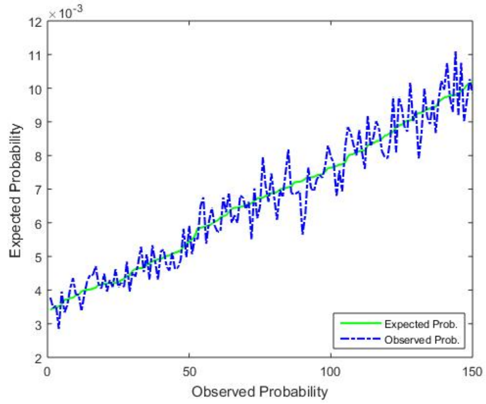


Figure 2:(a) Average accuracy of FBS for Pop1

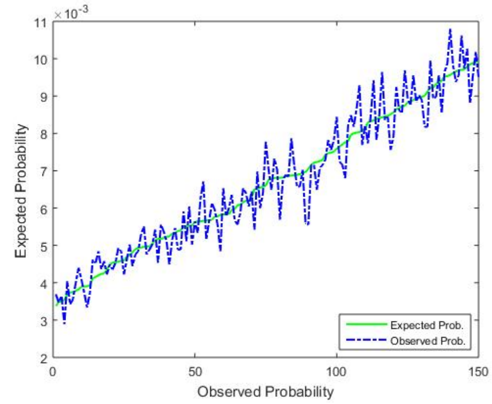


Figure 2:(b) Average accuracy of FBS for Pop2

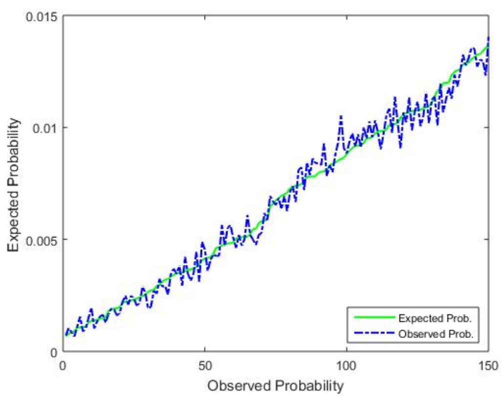


Figure 3:(a) Average accuracy of FPS for Pop1

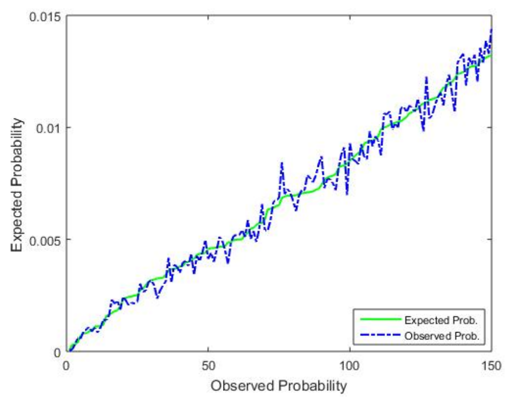


Figure 3:(b) Average accuracy of FPS for Pop2

i	Classes	φ_i	O_i	χ^2	Classes	φ_i	O_i	χ^2
1	1-25	15.145	15.380	0.864	1-44	15.089	14.706	0.782
2	26-45	15.238	15.006	0.902	45-65	14.969	14.773	0.917
3	46-63	15.373	15.600	0.842	66-81	15.335	15.300	0.941
4	64-79	15.431	15.446	0.758	82-95	15.513	16.020	1.053
5	80-93	14.618	14.000	0.803	96-106	14.313	14.146	1.207
6	94-106	14.834	14.886	0.887	107-116	14.749	14.813	0.909
7	107-118	14.908	15.340	0.993	117-125	14.295	14.526	0.790
8	119-129	14.450	14.500	0.868	126-134	15.370	14.966	0.958
9	130-140	15.337	15.226	1.071	135-142	14.808	14.826	0.889
10	141-150	14.661	14.613	0.974	143-150	15.553	15.920	1.236

Table 2: *Classes C_i in Pop2 with expected (φ_i) and observed (O_i) number of individuals for FBS and FPS*

4. Performance evaluation

In this section, the performance of the proposed FBS algorithm has been evaluated and compared with other selection operators. The simulation study for selection operators is performed on MATLAB software. Basic information about benchmark functions and the crossover operators has been provided in the next subsection.

4.1. Testing methodology

The experiment on fourteen different benchmark functions has been set up to compare the proposed algorithm with other competing selection schemes. These problems have different difficulty levels and multi-modality. The problems with their essential information are summarized in the appendix. In addition, results are compared using three different crossover operators, namely Logistic crossover (LogX), Simulated Binary crossover (SBX), and Laplace crossover (LX). The Makinen, Periaux, and Toivanen mutation (MPTM) is used as a mutation operator throughout the study. Experiments are conducted in three groups as four selection schemes are examined with three crossover operators using the same mutation operator. The parameter settings used in our simulation study are presented in Table 3

Parameter	Settings
Representation	Real
Population Size	300
Crossover Schemes	LogX, SBX and LX
Crossover Probability	0.75
Mutation Operator	MPTM
Mutation Rate	5%
Maximum generation	1000
Number of Dimensions	30

Table 3: *Parameter settings used for GA*

5. Results and discussion

The simulation results for competing selection schemes are given in Tables 4-6. Results for Logx-MPTM are displayed in Table 4. The Logx is replaced with the SBX crossover operator used in conjunction with the MPTM mutation operator in Table 5. Table 6 shows the simulation results for LX-MPTM.

5.1. Criterion(i)-Average values, S.D and successful runs

Results are compared based on average objective function value, S.D and the number of successful runs. The average objective function value shows the efficiency of an algorithm in providing nearby optimal results and S.D shows the stability of the proposed operator. The successful simulation is indicated through successful runs i.e., a run that provides the result within 5% of the optimal value. The performance of the proposed algorithm is evaluated on fourteen benchmark functions and results are compared with three other competing selection schemes (i.e., FPS, LRS, and BTS). The simulation results show less average value for the FBS operator on all benchmark functions except P3 which shows the less average value for the BTS operator but the results are more stable for the FBS operator. It solves all the problems with 100% successful rate than other selection operators.

In Table 6, the performance is evaluated with LX-MPTM on fourteen benchmark functions. On comparing the results with other selection schemes, FBS show less average objective function value and S.D. The successful rate of FBS remains 100% as with other crossover operators. Therefore, the proposed FBS outperforms other selection schemes based on the first criteria. Also, the results for LogX-MPTM manifest nearby optimal results (Table 4) than SBX-MPTM (Table 5) and LX-MPTM (Table 6) [12]. In Table 7, the proposed operator is further compared with the latest selection schemes [10, 7, 9]. The study findings manifest the better performance of FBS in producing near-optimal results. The less standard deviation value shows the stability of the proposed operator. Further, it successfully solved all the problems.

5.2. Criterion(ii)-Performance Index

The other criteria for performance evaluation is the performance index (PI). The PI is calculated in the following manner:

$$PI = \frac{1}{N_p}\sum_{i=1}^{N_P}(t_1\alpha_1^i + t_2\alpha_2^i + t_3\alpha_3^i) \tag{9}$$

where

$$\alpha_1^i = \frac{Sr^i}{Tr^i}$$

$$\alpha_2^i = \frac{Mf^i}{Lmf^i}$$

$$\alpha_3^i = \frac{Sf^i}{Lsf^i}$$

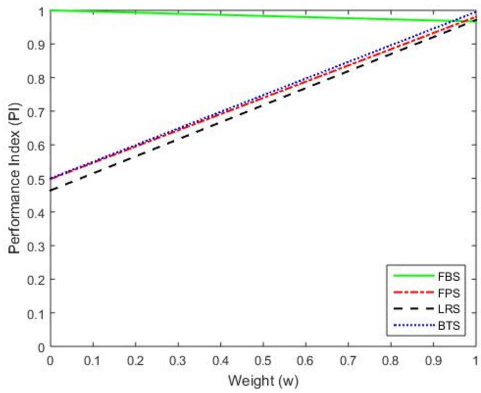
where $i = 1, 2, ..., N_p$; Sr^i is the number of successful runs of i^{th} problem; Tr^i is the total number of runs of i^{th} problem; Mf^i is the mean objective function value of i^{th} problem; Lmf^i is the least mean objective function value of i^{th} problem; Sf^i is the standard deviation of i^{th} optimization problem; Lsf^i is the least standard deviation value among all GAs of i^{th} optimization problem and N_p is the total number of problems analyzed.
t_1, t_2 and t_3 are the weights assigned to successful runs, mean objective function value and the standard deviation of the objective function value respectively. Same weights are assigned to two terms at a time so that the behavior of PI can be easily analyzed. The following three cases are considered here:

Case 1: $t_1 = w,\ t_2 = t_3 = \frac{1-w}{2}, \qquad 0 \le w \le 1$

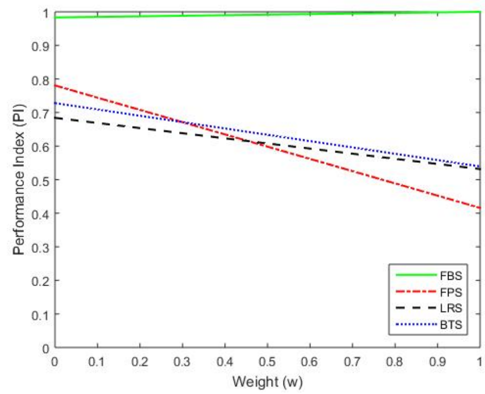
Case 2: $t_2 = w,\ t_1 = t_3 = \frac{1-w}{2}, \qquad 0 \le w \le 1$

Case 3: $t_3 = w,\ t_1 = t_2 = \frac{1-w}{2}, \qquad 0 \le w \le 1$

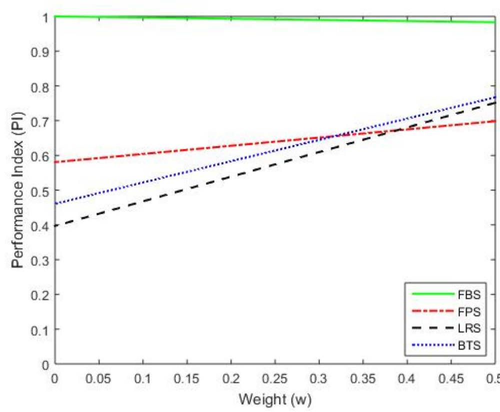
The PI measures the relative performance of FBS with FPS, LRS, and BTS simultaneously. Figure 4a shows the PI when S.D and successful runs are assigned equal weights. In Figure 4b, PI is computed for S.D by assigning equal weights to average values and successful runs. Lastly, Figure 4c displays the PI for successful runs with equal weight given to average values and S.D. All three cases are considered to show the superior performance of FBS. GA with FPS and BTS converges more rapidly due to high selection pressure. On the other hand, FBS converges to the optimal solution by taking care of the selection pressure and the population diversity.



(a) *PI when SD and successful runs are assigned equal weights*



(b) *PI when average and successful runs are assigned equal weights*



(c) *PI when average and SD are assigned equal weights*

Figure 4: PI measures

5.3. Criterion(iii)- convergence

The GA convergence performed on P9 (Axis parallel hyper ellipsoid) is shown in Figures 5-7. FPS and BTS converge too quickly at the starting generations due to high selection pressure. On the other hand, FBS produces lower average results and provides a better trade-off between selection pressure and population diversity. It can be analyzed on any test problem for all competing selection strategies.

6. Conclusion

The two problems which are mainly addressed by GA algorithms are exploration and exploitation. The FPS technique lacks population diversity (exploration) while LRS has a deficiency of selection pressure (exploitation). This paper proposed a new fitness-based selection operator which maintains a better trade-off between exploitation and exploration using two step procedure. Firstly, individuals are arranged in ascending order of their fitness values. This is mainly to overcome the fitness scaling issue. Secondly, probabilities are assigned to each individual by the proposed FBS operator. For performance evaluation, an extensive simulation

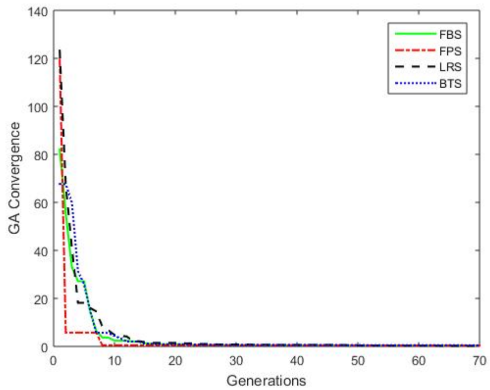


Figure 5: Convergence of GA with LogX-MPTM for P9

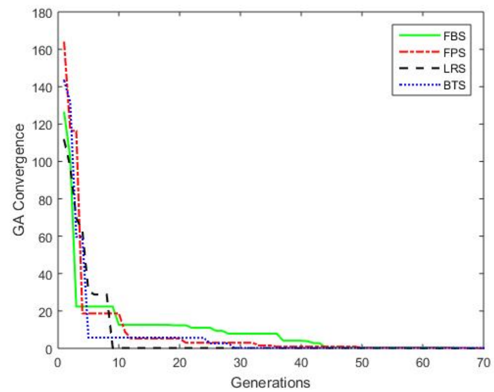


Figure 6: Convergence of GA with SBX-MPTM for P9

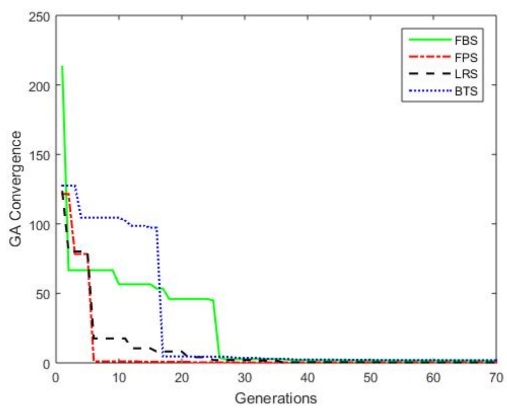


Figure 7: Convergence of GA with LX-MPTM for P9

study has been conducted using the conventional selection operators with different crossover schemes. Results demonstrate the superior performance of the proposed operator in comparison with the other competing selection schemes. There is also a negligible difference between the expected and observed number of individuals, as indicated by χ^2 goodness of fit test. This study proves that the proposed operator provides nearby optimal results and might be more effectively applied to other evolutionary problems.

7. Conflict of interest statement

There is no conflict of interest by authors.

Problem	Selection Scheme	Average	Standard deviation	Successful Runs
New 13	FBS	**2.70E-09**	**3.57E-09**	**30**
	FPS	6.06E-02	2.35E-01	28
	LRS	6.06E-02	2.35E-01	28
	BTS	3.96E-09	8.63E-09	30
Greiwank	FBS	**1.05E-02**	**1.18E-02**	**30**
	FPS	1.80E-02	1.34E-02	30
	LRS	1.46E-02	2.31E-02	26
	BTS	2.80E-02	2.53E-02	28
Cosine Mixture	FBS	-3.00E+00	1.64E-12	**30**
	FPS	-3.00E+00	4.21E-13	30
	LRS	-3.00E+00	4.74E-10	30
	BTS	-3.00E+00	2.15E-14	30
Brown	FBS	**7.87E-11**	**1.57E-10**	**30**
	FPS	4.31E-10	1.18E-09	30
	LRS	1.30E-10	1.38E-10	30
	BTS	1.50E-10	3.11E-10	30
Generalized1	FBS	**2.59E-11**	**2.90E-11**	**30**
	FPS	4.15E-11	5.11E-11	30
	LRS	1.41E-10	3.30E-10	30
	BTS	4.86E-11	7.86E-11	30
Generalized2	FBS	**2.84E-10**	6.56E-10	**30**
	FPS	4.00E-10	1.25E-09	30
	LRS	3.60E-10	4.72E-10	30
	BTS	5.65E-10	1.09E-09	30
Sphere	FBS	**1.22E-14**	**3.11E-14**	**30**
	FPS	1.12E-13	3.86E-13	30
	LRS	2.70E-14	5.85E-14	30
	BTS	4.32E-13	1.54E-12	30
New 25	FBS	**3.55E-10**	**3.94E-10**	**30**
	FPS	1.67E-09	3.68E-09	30
	LRS	5.31E-10	8.82E-10	30
	BTS	6.55E-10	1.23E-09	30
Hyper Ellipsoid	FBS	**1.10E-08**	**1.31E-08**	**30**
	FPS	2.62E-08	4.68E-08	30
	LRS	1.22E-08	3.05E-08	28
	BTS	1.24E-08	2.15E-08	30
LevyMount1	FBS	**5.61E-11**	**1.08E-10**	**30**
	FPS	8.43E-11	1.72E-10	30
	LRS	7.17E-11	1.41E-10	30
	BTS	5.95E-11	6.50E-11	30
LevyMount2	FBS	**2.91E-10**	**3.46E-10**	**30**
	FPS	6.69E-10	1.98E-09	30
	LRS	9.77E-10	3.18E-09	30
	BTS	1.83E-09	5.02E-09	30

Table 4: *Study findings of competing selection strategies for LogX-MPTM*

Problem	Selection Scheme	Average	Standard deviation	Successful Runs
Rosenbrock	FBS	**3.39E-04**	**4.48E-04**	26
	FPS	1.20E-03	2.20E-03	24
	LRS	1.60E-03	3.20E-03	26
	BTS	3.55E-04	5.78E-04	30
step	FBS	**4.46E-09**	**6.02E-09**	**30**
	FPS	7.44E-09	9.14E-09	30
	LRS	7.20E-09	1.14E-08	30
	BTS	1.06E-08	1.96E-08	30
Powersums	FBS	**1.44E-92**	**5.15E-92**	**30**
	FPS	9.98E-84	3.87E-83	30
	LRS	3.46E-86	1.34E-85	30
	BTS	5.46E-83	1.47E-82	30

Table 4(ii): *Table 4 continued*

Problem	Selection Scheme	Average	Standard deviation	Successful Runs
New 13	FBS	**1.85E-04**	**2.95E-04**	**30**
	FPS	2.52E-04	3.63E-04	30
	LRS	2.33E-04	3.94E-04	30
	BTS	2.86E-01	1.11E+00	28
Greiwank	FBS	**8.37E-02**	1.38E-01	**18**
	FPS	9.76E-02	1.25E-01	16
	LRS	1.47E-01	1.44E-01	8
	BTS	1.47E-01	1.99E-01	10
Cosine Mixture	FBS	-3.00E+00	**8.63E-06**	**30**
	FPS	-3.00E+00	6.71E-05	30
	LRS	-3.00E+00	2.03E-05	30
	BTS	-3.00E+00	8.34E-05	30
Brown	FBS	**5.15E-06**	**9.80E-06**	**30**
	FPS	9.72E-06	1.50E-05	30
	LRS	1.05E-05	3.20E-05	30
	BTS	6.29E-06	1.06E-05	30
Generalized1	FBS	**6.74E-07**	**9.12E-07**	**30**
	FPS	1.28E-06	2.57E-06	30
	LRS	1.68E-06	2.47E-06	30
	BTS	9.50E-07	2.24E-06	30
Generalized2	FBS	**1.60E-03**	**4.10E-03**	**30**
	FPS	3.40E-03	5.90E-03	30
	LRS	2.30E-03	4.80E-03	30
	BTS	2.80E-03	5.80E-03	30
Sphere	FBS	**1.73E-05**	**3.95E-05**	**30**
	FPS	6.26E-05	1.07E-04	30
	LRS	4.06E-05	9.63E-05	30
	BTS	8.03E-05	1.98E-04	30

Table 5: *Study findings of competing selection strategies for SBX-MPTM*

Problem	Selection Scheme	Average	Standard deviation	Successful Runs
New 25	FBS	**8.94E-06**	**1.81E-05**	**30**
	FPS	2.14E-05	3.33E-05	30
	LRS	8.68E-05	2.35E-04	30
	BTS	5.00E-05	1.17E-04	30
Hyper Ellipsoid	FBS	**4.02E-04**	**1.10E-03**	**30**
	FPS	4.51E-04	1.40E-03	30
	LRS	4.43E-04	1.30E-03	30
	BTS	1.30E-03	2.40E-03	30
LevyMount1	FBS	**1.40E-06**	**2.67E-06**	**30**
	FPS	2.10E-06	3.62E-06	30
	LRS	6.04E-06	9.52E-06	30
	BTS	8.93E-06	3.07E-05	30
LevyMount2	FBS	**1.00E-03**	**4.00E-03**	**30**
	FPS	4.00E-03	1.06E-02	30
	LRS	2.50E-03	6.60E-03	30
	BTS	7.00E-03	1.08E-02	30
Rosenbrock	FBS	**1.81E-01**	**4.07E-01**	22
	FPS	4.99E+00	1.84E+01	24
	LRS	1.92E+00	7.00E+00	20
	BTS	2.86E+00	7.71E+00	20
Step	FBS	**2.40E-05**	**3.84E-05**	**30**
	FPS	7.28E-05	1.84E-04	30
	LRS	4.99E-05	1.62E-04	30
	BTS	1.82E-04	3.46E-04	30
Powersums	FBS	**5.67E-83**	**2.20E-82**	**30**
	FPS	1.55E-74	6.01E-74	30
	LRS	1.08E-82	3.57E-82	30
	BTS	1.54E-82	5.96E-82	30

Table 5(ii): *Table 5 continued*

Problem	Selection Scheme	Average	Standard deviation	Successful Runs
New 13	FBS	**2.10E-03**	4.20E-03	**30**
	FPS	4.30E-03	6.10E-03	30
	LRS	2.80E-03	2.50E-03	30
	BTS	2.30E-03	5.00E-03	30
Greiwank	FBS	**2.84E-01**	**3.02E-01**	**6**
	FPS	3.42E-01	3.24E-01	6
	LRS	3.76E-01	3.60E-01	6
	BTS	3.95E-01	3.86E-01	2
Cosine Mixture	FBS	**-3.00E+00**	**1.19E-04**	**30**
	FPS	-3.00E+00	2.21E-04	30
	LRS	-3.00E+00	1.57E-04	30
	BTS	-3.00E+00	1.91E-04	30

Table 6: *Study findings of competing selection strategies for LX-MPTM*

Problem	Selection Scheme	Average	Standard deviation	Successful Runs
Brown	FBS	**4.26E-05**	**3.50E-05**	**30**
	FPS	5.22E-05	6.06E-05	30
	LRS	5.97E-05	9.43E-05	30
	BTS	8.36E-05	9.81E-05	30
Generalized1	FBS	**2.30E-05**	**2.61E-05**	**30**
	FPS	3.60E-05	6.29E-05	30
	LRS	4.42E-05	8.09E-05	30
	BTS	3.94E-05	7.95E-05	30
Generalized2	FBS	**1.77E-04**	**2.07E-04**	**30**
	FPS	3.45E-04	5.51E-04	30
	LRS	2.50E-03	9.30E-03	30
	BTS	2.40E-03	8.80E-03	30
Sphere	FBS	2.55E-04	4.38E-04	30
	FPS	4.80E-04	6.22E-04	30
	LRS	3.56E-04	6.30E-04	30
	BTS	4.30E-04	5.10E-04	30
New 25	FBS	**2.06E-04**	**3.20E-04**	**30**
	FPS	3.90E-04	8.53E-04	30
	LRS	2.32E-04	4.56E-04	30
	BTS	3.93E-04	5.82E-04	30
Hyper Ellipsoid	FBS	**2.90E-03**	**4.70E-03**	**30**
	FPS	6.20E-03	6.20E-03	30
	LRS	6.10E-03	5.90E-03	30
	BTS	4.70E-03	6.40E-03	30
LevyMount1	FBS	**3.24E-05**	4.07E-05	**30**
	FPS	3.52E-05	6.57E-05	30
	LRS	5.24E-05	5.81E-05	30
	BTS	7.43E-05	1.62E-04	30
LevyMount2	FBS	**1.87E-04**	**1.89E-04**	**30**
	FPS	3.30E-03	1.17E-02	30
	LRS	2.55E-01	9.89E-01	28
	BTS	2.00E-03	7.20E-03	30
Rosenbrock	FBS	**1.15E+00**	**1.60E+00**	1
	FPS	3.63E+00	8.46E+00	4
	LRS	4.72E+00	9.60E+00	0
	BTS	3.20E+00	6.66E+00	2
step	FBS	**2.69E-02**	**3.42E-02**	**20**
	FPS	1.02E-01	1.76E-01	16
	LRS	1.14E-01	2.50E-01	16
	BTS	7.25E-02	1.04E-01	20
Powersums	FBS	**1.69E-84**	**4.95E-84**	**30**
	FPS	1.80E-78	6.96E-78	30
	LRS	8.26E-81	3.19E-80	30
	BTS	3.84E-83	1.49E-82	30

Table 6(ii): *Table 6 continued*

Problem	Selection Scheme	Average	Standard deviation	Successful Runs
New 13	FBS	**2.70E-09**	**3.57E-09**	**30**
	SBS	1.40E-03	1.20E-03	30
	SWS	1.40E-03	1.70E-03	30
	SRS	1.40E-03	1.80E-03	30
Greiwank	FBS	**1.05E-02**	**1.18E-02**	**30**
	SBS	2.00E-01	1.38E-01	6
	SWS	1.99E-01	1.63E-01	6
	SRS	1.91E-01	1.85E-01	8
Cosine Mixture	FBS	-3.00E+00	**1.64E-12**	**30**
	SBS	-3.00E+00	3.29E-04	30
	SWS	-3.00E+00	3.19E-04	30
	SRS	-3.00E+00	5.73E-04	30
Brown	FBS	**7.87E-11**	**1.57E-10**	**30**
	SBS	3.25E-05	2.78E-05	30
	SWS	4.92E-05	8.18E-05	30
	SRS	5.79E-05	8.25E-05	30
Generalized1	FBS	**2.59E-11**	**2.90E-11**	**30**
	SBS	1.13E-05	9.92E-06	30
	SWS	1.78E-05	2.17E-05	30
	SRS	2.78E-05	3.79E-05	30
Generalized2	FBS	**2.84E-10**	6.56E-10	**30**
	SBS	8.64E-05	9.26E-05	30
	SWS	7.58E-05	7.84E-05	30
	SRS	6.84E-05	9.08E-05	30
Sphere	FBS	**1.22E-14**	**3.11E-14**	**30**
	SBS	3.49E-04	3.45E-04	30
	SWS	3.16E-04	3.74E-04	30
	SRS	2.69E-04	4.50E-04	30
New 25	FBS	**3.55E-10**	**3.94E-10**	**30**
	SBS	1.91E-04	1.68E-04	30
	SWS	2.41E-04	2.52E-04	30
	SRS	2.83E-04	2.27E-04	30
Hyper Ellipsoid	FBS	**1.10E-08**	**1.31E-08**	**30**
	SBS	8.40E-03	9.00E-03	30
	SWS	3.30E-03	2.90E-03	30
	SRS	7.30E-03	7.70E-03	30
LevyMount1	FBS	**5.61E-11**	**1.08E-10**	**30**
	SBS	6.23E-05	1.05E-04	30
	SWS	5.69E-05	7.28E-05	30
	SRS	6.23E-05	1.05E-04	30
LevyMount2	FBS	**2.91E-10**	**3.46E-10**	**30**
	SBS	1.39E-04	2.08E-04	30
	SWS	2.66E-05	3.74E-05	30
	SRS	1.39E-04	2.08E-04	30

Table 7: *Study findings of SRS, SWS and SBS for LogX-MPTM*

Problem	Selection Scheme	Average	Standard deviation	Successful Runs
Rosenbrock	FBS	**3.39E-04**	**4.48E-04**	26
	SBS	1.80E+00	3.45E+00	10
	SWS	2.44E+00	7.61E+00	4
	SRS	2.58E+00	7.87E+00	2
step	FBS	**4.46E-09**	**6.02E-09**	**30**
	SBS	3.88E-02	5.05E-02	24
	SWS	3.05E-02	4.58E-02	24
	SRS	5.54E-02	5.59E-02	16
Powersums	FBS	**1.44E-92**	**5.15E-92**	**30**
	SBS	9.29E-75	3.60E-74	30
	SWS	1.56E-66	6.06E-66	30
	SRS	7.18E-72	2.69E-71	30

Table 7(ii): *Table 7 continued*

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