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Stochastic Strength Analyses of Screws for Femoral Neck Fractures

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Abstract: This paper represents a multidisciplinary approach to biomechanics (medicine engineering and mathematics) in the field of *collum femoris* fractures, i.e., of osteosyntheses with femoral/cancellous screws with full or cannulated cross-sections. It presents our new numerical model of femoral screws together with their stochastic (probabilistic, statistical) assessment. In the first part of this article, the new simple numerical model is presented. The model, based on the theory of planar (2D) beams on an elastic foundation and on 2nd-order theory, is characterized by rapid solution. Bending and compression loadings were used for derivation of a set of three 4th-order differential equations. Two examples (i.e., a stainless-steel cannulated femoral screw and full cross-section made of Ti6Al4V material) are presented, explained, and evaluated. In the screws, the internal shearing forces, internal normal forces, internal bending moments, displacement (deflections), slopes, and mechanical stresses are calculated using deterministic and stochastic approaches. For the stochastic approach and a "fully" probabilistic reliability assessment (which is a current trend in science), the simulation-based reliability assessment method, namely, the application of the direct Monte Carlo Method, using Anthill software, is applied. The probabilities of plastic deformations in femoral screws are calculated. Future developments, which could be associated with different configurations of cancellous screws, nonlinearities, experiments, and applications, are also proposed.

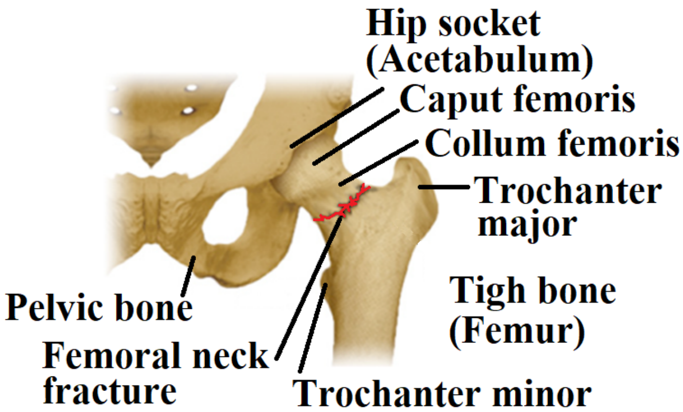
Keywords: biomechanics; *collum femoris*; cancellous screws; femoral neck fracture; strength analyses; deflections; beams on elastic foundation; stochastic approach; probability; reliability; mechanical stress assessment

1. Introduction

Femoral fractures rank among the most commonly observed fractures in traumatology and orthopedics; see [1].

Proximal femoral neck fractures (PFN fractures, *collum femoris* fractures) are typical intracapsular fractures representing a significant clinical problem; see Figure 1.

While the femur, i.e., *os femoris*, is the strongest bone in the human body, the *collum femoris* is the weakest part of the femoral bone. PFN fractures disrupt the integrity of the hip joint, i.e., *articulatio coxae*, and are associated with increased risk of avascular necrosis and other problems, with possibly relatively high patient morbidity and mortality. Elderly people and, in particular, osteoporotic women are at the greatest risk of PFN fractures caused by sudden falls, i.e., small-force injuries and low-energy trauma. Another significant group of patients is represented by younger people injured as a result of motor vehicle accidents, i.e., high-force injuries and high-energy trauma.



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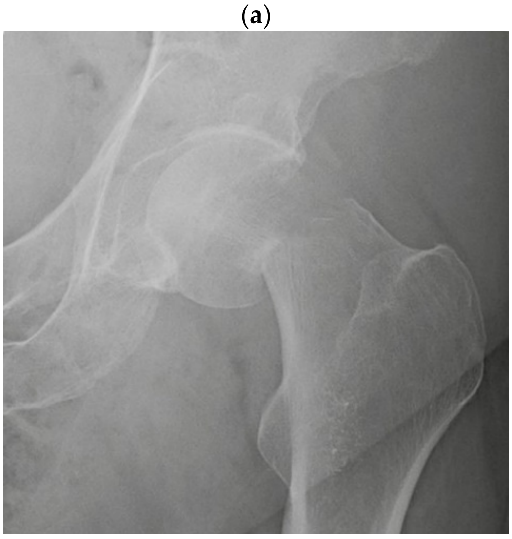

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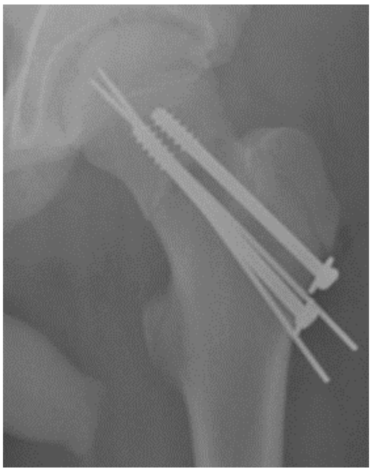
Figure 1. (**a**) *Articulatio coxae* and its parts with *collum femoris* and *collum femoris* fracture. (**b**) Rtg. snapshot of the displaced subcapital femoral neck fracture.

According to the international standard Arbeitsgemeinschaft für Osteosynthesefragen (AO) classification, the PFN fractures belong to Class 31. From an anatomical point of view, there are three basic types of these fractures, i.e., subcapital, mediocervical (i.e., transcervical), and basicervical; see [2].

However, for descriptions of PFN fractures, Pauwel's classification and Garden classification are also applied. Pauwel's classification classifies PFN according to the orientation and direction (i.e., type I, II, and III). Garden classification classifies PFN according to the amount or degree of displacement (i.e., type I, II, III, and IV). For more information see [3,4].

Usually, a plain radiograph is a first-line examination in patients with suspected PFN fractures. Hence, the diagnosis of PFN fractures is generally made radiographically with orthogonal radiographs of the hip. Surgical treatment predominates, mostly with open reduction and internal fixation (ORIF) or arthroplasty, usually depending on the age of the patient. However, the medical point of view is not the main subject of this article, and for more information on this perspective, please refer to medical literature [1–12].

The application of femoral screws, i.e., of lag spongious/cancellous screws, is a possible minimally invasive treatment method for the treatment of PFN fractures. In our case, cancellous screws produced by MEDIN a.s. were used for this work; see [13]. These screws are made up of either stainless steel or Ti6Al4V materials; see [1] and Figure 2.



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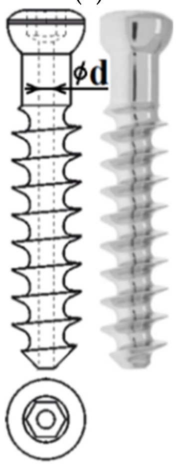

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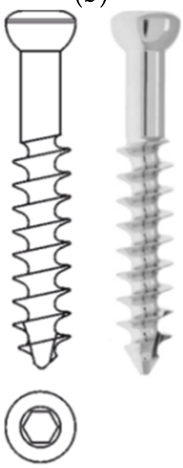


(**c**)

Figure 2. (**a**) Osteosynthesis with two cannulated femoral (cancellous) screws; (**b**) femoral screws with a cannulated cross-section (not-to-scale); (**c**) femoral screws with a full cross-section (not-to-scale).

This paper aims to perform deformation and strength analyses and assessments of various femoral screws and, subsequently, to evaluate the results by (i) a deterministic approach, see [1], and (ii) a probabilistic, i.e., stochastic, approach based on the simulation-based reliability assessment (SBRA) method presented in this article. The SBRA method, a typical application of a direct Monte Carlo method and probabilistic reliability assessment, is a modern and innovative approach applied to mechanical structures in engineering and physics. Inputs and outputs are of stochastic quantities.

Other possible approaches, based on the finite element method etc., are reviewed in our previous papers; see [1,14] and other papers [15–18]. However, those approaches are not based on stochastic quantities.

The development of some relatively easily applicable biomechanical numerical solutions presented in this article leads to possible improvements in the quality and safety of PFN fracture treatment. For example, the results can lead to alterations of dimensions, methods of insertion, or the number of cancellous screws used during the surgery. Application of the stochastic solution and stochastic evaluation is a new trend in this field of multidisciplinary science.

2. Limitations

Although this problem was previously tackled using the finite element method, i.e., 3D model with deformation and stress analyses, see [1,10], the presented study focuses on a planar (2D) model based on the screw, i.e., beam, resting on an elastic foundation, i.e., on the *collum femoris* bone. This novel approach is simpler, and its solution is quicker, enabling a better generation of random real inputs, such as the loading forces F, F_m, F_1, F_2, material properties of screws E, length of screws L, L_1, L_2, cross-section A, principal quadratic moment of cross-sectional area J_{ZT}, insertion angle of screws $\propto$, and the stiffness characteristics k of the collum femoris–screw interaction.

The beam is a typical, and relatively easy to understand, engineering simplification for long and narrow structures such as the cancellous screw in question.

Stiffness characteristics of the femur are substituted by the most popular Winkler's (bilateral elastic) foundation; see [19–22].

Hence, it is not a problem to conduct millions of random simulations (calculations) in real time using the direct Monte Carlo method, i.e., stochastic/probabilistic simulation and nature of reality. For more information, see [1,21,23–27].

In this article, the solved model presents the results for full or cannulated screws inserted in parallel positions, i.e., the easiest mathematical/mechanical case. However, this method is applicable for any position of the cancellous screw. Changes of angles $\propto$ and length L, see Figure 3, enable us to simply change the screw positions in the model due to patient anatomy, thus evaluating appropriate, less appropriate or inappropriate cancellous screw positions for the surgery. For more information, see [1].

The influence of possible dynamic effects is reflected in the dynamic coefficient k_{dyn}, which is a typical engineering approach and easy application of the dynamics. In our case, $k_{dyn} \in \langle 1; 4 \rangle$, which means that the static force can be increased up to four times.

The materials of the cancellous screws are isotropic, homogeneous, and linear, representing another typical and widely accepted engineering approach in mechanics/biomechanics.

From the traumatology/orthopedics perspective, a relatively large amount of information and statistical evaluations of treatment methods is available. From an engineering/biomechanical perspective, however, there is a relative absence of numerical models which would enable us to evaluate the appropriateness of screw positions or the selection of operating techniques from a biomechanical point of view. Hence, there is not enough information about mechanical stresses, deformations, or reliability assessment of osteosyntheses in PFN fractures; see [1].

The use of cancellous screws for PFN fracture treatment is limited by the quality of bone and type of fracture. Cancellous screws are usually applied in the treatment of subcapital and mediocervical fractures. In our case, three cancellous screws were applied. In some patients, two screws can be also applied (usually with different dimensions and "small" changes of elastic characteristics k).

Still, the approaches and results presented in this paper can also be applied to other types of screws, bones, and fractures, and even for other types of screw joints in engineering.

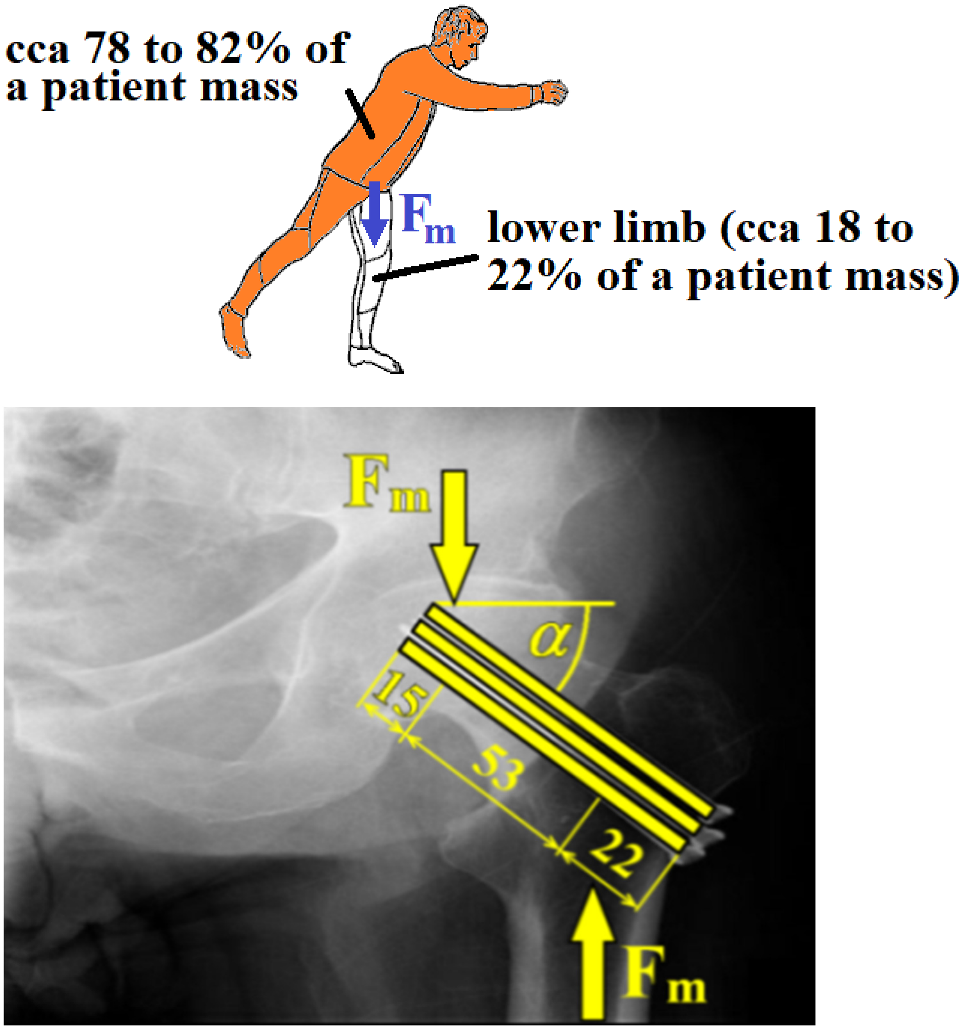


Figure 3. Loading of femoral (cancellous) screws and an example of three femoral screws and their approximation via parallel beams on an elastic foundation (dimensions are in millimeters).

3. Materials and Methods

As mentioned above, the beams on elastic foundations are frequently used in many types of engineering applications. The linear or nonlinear elastic foundation can also be applied if a physical object, such as an implant or bone, is supported or embedded in a continuum. This leads to a suitable approximation/simplification of mechanical contacts; see [1,19–22,25,28,29]. Therefore, from the biomechanical perspective, the cancellous screws are, in this paper, described and solved as beams on an elastic foundation.

From the engineering/mechanical/biomechanical point of view, the new numerical model is derived from and based on 2nd-order theory and on the theory of 2D beams on an elastic bilateral (Winkler's) foundation. This leads to a set of three 4th-order linear differential equations:

$$EJ_{ZT}\frac{d^4v_{\mathrm{i}}}{dx_{\mathrm{i}}^4} - N\frac{d^2v_{\mathrm{i}}}{dx_{\mathrm{i}}^2} + kv_{\mathrm{i}} = 0 \tag{1}$$

for calculated deflections (displacements) v_{i}, $\mathrm{i} = 1,\ 2,$ and 3 in global coordinate systems $x_1 \in (0; \mathrm{L}_1)$, $x_2 \in (\mathrm{L}_1; \mathrm{L}_2)$; $x_3 \in (\mathrm{L}_2; \mathrm{L}_3)$, see Figure 4, where the bone–screw interaction is approximated by the elastic foundation via parameter k.

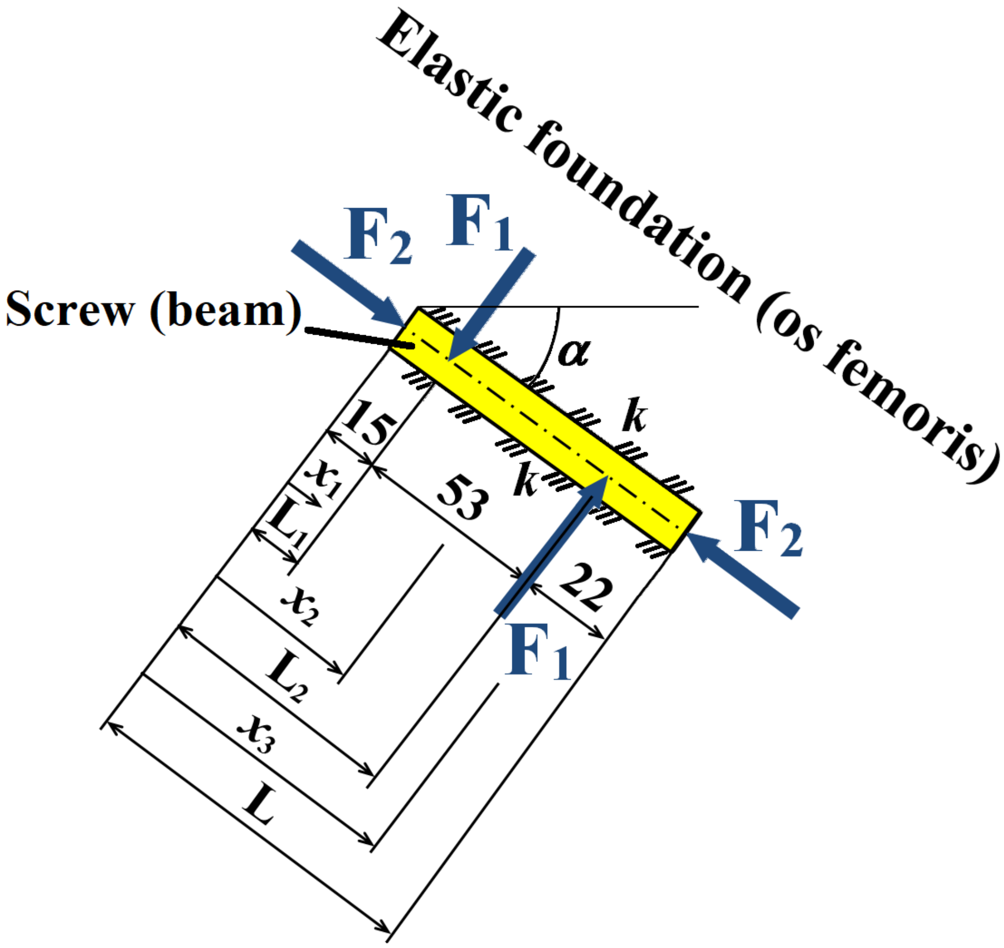


Figure 4. Loading of a single femoral screw and defined global coordinate systems x_i and dimensions L_i.

According to [21] and general mathematical solution of linear equations, the general solutions of differential Equation (1) are

$$v_i = e^{\omega_I x_i}[A_{1i}\cos(\omega_R x_i) + A_{2i}\sin(\omega_R x_i)] + e^{-\omega_I x_i}[A_{3i}\cos(\omega_R x_i) + A_{4i}\sin(\omega_R x_i)] \quad (2)$$

where parameters

$$\omega_R = \sqrt{\omega^2 + \frac{|N|}{4EJ_{ZT}}},\ \omega_I = \sqrt{\omega^2 - \frac{|N|}{4EJ_{ZT}}} \text{ and } \omega = \sqrt[4]{\frac{k}{4EJ_{ZT}}} \quad (3)$$

Equation (2) contain twelve integral constants $A_{1i}, \ldots, A_{4i}$.

Hence, the femoral screw is resting on an elastic foundation prescribed by elastic stiffness k; see [1,21]. By changing the stiffness k, it is simply possible to consider a fracture or a healthy bone or even to fit it to specific bones, such as healthy or osteoporotic bone.

Three screws of the length L were applied in parallel positions on the elastic foundation, i.e., in the *os femoris*, and were loaded by the total quasi-dynamical force F_m acting on the direction of the femoral screw angle ∝; see Figure 3.

However, the real variability of all inputs/outputs is taken into account by the probabilistic/stochastic/statistical approach, i.e., the SBRA method.

The SBRA method (in our case, the application of Anthill sw), was developed to use available personal computers for a qualitative new improvement of the reliability assessment of structures. Its application allows desired transitioning from the deterministic to the probabilistic concept of reliability assessment of the natural phenomena, i.e., transitioning from deterministic reliability assessment to probabilistic assessment of structures or systems. With the SBRA method, the calculated probability of failure is estimated according to the theory of limit states [21,23–27,30,31].

Reference [32] also investigated femoral screws as beams without elastic foundations. However, the solution in that paper is different, being performed for only one loading force and neglecting the influence of axial forces; besides, the elastic foundation is only mentioned but not used. The accuracy of this solution is, in our opinion, not sufficient.

Typical shapes and dimensions of the cancellous screws are presented in [13].

The corrosion-resistant, i.e., stainless, steels (for example DIN 1.4441–316 L medical, AISI 316 L, ISO 5832-1, formerly standard ČSN 17 350 in the Czech Republic) used nowadays to produce implants are primarily high-alloy austenitic steels with high Cr, Ni, and Mo content and low carbon content; see [33].

Pure titanium and its alloys (for example Ti6Al4V, ISO 5832-3, see [33,34]) usually have good mechanical properties and inertness, with a high degree of corrosion resistance—both when exposed to air and in the chemically aggressive environment of the human body.

Titanium alloys are more suitable for traumatology/orthopedics implants than stainless steels because in terms of biocompatibility, they are more inert. Titanium alloys can be surface-anodized (i.e., the titanium oxide can be electrolytically produced on the surface of the implant), see [33].

The mass of the human body resting on the hip joint m is the full patient mass without one lower limb (i.e., 78–82% of the full patient mass; see Figure 3). This fact (probabilistic value) is taken into consideration by the coefficient k_m.

The formula for the total force acting on the screws is

$$F_m = m\, k_m k_{dyn} g \tag{4}$$

including dynamical effects, i.e., the coefficient k_{dyn}, and gravitational acceleration g. Division of F_m into three screws (beams) is explained in [1].

In one beam, the force F, see Figure 4, is defined by the expressions

$$F = F_m/n,\ F_1 = F\cos(\propto),\ F_2 = F\sin(\propto), \tag{5}$$

where n is the coefficient of inequality in the division of forces. Coefficient $n \in (2;3)$ respects possible variations of maximal and minimal values of force F_m. Coefficient n is partially connected with the quality of a bone and with the quality of possible screw insertions. Force F_1 is the tangential force and F_2 is the axial force; see Figure 4. For more information, see [1].

The typical diameter of the femoral screws is the shank diameter D, which is used in the solutions below. However, the cannulated femoral screws also have their inner diameter d; see Figure 2.

Let us solve one femoral screw of a given length L (i.e., a beam on an elastic foundation) presented in Figures 3 and 4. The vertical displacement $v_i = v(x_i)$, see Equation (2), must be solved in three defined sections x_i, see Figure 4. Hence, three differential equations must be solved. This, in turn, requires the solution of twelve constants of integration $A_{1i}, \ldots, A_{4i}$ via twelve boundary conditions at points $x_1 = 0$ m, $x_1 = x_2 = L_1$, $x_2 = x_3 = L_2$, and $x_3 = L$. The boundary conditions, therefore, are

$$M_{o1}(x_1 = 0) = 0\,,\ T_1(x_1 = 0) = 0\,, \tag{6}$$

$$\left.\begin{array}{c} v_1(x_1 = L_1) - v_2(x_2 = L_1) = 0, \\ \frac{dv_1}{dx_1}(x_1 = L_1) - \frac{dv_2}{dx_2}(x_2 = L_1) = 0\,, \\ M_{o1}(x_1 = L_1) - M_{o2}(x_2 = L_1) = 0, \\ T_1(x_1 = L_1) - T_2(x_2 = L_1) = F_1, \end{array}\right\} \tag{7}$$

$$\left.\begin{aligned} v_2(x_2 = \mathrm{L}_2) - v_3(x_3 = \mathrm{L}_2) = 0\,, \\ \tfrac{dv_2}{dx_2}(x_2 = \mathrm{L}_2) - \tfrac{dv_3}{dx_3}(x_3 = \mathrm{L}_2) = 0\,, \\ M_{o2}(x_2 = \mathrm{L}_2) - M_{o3}(x_3 = \mathrm{L}_2) = 0, \\ T_2(x_2 = \mathrm{L}_2) - T_2(x_3 = \mathrm{L}_2) = -\mathrm{F}_1, \end{aligned}\right\} \quad (8)$$

$$M_{o3}(x_3 = \mathrm{L}) = 0\,,\ T_3(x_3 = \mathrm{L}) = 0. \quad (9)$$

Hence, based on Equations (6)–(9), it is possible to derive and solve a set of twelve linear equations which can be expressed in matrix form as

$$\{\mathbf{A}\} = [\mathbf{M}]^{-1} \times \{\mathbf{B}\} \quad (10)$$

The constants $\{\mathbf{A}\} = \{\mathrm{A}_{1\mathrm{i}}, \ldots, \mathrm{A}_{4\mathrm{i}}\}^{\mathrm{T}}$ can be acquired by solving a set of linear equations; see [1]. This presented analytical approach is easy to solve and evaluate.

Hence, shearing forces T_{i}, bending moments $M_{o\mathrm{i}}$, normal forces N, slopes $\frac{dv_{\mathrm{i}}}{dx_{\mathrm{i}}}$, and displacements v_{i} can be evaluated over the whole length of the femoral screw (beam). The definitions of mechanical stresses and their maximal values σ_{MAX}, σ_{MAX1}, σ_{MAX2}, and τ_{MAX}, see Figure 5, are based on combined loadings, and are explained in [1].

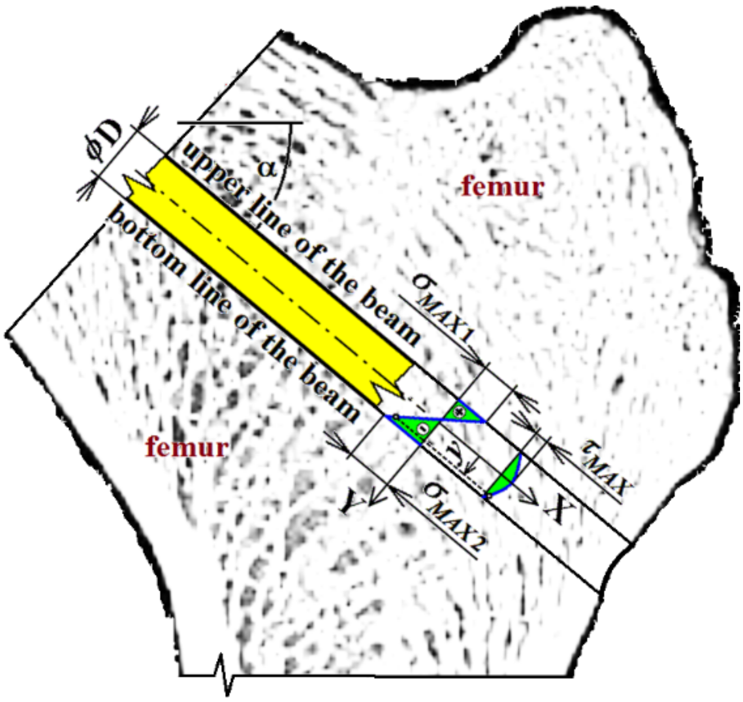


Figure 5. Mechanical stress evaluation in the cancellous screw (beam) with full cross-section, where D is the pitch diameter.

The calculated maximal stress values σ_{MAX} are derived from the bending and compression stress state and are found in the points of the expected maximum, i.e., in the areas of mechanical contact between the cancellous screw and femoral bone. Thus,

$$\sigma_{MAX1} = \frac{N}{A} - \frac{M_{oMAX}}{W_o},\ \sigma_{MAX2} = \frac{N}{A} + \frac{M_{oMAX}}{W_o} \text{ and } \sigma_{MAX} = \max(|\sigma_{MAX1}|,\ |\sigma_{MAX2}|) \quad (11)$$

The σ_{MAX} is present in a bottom line of the screw as shown in Figure 5.

Examples of the deterministic solutions and results are presented in [1]; additional examples of both deterministic and probabilistic approach are presented in the following text.

4. Probabilistic Inputs

As mentioned above, our new probabilistic approach (SBRA method) offers simple but fast and acceptable solutions to the presented problem.

Probabilistic inputs and outputs can be represented and evaluated by histograms, i.e., by graphs that display the distribution of our data. Histograms are good approximate representations of the statistic distributions of numerical data and give us a rough sense of the density of the underlying distribution of the data (see Figure 6 for an example).

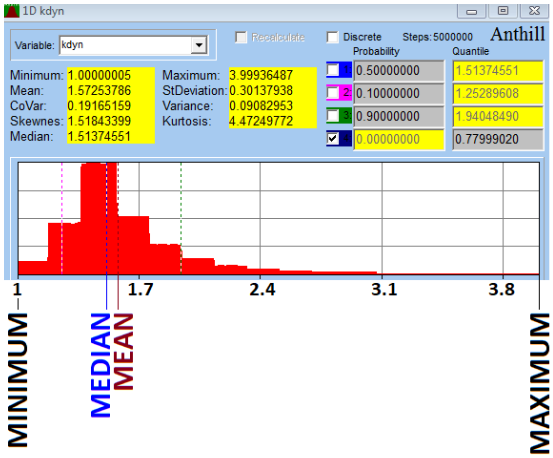


Figure 6. Histogram of the coefficient $k_{dyn} \in (1;4)$; Anthill software.

A deterministic solution, i.e., an example of one Monte Carlo random simulation with inputs defined in Table 1, is presented in Figures 7–9.

Table 1. Input parameters for 1 Monte Carlo simulation (cannulated cross-section, stainless steel).

Input Parameters (for Figures 7–9)	D = 0.005 m, d = 0.0018 m, $E = 2.1 \times 10^{11}$ Pa, g = 9.807 ms^{-2}, $k = 2.2222 \times 10^{7}$ Pa, $k_{dyn} = 1.4$, $k_m = 0.814$, L = 0.09 m, $L_1 = 0.015$ m, m = 120 kg, $L_2 = 0.068$ m, n = 3, $R_e = 10^3$ MPa, ∝ = 50 deg

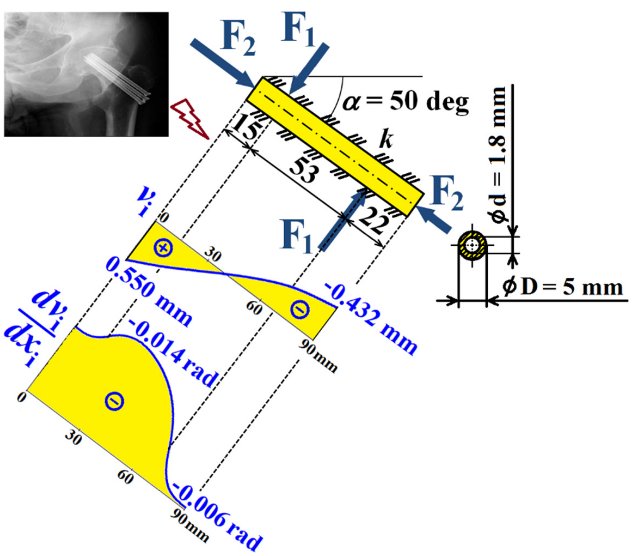


Figure 7. Displacement v_i and slope $\frac{dv_i}{dx_i}$ along the length of the screw in one cancellous screw (cannulated cross-section, stainless steel, result of a single Monte Carlo simulation).

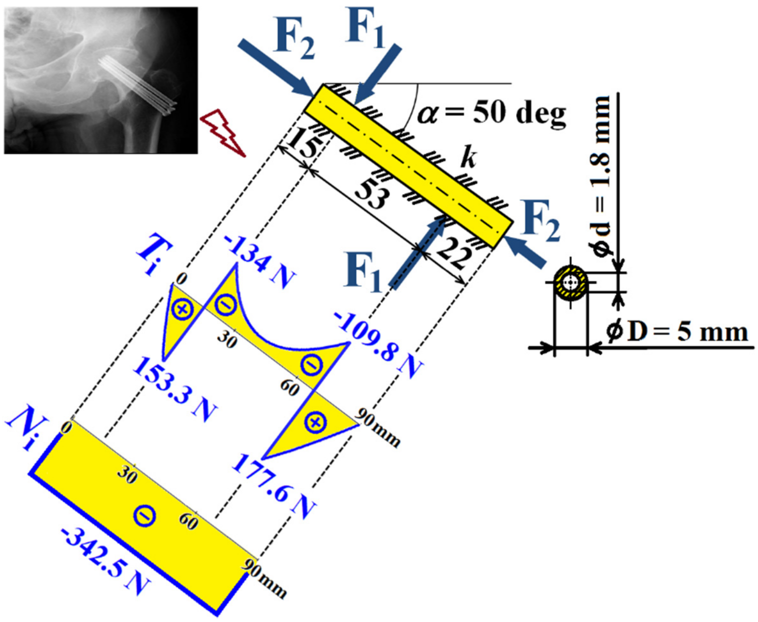


Figure 8. The shearing force T_i and normal force N_i along the length of a cancellous screw (cannulated cross-section, stainless steel, result of a single Monte Carlo simulation).

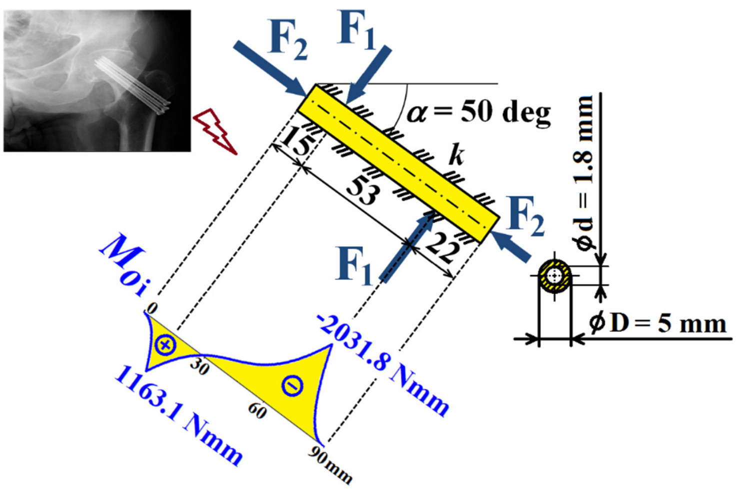


Figure 9. The bending moment M_{oi} along the length of a cancellous screw (cannulated cross-section, stainless steel, result of a single Monte Carlo simulation).

For the stochastic simulations (Anthill sw, cannulated screw, Ti6Al4V material), six stochastic inputs and eight deterministic inputs were used; see Table 2.

Table 2. Input parameters for probabilistic calculations (cannulated screw, Ti6Al4V material).

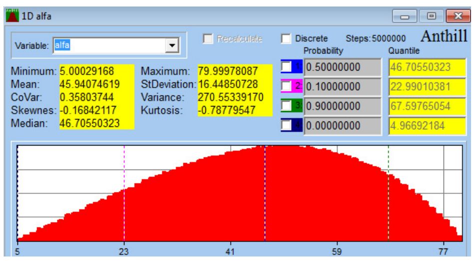
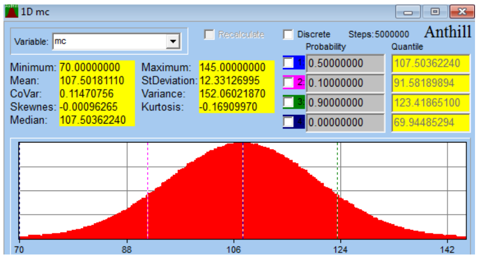
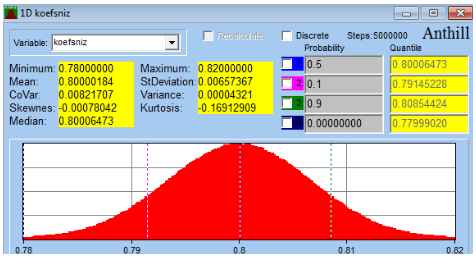
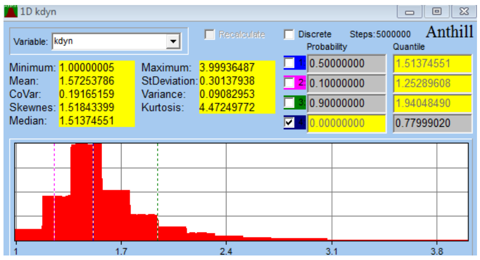
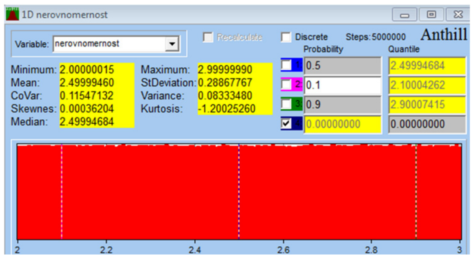
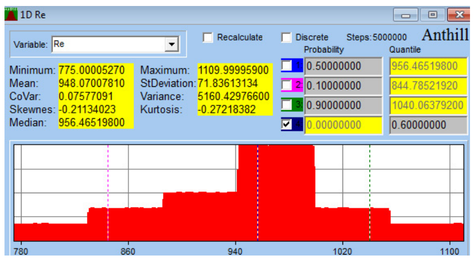
Input Variable	Minimum	Maximum	Mean	Median	Standard Deviation	Bounded (Truncated) Histogram
∝/deg/ Cancellous screw angle	5	80	45.94	46.71	16.45	

Table 2. *Cont.*

Input Variable	Minimum	Maximum	Mean	Median	Standard Deviation	Bounded (Truncated) Histogram
m/kg/ Entire mass of a patient	70	145	107.50	107.50	12.33	
k_m/1/ Mass reduction coefficients	0.78	0.82	0.80	0.80	6.57×10^{-3}	
k_{dyn}/1/ Dynamic force coefficient	1	4	1.57	1.51	0.301	
n/1/ Coefficient of inequality in the division of force F	2	3	2.5	2.5	0.289	
R_e/MPa/ Yield stress of Ti6Al4V material of cancellous screws	775	1110	948.07	956.47	71.84	

Other input parameters are constant, i.e., D = 5 mm, d = 1.8 mm, $E = 1.138 \times 10^{11}$ Pa, $k = 2.2222 \times 10^{7}$ Pa, L_1 = 15 mm, L_2 = 68 mm, L = 90 mm, g = 9.807 ms^{-2}.

5. Probabilistic Solution and Probabilistic Reliability Assessment

Based on the parameters presented in Table 2, the probabilistic (stochastic) solution was performed for 5×10^6 random Monte Carlo simulations using the Anthill sw.

Some examples of calculated output histograms are presented in Figures 10 and 11.

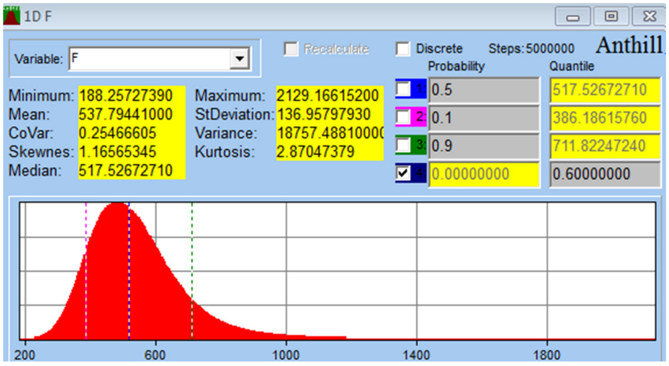


Figure 10. Histogram of the calculated force F $\in$ (188.3; 2129.2) N acting on a cancellous screw (Anthill software, cannulated screw, Ti6Al4V material).

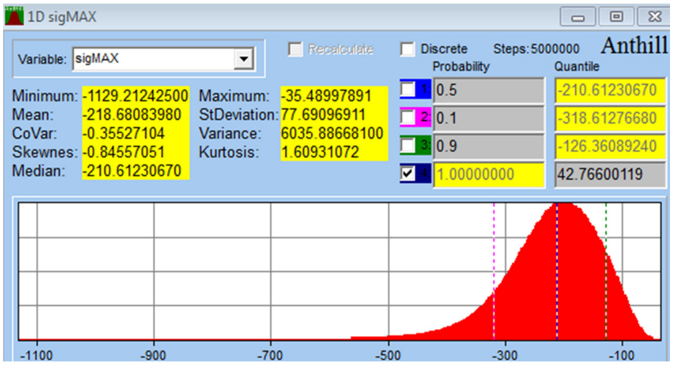


Figure 11. Histogram of the calculated maximal global compression stress $\sigma_{MAX2} \in (-1129.21; -35.49)$ MPa acting on a cancellous screw (Anthill software, cannulated screw, Ti6Al4V material).

Using the SBRA method (Anthill sw), the probabilistic reliability function *RF* can be defined as

$$RF = \mathrm{R_e} - \sigma_{MAX} \quad (12)$$

where maximal global stress

$$\sigma_{MAX} = |\sigma_{MAX2}| \in (35.49;\ 1129.21)\ \mathrm{MPa} \quad (13)$$

is defined in Figure 5 and in Equation (11). The histogram of *RF* is presented in Figure 12.

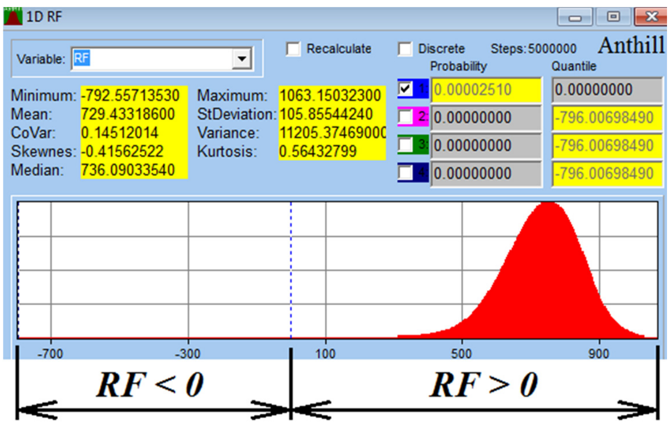


Figure 12. Histogram of the calculated reliability function *RF* $\in$ (−792.56; 1063.15) MPa for a cancellous screw (Anthill software, cannulated screw, Ti6Al4V material).

If situations when $RF \leq 0$ occur, the yield limit is reached in the cancellous screw (i.e., an undesirable or unsafe situation occurs), while as long as $RF > 0$, the situations are safe.

In physics, mechanics, biomechanics, and structural reliability assessment, the concepts of limit states separating multidimensional domains of random (stochastic) variables into "safe" and "unsafe" domains have been generally accepted. Therefore, the probability of failure or of an undesirable situation (in other words, the probability that the yield limit R_e exceeds σ_{MAX}) is the probability of $P_{RF\leq 0}$; see Figures 12 and 13 and [24].

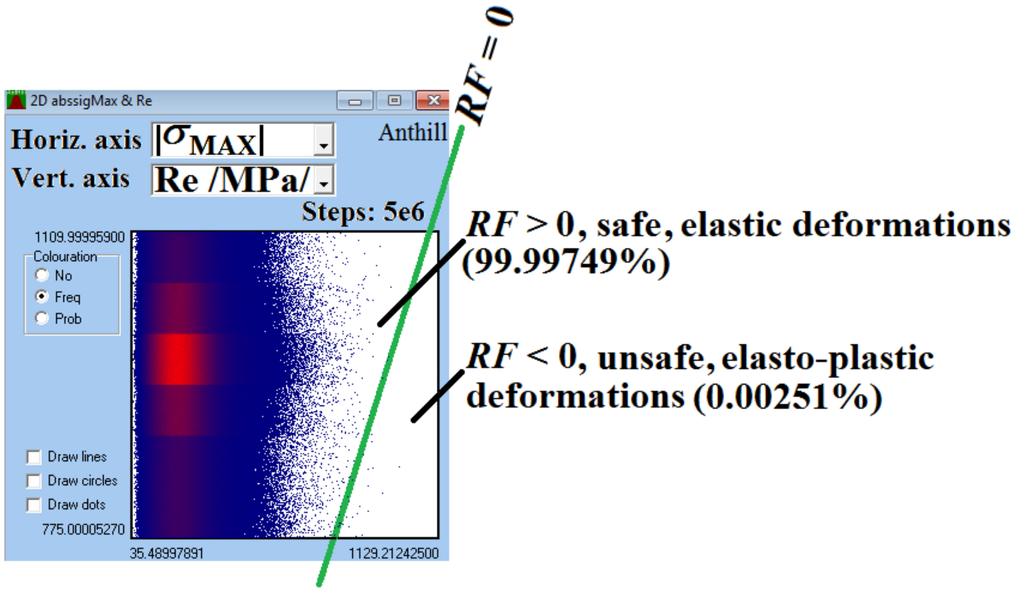


Figure 13. Two-dimensional histogram of the calculated reliability function *RF* and probability evaluation for a cancellous screw (Anthill software, cannulated screw, Ti6Al4V material).

The presented results for a cannulated cancellous screw inserted in the *collum femoris* imply the biomechanical probability of failure to be $P_{RF\leq 0} = 2.51 \times 10^{-5} = 0.00251\%$. It should be, however, noted that the calculated probability $P_{RF\leq 0}$ is caused mainly by the *collum femoris* overloading (i.e., it accounts for mechanical/biomechanical problems). Other possible problems, such as the development of pathological problems during the treatment, surgical mistakes, etc., are not taken into the account. Hence,

$$P_{unsuccessful} \geq P_{RF\leq 0} \tag{14}$$

In other words, the total probability of unsuccessful treatment $P_{unsuccessful}$ of *collum femoris fractura* is greater than our calculated $P_{RF\leq 0}$. Calculation of the overall probability of unsuccessful treatment encompassing all possible nonmechanical effects, i.e., $P_{unsuccessful}$, is, however, not the aim of this article. Such a value should be rather acquired through a long-term statistical evaluation of medical cases than by an engineering calculation. Our $P_{RF\leq 0}$ is only a partial parameter responsible only for a minor part of osteosynthesis failures.

6. Discussion

As mentioned above, treatment of PFN (*collum femoris*) fractures represents an ongoing problem in traumatology, orthopedics, and in rehabilitation medicine. The application of cancellous screws, i.e., femoral or lag spongious screws, made of titanium alloys or stainless steels is a possible and popular method for the treatment of these fractures. The medical perspectives of these fractures and their treatments are explained at the beginning of this article and in [1,3].

Biomechanical research is highly desirable for improving the quality and reliability of fracture treatment. To model the placement of the screw in the femoral neck, a planar (2D) model of a beam on the elastic foundation (femoral bone) according to Winkler was used. This model, i.e., derivation and solution of a set of differential equations using a deterministic approach based on the static theory of the 2nd order, was presented in this article.

Our long-standing cooperation with medical experts led to the development of the presented innovative probability-based stochastic, i.e., statistical, approach (SBRA method, direct Monte Carlo method, Anthill sw) that was used as a significant improvement of the results of our previously published research. In this approach, real random inputs, typical for nature and the patients, are taken into account using bounded histograms.

One of the principal results is, therefore, the determination of the probability of occurrence of an undesirable situation from the perspective of mechanics/biomechanics, i.e., calculation of the probability of exceeding the screw yield strength. This "biomechanical" probability, calculated to be $P_{RF\leq 0} = 2.51 \times 10^{-5}$, is acceptable and can be further evaluated and developed in association with unsuccessful treatment results, i.e., with pathological changes during osteosynthesis healing, surgical mistakes, noncompliance with the treatment algorithms, associated diseases, etc. Hospitals are continuously statistically evaluating their successful or unsuccessful treatments, and the data could be associated with our biomechanical results. For example, for future extension, the bone-healing model presented in [35] could be applied for mechanobiological rules in the ossification of femoral fractures under differing biomechanical conditions and varying anatomy and fracture types.

Our models presented in this article and [1] can also be used for calculating/assessing general, inappropriate, or unacceptable positions of cancellous screws. Changes can be made to the length L, angle $\propto$, number of screws, parallel or nonparallel positions of screws, type of fractures, screws that can or cannot be in contact with the femoral neck cortex, etc. Our model can be further extended and applied in the mechanics of joints in engineering (for example, in wooden structures, etc.).

In this article, the two solutions, i.e., example of one deterministic calculation and one stochastic solution, are performed for cancellous screws with cannulated cross-sections made of stainless steel or Ti6Al4V material. Additional specific deterministic solutions are presented in [1,14].

The presented approach represents a rapid solution to a stochastic "real-world" modeling of a biomechanical task and represents a valid alternative to computationally and temporally more demanding and more complicated calculations using the finite element method; see Figure 14 for an example. A comparison of the results acquired by the finite element method and the method presented in this paper demonstrates the accuracy of our simple 2D model; see [1,14]. Figure 14 shows stress distribution in three cancellous screws for static loading in a large 3D model of the femur with applications of mechanical contacts. The maximum stresses are in good agreement with our 2D stochastic beam model presented in this article. Thus, the maximum stress of 153.28 MPa acquired via the finite element method (see Figure 14) lies in the maximum stress interval of Equation (13).

More detailed calculations on the topic of femoral neck fracture osteosynthesis using the finite element method and/or experimental solution will be presented in the next continuation of our research.

If needed, our model can be expanded to include nonlinear tasks, i.e., respecting the nonlinear behavior of the elastic/nonelastic substrate, geometry, and material. More information can be found in the work of the principal author [19–21] and references [28,29].

There are other opportunities for the future development of screws for osteosyntheses in surgery, traumatology, and orthopedics. These include experimental work and numerical simulations to improve the quality and reliability assessment of screw placement within the bone. Our present, as well as future, research of osteosynthetic screws in the internal and external fixation (see Figure 15) is based on our previous work (for example, see [1,11,23,33,36–38]). Figure 15 shows an angularly stable plate with locking bone screws removed from the human wrist. This plate is covered by body liquids, such as *haema*, *exudate*, and pieces of tissues. Subsequent biochemical, biomechanical, and material laboratory testing of the extracted screws revealed information important for designing new screws;

see [33]. The long-term goal is to improve, in cooperation with medical staff, the quality and safety of osteosynthesis where the screws play the main part.

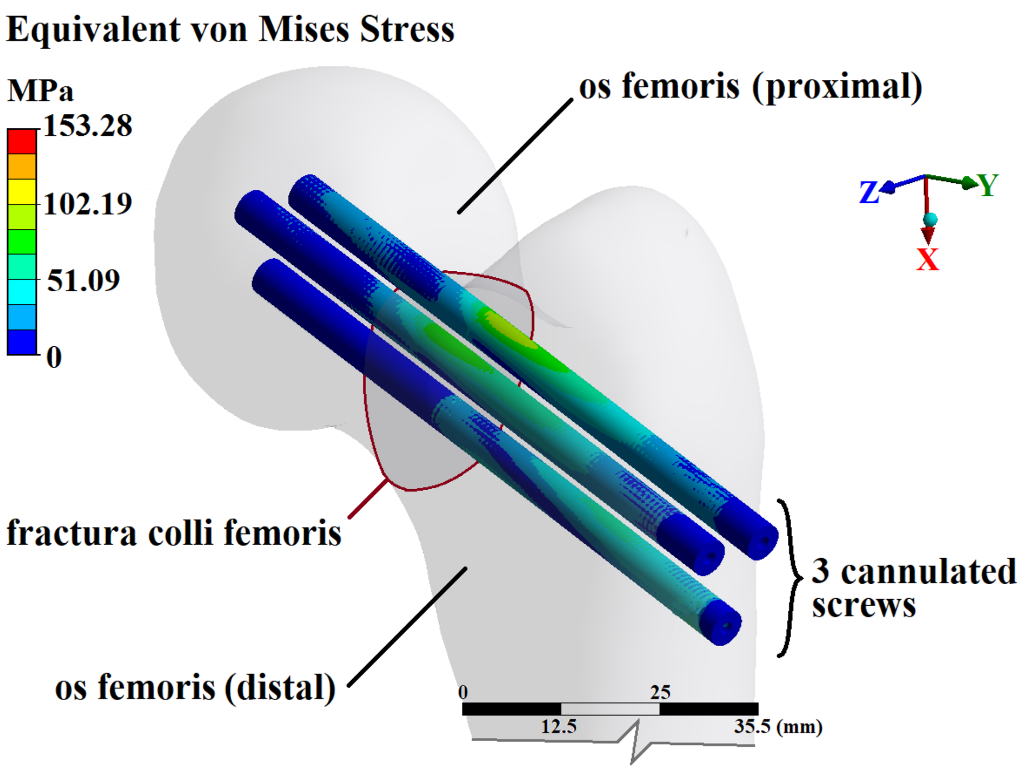


Figure 14. Simple 3D finite element solution of three cannulated cancellous screws in the femur for verification of the presented results (Ansys software).

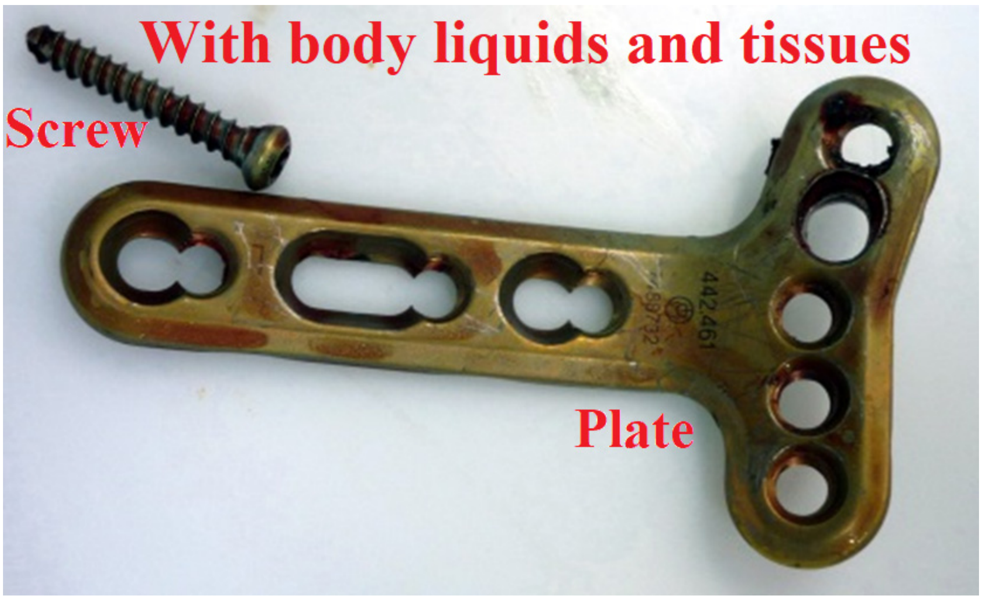


Figure 15. Angularly stable plate with locking bone screws removed from the human wrist.

The study of dynamic properties of screws and bone interaction via experiments can also be applied, and screws or their parts can be made of composites as well; see [39].

7. Conclusions

This paper, as well as our previous one [1], discusses and solves basic medical perspectives on *collum femoris* fractures with the focus on their treatment via cancellous, i.e., femoral, screws, in particular of subcapital, mediocervical fractures.

The new, simple, deterministic, and stochastic (probabilistic) 2D model of a cancellous screw in the femur as a beam on an elastic bilateral Winkler foundation, along with the 2nd-order theory of small deformations, i.e., bending and compression loading, were applied in the beams.

Materials, dimensions, loading, differential equations and their solutions, evaluations, etc., are described.

Important biomechanical stochastic evaluations, i.e., evaluations of normal forces, shearing forces, bending moments, deflections, slopes, stresses, probabilistic reliability assessment, and the probability of failure were carried out.

Compared to the finite element method, our computational model as a whole is characterized by a quick and easy solution and high variability of possible screw insertion positions.

One example of probabilistic reliability assessment of the undesirable situation (i.e., probability of plastic deformations) with results $P_{RF \leq 0} = 2.51 \times 10^{-5}$ was presented and discussed in this paper.

According to the results, from the biomechanical point of view, the cancellous screws are safe enough and are recommended as a suitable and safe osteosynthetic tool for fracture treatment.

Other possibilities for future research and developments, such as material nonlinearities, and experiments are discussed.

Hence, together with our former work, this article has presented new ideas and methods and has demonstrated their practical applications in biomechanical engineering, centered around a new simple approach to the solution of cancellous screws with applications in the field of traumatology and orthopedics.

Applications of stochastic mechanics, together with probabilistic reliability assessment in biomechanics, are still rare but are in accordance with modern and innovative trends of science.

According to our presented simple model, the relatively complicated problems of geometry, dimensions, configurations, way of insertion, reliability assessment, etc., for cancellous screws can be solved.

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List of Symbols

$A_{1i}, \ldots, A_{4i}, \{\mathbf{A}\}$	Integral constants and vector of integral constants (m). They are related to boundary conditions.
$\{\mathbf{B}\}$	Vector of the left side (1)
D, d	Outer (shank) and inner (cannulation) diameter of cancellous screw (m) (input variables)
E	Elastic modulus of cancellous screw material (Pa) (input variable)
F	Quasi-dynamic force acting on one cancellous screw (N)
F_m	Total loading quasi-dynamic force acting in *caput femoris* (N)
F_1, F_2	Tangential and axial force acting in one cancellous screw (N)
g	Gravitational acceleration (ms^{-2}) (input variable)
i = 1, 2 and 3	Index of a section of the cancellous screw
J_{ZT}	Principal quadratic moment of the cross-sectional area of cancellous screw (m^4)
k	Elastic foundation stiffness, i.e., femoral bone stiffness (Nm^{-2}) (input variable)
k_{dyn}, k_m	Dynamic force and mass reduction coefficients (1) (input variables)
L, L_1, L_2	Total length and local lengths of cancellous screws (m) (input variables)
$[\mathbf{M}]$	Matrix of equations (1)
M_{oi}, M_{oMAX}	Internal bending moments in sections of the screw and the absolute value (Nm) of the maximal bending moment in the screw
m	Patient mass (kg) (input variable)
N	Internal normal (axial) force in the cancellous screw (N)
n	Coefficient of inequality in the division of force (F) (1) (input variable)
$P_{RF\leq 0}$	Probability of biomechanical failure
$P_{unsuccessful}$	Probability of unsuccessful treatment
PFN	Proximal femoral neck
R_e	Yield stress of the material of cancellous screws (MPa) (input variable)
RF	Probabilistic (stochastic) reliability function (MPa)
T_i, T_{MAX}	Internal shearing forces in sections of the cancellous screw and absolute value (N) of the maximal shearing force
v_i, v_{MAX}	Deflections (i.e., vertical displacements) in sections of the cancellous screw and their maximum (m)
$\frac{dv_i}{dx_i}$	Slope of the cancellous screw (rad)
x_1, x_2, x_3, x_i	Cartesian coordinates in sections (m)
$\propto$	Cancellous screw angle (deg)(input variable)
$\sigma_{MAX}, \sigma_{MAX1}, \sigma_{MAX2}$	Global maximal and maximal normal mechanical stresses in the cancellous screw (MPa)
τ_{MAX}	Maximal shear mechanical stress in the cancellous screw (MPa)
$\omega, \omega_R, \omega_I$	Parameters of the numerical solutions (m^{-1})

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