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A "dual" model of a massive spin-$\frac{1}{2}$ point particle, and a theoretical explanation for the effect of "maximal parity-violation"

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The open question left by elementary particle physics about the origin of the so-called "maximal parity-violation" is dealt with. It is argued that a straightforward theoretical answer could be found within a new relativistic quantum field formalism being a strictly covariant fermion–antifermion extension of the usual one for massive fermions. This formalism can account naturally for the effect at issue, in such a way as even to restore both parity and charge-conjugation symmetries: it spontaneously provides a true "chiral field" approach, which points to the existence of a pseudoscalar (extra) charge variety anticommuting with the scalar (ordinary) one and just underlying the "maximally parity-violating" phenomenology. A "dual" − either "Dirac" or "chiral" − fermion–antifermion model may be introduced accordingly, which redepicts any massive spin-$\frac{1}{2}$ point fermion and related antifermion as two particles being in turn able to manifest themselves like a sheer pair of scalar- or pseudoscalar-charge conjugated eigenstates, with ordinary mirror symmetry being truly respected in either case. The zero-mass limit would bring this internal duality to its extreme consequences: what would be left is a strict (one-helicity) "chiral" particle model, which universally redefines a massless spin-$\frac{1}{2}$ fermion and its own antifermion as two mere pseudoscalar-charge eigenstates being (by nature) the mirror images of each other and at most carrying scalar (additional) charges bound to a maximal uncertainty in sign. Such an outcome may be expected to lead to a better understanding of the origin itself of fermion masses, since it can be stated that only by acquiring a mass, and by gaining an extra helicity freedom degree, a fermion is further enabled to appear as a "Dirac" particle (i.e., an actual scalar-charge eigenstate).

1   Introduction

As is well-known, what underlies the "maximally parity-violating" weak phenomenology of fermions[1,2] is certainly one of the most intriguing basic questions left unanswered by the physics of the past century.[3] The Standard Model itself,[4−6] despite its undoubted achievements, can only allow for that peculiar kind of phenomenology by means of suitable *ad hoc* prescriptions such as the $V - A$ approach[7−9] and the neutrino two-component scheme,[10−13] without being able to provide any theoretical justification for such an "oddness" of Nature.

In this paper it is stressed that a fundamental insight into the "maximal parity-violation" effect can simply be got if one relies upon a strictly covariant fermion–antifermion generalization of the usual relativistic quantum field theory for massive spin-$\frac{1}{2}$ fermions. On passing to the enlarged formalism in question,[14] an extra, *pseudoscalar* variety of charges naturally emerges (by "charge," of course, any single additive internal quantum number is here meant) which has the remarkable property of *anticommuting* with the customary, *scalar* variety. This corresponds to the gained possibility of rigorously introducing also a "chiral" (besides the usual "Dirac") type of massive spin-$\frac{1}{2}$ fields, within an overall fermion–antifermion scheme that may include, on the *same* footing, both a *covariant* pair of "scalar-charge conjugated" Dirac fields and a *covariant* pair of "pseudoscalar-charge conjugated" chiral fields. Such a formalism naturally points to a *dual* (either "Dirac" or "chiral") model of a massive fermion–antifermion pair, according to whether the scalar or pseudoscalar charge variety is temporarily *superselected*. A model like this − founded on the internal coexistence of two *anticommuting* charge varieties, and formally supported by the replacement of the (ad hoc) Dirac-field "$V - A$" current with a (natural) chiral-field *pure* vector current − can account for the "maximally P-violating" phenomenology in such a way as to achieve a paradoxical *recovery* of P symmetry itself: it enables one to reread the well-known CP mirror symmetry inherent in the "$V - A$" formalism as just *a P mirror symmetry between fermions and antifermions that are momentarily looking like net pseudoscalar-charge, rather than net scalar-charge, conjugated particles.* The extreme consequences can be found in the zero-mass limit: what is automatically obtained is still a P-symmetric scheme in which, however, a massless spin-$\frac{1}{2}$ fermion and its own antifermion can only be thought of as two strict "chiral" particles, each one being just the *ordinary* (i.e., helicity-conjugated) mirror image of the other. This limiting scheme,

consisting of two mere *chirality-conjugated* (fermion and antifermion) two-component models, has already been shown both to be fully compatible with the $SU(2)_L \otimes U(1)$ electroweak formulation − provided that the "spurious" (right-handed) *fermion* $SU(2)_L$-singlets added to fermion $SU(2)_L$-doublets are properly reread as *antifermion* $SU(2)_L$-singlets − and to allow an equivalent $SU(2)_R \otimes U(1)$ formulation with, conversely, *fermion* $SU(2)_R$-singlets added to *antifermion* $SU(2)_R$-doublets.[15,16]

2   A natural, pure theoretical approach to the "maximal parity-violation" effect, with parity and charge-conjugation symmetries paradoxically recovered

We know that the Dirac quantum field formalism cannot provide a one-particle relativistic description: the associated Fock space is necessarily the sum of two pure positive-energy Fock spaces − referring (in Dirac's language) to "particles" and "holes" respectively − which are taken into each other by a suitable operation of "particle" $\rightleftharpoons$ "hole" conjugation. We also know that a (manifestly covariant) one-particle description may all the same be recovered, once the Stückelberg–Feynman general approach to the negative-energy problem is adopted:[17] the "hole" motion can then be reread as a negative-energy "particle" motion, *backwards* in time, and the Fock space above can likewise be recast as a *single* one for "particles" only, with energies now covariantly running over the *entire* spectrum of positive and negative eigenvalues. Thanks to this improved view, the "particle–hole" language has clearly lost its original motivations; yet, the use of such a language may still turn out to be convenient, if one wants to make somehow a distinction between a "fermion" picture (in which, i.e., one has "particle" = fermion and "hole" = antifermion) and an "antifermion" picture (in which, i.e., one conversely has "particle" = antifermion and "hole" = fermion).

Let $\mathcal{F}^\circ$ denote the (manifestly covariant) Stückelberg–Feynman Fock space in question, and let it particularly stand for a "fermion" Fock space (where, i.e., "particle" = fermion). This definition of $\mathcal{F}^\circ$ might lead one to wonder whether a *covariant* charge-conjugation operation can be introduced too, which may be able to turn $\mathcal{F}^\circ$ into another space being an "antifermion" Fock space (where, i.e., "particle" = antifermion). Such an operation should have the effect of *transforming a fermion as taken in its whole* (positive- and negative-energy) *spectrum, into the corresponding* (positive- and negative-energy) *antifermion*; it should then be, in principle, *not* just the same as the "particle" $\rightleftharpoons$ "hole"

conjugation (which, on the contrary, is a mere *non*covariant operation interchanging positive-definite-energy objects). The trouble is, however, that $\mathcal{F}^\circ$ can *itself* be recast as an "antifermion" Fock space, the reason being because, according to the Stüeckelberg–Feynman views, a complete set of $\mathcal{F}^\circ$ kets (bras) for fermions will clearly amount to a complete set of $\mathcal{F}^\circ$ bras (kets) for antifermions. So, at first sight, trying to define a covariant charge-conjugation operation would seem to be a trivial matter, since we cannot think of any *further* Fock space being the "covariant charge conjugate" of $\mathcal{F}^\circ$. This just corresponds to the fact that, in a symmetrized "particle"–"hole" standard description, one may indifferently put *either* "particle" = fermion (and "hole' = antifermion) *or* "particle" = antifermion (and "hole" = fermion).

Actually, the question can be set anew with the help of a careful re-examination of the Stüeckelberg–Feynman approach from a classical viewpoint. Let $-p^\mu = m(-u^\mu)$ ($\mu = 0, 1, 2, 3$; metric: $+ - --$) be the four-momentum of a negative-energy particle of proper (i.e., covariant) mass m (> 0) and four-velocity $-u^\mu = -dx^\mu/ds$ ($-dx^0 < 0$). Since the equivalent positive-energy antiparticle, with four-momentum p^μ, is merely going along the same world-line in the *opposite* direction, $ds \to -ds$, one has that the "slope" $-u^\mu$ of that world-line will be left *unaltered* by the Stüeckelberg–Feynman procedure, $(-dx^\mu)/ds = dx^\mu/(-ds)$. Strictly speaking, it should therefore be claimed that in replacing $-p^\mu$ for the particle with p^μ for the antiparticle, a net change of the *proper-mass* sign is only involved, $-p^\mu \to p^\mu \Longrightarrow m \to -m$. This does not appear to be a very surprising result: due to the *quadratic* character of the energy–momentum relationship $E^2 = \mathbf{p}^2 + m^2$ ($c = 1$), we may clearly associate both energy roots $\pm E$ with the same (positive) proper mass m, but we may just as well associate both proper-mass roots $\pm m$ with the same (positive) energy E. The fact is that the relative sign of energy and proper mass does depend on either sign of the time component $\pm u^0$ of four-velocity. So, the assignment of a proper mass $-m$ to the antiparticle cannot be said at all to clash with the "CPT theorem:" what *rigorously* follows from the validity of "CPT symmetry" is that a particle and its antiparticle must have identical *rest energies*, which does only mean that m^2, and *not m* itself, must be equal for them both. If these considerations are in particular applied to fermions, then it may be stated that a Dirac fermion and the related antifermion can *covariantly* be distinguished by the (opposite) signs of their proper masses. This really enables one to think also of a *non*trivial "covariant charge-

conjugation" operation for fermions themselves: it should just coincide with the pure internal operation of *proper-mass reversal*[18−21] (leaving both four-momentum and helicity unvaried). On the other hand, it is evident that the proper-mass sign in the Dirac equation is immaterial, so that the inversion of it (with both four-momenta and helicities left unchanged) has no effects on $\mathcal{F}^\circ$ states: this is also the reason why the usual (noncovariant) operation of "particle"$\rightleftharpoons$"hole" conjugation can all the same be defined without affecting proper mass. One is thereby led to conclude that the space $\mathcal{F}^\circ$ *as such* could not be an adequate Fock space for allowing proper-mass reversal to behave like an *actual* "covariant charge-conjugation" operation. What would really be needed is some *enlarged* Fock space that may be able also to make an *explicit* formal distinction between a "fermion" covariant Dirac picture, marked by the *positive* root of m^2, and an "antifermion" one, marked by the *negative* root of m^2. In other words, we have to *double* $\mathcal{F}^\circ$ by giving it some "label" that may actually tell us which one of the above alternative pictures is being considered.

For this purpose, it turns out appropriate to introduce two (orthogonal) unit internal state-vectors, $|\,f\,\rangle$ and $|\,\bar{f}\,\rangle$, which are eigenvectors of a (one-particle) proper-mass operator, M, with eigenvalues $+m$ and $-m$:

$$M|\,f\,\rangle = +m\,|\,f\,\rangle \ , \ M|\,\bar{f}\,\rangle = -m\,|\,\bar{f}\,\rangle. \tag{1}$$

Let S_{in} be the two-dimensional internal space that is spanned by such eigenvectors. We may then say that a "dressed" Fock space $\mathcal{F}$ can be built from the "bare" one $\mathcal{F}^\circ$, being such that

$$\mathcal{F} = \mathcal{F}^\circ \otimes S_{\mathrm{in}}. \tag{2}$$

As an effect of a "dressing" procedure like this, the starting complete set of $\mathcal{F}^\circ$ kets (bras) appears to be *doubled* into a "Dirac fermion" set, covariantly labeled by $|\,f\,\rangle$ ($\langle\,f\,|$), plus a "Dirac antifermion" one, covariantly labeled by $|\,\bar{f}\,\rangle$ ($\langle\,\bar{f}\,|$), with an energy range still including, in either case, both positive and negative eigenvalues. In such a framework, we may strictly think of a "covariant charge-conjugation" as represented by a unitary and Hermitian operator, C_{cov}, properly acting in S_{in} and trivially behaving (just like an identity operator) in $\mathcal{F}^\circ$:

$$C_{\mathrm{cov}}|\,f\,\rangle = |\,\bar{f}\,\rangle \ , \ C_{\mathrm{cov}}|\,\bar{f}\,\rangle = |\,f\,\rangle \quad (C_{\mathrm{cov}}^{-1} = C_{\mathrm{cov}}^\dagger = C_{\mathrm{cov}}). \tag{3}$$

We clearly have that C_{cov} anticommutes with M, in line with the fact that it primarily works as a *proper-mass conjugation* operator. By the way, note that throughout this paper, to avoid misunderstandings, the sheer symbol C is still used *just* to refer to the ordinary (noncovariant) charge-conjugation, while the self-explaining new symbol C_{cov} is expressly utilized to refer to the "covariant" charge-conjugation in question.

The Fock-space *doubling* brought in by (2) has a meaning which can be conveniently rendered in the "particle–hole" language: it leads to a generalized description distinctly including *both* of the equally admissible Dirac pictures that can be obtained by choosing either "particle" = fermion (and "hole" = antifermion) or "particle" = antifermion (and "hole" = fermion). These − the former amounting to a pure *fermion* picture (with both positive and negative energies) and the latter to a pure *antifermion* picture "covariantly conjugated" to the *fermion* one − are just distinguished by the (discordant) sign of proper mass. So, a pair of Dirac free-field equations like

$$i\gamma^{\mu}\partial_{\mu}\psi_f = +m\,\psi_f \;\; , \;\; i\gamma^{\mu}\partial_{\mu}\psi_{\bar{f}} = -m\,\psi_{\bar{f}} \tag{4}$$

($\hbar = c = 1$; $\gamma^{0\dagger} = \gamma^0, \gamma^{k\dagger} = -\gamma^k, k = 1, 2, 3$) is to be associated with them, where $\psi_{\bar{f}}$ should consistently stand for the *proper-mass conjugate* counterpart of ψ_f. Let $u_f(\mathbf{p})$ and $u_{\bar{f}}(\mathbf{p})$ be, accordingly, two positive-energy eigenspinors satisfying the equations

$$\gamma^{\mu}p_{\mu}u_f = +m\,u_f \;\; , \;\; \gamma^{\mu}p_{\mu}u_{\bar{f}} = -m\,u_{\bar{f}}\,. \tag{5}$$

For $\mathbf{p} = 0$, one clearly obtains

$$\gamma^0\,(+m)\,u_f(0) = E\,u_f(0) \;\; , \;\; \gamma^0\,(-m)\,u_{\bar{f}}(0) = E\,u_{\bar{f}}(0)\,, \tag{6}$$

where E (> 0) is the related energy eigenvalue, such that $E = m$. If this substitution is made, it is immediate then to see that

$$\gamma^0\,u_f(0) = u_f(0) \;\; , \;\; \gamma^0\,u_{\bar{f}}(0) = -u_{\bar{f}}(0)\,. \tag{7}$$

One therefore has that due to the discordant signs of the associated proper masses, the opposite-intrinsic-parity requirement for the eigenspinors $u_f(\mathbf{p})$ and $u_{\bar{f}}(\mathbf{p})$ can be automatically fulfilled by applying *one and the same* parity matrix (say, γ^0) to them both. This is not the

case of two mere "particle" and "hole" conjugated eigenspinors, which are coincident solutions of just either one of Eqs. (5) and then require two *discordant* parity representations (say, γ^0 and $-\gamma^0$, respectively) for them to be assigned opposite intrinsic parities. As both of the field equations (4) are in turn allowable in the framework of the "bare" Fock space $\mathcal{F}^\circ = \mathcal{F}^\circ(|m|)$, we may build on the whole a *double*-structured, "undressing" field operator, $\Psi(x)$ ($x \equiv x^\mu$), which just reduces to $\psi_f(x)$ or $\psi_{\bar{f}}(x)$ according to whether applying to $\mathcal{F}^\circ$ states coupled to $|f\rangle$ or $|\bar{f}\rangle$: it looks like

$$\Psi(x) = \psi_f(x)\langle f| + \psi_{\bar{f}}(x)\langle \bar{f}|, \tag{8}$$

and obeys the *generalized Dirac equation*

$$i\gamma^\mu \partial_\mu \Psi(x) = \Psi(x)M, \tag{9}$$

M being the proper-mass operator defined by (1). This can be strictly said to be a *covariant fermion–antifermion field*. Besides being still a Lorentz four-spinor, it is also a (bra) *vector* in the internal space S_{in}, with $\psi_f(x)$ and $\psi_{\bar{f}}(x)$ correspondingly acting as its *components* relative to the orthonormal basis $(\langle f|, \langle \bar{f}|)$. Let $\psi_f(x)$ and $\psi_{\bar{f}}(x)$ be in particular called the "Dirac" (fermion and antifermion) components of $\Psi(x)$ in S_{in}, the former annihilating (either positive or negative energy) "Dirac" fermion states covariantly marked by $|f\rangle$, and the latter annihilating (either positive or negative energy) "Dirac" antifermion states covariantly marked by $|\bar{f}\rangle$. If we take account of (3), we can also define the C_{cov} counterpart of $\Psi(x)$ as

$$\Psi^{(C_{\text{cov}})}(x) \equiv \Psi(x)C_{\text{cov}} = \psi_f(x)\langle \bar{f}| + \psi_{\bar{f}}(x)\langle f| \tag{10}$$

and introduce the adjoints of Ψ and $\Psi^{(C_{\text{cov}})}$, such that

$$\bar{\Psi}(x) = |f\rangle \bar{\psi}_f(x) + |\bar{f}\rangle \bar{\psi}_{\bar{f}}(x) \tag{11}$$

($\bar{\psi} = \psi^\dagger \gamma^0$) and $\bar{\Psi}^{(C_{\text{cov}})}(x) \equiv C_{\text{cov}}^\dagger \bar{\Psi}(x)$. From a comparison, e.g., of (10) with (8), it is immediate to realize that applying C_{cov} to both $\Psi(x)$ and $\bar{\Psi}(x)$ can equivalently be implemented by prescribing

$$C_{\text{cov}}: \quad \psi_f(x) \rightleftharpoons \psi_{\bar{f}}(x), \quad \bar{\psi}_f(x) \rightleftharpoons \bar{\psi}_{\bar{f}}(x). \tag{12}$$

This just leads us to state that $\psi_{\bar{f}}(x)$ should be *covariantly* obtained from $\psi_f(x)$ by simply demanding the proper-mass reversal $m \rightarrow -m$ in the Dirac equation obeyed by $\psi_f(x)$. So, after all, one may write (apart from a phase factor):

$$\psi_{\bar{f}}(x) = \gamma^5 \psi_f(x) \quad , \quad \bar{\psi}_{\bar{f}}(x) = -\bar{\psi}_f(x)\gamma^5 \tag{13}$$

($\bar{\psi} = \psi^\dagger \gamma^0$; $\gamma^5 \equiv i\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3$). Such an outcome clearly calls for some explanatory comments. In view of (13) − and in accordance with the fact that C_{cov} is essentially defined in S_{in} − one has that the Fourier expansions of $\psi_f(x)$ and $\psi_{\bar{f}}(x)$ will share a *unique* type of "particle" annihilation operators, say, $a(\mathbf{p}, \sigma)$, as well as a *unique* type of "hole" creation operators, say, $a^{\text{h}\dagger}(\mathbf{p}, \sigma)$ (σ being the helicity variable). This appears to be admissible, for the simple reason that $\psi_f(x)$ and $\psi_{\bar{f}}(x)$ belong to two *alternative* pictures − marked by $|f\rangle$ and $|\bar{f}\rangle$, respectively − each being *independently* able to describe the creation or annihilation of a "particle"–"hole" pair (though with an interchange of what is meant by "particle" and "hole" in the other picture): according to whether $\psi_f(x)\langle f|$ or $\psi_{\bar{f}}(x)\langle\bar{f}|$ is in turn working, the same "particle" annihilation operator a ("hole" creation operator $a^{\text{h}\dagger}$) may well be assumed to annihilate a positive-energy *fermion* or *antifermion* (create a positive-energy *antifermion* or *fermion*) without any actual possibility of interference. The point is that both "particle" annihilation (creation) and "hole" creation (annihilation) operators can be defined *regardless of* whether "particle" ("hole") may just be referring to a fermion (antifermion) or an antifermion (fermion): as an example, if $|1_{\mathbf{p},\sigma}\rangle$ denotes an occupied "particle" state, then only $|1_{\mathbf{p},\sigma}\rangle\,|f\rangle$ will now be specifically standing for an occupied positive-energy (Dirac) *fermion* state. Such an "ambivalence" can be made more explicit by setting

$$a(\mathbf{p}, \sigma) = a(\mathbf{p}, \sigma; |m|) \quad , \quad a^{\text{h}\dagger}(\mathbf{p}, \sigma) = a^{\text{h}\dagger}(\mathbf{p}, \sigma; |m|) \tag{14}$$

for both $\psi_f(x)$ and $\psi_{\bar{f}}(x)$, so that one symmetrically obtains

$$\psi_f(x) \equiv \psi(x; m) \quad , \quad \psi_{\bar{f}}(x) \equiv \psi(x; -m) \tag{15}$$

(as just a direct expression of the fact that C_{cov} behaves in $\mathcal{F}^\circ$ like an identity). Of course, $\psi_{\bar{f}}(x)$ has nothing to do with the charge-conjugate field that can (noncovariantly) be obtained from $\psi_f(x)$ by applying the usual operation of "particle" $\rightleftharpoons$ "hole" conjugation (i.e., $a \rightleftharpoons a^{\text{h}}$, $a^{\text{h}\dagger} \rightleftharpoons$

$a^\dagger$): this latter charge-conjugate field type is independently definable both for $\psi_f(x)$ and $\psi_{\bar{f}}(x)$ itself, and can be encountered just *within* either single picture above, as a result of normal ordering. Also, note that due to the *covariant* eigenvalues m and $-m$ marking the pictures in question, an *unambiguous* distinction between "particle" and "hole" now follows the choice of either picture: for instance, if the "fermion" Dirac picture (marked by m) is adopted, then the "hole" label is automatically left assigned to the antifermion, and no ambiguity can arise even when normal ordering is applied. Allowing for (13), one may compactly write

$$\Psi^{(C_{\mathrm{cov}})}(x) = \gamma^5 \Psi(x) \; , \quad \bar{\Psi}^{(C_{\mathrm{cov}})}(x) = -\bar{\Psi}(x)\gamma^5 \,. \tag{16}$$

These equations strictly define the role now played by γ^5 *as the γ-matrix representing C_{cov}*. Note, on the other hand, that from the requirement of invariance of (9) under space inversion $x^\mu \to x_\mu$, one may still infer the P mirror counterparts of Ψ and $\bar{\Psi}$ as those fields looking (apart from phase factors) like

$$\Psi^{(P)}(x_\mu) = \gamma^0 \Psi(x^\mu) \, , \quad \bar{\Psi}^{(P)}(x_\mu) = \bar{\Psi}(x^\mu)\gamma^0 \,. \tag{17}$$

To see the real advantages of this (apparently redundant) formalism, a better insight into the properties of the internal space S_{in} is needed. Consider the new S_{in} basis obtained from the "Dirac" one $\big(|f\rangle, |\bar{f}\rangle\big)$ by carrying out the rotation

$$|f\rangle = \frac{1}{\sqrt{2}}\big(|f^{\mathrm{ch}}\rangle + |\bar{f}^{\mathrm{ch}}\rangle\big) \; , \quad |\bar{f}\rangle = \frac{1}{\sqrt{2}}\big(-|f^{\mathrm{ch}}\rangle + |\bar{f}^{\mathrm{ch}}\rangle\big). \tag{18}$$

The peculiar feature of such a basis is that C_{cov} is made *diagonal* in it:

$$C_{\mathrm{cov}}|f^{\mathrm{ch}}\rangle = -|f^{\mathrm{ch}}\rangle \; , \quad C_{\mathrm{cov}}|\bar{f}^{\mathrm{ch}}\rangle = |\bar{f}^{\mathrm{ch}}\rangle \,. \tag{19}$$

In view of (16), the C_{cov} eigenvalues may just be said to provide the "chiralities" of the associated (either positive- or negative-energy) Fock states covariantly labeled by $|f^{\mathrm{ch}}\rangle$ and $|\bar{f}^{\mathrm{ch}}\rangle$. A similar (unitary and Hermitian) operator, say, P_{in}, can clearly be introduced in S_{in}, which, vice versa, is diagonal in the basis $\big(|f\rangle, |\bar{f}\rangle\big)$ and has the property of interchanging $|f^{\mathrm{ch}}\rangle$ and $|\bar{f}^{\mathrm{ch}}\rangle$:

$$P_{\mathrm{in}}|f^{\mathrm{ch}}\rangle = |\bar{f}^{\mathrm{ch}}\rangle \; , \quad P_{\mathrm{in}}|\bar{f}^{\mathrm{ch}}\rangle = |f^{\mathrm{ch}}\rangle \quad (P_{\mathrm{in}}^{-1} = P_{\mathrm{in}}^\dagger = P_{\mathrm{in}}). \tag{20}$$

Since

$$P_{\text{in}}|f\rangle = |f\rangle \ , \ \ P_{\text{in}}|\bar{f}\rangle = -|\bar{f}\rangle \,, \tag{21}$$

it is appropriate to interpret P_{in} (apart from a phase constant $\eta = \pm 1$) as an "internal parity" covariant operator. As far as the only positive-energy spectrum is concerned, the P_{in} eigenvalues drawn from (21) may well be assumed to reproduce the *intrinsic parities* (i.e., the zero-momentum P eigenvalues) of the Dirac fermion and antifermion. Such a coincidence can no longer be pursued when also the negative-energy spectrum is included, since the intrinsic parity of a spin-$\frac{1}{2}$ particle (unlike the "internal parity" of it) is not a strict *covariant* eigenvalue and changes sign on passing to negative energies. The fact is that parity P is now to be taken as an operator defined in the whole Fock space (2), with an "external" representation, P_{ex}, properly acting on $\mathcal{F}^\circ$ vectors, and an "internal" one, P_{in}, properly acting on S_{in} vectors: one should write, e.g.,

$$\Psi^{(P)}(x_\mu) = P_{\text{ex}}^\dagger \psi_f(x_\mu) P_{\text{ex}} \langle f|P_{\text{in}} \ + \ P_{\text{ex}}^\dagger \psi_{\bar{f}}(x_\mu) P_{\text{ex}} \langle \bar{f}|P_{\text{in}} \,, \tag{22}$$

where a comparison with (17) and (21) shows (in full accordance with the anticommutation relation $\gamma^0\gamma^5 + \gamma^5\gamma^0 = 0$) that

$$P_{\text{ex}}^\dagger \psi_f(x_\mu) P_{\text{ex}} = \gamma^0 \psi_f(x^\mu) \ , \ \ P_{\text{ex}}^\dagger \psi_{\bar{f}}(x_\mu) P_{\text{ex}} = -\gamma^0 \psi_{\bar{f}}(x^\mu) \,. \tag{23}$$

On passing to the new basis $\left(|f^{\text{ch}}\rangle, |\bar{f}^{\,\text{ch}}\rangle \right)$, which may be called the "chiral" basis in S_{in}, the field $\Psi(x)$ and its adjoint will read

$$\Psi(x) = \chi_f(x) \langle f^{\text{ch}}| + \chi_{\bar{f}}(x) \langle \bar{f}^{\,\text{ch}}| \ , \ \ \bar{\Psi}(x) = |f^{\text{ch}}\rangle \bar{\chi}_f(x) + |\bar{f}^{\,\text{ch}}\rangle \bar{\chi}_{\bar{f}} \tag{24}$$

$(\bar{\chi} = \chi^\dagger \gamma^0)$ with

$$\chi_f(x) \equiv \frac{1}{\sqrt{2}}(1 - \gamma^5)\psi_f(x) \ , \ \ \chi_{\bar{f}}(x) \equiv \frac{1}{\sqrt{2}}(1 + \gamma^5)\psi_{\bar{f}}(x) \tag{25}$$

and

$$\psi_f = \frac{1}{\sqrt{2}}(\chi_f \ + \ \chi_{\bar{f}}) \ , \ \ \psi_{\bar{f}} = \frac{1}{\sqrt{2}}(-\chi_f \ + \ \chi_{\bar{f}}) \,. \tag{26}$$

So, in the enlarged framework provided by (2), two (massive) "chiral fields," χ_f and $\chi_{\bar{f}}$, can spontaneously be introduced, just having *opposite* chiralities and being *on the same footing* as the two "covariant charge-conjugated" Dirac fields ψ_f and $\psi_{\bar{f}}$. They may themselves be said to be

"covariantly conjugated" to each other, but with P_{in} taking the place of C_{cov}: if (20) is taken into account, then, by an inspection of (24), it is immediate to see that applying P_{in} to both Ψ and $\bar{\Psi}$ can equivalently be accomplished by prescribing

$$P_{\text{in}} : \quad \chi_f(x) \;\rightleftharpoons\; \chi_{\bar{f}}(x) \;, \;\; \bar{\chi}_f(x) \;\rightleftharpoons\; \bar{\chi}_{\bar{f}}(x) \,. \tag{27}$$

It turns out evident, on the other hand, that C_{cov} is conversely acting *as if*

$$C_{\text{cov}} \;: \quad \begin{cases} \chi_f(x) \to -\chi_f(x) \,, \, \chi_{\bar{f}}(x) \to \chi_{\bar{f}}(x) \\ \bar{\chi}_f(x) \to -\bar{\chi}_f(x) \,, \, \bar{\chi}_{\bar{f}}(x) \to \bar{\chi}_{\bar{f}}(x) \,. \end{cases} \tag{28}$$

The general meaning of (27) can be gathered with the help of (13). If we take, e.g., the *intrinsic* mirror counterparts of χ_f and $\chi_{\bar{f}}$, defined as their respective chirality-conjugate versions

$$\xi_f(x) \equiv \frac{1}{\sqrt{2}}(1 + \gamma^5)\psi_f(x) \;, \;\; \xi_{\bar{f}}(x) \equiv \frac{1}{\sqrt{2}}(1 - \gamma^5)\psi_{\bar{f}}(x) \,, \tag{29}$$

we see that

$$\xi_f(x) = \chi_{\bar{f}}(x) \;, \;\; \xi_{\bar{f}}(x) = -\chi_f(x) \,. \tag{30}$$

Thus, no further independent pair of chiral fields can be obtained by chirality inversion; and in view of (30) it may be argued that χ_f ($\bar{\chi}_f$) and $\chi_{\bar{f}}$ ($\bar{\chi}_{\bar{f}}$) *themselves* are merely acting as the intrinsic mirror counterparts of each other.

All that *a fortiori* makes sense in the zero-mass limit, which clearly reduces both (fermion and antifermion) Dirac fields (26) to simple mixtures of a pure *left-handed* fermion and a pure *right-handed* antifermion (chiral) field.[22,16] Under such special circumstances, only the two fields χ_f ($\bar{\chi}_f$) and $\chi_{\bar{f}}$ ($\bar{\chi}_{\bar{f}}$) (actually being the *pure chirality-conjugates* of each other) may thus be said still to represent a pair of mutually "charge-conjugated" spin-$\frac{1}{2}$ fields.

By virtue of (26), the "maximally P-violating" *Dirac-field $V - A$ current* apparently entering into both lepton and quark pure weak phenomenologies[23] can now find a quite *natural* theoretical room in the straightforward form of a *chiral-field V current*: using the subscripts a, b to specify the point-fermion types involved in the current, one obtains

$$\bar{\psi}_{\text{b}}\gamma^\mu(1 - \gamma^5)\psi_{\text{a}} \equiv \bar{\chi}_{\text{b}}\gamma^\mu\chi_{\text{a}} \,. \tag{31}$$

A similar conclusion could be drawn also for an equivalent $V + A$ current in terms of the respective "covariant charge-conjugate" Dirac antifermion fields $\psi_{\bar{\text{a}}} = \gamma^5 \psi_{\text{a}}$ and $\bar{\psi}_{\bar{\text{b}}} = -\gamma^5 \bar{\psi}_{\text{b}}$:

$$\bar{\psi}_{\bar{\text{b}}} \gamma^\mu (1 + \gamma^5) \psi_{\bar{\text{a}}} \equiv \bar{\chi}_{\bar{\text{b}}} \gamma^\mu \chi_{\bar{\text{a}}}. \tag{32}$$

So, a pure weakly-interacting point fermion may strictly be referred to as a "chiral" fermion, on which, in view of (27), P_{in} itself will play a "covariant charge-conjugation" role quite similar to the one played by C_{cov} on a "Dirac" fermion: it will give rise to a new, equally allowable chiral-particle description (covariantly conjugated to the starting one) where the original associations "particle" = fermion and "hole" = antifermion appear to be interchanged. This seems even to lead to a *recovery* of P symmetry, as follows from the fact that either in the new covariant form (31) or (32) the parity matrix γ^0 is to be applied *directly* to χ and $\bar{\chi}$, rather than (as usual) to ψ and $\bar{\psi}$:

$$P : \quad \chi(x^\mu) \to \gamma^0 \chi(x_\mu) \ , \ \ \bar{\chi}(x^\mu) \to \bar{\chi}(x_\mu) \gamma^0 \,. \tag{33}$$

More precisely, setting − as prescribed by (2) − $P = P_{\text{in}} P_{\text{ex}} \ (= P_{\text{ex}} P_{\text{in}})$ (P_{ex} standing for the "external" parity, properly defined in $\mathcal{F}^\circ$, and P_{in} for the "internal" parity, properly defined in S_{in}) one can see that the peculiar left–right spatial asymmetry shown by the pure weak couplings is here accounted for as a mere (maximal) P_{ex} violation, which, suitably combined with a (maximal) P_{in} violation, does not prevent P itself from being left, on the whole, a symmetry operation. A simple comparison of (27) with (33), via (30) and (29), reveals that *applying P_{ex} to a chiral-field V current is quite the same as usually applying P to the corresponding Dirac-field $V - A$ current.* If by P_{st} we denote an operator just reproducing the "standard" formal way of applying P according to the $V - A$ scheme, we may then put $P_{\text{ex}} = P_{\text{st}}$ and $P = P_{\text{in}} P_{\text{st}}$. Of course, as long as only a current of the (Dirac) type $\bar{\psi} \gamma^\mu \psi$ is taken into account, the effect of P strictly coincides with the effect of P_{st} , so that no distinction can yet be made between P and P_{st} : a glance at both (27) and (26) points out that the P_{in} behavior is irrelevant to such a current. Quite different is the case of the chiral-field currents (31) and (32) taken alone, since they, on the contrary, are turned into *each other* as an effect of P_{in} : thus also P_{in} (and not only P_{ex}) is maximally violated by them, the overall result being that they may still be singly P-invariant (despite their P_{ex}-violating character). This ultimately means

that *applying what is only an "external" parity operation* (as it is usually done) *would not exhaust the real effects produced by space reflection on a pure weakly-interacting fermion system*: some "internal" nontrivial effects (really able to restore ordinary mirror symmetry) would be indeed neglected, which should involve a yet unexplored, complementary aspect of the intrinsic nature itself of spin-$\frac{1}{2}$ point fermions. The physical contents of these further effects of space inversion will be made clear in the next section, where just a *dual* (either "Dirac" or "chiral") massive fermion model, generally based on the coexistence of two *anticommuting* (scalar and pseudoscalar) charge varieties, is outlined.

Such a restitution of P symmetry to the "maximally P-violating" phenomenology is consistently supplemented by a parallel restitution of C symmetry, C being the ordinary (noncovariant) charge conjugation. The key-novelty is afforded again by (26), which similarly requires, either for the chiral-field current (31) or (32), a *direct* application of C to χ and $\bar{\chi}$. Using the symbol C also to denote the associated 4×4 unitary matrix, we have, e.g., that applying C to (31) now means making the substitutions

$$\chi_\mathrm{a} \to \chi_\mathrm{a}^{(C)} = C\widetilde{\chi}_\mathrm{a}^\dagger \ , \ \ \bar{\chi}_\mathrm{b} \to \bar{\chi}_\mathrm{b}^{(C)} = \widetilde{\chi}_\mathrm{b}C^\dagger\gamma^0 \tag{34}$$

where $\widetilde{\chi}$ stands for the transpose of χ and C is, as usual, such that

$$C\widetilde{\gamma}^{\mu\dagger}C^\dagger = -\gamma^\mu \ , \ \ C\widetilde{\gamma}^{5\dagger}C^\dagger = -\gamma^5 \, . \tag{35}$$

In this way C will also induce *chirality inversion* besides ordinarily acting on Dirac fields; which clearly ensures C symmetry to be restored (provided that normal ordering is applied): as an effect of (34), it will turn out that the "charge conjugate" of (31) is indeed a $V + A$ (rather than a $V - A$) "hole" current. Since (34) generally implies

$$\psi_{\bar{f}}^{(C)}(x) = -\gamma^5\psi_f^{(C)}(x) \ , \ \ \bar{\psi}_{\bar{f}}^{(C)}(x) = \bar{\psi}_f^{(C)}(x)\gamma^5 \, , \tag{36}$$

it is immediate to see, by a comparison with (13), that the C operator may now be written down as $C = P_\mathrm{in}C_\mathrm{st}$ ($=C_\mathrm{st}P_\mathrm{in}$) with C_st exactly reproducing the "standard" formal way of applying C according to the $V - A$ scheme. This is just how C is to be represented in the "dressed" Fock space (2). Of course, no real distinction can yet emerge between C and C_st, as far as their effects on a current like $\bar{\psi}\gamma^\mu\psi$ are concerned:

the reason is because by writing $\psi \rightarrow C\widetilde{\psi}^{\dagger}$ one may indifferently mean not only

$$\frac{1}{2}(1 \mp \gamma^5)\psi \longrightarrow \frac{1}{2}(1 \mp \gamma^5)C\widetilde{\psi}^{\dagger}$$

(with $C = C_{\mathrm{st}}$ as in the usual formalism) but also

$$\frac{1}{2}(1 \mp \gamma^5)\psi \longrightarrow \frac{1}{2}C\big[(1 \mp \widetilde{\gamma}^{5\dagger})\widetilde{\psi}^{\dagger}\big] = \frac{1}{2}(1 \pm \gamma^5)C\widetilde{\psi}^{\dagger}$$

(with $C = P_{\mathrm{in}}C_{\mathrm{st}}$ as in the new formalism). The C_{st} and $P_{\mathrm{in}}C_{\mathrm{st}}$ effects are made fully distinguishable, on the contrary, just when a single chiral-field current like (31) or (32) is involved: C_{st} will act in such a way as to be (maximally) violated, while $P_{\mathrm{in}}C_{\mathrm{st}}$ in such a way as to be left a symmetry operation.

Hence, in the enlarged framework here considered, it is only when dealing with a Dirac-field V current that we may really put $P = P_{\mathrm{st}}$ and $C = C_{\mathrm{st}}$. Note, nevertheless, that in this same framework it always results

$$CP = C_{\mathrm{st}}P_{\mathrm{st}} = (CP)_{\mathrm{st}} \tag{37}$$

whether a Dirac- or chiral-field V current is individually involved. If it is further considered that "chiral" fermions would also be manifest C_{cov} eigenstates which are taken by P_{in} into their antifermion counterparts, it may therefore be guessed that the recovered P mirror symmetry for "chiral" fermions should essentially amount to CP symmetry itself, with C thus really acting on them just like an *identity*. A direct confirmation can be obtained by singly applying C $(= P_{\mathrm{in}}C_{\mathrm{st}})$ and P $(= P_{\mathrm{in}}P_{\mathrm{st}})$ to a "dressed" Fock state of the type $|1_{\mathbf{p},\sigma}\rangle|f^{\mathrm{ch}}\rangle$, with $|1_{\mathbf{p},\sigma}\rangle$ denoting an usual occupied "particle" (= fermion) state of momentum $\mathbf{p}$ and helicity σ in $\mathcal{F}^{\circ}$. The fact is that

$$C\,|1_{\mathbf{p},\sigma}\rangle|f^{\mathrm{ch}}\rangle = |1_{\mathbf{p},\sigma}^{(\mathrm{h})}\rangle|\bar{f}^{\mathrm{ch}}\rangle \quad , \quad P\,|1_{\mathbf{p},\sigma}\rangle|f^{\mathrm{ch}}\rangle = |1_{-\mathbf{p},-\sigma}\rangle|\bar{f}^{\mathrm{ch}}\rangle, \tag{38}$$

where both transformed "dressed" Fock states are also belonging to a *new* picture (marked by $|\bar{f}^{\mathrm{ch}}\rangle$ and covariantly conjugated to the starting one) in which by "particle" the corresponding "chiral" antifermion is meant: so, the occupied "hole" state $|1_{\mathbf{p},\sigma}^{(\mathrm{h})}\rangle|\bar{f}^{\mathrm{ch}}\rangle$ is nothing but the original *fermion* state as re-expressed in such a picture, while the occupied (mirror) "particle" state $|1_{-\mathbf{p},-\sigma}\rangle|\bar{f}^{\mathrm{ch}}\rangle$ is rather a "chiral" *antifermion* (and no longer a "chiral" *fermion*) state. As will be shown in the next

section, these can more straightforwardly be seen to correspond to a $C = 1$ and a $P = CP = (CP)_{\mathrm{st}}$ effect, if an explicit use is made of a symmetrized "particle–hole" formalism. All that, of course, makes sense, provided that a "Dirac" and a "chiral" fermion are supposed to embody two mere *complementary* and *mutually exclusive* internal attitudes of one and the same spin-$\frac{1}{2}$ point particle. But which would be the reasons for the coexistence of two such (apparently incompatible) fermion natures? Answering this question may indeed be decisive for tracing the origin itself of what is known as the "maximal parity-violation" effect.

3   Anticommuting, scalar and pseudoscalar, varieties of charges, and a dual – either "Dirac" or "chiral" – model of a massive spin-$\frac{1}{2}$ point particle

An insight into the above results can be gained by introducing two general one-particle "charge" operators, Q and Q^{ch}, the former being diagonal in the "Dirac" S_{in}-basis $\left(\,|\,f\rangle, |\,\bar{f}\,\rangle\,\right)$ (with eigenvalues $\pm q$) and the latter in the "chiral" S_{in}-basis $\left(\,|\,f^{\mathrm{ch}}\rangle, |\,\bar{f}^{\,\mathrm{ch}}\rangle\,\right)$ (with eigenvalues $\pm q^{\mathrm{ch}}$). In view of (3), (19), (20), and (21), one has

$$C_{\mathrm{cov}}Q = -QC_{\mathrm{cov}} \quad , \quad P_{\mathrm{in}}Q = QP_{\mathrm{in}} \tag{39}$$

$$P_{\mathrm{in}}Q^{\mathrm{ch}} = -Q^{\mathrm{ch}}P_{\mathrm{in}} \quad , \quad C_{\mathrm{cov}}Q^{\mathrm{ch}} = Q^{\mathrm{ch}}C_{\mathrm{cov}} \,. \tag{40}$$

Hence, Q is a *scalar* quantity anticommuting with C_{cov}, while Q^{ch} is a *pseudoscalar* quantity anticommuting with P_{in}; so that C_{cov} and P_{in} may strictly be said to stand for a *scalar*- and a *pseudoscalar-charge* conjugation operator, respectively. It may be asserted, moreover, that the "Dirac" internal states $\left(\,|\,f\rangle, |\,\bar{f}\,\rangle\,\right)$ should typically behave like a *net* pair of scalar-charge conjugated eigenstates – see Eqs. (3) – while the "chiral" ones $\left(\,|\,f^{\mathrm{ch}}\rangle, |\,\bar{f}^{\,\mathrm{ch}}\rangle\,\right)$ like a *net* pair of pseudoscalar-charge conjugated eigenstates – see Eqs. (20). This is to be related to the fact Q and Q^{ch} are themselves two *anticommuting* operators,

$$QQ^{\mathrm{ch}} + Q^{\mathrm{ch}}Q = 0, \tag{41}$$

whose squares clearly satisfy the commutation relations

$$\left[Q^2, Q^{\mathrm{ch}}\right] = \left[Q^{\mathrm{ch}2}, Q\right] = 0 \,. \tag{42}$$

Either Q or Q^{ch}, if applied (from the right) to the fermion–antifermion field $\Psi(x)$, is automatically able to *superselect* that internal

representation of $\Psi(x)$ – either (8) or (24) – which may diagonalize it. Hence it can be argued that the same massive spin-$\frac{1}{2}$ point fermion and related antifermion may both display, in principle, a *dual* intrinsic nature as an alternate pair of *superselected* Q or Q^{ch} eigenstates.[24] If so, then, e.g., the "true" operation of fermion $\to$ antifermion covariant conjugation should strictly be identified with $C_{\mathrm{cov}}P_{\mathrm{in}}$, although one has that $C_{\mathrm{cov}}P_{\mathrm{in}}$ is just reducible to C_{cov} when acting on $|\,f\rangle$ and to P_{in} when acting on $|\,f^{\mathrm{ch}}\rangle$:

$$C_{\mathrm{cov}}P_{\mathrm{in}}|\,f\rangle = C_{\mathrm{cov}}|\,f\rangle \;\;,\;\; C_{\mathrm{cov}}P_{\mathrm{in}}|\,f^{\mathrm{ch}}\rangle = P_{\mathrm{in}}|\,f^{\mathrm{ch}}\rangle \tag{43}$$

($C_{\mathrm{cov}}P_{\mathrm{in}} = -P_{\mathrm{in}}C_{\mathrm{cov}}$). In the former case the fermion would behave *as if* it were a pure scalar-charge (i.e., "Dirac") particle, while in the latter *as if* it were a pure pseudoscalar-charge (i.e., "chiral") particle. Yet it would be in *either* (and not only in the former) case that P and C_{cov} symmetries can be singly respected,[25] the difference being merely that P and C_{cov} would result to play *interchanged* internal roles on passing from one to the other case. Actually, Eqs. (43) show that a "chiral" fermion as compared with a "Dirac" one would conversely stand for a C_{cov} (and not an intrinsic P) eigenstate which instead is turned by P into the corresponding "chiral" antifermion: in such a case, therefore, P itself (thanks to P_{in}) would take the place of C_{cov} as a "covariant charge-conjugation" operation.

On the other hand, coming back to the "dressed" version, $C = P_{\mathrm{in}}C_{\mathrm{st}}$, of the ordinary (noncovariant) charge conjugation C as redefined in the Fock space (2), we clearly have that the "bare" contribution C_{st} will interchange "particle" and "hole" no matter whether a "Dirac" or "chiral" fermion (antifermion) is being dealt with. So, according to the case considered, C_{st} may in turn be said to be the noncovariant analogue of C_{cov} and P_{in}, respectively. What characterizes C_{st} (or $CP = C_{\mathrm{st}}P_{\mathrm{st}}$) is just its noncovariant behavior, which allows it to perform an actual fermion $\rightleftharpoons$ antifermion conjugation in the framework of *one and the same* (covariant) picture with either "particle" = fermion (and "hole" = antifermion) or "particle" = antifermion (and "hole" = fermion). This cannot happen for C_{cov} (in the "Dirac" case) and for P_{in} (in the "chiral" case) since "covariant charge-conjugation" by definition implies the change from the former picture (properly in terms of positive- and negative-energy *fermions*) to the latter (properly in terms of positive- and negative-energy *antifermions*) or vice versa. Such a distinction is in particular essential to understand how the P ($= P_{\mathrm{in}}P_{\mathrm{st}}$)

mirror image of a "chiral" fermion may already amount to a CP mirror image of it: due to the P_{in} contribution, applying P to a "chiral" fermion will also mean going over to a *new* "particle–hole" picture where one has "particle" = antifermion (rather than "particle" = fermion) and where a "particle" space-inverted state correspondingly stands for an *antifermion* (rather than still for a fermion) state. Such an equivalence between the recovered P mirror symmetry and the well-known CP mirror symmetry can be given a more straightforward (although only *effective*) representation, if use is made of a symmetrized "particle–hole" formalism (as obtained via normal ordering). A formalism like this is no longer strictly covariant but gives the opportunity of evaluating the C and P individual effects even without passing to a new "particle–hole" picture. To see it, consider also the "hole" version of transformation (26),

$$\psi_f^{(\text{h})} = \frac{1}{\sqrt{2}}(\chi_f^{(\text{h})} + \chi_{\bar{f}}^{(\text{h})}) \ , \ \ \psi_{\bar{f}}^{(\text{h})} = \frac{1}{\sqrt{2}}(-\chi_f^{(\text{h})} + \chi_{\bar{f}}^{(\text{h})}), \qquad (44)$$

where the fields $\psi_f^{(\text{h})}, \psi_{\bar{f}}^{(\text{h})}$ and $\chi_f^{(\text{h})}, \chi_{\bar{f}}^{(\text{h})}$ are obtained from the corresponding fields $\psi_f, \psi_{\bar{f}}$ and $\chi_f, \chi_{\bar{f}}$ by simply making (in their single Fourier expansions) the substitutions $a \to a^{\text{h}}$, $a^{\text{h}\dagger} \to a^{\dagger}$. Take then, e.g., the normally ordered "particle–hole" chiral-field picture with "particle" = fermion (and "hole" = antifermion): there will symmetrically enter both the (negative chirality) "particle" field χ_f and the (positive chirality) "hole" field $\chi_{\bar{f}}^{(\text{h})}$ (along with their adjoints $\bar{\chi}_f$ and $\bar{\chi}_{\bar{f}}^{(\text{h})}$). It follows that the operation

$$P_{\text{in}} : \quad \chi_f(\bar{\chi}_f) \longrightarrow \chi_{\bar{f}}(\bar{\chi}_{\bar{f}}) \ , \ \ \chi_{\bar{f}}^{(\text{h})}(\bar{\chi}_{\bar{f}}^{(\text{h})}) \longrightarrow \chi_f^{(\text{h})}(\bar{\chi}_f^{(\text{h})}) \qquad (45)$$

cannot globally be distinguished from

$$C_{\text{st}} : \quad \chi_f(\bar{\chi}_f) \longrightarrow \chi_f^{(\text{h})}(\bar{\chi}_f^{(\text{h})}) \ , \ \ \chi_{\bar{f}}^{(\text{h})}(\bar{\chi}_{\bar{f}}^{(\text{h})}) \longrightarrow \chi_{\bar{f}}(\bar{\chi}_{\bar{f}}), \qquad (46)$$

so that P_{in} itself may equivalently be ascribed an *effective* behavior like $P_{\text{in}} = C_{\text{st}}$ (as if it were defined directly in the "bare" Fock space $\mathcal{F}^{\circ}$). Analogous, *effective* behaviors on $\mathcal{F}^{\circ}$ states may then be thought of, after all, for C $(=P_{\text{in}}C_{\text{st}})$ and P $(=P_{\text{in}}P_{\text{st}})$, which are just like $C = 1$ and $P = CP = C_{\text{st}}P_{\text{st}}$.

With the help of both (1) and (21), Eq. (9) can be recast into the more convenient form

$$i\gamma^{\mu}\partial_{\mu}\Psi(x) = |m|\Psi^{(P_{\text{in}})}(x) \qquad (47)$$

where
$$\Psi^{(P_{\mathrm{in}})}(x) \equiv \Psi(x)\, P_{\mathrm{in}} = \psi_f(x)\,\langle f| \, - \, \psi_{\bar f}(x)\,\langle \bar f|\,. \tag{48}$$

A field equation like (47) is actually derivable from the Hermitian free Lagrangian

$$\mathcal{L}(\Psi, \Psi^{(P_{\mathrm{in}})}, \bar\Psi, \bar\Psi^{(P_{\mathrm{in}})}, \ldots; |m|\,) = \frac{1}{4}\Big[i(\bar\Psi\gamma^\mu\partial_\mu\Psi \,+\, \bar\Psi^{(P_{\mathrm{in}})}\gamma^\mu\partial_\mu\Psi^{(P_{\mathrm{in}})}) \,+\, \mathrm{H.c.}\Big]$$
$$-\frac{1}{2}|m|\,(\bar\Psi\Psi^{(P_{\mathrm{in}})} \,+\, \bar\Psi^{(P_{\mathrm{in}})}\Psi) \tag{49}$$

where $\bar\Psi^{(P_{\mathrm{in}})} = P_{\mathrm{in}}\bar\Psi$. In (49), one has that $\Psi, \Psi^{(P_{\mathrm{in}})}, \bar\Psi$ and $\bar\Psi^{(P_{\mathrm{in}})}$ are *single* field variables; by this it is understood that χ_f and $\chi_{\bar f}$ are subject *from the beginning* to the "chiral condition" (25) (automatically fixing the link between ψ_f and $\psi_{\bar f}$). A glance at (49) shows that $\mathcal{L}$ is not only manifestly P_{in}-invariant, but also P-invariant, as can be immediately checked by applying the usual effective prescription

$$P: \quad \partial_\mu \rightarrow \partial^\mu \ , \ \ \gamma^\mu \rightarrow \gamma^0\gamma^\mu\gamma^0\,. \tag{50}$$

This (fully covariant) outcome is *independent* of the special S_{in} representation chosen for the fields $\Psi, \Psi^{(P_{\mathrm{in}})}, \bar\Psi$ and $\bar\Psi^{(P_{\mathrm{in}})}$; so one has that parity invariance consistently holds *even when the chiral S_{in} representation is adopted*. Quite a similar remark applies to the (substantial) invariance of $\mathcal{L}$ under the ordinary operation of charge conjugation,

$$\Psi \rightarrow C\widetilde\Psi^\dagger \ , \ \ \Psi^\dagger \rightarrow \widetilde\Psi C^\dagger \ \ ; \ \ \Psi^{(P_{\mathrm{in}})} \rightarrow C\widetilde\Psi^{(P_{\mathrm{in}})\dagger} \ , \ \ \Psi^{(P_{\mathrm{in}})\dagger} \rightarrow \widetilde\Psi^{(P_{\mathrm{in}})}C^\dagger\,, \tag{51}$$

with the matrix C fulfilling the usual conditions (35).

Global phase invariance of the Lagrangian (49) yields a manifestly P_{in}-invariant, conserved free current like

$$J \equiv J^\mu = \frac{1}{2}\big[\bar\Psi\gamma^\mu\Psi \,+\, \bar\Psi^{(P_{\mathrm{in}})}\gamma^\mu\Psi^{(P_{\mathrm{in}})}\big]\,. \tag{52}$$

By use of the closure relation $|\,f\,\rangle\langle f|\, + \,|\,\bar f\,\rangle\langle\,\bar f|\, = 1$ (throughout this paper the identity operator in S_{in} is simply denoted by 1) such a current can essentially be reduced to the "bare" form

$$J^\mu = \bar\psi_f\gamma^\mu\psi_f = \bar\psi_{\bar f}\,\gamma^\mu\psi_{\bar f} = \frac{1}{2}\big[\,\bar\chi_f\gamma^\mu\chi_f \,+\, \bar\chi_{\bar f}\,\gamma^\mu\chi_{\bar f}\,\big] \tag{53}$$

acting in the strict Fock space $\mathcal{F}^\circ$. Likewise one has $J = J^{(C_{\mathrm{cov}})}$, and this corresponds to the fact that (52) is chirality-invariant. The form (53) can be suitably "dressed" to give the two distinct, *scalar-* and *pseudoscalar-*charge, conserved free currents

$$\mathcal{J}^{(Q)} = QJ = JQ \ , \ \ \mathcal{J}^{(Q^{\mathrm{ch}})} = Q^{\mathrm{ch}}J = JQ^{\mathrm{ch}}, \tag{54}$$

which act in the whole space (2) and are, according to (39) and (40), such that

$$C_{\mathrm{cov}} \, \mathcal{J}^{(Q)} = -\mathcal{J}^{(Q)}C_{\mathrm{cov}} \ \ , \ \ P_{\mathrm{in}}\mathcal{J}^{(Q)} = \mathcal{J}^{(Q)}P_{\mathrm{in}} \tag{55}$$

$$P_{\mathrm{in}}\mathcal{J}^{(Q^{\mathrm{ch}})} = -\mathcal{J}^{(Q^{\mathrm{ch}})}P_{\mathrm{in}} \ \ , \ \ C_{\mathrm{cov}} \, \mathcal{J}^{(Q^{\mathrm{ch}})} = \mathcal{J}^{(Q^{\mathrm{ch}})}C_{\mathrm{cov}} \ . \tag{56}$$

These can also (more properly) be inferred by exploiting the invariance of (49) under two individual kinds of global gauge transformations applying to S_{in} vectors, such that

$$\Psi \to \Psi e^{iQ\alpha} \ , \ \bar{\Psi} \to e^{-iQ\alpha}\bar{\Psi} \ ; \ \Psi^{(P_{\mathrm{in}})} \to \Psi^{(P_{\mathrm{in}})}e^{iQ\alpha} \ , \ \bar{\Psi}^{(P_{\mathrm{in}})} \to e^{-iQ\alpha}\bar{\Psi}^{(P_{\mathrm{in}})} \tag{57}$$

and

$$\Psi \to \Psi e^{iQ^{\mathrm{ch}}\beta} , \bar{\Psi} \to e^{-iQ^{\mathrm{ch}}\beta}\bar{\Psi} \ ; \ \Psi^{(P_{\mathrm{in}})} \to \Psi^{(P_{\mathrm{in}})}e^{iQ^{\mathrm{ch}}\beta} , \bar{\Psi}^{(P_{\mathrm{in}})} \to e^{-iQ^{\mathrm{ch}}\beta}\bar{\Psi}^{(P_{\mathrm{in}})}, \tag{58}$$

respectively (α and β being two constant real angles).[14] In particular, note that the invariance with respect to the substitutions (58) may hold also for the mass sector of (49) by virtue of the chiral condition (25). The *vector* and *axial-vector* behaviors of $\mathcal{J}^{(Q)}$ and $\mathcal{J}^{(Q^{\mathrm{ch}})}$ can be explicitly checked as follows:

$$\begin{cases} P^\dagger \mathcal{J}^{(Q)\mu}(x_\nu)P = (P_{\mathrm{in}}^\dagger Q P_{\mathrm{in}})[P_{\mathrm{ex}}^\dagger J^\mu(x_\nu)P_{\mathrm{ex}}] = \mathcal{J}_\mu^{(Q)}(x^\nu) \\ P^\dagger \mathcal{J}^{(Q^{\mathrm{ch}})\mu}(x_\nu)P = (P_{\mathrm{in}}^\dagger Q^{\mathrm{ch}} P_{\mathrm{in}})[P_{\mathrm{ex}}^\dagger J^\mu(x_\nu)P_{\mathrm{ex}}] = -\mathcal{J}_\mu^{(Q^{\mathrm{ch}})}(x^\nu) \ . \end{cases} \tag{59}$$

The corresponding behaviors of their normally ordered versions under the (ordinary) charge-conjugation C are

$$C^\dagger (: \mathcal{J}^{(Q)} :) \, C = -(: \mathcal{J}^{(Q)} :) \ \ , \ \ C^\dagger (: \mathcal{J}^{(Q^{\mathrm{ch}})} :) \, C = (: \mathcal{J}^{(Q^{\mathrm{ch}})} :), \tag{60}$$

with $C = P_{\mathrm{in}}C_{\mathrm{st}}$ and

$$C_{\mathrm{st}}^\dagger Q \, C_{\mathrm{st}} = Q \ , \ \ C_{\mathrm{st}}^\dagger Q^{\mathrm{ch}} \, C_{\mathrm{st}} = Q^{\mathrm{ch}} \ , \ \ C_{\mathrm{st}}^\dagger (: J :) \, C_{\mathrm{st}} = -(: J :). \tag{61}$$

Note, on the other hand, that both $(:\mathcal{J}^{(Q)}:)$ and $(:\mathcal{J}^{(Q^{\mathrm{ch}})}:)$ behave *identically* under CP $(=C_{\mathrm{st}}P_{\mathrm{st}})$.

Concerning the scalar-charge current $\mathcal{J}^{(Q)}$ the "dressing" one-particle charge operator relevant to it may be expressed as

$$Q = qP_{\mathrm{in}} = q\left(\,|f\rangle\langle f|\,-\,|\bar{f}\rangle\langle\bar{f}|\,\right), \tag{62}$$

$q\,(=\mp|q|)$ being the given Q eigenvalue associated with $|f\rangle$. By substituting (62), a pair of *covariant charge-conjugated* (and only *in turn* working) currents can be seen to be included in $\mathcal{J}^{(Q)}$: they are the "Dirac"-fermion current, marked by $|f\rangle\langle f|$ and the "Dirac"-antifermion one, marked by $|\bar{f}\rangle\langle\bar{f}|$, the former being associated with a proper-mass root $+m$ and the latter with a proper-mass root $-m$. Of course, *both* alternative pictures in terms of such currents are consistent with QED, since a Dirac bilinear form like $\bar{\psi}\,\gamma^{\mu}\psi$ is left unvaried by proper-mass reversal $\psi \to \gamma^{5}\psi$, $\bar{\psi} \to -\bar{\psi}\gamma^{5}$. Under normal ordering, either the fermion or antifermion sector of $\mathcal{J}^{(Q)}$ can itself be recast into a *complete* (antisymmetric) "particle + hole" current (marked by a *single* proper-mass sign): in the former case, one is choosing "particle" = fermion (and "hole" = antifermion) whereas in the latter, "particle" = antifermion (and "hole" = fermion). The Lagrangian (49) can be made invariant also under local U(1) transformations generated by Q, provided that a minimal coupling term like $-\mathcal{J}^{(Q)}A$ is inserted into it, with $A \equiv A_{\mu}$ being a (massless) *vector* field, such that

$$P_{\mathrm{in}}^{\dagger}\,A_{\mu}(x_{\nu})\,P_{\mathrm{in}} = A_{\mu}(x_{\nu}) \quad , \quad P_{\mathrm{ex}}^{\dagger}\,A_{\mu}(x_{\nu})\,P_{\mathrm{ex}} = A^{\mu}(x^{\nu})\,. \tag{63}$$

Note, however, that the presence of such a coupling breaks the original covariance of (49) under rotations in S_{in}. The term in question is *double*-structured as well: it actually merges two *equivalent*, and only *in turn* available, coupling terms, which singly involve the fermion and antifermion covariant currents embodied in $\mathcal{J}^{(Q)}$. It is worth also pointing out that $\mathcal{J}^{(Q)}A$ itself is left unchanged by the chirality transformation $\psi \to \gamma^{5}\psi$, $\bar{\psi} \to -\bar{\psi}\gamma^{5}$. So, one may always think of (3) as a *symmetry* operation represented by γ^{5}: it now stands manifestly for a *scalar-charge covariant conjugation*, which applies to the *whole* interacting system (A_{μ} included) and yields

$$C_{\mathrm{cov}}^{\dagger}(\mathcal{J}^{(Q)\mu}A_{\mu})\,C_{\mathrm{cov}} = (-\mathcal{J}^{(Q)\mu})(-A_{\mu})\,. \tag{64}$$

One has, on the other hand, that γ^5-invariance holds also for the single alternative (fermion and antifermion) couplings included in $\mathcal{J}^{(Q)}A$. This corresponds to the individual invariance of them under the "bare" covariant operation

$$C_q : \quad q \rightleftharpoons -q \ , \ m \rightleftharpoons -m \ , \ A_\mu \rightleftharpoons -A_\mu \,. \tag{65}$$

Consider now the pseudoscalar-charge current $\mathcal{J}^{(Q^{\mathrm{ch}})}$, with

$$Q^{\mathrm{ch}} = -q^{\mathrm{ch}} C_{\mathrm{cov}} = q^{\mathrm{ch}} \left(|f^{\mathrm{ch}}\rangle\langle f^{\mathrm{ch}}| - |\bar{f}^{\mathrm{ch}}\rangle\langle \bar{f}^{\mathrm{ch}}| \right), \tag{66}$$

q^{ch} $(= \mp|q^{\mathrm{ch}}|)$ being the given Q^{ch} eigenvalue associated with $|f^{\mathrm{ch}}\rangle$. At first sight, no *individual* chiral-field currents seem to be involved in the fermion and antifermion sectors, $q^{\mathrm{ch}}|f^{\mathrm{ch}}\rangle J\langle f^{\mathrm{ch}}|$ and $-q^{\mathrm{ch}}|\bar{f}^{\mathrm{ch}}\rangle J\langle \bar{f}^{\mathrm{ch}}|$, of it. Despite this, the virtual existence of *superselected* roles for such currents can still be recognized if $\mathcal{J}^{(Q^{\mathrm{ch}})}$ is suitably rewritten in the form

$$\mathcal{J}^{(Q^{\mathrm{ch}})} = \frac{1}{2} q^{\mathrm{ch}} \left(\mathcal{J}^{\mathrm{ch}} - \mathcal{J}^{\mathrm{ch}(P_{\mathrm{in}})} \right), \tag{67}$$

where

$$\mathcal{J}^{\mathrm{ch}} = |f^{\mathrm{ch}}\rangle \bar{\chi}_f \, \gamma^\mu \chi_f \langle f^{\mathrm{ch}}| - |\bar{f}^{\mathrm{ch}}\rangle \bar{\chi}_{\bar{f}} \, \gamma^\mu \chi_{\bar{f}} \langle \bar{f}^{\mathrm{ch}}| \tag{68}$$

and $\mathcal{J}^{\mathrm{ch}(P_{\mathrm{in}})} = P_{\mathrm{in}}^\dagger \mathcal{J}^{\mathrm{ch}} P_{\mathrm{in}}$. The fact is that as long as a mass term is present in the Lagrangian (49), neither $\mathcal{J}^{\mathrm{ch}}$ nor $\mathcal{J}^{\mathrm{ch}(P_{\mathrm{in}})}$ may be independent and divergenceless, and only $\mathcal{J}^{(Q^{\mathrm{ch}})}$ as a whole may really turn out to be a *conserved* pseudoscalar-charge current.[14] Strictly speaking, and still in agreement with the standard electroweak formulation, one is thus led to conclude that *pure chiral-field-current gauge couplings cannot be conceived for originally massive particles*.

4 Concluding remarks

The internal coexistence of two, scalar and pseudoscalar, *anticommuting* charge varieties is the main distinctive feature of the generalized spin-$\frac{1}{2}$ particle model here discussed. It is a natural prediction of the "dressed" fermion–antifermion quantum field formalism summed up in Secs. 2,3 and implies that any pair of scalar-charge (pseudoscalar-charge) conjugated eigenstates can be simultaneously compatible only with *null* expectation values of pseudoscalar (scalar) charges. Because of it, therefore, the activation of some *superselective* inner mechanism is now generally needed in order that a charge may give rise to a local gauge coupling.

In the case of a nonzero-mass fermion, such a mechanism is automatically switched on by the explicit involvement of a *one-particle operator* representing the charge (and being defined in the "dressing" fermion–antifermion covariant internal space S_{in}). As an example, if Q is the one-particle operator associated with a given scalar charge, then applying Q (from the right) to the unified covariant fermion–antifermion field $\Psi(x)$ has just the result of *superselecting* the "Dirac" representation of $\Psi(x)$ in S_{in} (i.e., the one in which Q itself is made diagonal).

On passing to the zero-mass limit, however, the fermion model in question loses its *dual* character, due to the fact that only a "chiral" (and not a "Dirac") type of a massless spin-$\frac{1}{2}$ particle–antiparticle pair is now made admissible, with the antiparticle internally looking just like the *mirror image* of the particle or vice versa. This unavoidably leads to a natural superselection rule for pseudoscalar charges, versus a natural *anti*-superselection rule (characterized by *null* expectation values) for scalar charges. Under such constraints, it can be shown that *any scalar charge carried by a massless spin-$\frac{1}{2}$ fermion (antifermion) would be strictly prevented from generating a local gauge coupling, unless the fermion (antifermion) itself is put in a position to acquire mass.* To prove it, one should merely allow for the general transformation (26) covariantly defining the pair of "scalar-charge conjugated" eigenfields $\psi_f, \psi_{\bar{f}}$ in terms of the "pseudoscalar-charge conjugated" eigenfields $\chi_f, \chi_{\bar{f}}$. As long as mass is absent, the former ones can only stand for two *mixtures* of the latter ones, so that (differently from the nonzero-mass case) there can be no mechanism diagonalizing a scalar charge (and allowing it to generate a local gauge coupling).

All that, if applied to the electroweak scheme, sets *anew* the question about spontaneous symmetry-breaking (SSB) and fermion masses. According to standard views, there seems to be no reason (inherent in electroweak dynamics) for the appearance of fermion masses: the Higgs couplings to fermions are just inserted *ad hoc*, so that theory may fit in with experience. On passing to the new views, the scenario is reversed; and the fact that every real electrically-charged pointlike fermion turns out to be also a *massive* particle can no longer be taken as occurring by chance. Now, on the contrary, the appearance of fermion masses (via SSB) should enter as an *essential internal requirement* of the electroweak model: without it, no actual scalar-charge eigenstates (with eigenvalues different from zero) could ever be obtained, and no scalar charges (such as, e.g., the electric and color ones) would ever be able to generate local

gauge couplings. Such a viewpoint would more properly demand some SSB mechanism no longer presupposing an "external" origin (connected with the Higgs-boson existence). This is indeed enforced by the fact that according to the quantum field formalism here relied upon, *even a massless spin-$\frac{1}{2}$ fermion, and not only a massless spin-1 boson, needs to gain an extra helicity freedom degree in order to be made massive.* If so, then the generation of a fermion mass should likewise be expected to be obtained merely from the "absorption" of a suitable would-be-Goldstone boson,[16,26] rather than from the coupling to a further, yet undiscovered, real particle.

Such conclusions, which are addressed to both electrically-charged leptons and quarks, could just as well apply, in principle, to neutrinos. This is because the general new notion of a "chiral" particle is not confined to the zero-mass case, and the two-component formalism for a massless neutrino is now also the natural quantum field formalism common to *all* leptons and quarks at the zero-mass primary stage.

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