

The role of general relativity in the evolution of low-mass X-ray binaries

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ABSTRACT

We study the evolution of low-mass X-ray binaries hosting a neutron star and of millisecond binary radio pulsars using numerical simulations that take into account the detailed evolution of the companion star, of the binary system, and of the neutron star. According to general relativity, when energy is released during accretion or due to magnetodipole radiation during the pulsar phase, the system loses gravitational mass. Moreover, the neutron star can collapse to a black hole if its mass exceeds a critical limit, which depends on the equation of state of ultradense matter and is typically $\sim 2 M_{\odot}$. These facts have some interesting consequences. (i) In a millisecond radio pulsar the mass-energy is lost with a specific angular momentum that is smaller than the specific angular momentum of the system, resulting in a positive contribution to the orbital period derivative. If this contribution is dominant and can be measured, we can extract information about the moment of inertia of the neutron star, since the energy loss rate depends on it. Such a measurement can help to put constraints on the equation of state of ultradense matter. (ii) In low-mass X-ray binaries below the bifurcation period (~ 18 h), the neutron star survives the ‘period gap’ only if its mass is smaller than the maximum non-rotating mass when the companion becomes fully convective and accretion pauses. Since in such evolutions $\sim 0.8 M_{\odot}$ can be accreted on to the neutron star, short-period ($P \leq 2$ h) millisecond X-ray pulsars such as SAX J1808.4 – 3658 can be formed only if either a large part of the accreting matter has been ejected from the system, or the equation of state of ultradense matter is very stiff. (iii) In low-mass X-ray binaries above the bifurcation period, the mass-energy loss lowers the mass transfer rate. As a side effect, the inner core of the companion star becomes ~ 1 per cent bigger than in a system with a non-collapsed primary. As a result of this difference, the final orbital period of the system is 20 per cent longer than if the mass-energy loss effect is not taken into account.

Key words: relativity – binaries: close – stars: individual: SAX J1808.4 – 3658 – stars: neutron – pulsars: general – X-rays: binaries.

1 INTRODUCTION

Low-mass X-ray binaries (LMXBs) are systems consisting of a neutron star (NS) with a relatively weak magnetic field ($< 10^{10}$ G) accreting from a low-mass ($\sim 1 M_{\odot}$) companion star. When the companion star fills its Roche lobe, it transfers mass to the NS. The companion fills its Roche lobe either because it expands due to nuclear evolution or because the lobe shrinks owing to orbital angular momentum losses caused by gravitational radiation and magnetic braking. The matter flowing from the inner Lagrangian point towards the NS forms a Keplerian accretion disc around it. The NS is spun up by the accreting matter to an equilibrium period that is roughly equal to the Keplerian frequency at the inner rim of the

accretion disc (Ghosh & Lamb 1979). Once accretion ends, the NS can light up as a rapidly rotating magnetodipole (radio pulsar): this is the so-called recycling scenario for the formation of millisecond pulsars (Bhattacharya & van den Heuvel 1991).

The secular evolution of LMXBs can follow two very different paths, according to the evolutionary stage of the companion at the start of the mass transfer. If the orbital period of the binary at the beginning of the mass transfer is large, the evolution of the LMXB begins when the companion evolves off the main sequence, the main driving mechanism for mass transfer is nuclear evolution, and the system will evolve towards large orbital periods. Such systems are said to be above the bifurcation period (Tutukov et al. 1985; Ergma 1996). If the orbital period of the system at the onset of the mass transfer is small (i.e. if it is below the bifurcation period), the companion is relatively unevolved, and the only important mechanism driving mass transfer is systemic angular momentum loss (AML)

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due to magnetic braking and gravitational radiation. This type of evolution is similar to the classical evolution of cataclysmic binaries. The orbital period becomes shorter and shorter, and the system may experience a period gap when magnetic braking stops being effective (when the secondary becomes fully convective) and ultimately reaches a minimum period just before hydrogen burning is extinguished (Paczynski & Sienkiewicz 1981; Rappaport, Joss & Webbink 1982). Beyond the period minimum (which depends on the evolutionary stage of the initial model), the donor's radius begins increasing, following the mass–radius relation for degenerate stars, and the orbital period will increase again, driven by gravitational radiation alone. In fact, the distribution of orbital periods of LMXBs, based on relatively few systems (Liu, van Paradijs & van den Heuvel 2001), does not show a period gap as clear as that of CVs, but a more general limitation in the number of systems below ~ 4 h. It may well be that the distribution of initial periods of LMXBs favours the evolution of more evolved donors (Nelson & Rappaport 2003), and population synthesis results show that several LMXB systems may be descendants of intermediate-mass secondaries, after a phase of thermal time-scale mass transfer (Podsiadlowski, Rappaport & Pfahl 2002), but the evolutionary path towards shorter periods is mainly followed by evolutions starting from unevolved secondaries. For these systems, the donors have the same structure as the donors in CVs, and in principle there is no reason why they should not pass through a similar period gap, possibly having a smaller width, as a result of the fact that the accreting component is more massive ($1.35 M_{\odot}$) than the typical white dwarf in CVs ($\sim 0.6 M_{\odot}$) (Podsiadlowski et al. 2002).

Studies of the evolution of LMXBs usually disregard the compact nature of the primary. This is not legitimate, since once the matter is transferred from the companion to the NS it releases mass-energy as a result of the strong gravitational binding of the NS, and therefore its gravitational mass (i.e. the ‘charge’ of the gravitational force) decreases once it is accreted on to the NS. Thus, even if the mass transfer is conservative, *the total gravitational mass of a LMXB decreases* when matter is transferred from the companion on to the NS: it has been suggested that this could have observable effects (Alécian & Morsink 2004). Moreover, if enough mass is accreted on to the primary, it can collapse to a black hole (Lavagetto et al. 2004, hereafter Paper I). We should, then, take into account both of these effects if we want to study the evolution of a LMXB accurately.

In this paper we will first introduce the evolution equations for the binary system, including the effects of general relativity (Section 2). Then we will show how the mass-energy loss is potentially measurable in binary millisecond radio pulsars (MSPs), and how this measurement can put constraints on the equation of state (EOS) of NSs (Section 3). In Section 4 we will show under which conditions the NS in systems below the bifurcation period can survive the ‘period gap’. Finally, in Section 5 we will study the evolution of a system above the bifurcation period, showing how mass-energy loss alters the evolution of the system.

2 THE EVOLUTION EQUATIONS

We implemented a simulation code that simultaneously includes the stellar evolution of the companion, the evolution of the binary system, and the evolution of the NS under the effect of accretion in order to simulate accurately the evolution of a LMXB. Our evolution code couples the routines of the ATON code (D’Antona, Mazzitelli & Ritter 1989), updated with the physical inputs described in Ventura et al. (1998), which accounts for the stellar evolution and the binary

evolution of the system, and routines accounting for the evolution of the NS (which is considered to be fully relativistic) under accretion.¹

If the primary of the system is a NS, its gravitational mass will be given by the sum of the baryonic mass (i.e. the number of baryons N times the average bare mass of the baryons m_B) and of the potential and kinetic energies divided by c^2 , which are non-negligible since the gravitational binding energy is large for matter as dense as NS matter. We know that for any given equilibrium configuration of a NS we have on a general basis (Bardeen 1970) that

$$M_G = M_G(M_B, J), \quad (1)$$

where we denote the gravitational mass of the star by M_G , its baryonic mass by M_B , and its intrinsic angular momentum by J . The gravitational mass accreted per unit time then depends both on the number of baryons accreted and on the accreted angular momentum (Bardeen 1970):

$$\dot{M}_G = \left(\frac{\partial M_G}{\partial M_B} \right)_J \dot{M}_B + \left(\frac{\partial M_G}{\partial J} \right)_{M_B} \dot{J}, \quad (2)$$

where

$$\left(\frac{\partial M_G}{\partial J} \right)_{M_B} = \frac{\omega_{NS}}{c^2}, \quad (3)$$

$$\left(\frac{\partial M_G}{\partial M_B} \right)_J = \Phi,$$

where ω_{NS} is the NS spin frequency, c is the speed of light, and Φ is the energy needed to bring a unit mass from infinity to the pole of the star.

Therefore equation (2) becomes

$$\dot{M}_G = \Phi \dot{M}_B + \frac{\omega_{NS}}{c^2} \dot{J}. \quad (4)$$

When matter is transferred from the companion on to the NS, and the mass transfer is conservative, we have

$$\dot{M}_B = -\dot{M}_c, \quad (5)$$

where M_c is the mass of the companion. It is useful to rewrite equation (4) using equation (5) as (Alécian & Morsink 2004)

$$\dot{M}_G = -(1 - \beta)\dot{M}_c, \quad (6)$$

where

$$0 < \beta = 1 - \Phi - \frac{\omega_{NS}}{c^2} \frac{J}{M_B} < 1.$$

According to general relativity, the binding energy of the accreting matter results in a mass deficit. The gravitational mass lost in accretion is released from the system (mainly as X-rays) and carries away a specific orbital angular momentum that we assume to be equal to the specific orbital angular momentum of the NS:

$$\left(\frac{\dot{L}}{\dot{L}} \right)_{\beta} = \frac{1}{L} \beta \dot{M}_c \frac{2\pi}{P} \left(a \frac{M_c}{M_{tot}} \right)^2, \quad (7)$$

where P is the orbital period of the binary system and a is the orbital separation. Using Kepler’s law,

$$\frac{2\pi}{P} = \left(\frac{GM_{tot}}{a^3} \right)^{1/2},$$

¹ For a detailed description of the fully relativistic study of the response of the NS to the accretion of matter, see Paper I.

where $M_{\text{tot}} = M_c + M_G$, and the expression for the orbital angular momentum,

$$L = M_c M_G \left(\frac{Ga}{M_{\text{tot}}} \right)^{1/2}, \quad (8)$$

we can rewrite equation (7) as

$$\left(\frac{\dot{L}}{L} \right)_\beta = \beta \dot{M}_c \frac{M_c}{M_G M_{\text{tot}}} = \beta \frac{\dot{M}_c}{M_c} \frac{q^2}{1+q}, \quad (9)$$

where $q = M_c/M_G$. The total variation of the orbital angular momentum L will then be equal to the sum of the systemic orbital angular momentum losses L_{sys} and of the angular momentum losses due to the relativistic mass deficit \dot{L}_β . We can therefore write

$$\begin{aligned} \left(\frac{\dot{L}}{L} \right)_{\text{sys}} + \left(\frac{\dot{L}}{L} \right)_\beta &= \frac{1}{2} \frac{\dot{a}}{a} + \frac{\dot{M}_G}{M_G} + \frac{\dot{M}_c}{M_c} - \frac{1}{2} \frac{\dot{M}_G + \dot{M}_c}{M_G + M_c} \\ &= \frac{1}{2} \frac{\dot{a}}{a} + (1-q) \frac{\dot{M}_c}{M_c} + \beta \left[q - \frac{1}{2} \frac{q}{1+q} \right] \frac{\dot{M}_c}{M_c}, \end{aligned} \quad (10)$$

where we have made use of equation (6). If we now substitute equation (9) into equation (10), we can write the derivative of the orbital separation as

$$\frac{\dot{a}}{a} = 2 \left(\frac{\dot{L}}{L} \right)_{\text{sys}} - 2(1-q) \frac{\dot{M}_c}{M_c} - \beta \frac{q}{1+q} \frac{\dot{M}_c}{M_c}. \quad (11)$$

The last term on the right is due to the relativistic mass deficit: it is relevant only in compact systems where β is non-negligible. Using Kepler's Law, we can write for the evolution of the orbital period

$$\begin{aligned} \frac{\dot{P}}{P} &= \frac{3}{2} \frac{\dot{a}}{a} - \frac{1}{2} \frac{\dot{M}_G + \dot{M}_c}{M_G + M_c} \\ &= 3 \left(\frac{\dot{L}}{L} \right)_{\text{sys}} - 3(1-q) \frac{\dot{M}_c}{M_c} - 2\beta \frac{q}{1+q} \frac{\dot{M}_c}{M_c}, \end{aligned} \quad (12)$$

where the third term appears due to the relativistic mass deficit. This term is positive since $-\dot{M}_c$ is positive: this means that, as mass is transferred, an additional positive contribution to the orbital period derivative is present in relativistic systems. One should keep in mind that the mass transfer rate \dot{M}_c depends upon the whole evolution of the binary system: in general it is a function of the nuclear evolution of the companion and of the orbital separation. In order to evaluate quantitatively the influence of the evolution of the NS on the evolution of a system we must carry out detailed numerical simulations of the binary system, because of the non-linearity of the equations. It is straightforward to extend these equations to the case of non-conservative mass transfer.

In the following sections we will show how these effects, together with the evolution of the compact object, can have observable effects and alter the secular evolution of LMXBs.

3 OBSERVABLE EFFECTS OF GENERAL RELATIVITY

We can ask ourselves if the effects of relativity on the orbital parameters of the binary system are observable. In theory, any binary system about which we have enough information can reveal the effects described in the preceding section. However, since we do not know much about most binary systems, and since various effects can overlap (see Section 5), we have little chance of observing directly those effects that play a role only in the secular evolution of the system. Alécian & Morsink (2004) argued that, if the orbital

period derivative of an accreting LMXB can be measured, it will allow information on the structure of the NS, such as its mass and its gravitational binding energy, to be obtained. This effect is measurable only if the mass transfer is driven exclusively by the emission of gravitational radiation: there are too many uncertainties on the amount of angular momentum lost due to magnetic braking and on the nuclear evolution of the companion to allow the relativistic effects in the orbital evolution to be separated in such cases. This limits the range of systems that are interesting to close LMXBs (with $P \lesssim 2$ h) with a main-sequence companion. In this situation, the mass loss from the system will result in a potentially observable modification of the orbital period derivative. As Alécian & Morsink point out, however, uncertainties in the physics of binary evolution and of mass accretion make it difficult to separate this effect on the orbital period from others. Moreover, it is impossible to infer the mass accretion rate in a LMXB from its observed X-ray luminosity with the accuracy that is needed to extract information in their model. It is therefore unlikely that this effect can be used to investigate the EOS of ultradense matter.

When accretion on to the primary ends, however, the NS lights up as a radio pulsar, and it brakes down due to rotating magnetodipole emission. The NS loses some of its gravitational mass because it is radiating away energy. We can then write equation (12) in the form

$$\frac{\dot{P}}{P} = -2 \frac{\dot{M}_G}{M_c} \frac{q}{1+q} + \frac{\dot{P}_{\text{GW}}}{P}, \quad (13)$$

where $\dot{P}_{\text{GW}}/P = 3\dot{L}_{\text{GW}}/L$ is the orbital period derivative due to the emission of gravitational waves from the binary system, which we can write as

$$\begin{aligned} \dot{P}_{\text{GW}} &= -\frac{192\pi}{5c^5} \left(\frac{2\pi G}{P} \right)^{5/3} \frac{M_G M_c}{M_{\text{tot}}^{1/3}} \\ &\times \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) (1-e)^{-7/2} \end{aligned} \quad (14)$$

(Lorimer 2001), where e is the eccentricity of the system. It is then evident that, when energy is released from the MSP, the mass deficit yields a positive contribution to the orbital period derivative, opposite to the contribution of gravitational wave emission. Since the effect of gravitational mass loss is $\propto P$, while the effect of gravitational wave emission is $\propto P^{-5/3}$, the former effect will be dominant in systems with large enough orbital periods (say $P \geq 6$ h), while the latter will become more relevant in systems with short periods.

The variation of the gravitational mass is equal to the spin-down energy of the pulsar, divided by c^2 . To a good approximation we can write

$$\dot{M}_G = \frac{I}{c^2} \omega_{\text{NS}} \dot{\omega}_{\text{NS}}, \quad (15)$$

where I is the moment of inertia of the NS. Esposito & Harrison (1975) noticed after the discovery of the first binary pulsar, PSR 1913+16, that the mass deficit can alter the orbit of a binary pulsar. They found, however, that its effect on the orbital evolution of PSR 1913+16 was negligible. Today we know many MSPs that have periods below 5 ms and whose spin-down mass loss is large (see equation 15). In these systems, the effect can be orders of magnitude stronger than the effect of gravitational waves. When this is the case, measuring the orbital period derivative of the binary system can help to put strong constraints on the EOS of NSs. In many cases it is possible to measure both the spin frequency and its derivative in a radio pulsar with high precision. Moreover, the absence of mass transfer cuts down any uncertainty in the binary evolution model. The orbital period derivative depends then only on measured

quantities (ω_{NS} , $\dot{\omega}_{NS}$ and e), on the masses of the two stars, and on the moment of inertia of the neutron star (see equations 13, 14 and 15). We can obtain information on the two masses from the mass function

$$f(M) = \frac{M_c^3 \sin^3 i}{M_{\text{tot}}^2},$$

which is measurable in binary radio pulsars with very good precision. Using it, we can impose constraints on the moment of inertia of the NS. Since the moment of inertia depends strongly on the EOS of the NS (Cook et al. 1994), the detection of these effects will allow us to discriminate between various EOSs on a solid observational basis.

We demonstrate this method with an example: suppose that we observe a system with an orbital period $P = 8$ h, a spin period $P_s = 2$ ms, $\dot{P}_s = 3 \times 10^{-19}$, a mass function of $5 \times 10^{-3} M_{\odot}$, and that we have measured the orbital period derivative to be $+2.5 \times 10^{-14}$. In Fig. 1 we plot the values of the masses of the two stars that are compatible with this value of the orbital period derivative, with the hypothesis that the NS is governed by the pure neutron EOS defined by Pandharipande, named EOS A in the classic catalogue by Arnett & Bowers (1977), or by the realistic hadronic EOS defined by Baldo, Bombaci & Burgio (1997), which we label EOS BBB. The mass function imposes the condition that the values of the two masses should be above the dotted line in Fig. 1. The figure tells us that, given the mass function and the orbital period derivative we assumed, the pulsar cannot be described by EOS A.

Now that we have shown how promising this effect is in principle, we can ask which, among the known MSPs, is the best candidate for detecting such an effect. The most promising object that we have found is PSR J0218+4232, which comprises a NS spinning at 2.3 ms in orbit with a white dwarf companion, with an orbital period of 2 d. The mass ratio is measured to be $q = 0.13 \pm 0.04$ (Bassa et al. 2003).

Owing to its strong power output, the pulsar loses gravitational mass at the rate of $4 \times 10^{-12} I_{45} M_{\odot} \text{yr}^{-1}$, where I_{45} is the moment

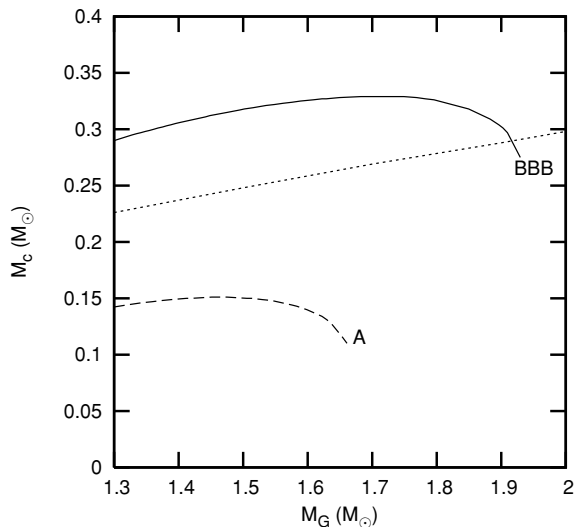


Figure 1. Allowed values of the mass of the primary versus the mass of the companion (both in solar masses) for a system orbital period of $P = 8$ h, a spin period of $P_s = 2$ ms, $\dot{P}_s = 3 \times 10^{-19}$, and an orbital period derivative of 2.5×10^{-14} . The two lines in the figure are for NSs with EOS A (dashed line) and NSs with EOS BBB (solid line). The dotted line indicates the lower limit on the companion mass obtained if the mass function is $5 \times 10^{-3} M_{\odot}$.

of inertia of the NS in units of 10^{45}g cm^2 . The corresponding orbital period derivative, according to equation (13), becomes $\dot{P}_{\text{orb}} = 2.5 \times 10^{-14} I_{45}$. The variation in the orbital period is derived from the measure of the periastron time delay. The effect of an orbital period derivative on the periastron arrival time is given by

$$\Delta T_{\text{per}} \simeq 0.5 \frac{\dot{P}}{P} \Delta T_{\text{obs}}^2. \quad (16)$$

We find that we will need $117/I_{45}^{1/2}$ yr of observation to detect a delay of 1 s, so this effect is impractical to measure for this pulsar, although period derivatives of the same order of magnitude have been detected (see e.g. Nice, Splaver & Stairs 2004), but in pulsars with short orbital periods. Since the relativistic effect becomes dominant in pulsars with a large enough orbital period, we should look for a pulsar with a higher spin-down energy – and therefore with a higher orbital period derivative for the same orbital period – in order to measure this effect in a shorter observation time. Such a measurement is, then, unrealistic for currently known millisecond pulsars. If, however, we could find a pulsar with sufficiently high spin-down power it would become observable in a decade, becoming therefore feasible.

4 HOW ARE SHORT-PERIOD LMXBs FORMED?

Let us consider a binary system that is below the bifurcation period, and comprises a companion of $1 M_{\odot}$ and a primary with an initial gravitational mass of $1.35 M_{\odot}$ – a mass that appears to be typical of isolated NSs (Thorsett & Chakrabarty 1999). The orbital period of the binary at the onset of the mass transfer is ~ 8 h. In the standard scenario, the system transfers mass because it loses angular momentum due to the magnetic braking of the companion, at a rate large enough to push the star well out of thermal equilibrium for periods shorter than ~ 4 h. When the companion becomes fully convective (typically at an orbital period of ~ 3 h), magnetic braking is thought to stop, the companion recovers thermal equilibrium, and the mass transfer ceases (Spruit & Ritter 1983). The binary system will continue to shrink due to gravitational wave emission, until it reaches an orbital period of ~ 2 h, when mass transfer resumes. The evolution of this system in the period versus accretion-rate plane is shown in Fig. 2. We used the magnetic braking law of Verbunt & Zwaan (1981) with a braking index of 0.5. In doing this first simulation, we neglected any evolution of the NS.

What happens to the system if we take into account the evolution of the primary? The fate of the NS in the period gap happens to be very interesting. An average $\sim 0.8 M_{\odot}$ has been accreted, and the NS is therefore spinning rapidly (see Paper I), and will light up as a millisecond radio pulsar since accretion has stopped. Two effects of general relativity are important in this phase.

(i) The loss of gravitational mass from the pulsar yields an additional positive contribution to the orbital period derivative (see the preceding section). This additional effect counteracts the shrinking of the orbit due to the gravitational wave emission, thus increasing the duration of the detached phase of the system. This increase can vary strongly, depending on the spin-down energy of the primary and on q (see equation 13).

(ii) If the pulsar is supramassive (i.e. its mass exceeds the maximum non-rotating mass), it will collapse silently to a black hole, once it loses enough angular momentum.

For the NS to survive the gap, the time-scale of the collapse (T_c) must be larger than the time-scale needed for the system to ‘cross’

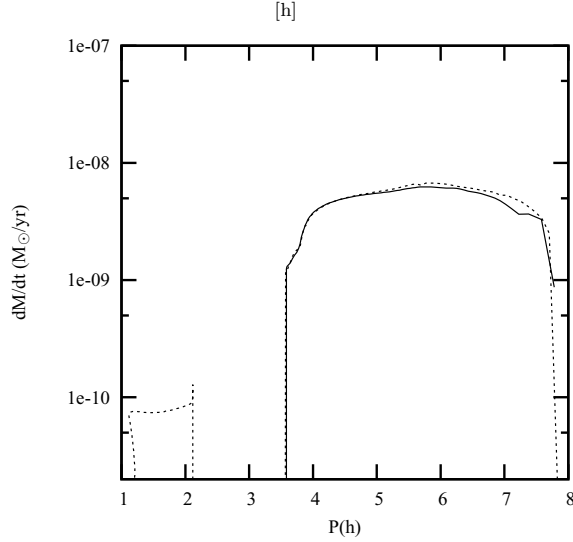


Figure 2. Mass accretion rate as a function of the orbital period for a system below the bifurcation period. The companion is a Population I star of $1 M_{\odot}$. We show the evolution of the system not taking into account the evolution of the NS (dotted line), and the evolution of the system when we include the evolution of the primary (solid line): the companion transfers most of its mass as a result of magnetic braking, passing from $P \sim 8$ h to $P \sim 3.5$ h when the star becomes fully convective and mass transfer stops. The mass transfer then resumes once the system is close enough, at $P \sim 2$ h in the system with an unevolved primary. When we take into account the evolution of the primary, the system evolves similarly to the classical one, but when the companion becomes fully convective the NS is supramassive and therefore it collapses to a black hole before accretion can resume. As can be seen from the figure, no real difference is present in the evolution above the period gap, even if the relativistic mass deficit is taken into account. For the sake of clarity, data were smoothed clean from the numerical noise.

the period gap (T_g). Otherwise, the NS will collapse to a black hole before the mass transfer resumes. This happens, for example, in our simulated system if the primary is a NS governed by EOS BBB, as can be seen in Fig. 2.

The collapse time T_c is defined by the equation

$$J_{\text{in}} - J_{\text{crit}} = - \int_0^{T_c} \dot{J} dt, \quad (17)$$

where J_{in} is the angular momentum at the beginning of the detached phase, J_{crit} is the critical angular momentum below which the star collapses, and \dot{J} is the angular momentum lost during the pulsar phase. Combining equation (4) and the formula for the energy released by a magnetodipole rotator in general relativity (Rezzolla & Ahmedov 2004), \dot{J} is given by

$$\dot{J} = - \frac{2}{3c^3} \mu^2 \omega_{\text{NS}}^3 \left(\frac{f}{N^2} \right)^2, \quad (18)$$

where

$$N = (1 - 2\chi)^{1/2}, \quad \chi = \frac{GM_G}{c^2 R_{\text{NS}}},$$

$$f = \frac{3}{8} \chi^{-3} [\log N^2 + 2\chi(1 + 2\chi)]. \quad (19)$$

The gap time T_g is defined by the equation

$$\Delta P_{\text{gap}} = - \int_0^{T_g} \dot{P} dt, \quad (20)$$

where ΔP_{gap} is the amplitude of the period gap and \dot{P} is defined in equation (13). In most situations $\Delta P_{\text{gap}} \sim 1$ h, and the mass of the companion is $\lesssim 0.25 M_{\odot}$. Integration of equations (20) and (17) shows that $T_g > T_c$ if the NS is supramassive and if the dipole magnetic field of the neutron star exceeds 10^{26} G cm³. This result holds for a vast range of EOSs, from softer ones such as EOS A to the stiffer ones such as EOS L (Arnett & Bowers 1977), including recent, realistic EOSs such as EOS FPS (Lorenz et al. 1993), EOS APR (Akmal et al. 1998) and EOS BBB (Baldo et al. 1997), as long as we assume that the mass of the NS exceeds by at least $0.02 M_{\odot}$ the maximum non-rotating mass. The result holds independently of the angular momentum of the NS at the onset of the pulsar phase. This means that LMXBs that host a NS and have a period $\lesssim 2$ h, like all the millisecond X-ray pulsars known to date, cannot be supramassive if they have a non-negligible magnetic field. This result obviously holds only if the system is a NS–MS (main sequence) binary that has evolved through the period gap.

For instance, the surface magnetic field of the first millisecond X-ray pulsar discovered, SAX J1808.4 – 3658 (Wijnands & van der Klis 1998), has been estimated to be in the range $(1-5) \times 10^8$ G (Di Salvo & Burderi 2003). If this system has a MS companion, so that it evolved from longer orbital periods, it cannot be supramassive, as it survived the period gap. The primary will not be a supramassive NS only if one of the following conditions holds.

(i) A significant part of the accreting matter has been ejected from the system, and the mass transfer has therefore been non-conservative for most of the binary evolution.

(ii) The maximum non-rotating mass of the NS is very high, $\geq 2 M_{\odot}$ (i.e. the EOS of the NS is very stiff).

5 SECULAR EVOLUTION OF SYSTEMS ABOVE THE BIFURCATION PERIOD

In the preceding section, we showed how the evolution of the compact object can alter the evolution of a system below the bifurcation period. Now we will show how the evolution of the NS can alter the secular evolution of a system above the bifurcation period. In such a system, the mass transfer is driven by the nuclear evolution of the companion. The evolution of such wide systems is well understood (Webbink et al. 1983), and is thought to explain well the formation of binary MSPs with large orbital periods.

In order to show how this canonical evolution is altered when we take into account the evolution of the primary, we simulated a binary system whose companion is a $1.1 M_{\odot}$ Population I star. The initial mass of the primary is again chosen to be $1.35 M_{\odot}$. When the mass transfer begins the orbital period of the system is 11 d.

First of all, we carried out a simulation in which we disregarded the evolution of the primary, for which we assumed $M_G = M_B$. This system is denoted in the following as system A. We also simulated a system (denoted B in the following) with the same companion star, but this time taking into account the evolution of the primary. The NS was assumed to have a low surface magnetic field of 10^8 G, and the EOS was fixed to be EOS BBB, introduced in Section 3. In Paper I we showed that a weakly magnetized NS is easily spun-up to periods well below one millisecond, uncomfortably lower than the minimum observed period for a NS of 1.56 ms (Backer et al. 1982). There is mounting evidence that some mechanism, whose nature is still unclear, prevents NSs from spinning faster than 700 Hz (Chakrabarty et al. 2003). We therefore simulated a system identical to system B, but kept the spin frequency artificially below 700 Hz. We will refer to this as system C.

As mentioned earlier, the evolution of the mass of a NS differs from that of a non-collapsed star because of the discrepancy between the gravitational and the baryonic masses in NSs. According to equation (4) the gravitational mass of the NS will be smaller if the spin frequency and the angular momentum of the star are smaller for a given baryonic mass. Therefore, we expect that the mass of the NS in system C will be smaller than that of the NS in system B, and both will be smaller than that of system A. From our simulations we find, in fact, that the gravitational mass of the NS at the end of the evolution is $1.96 M_{\odot}$ in system A, $1.88 M_{\odot}$ (4 per cent less) in system B, and $1.79 M_{\odot}$ (9 per cent less) in system C. The higher mass deficit in system C is due both to the smaller angular momentum of the star (this is almost irrelevant, since in a neutron star the rotational energy is usually ~ 0.1 times the binding energy) and to the fact that when the NS spins more slowly it is more compact and therefore its binding energy is higher.

It is interesting to note that the evolution of the NS has other effects on the evolution of the binary system. We know that the final state of such systems depends heavily only on the mass of the inner core of the companion star (Webbink et al. 1983). Looking at equation (12) we see that the relativistic effect is dominant during the first phases of accretion, when $q \sim 1$: when matter is transferred, a relativistic system enlarges faster than a non-relativistic one. Consequently, the Roche lobe of the companion is larger, and the mass accretion rate will become smaller. As a consequence, relativistic systems have a longer time to evolve, and the mass of the core becomes slightly bigger: at the end of accretion, the companion in system A has a core of $0.305 M_{\odot}$. In system B the companion has a core that is $0.003 M_{\odot}$ heavier than in system A, while in system C, the core is $0.006 M_{\odot}$ heavier. As a consequence (see again Webbink et al. 1983), the orbital period at the end of the accretion process is 105 d in system A, 114.5 d (~ 9 per cent longer) in system B and 124 d (~ 18 per cent longer) in system C (as can be seen in Fig. 3).

This means that the net effect on the orbital period evolution of the system is stronger than the effect on the mass alone, but this is due to the small changes in the final core mass and not directly to the third term on the right-hand side of equation (12). Small changes in the

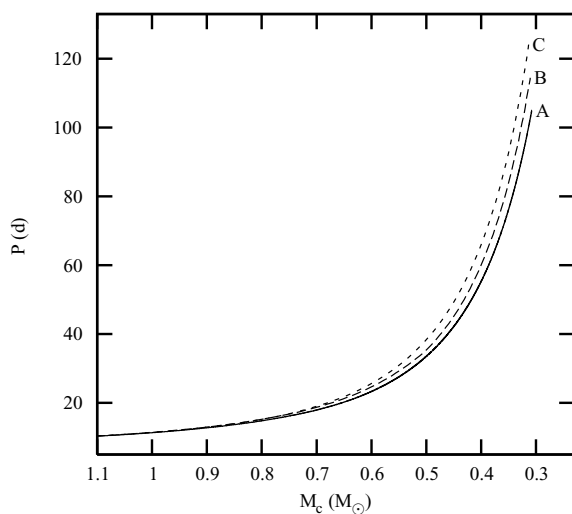


Figure 3. Orbital period (in days) of the binary system as a function of the mass of the companion (in solar masses). Letters indicate the evolutionary tracks for systems A, B and C. The evolution is followed only until the companion collapses to a white dwarf. The x -axis is inverted, so that the evolution of the systems goes from left to right.

mass of the core also account for significant variations in the orbital period at the end of mass transfer: this means that, although the effects we describe can change the evolution of a system from given initial conditions, they do not alter the scenario of the evolution of such systems.

6 CONCLUSIONS

In this paper we have shown that the presence of a NS, which can be described using general relativity, can have a big impact on the evolution of the binary system.

First of all we searched for potentially observable effects of the relativistic nature of the primary. We noticed that when the NS releases energy without accreting during the pulsar phase, it will lose gravitational mass. Therefore, a positive contribution to the orbital period derivative can dominate in certain situations over the negative contribution due to gravitational wave emission, resulting in an overall *positive* orbital period derivative. We have shown that the measurement of such a period derivative in a binary millisecond pulsar can allow us to put constraints on the EOS of ultradense matter on a solid observational basis.

Next we concentrated on the effects that the secular evolution of the primary can have on the evolution of binary systems, both below and above the bifurcation period. In systems starting below the bifurcation period a necessary condition for a NS to survive the period gap without collapsing to a black hole is for it to be non-supramassive. This means that the NSs in LMXBs with short periods, such as the millisecond X-ray pulsar SAX J1808.4 – 3658, cannot be supramassive if they evolved from longer periods. This implies either that the mass transfer is highly non-conservative or that the EOS of ultradense matter is stiff.

In systems starting above the bifurcation period, we have shown how the relatively small effect of general relativity on the orbital evolution can alter the evolution of the companion; this means that the total effect can be non-negligible. However these effects do not change the standard scenario for the evolution of large-period systems in any significant way.

In all, the relativistic mass deficit has some effect on the evolution of binary systems, but it is negligible if compared with the uncertainties that exist on other effects present in the standard theory of binary evolution. For example, various laws have been proposed for the magnetic braking mechanism, and the intensity of the effect has been questioned (see e.g. Ivanova & Taam 2003). However, the evolution of the primary – and the resulting mass deficit – can be important in studying the evolution of binary systems, both because it can give rise to an observable effect in MSPs and because it puts constraints on the evolution of systems below the bifurcation period, and therefore cannot be disregarded in evolutionary studies.

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