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PERFORMANCE OF DOLLAR-COST AVERAGING AND VALUE AVERAGING

INVESTMENT STRATEGIES – INTERNATIONAL COMPARISON

Bachelor thesis

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I have written this Bachelor thesis independently. Any ideas or data taken from other authors or other sources have been fully referenced.

Introduction

Investing in the equity market by households globally has been a critical part of preserving and accumulating capital for many decades. According to Federal Reserve data (2019), in America more than half of the population have some part of their assets in stocks, and the share of such residents is on the rise (McKinsey & Company, 2021). In Europe, despite the more modest involvement of households in investing in riskier assets, there is also a trend towards an increase in the number of capital market participants (Guiso, Haliassos, Jappelli, & Claessens, 2003; Invest Europe, 2019).

The main reason for the growth in the activity of private investors is the simplification of access to trading on the stock exchange. The so-called gamification of trade is taking place, seeking to democratise finance and empower small-time investors by expanding access to financial markets (van der Heide & Želinský, 2021). In addition, due to technological changes, the threshold amount for investment has decreased. Making a deal now does not require much effort. This process is especially noticeable in recent years when people locked up at home in a pandemic have discovered online trading possibilities (Jackson, 2021; Finch, 2021).

Brokers, exchanges and other financial intermediaries, in turn, seek to ride the current trend to attract new customers. Among other benefits, clients are attracted by providing them with "working" investment strategies, which often do not help to achieve the promised result (Graham, Zweig, & Buffett, 2006). Therefore, households face the difficult task of selecting the optimal investment strategy, implying minimising the probability of lifetime ruin (Young, 2004), which will not turn out to be another marketing trick. After all, one should remember that the primary goal of a financial intermediary is to maximise its profit by increasing the volume of trades (Andries, 2009) and not at all to increase the clients' welfare.

Although, in reality, there are investment strategies described in the scientific literature that could help a private investor make an optimal decision, the best among alternatives (Kennerley, Walton, Behrens, Buckley, & Rushworth, 2006). Such well-known strategies are Dollar-Cost Averaging (DCA) and Value Averaging (VA), allowing a private investor to benefit from the flexibility of periodic investment combined with the ability not to take on additional risks of choosing the right moment for investment (Xiong, Wang, Kou, & Xu, 2021). Furthermore, these strategies, in the conditions of long-term investment, were proven to show a positive result even in unfavourable market conditions (Malkiel, 2008), meaning that they are a wise choice for households when accumulating funds in the long run, such as pension savings, savings to finance children's education, etc.

This bachelor thesis aims to clarify the possible superiority in the rate of return of one of the selected investment strategies, Dollar-Cost Averaging and Value Averaging, in different international markets. In order to achieve the aim, the following research tasks were identified:

- Position Dollar-Cost Averaging and Value Averaging among other investment strategies;
- Outline advantages and disadvantages of Dollar-Cost Averaging and Value Averaging investment strategies;
- Provide an overview of the results of the application of Dollar-Cost Averaging and Value Averaging investment strategies in existing scientific research;
- Collect the necessary data for analysis and introduce chosen research methodology;
- Process data for quantitative analysis;
- Calculate the rate of return of Dollar-Cost Averaging and Value Averaging investment strategies in international markets applying developed research methodology;
- Analyse the results obtained by determining whether there is a difference in the rate of return when applying these investment strategies in various markets.

The novelty of this approach, which is trying to fill the existing research gap, lies in the fact that the analysis of the return of the above-mentioned investment strategies, based on historical data from the various international stock exchanges indexes, is strived to provide a generalised conclusion about the effectiveness of DCA and VA. Current scientific research primarily is based its analysis on historical data of the American S&P 500 index (Leggio & Lien, 2003), simulating data (Estes & Chen, 2010) or including one additional index as a representative of foreign stocks (Marshall, 2006), the results of which cannot be guaranteed to be implemented in another market.

1. Theoretical foundation of Dollar-Cost Averaging and Value Averaging investment strategies

1.1 Positioning of Dollar-Cost Averaging and Value Averaging among investment strategies

The Dollar-Cost Averaging strategy has been known to the world for many decades, first introduced in the second half of the twentieth century. This strategy gained wide popularity after the first publication of the book A Random Walk Down Wall Street, in which

Burton Malkiel (1973) stated that even though this technique is controversial, it nevertheless helps an investor avoid buying an asset at the wrong time.

The application of this strategy involves the regular investment, at set intervals (week, month, quarter), of the same amount of money in the risky asset, regardless of the price. Such an asset can be the shares of an Exchange-Traded Fund for a selected market index, with the purchase of which the investor remains within the limits of systematic, market risk while not losing in profitability (Adjei, 2009). Furthermore, the investor can determine the overall investment period.

Applying the DCA strategy is possible if there is already capital that needs to be invested. In this case, instead of the Lump-sum investment (a strategy when the entire amount is invested in a risky asset at a time), the strategy is used to reduce the risk of acquiring an asset at an unfavourable time (Brennan, Li, & Torous, 2005). During the investment process, the initial amount is in the form of a risk-free asset (cash, short-term government bonds) and is gradually converted into a risky one (Knight & Mandell, 1993). However, some scholars state that such an approach is ineffective (Knight & Mandell, 1993; Greenhut, 2006).

One of the DCA strategy properties sometimes noted by scholars is non-sequential (Constantinides, 1979; Statman, 1995). They imply that this strategy allows one to resist the general mood of other investors, whether positive or negative, by continuing to invest evenly. "No matter how pessimistic you are (and everybody else is), and no matter how bad the financial and world news is, you must not interrupt the plan or you will lose the important benefit of insuring that you buy at least some of your shares after a sharp market decline" (Malkiel, 2008, p. 358).

Value Averaging is a more modern development, having been mentioned for the first time in the article Value averaging: A new approach to accumulation, by Michael Edleson in 1988, evolved to the book later. This strategy was designed to improve the scheme of uniform investment when using DCA.

The VA strategy assumed that a target amount for a risky asset is set at the beginning of investment project, which must be achieved at the end of . Unlike the DCA strategy, the determination of the final aim is necessary here since the amount of money that needs to be invested for the set periods will depend on it. Upon reaching the end of each investment period, the target amount is achieved by buying or selling a risky asset class. If the target amount is exceeded, the risky asset is sold, and a reserve is created in the form of cash. When

the target amount is not exceeded, cash from the reserve is used to purchase a risky asset. After that, the investment period starts over.

For example, it is planned that in 5 years, the final amount of a risky asset (ETF shares tracking the S&P 500 index) should be $ 20 000. It is supposed to invest quarterly, then the invested amount for one period will be $ 1 000. Let us assume that the shares, which an investor bought at the beginning of the first investment period, have risen in price by the end of it and are now worth $ 1 100. This means that at the beginning of the second period, it is necessary to buy an asset for $ 900 so that the total amount invested in a risky asset is $ 2 000. If it turned out that there was a boom in the market, and by the end of the first period, the share price would be $ 2 100, then the investor would need to sell them in the amount of $ 100 and form a reserve fund.

A distinctive feature of this strategy is its ability to provide investors with sell signals, combining the properties of both a DCA strategy and portfolio rebalancing (Ramsey, 2010; Sirucek & Škatuĺárová, 2015).

The main properties of both DCA and VA strategies are presented in Table 1.

Table 1

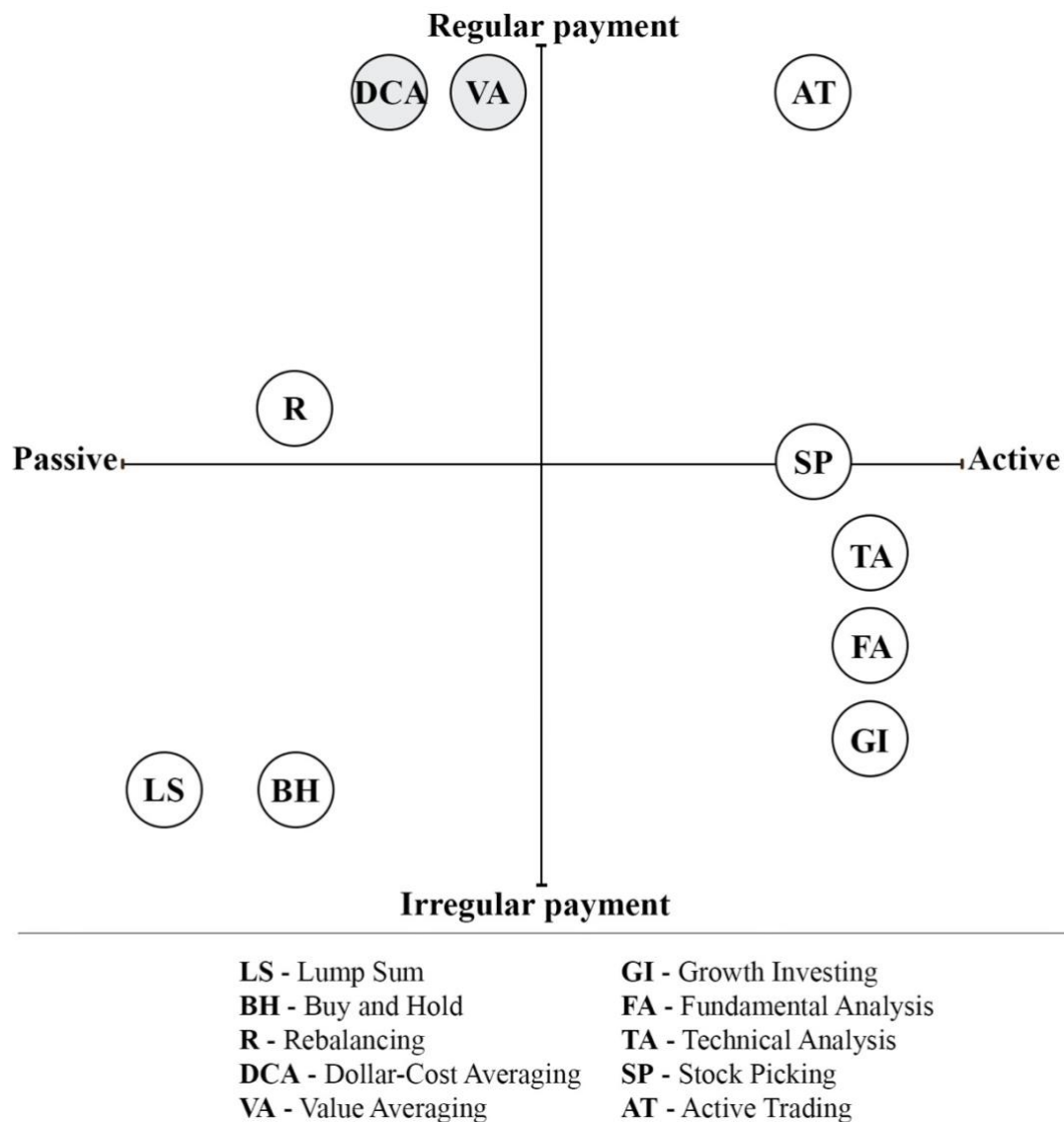*Properties of Dollar-Cost Averaging and Value Averaging investment strategies*

Properties	Dollar-Cost Averaging	Value Averaging
Regular investment	Yes	Yes
Active or passive strategy	Passive	Passive
Sequential or non-sequential	Non-sequential	Non-sequential
Formula investing	Yes	Yes
Assumes initial capital	Optional	No
Determination of the overall investment period	Optional	Yes
Amount invested	Constant	Not necessarily constant
Need to form a reserve (liquidity) fund	No	Yes
Sell Signals	No	Yes

Source: Compiled by author

Having studied the basic properties of DCA and VA investment strategies, it becomes possible to position these investment strategies among other popular ones (see Figure 1).

Figure 1

Dollar-Cost Averaging and Value Averaging investment strategies among other popular ones

LS - Lump Sum   GI - Growth Investing
BH - Buy and Hold   FA - Fundamental Analysis
R - Rebalancing   TA - Technical Analysis
DCA - Dollar-Cost Averaging   SP - Stock Picking
VA - Value Averaging   AT - Active Trading

Source: Compiled by author

Combining the features of passivity, namely full replication approach (non-reacting), when an investor is only required to copy a reference index (independently or with the help of an ETF) (Maginn, Tuttle, & Pinto, 2007), as well as the regularity since there is no need to invest a large amount of money simultaneously, allows DCA and VA to be easily used as a tool of a private investor. The difference in the properties of these strategies makes it possible to determine their advantages and disadvantages in comparison with each other, as well as against the background of other investment strategies. The specifics of others investment strategies are not going to get covered in the following work, though their nature might be intuitively understandable. If one is interested in more details, they can be found in relevant scientific literature.

1.2. Advantages and disadvantages of Dollar-Cost Averaging and Value Averaging investment strategies

As described earlier, the Value Averaging strategy is a modified version of the Dollar-Cost Averaging strategy. In this regard, this part will first consider the pros and cons characteristic of both strategies. Then the distinctive advantages and disadvantages of VA against the background of DCA will be provided.

First of all, the universality of the investment strategies under consideration can be noted (Brennan, Li, & Torous, 2005; Smith & Artigue, 2018). The standardised approach makes it easier to work with DCA and VA, avoiding the need for prior education and allowing it to be applied regardless of the investor's specific needs. However, this advantage hides a corresponding disadvantage: the individual characteristics of the investor are not taken into account (Brennan, Li, & Torous, 2005). Each investor is unique, pursues different goals in the stock market; and therefore, the contents of the investment portfolio will be different in each case. Thus, initial preparation is necessary to apply DCA and VA strategies. For example, it is required to determine the investor's risk profile, the necessary assets, their distribution in the portfolio etc. (Maginn, Tuttle, & Pinto, 2007).

DCA and VA strategies allow investors not to succumb to the general emotional upswing or downturn prevailing in the stock markets, showing the features of non-sequential strategies. Dollar-Cost Averaging and Value Averaging allow one to buy more units of an asset when the markets are depressed, and on the contrary, refrain from a large number of purchases when the markets tend to peak. (Constantinides, 1979; Brennan, Li, & Torous, 2005)

Moreover, based on the properties of DCA and VA, it can be noted that the use of these strategies does not require initial savings (Marshall, 2000; Panyagometh, 2013). This can undoubtedly be considered an advantage for private investors, especially for beginners who would like to apply these strategies as a way of accumulating initial capital. Although, such traits should complement the image of an investor as discipline and perseverance. This is because not all people have enough desire and patience for a uniform investment (Malkiel, 2008), which introduces its limitations for using these investment strategies.

The strategies under study simplify the activities of investors, allowing them not to concentrate on the search for the so-called "right time" for purchase (Smith & Artigue, 2018; Kirkby, Mitra, & Nguyen, 2020), implying the subsequent rapid growth of the selected asset. Investing according to the formula helps to maintain the average value of assets in the portfolio below the market price, which leads to an investor's profit.

In addition, such an investment approach allows one to reduce the regrets about buying at the "wrong time" when the price of securities has dropped sharply. So, according to the Prospect Theory of Kahneman and Tversky (1979), a person regrets the loss much more than he rejoices in the acquisitions. DCA and VA investment strategies positively correlate with behavioural finance, helping to avoid or at least minimise the regrets (aversion of regrets) (Statman, 1995). Using the strategy of evenly buying assets, a person will be easier to experience drawdowns in their value since they will have fewer regrets than if they had bought everything at one particular moment. This will allow one to stick to investment plans and not give in to panic at moments of crisis.

Table 2 summarises the main pros and cons of Dollar-Cost Averaging and Value Averaging investment strategies against the background of others.

Table 2

Advantages and disadvantages of Dollar-Cost Averaging and Value Averaging investment strategies against the background of others

Advantages	Disadvantages
• Universality • Allows being robust to market sentiment • No need to seek the "right time" to invest • Takes into account the features of behavioural finance • No need for initial savings	• The individual characteristics of the investor are not taken into account • Need for discipline to constant investments

Source: Compiled by author

It is also worth highlighting the advantages and disadvantages of Value Averaging compared to Dollar-Cost Averaging. The key improvement that Edleson (1988) brought was the accumulation of portfolio value despite market prices. This approach allows one to gain more from drawdowns in the market prices of selected assets compared to DCA since when using a VA investment strategy, the entire value of the portfolio subsides, and it is necessary to compensate for all the loss in the value, which makes an investor purchase a significantly larger number of securities at a relatively low price. While the DCA strategy will only help buy a little more units of the asset for a fixed amount. Conversely, with an increase in the price of an asset, DCA will reduce the number of acquired units of the asset when VA signals the investor to sell assets to keep the portfolio's value at the level corresponding to the planned schedule. Thus, VA theoretically enables to achieve a lower average value of an asset in a portfolio than DCA (Marshall, 2006).

Moreover, VA guarantees to finish investing with a pre-planned portfolio value. When using DCA, an investor can profitably sell securities only if the average value of the

purchased assets is lower than the current market value (Statman, 1995). This can lead to a delay in the timing of investment or potential losses if there is no way to wait out a depression in the stock market.

Nevertheless, the complications in the application of the VA strategy have also brought some adverse effects. Thus, when using VA, it is impossible to determine or predict in advance the amount needed to purchase assets since the volatility of market prices will affect the value of the portfolio (Malkiel, 2008; Leggio & Lien, 2003).

To conclude, both investment strategies under consideration may be suitable for most investors, both private and institutional, despite some limitations. Theoretically, VA is able to demonstrate better profitability in comparison with DCA due to the modified investment formula. To investigate this hypothesis, it is worth considering the studies already conducted.

1.3. Review of previous studies

The topic of studying various investment strategies is quite popular in scientific circles. However, the authors rarely apply DCA and VA strategies for comparison. Instead, they prefer to compare the investment strategy understudy with Buy and Hold (a strategy when a one-time purchase occurs of a part of assets with a fixed risk-free income, and the other part is invested in a risky asset) or with Lump sum. Even more rare is the comparison of more than two investment strategies (Leggio & Lien, 2003; Choe & Ban, 2020).

For analysis, most authors preferred to take data on the value of the Standard and Poor's 500 index due to the high quality of the data, as well as a long historical observation period (more than 150 years). In addition to the S&P 500 index, scientists also studied the indices of the country of origin in order to apply the results obtained in the conditions of the national market (Panyagometh, 2013; Majumdar, Puthiya, & Bendarkar, 2021). Nevertheless, there are barely enough studies in scientific publications that would compare the rate of return of DCA and VA in different stock markets. Marshall (2006) was developing in this direction conducting a comparative study using data from the American (S&P 500, Dow Jones) and English (FTSE) markets. Alternatively, for example, Ramsey (2010) performed an analysis of the American market (S&P 500) and the aggregate market of emerging markets (MSCI Emerging Markets), but the period tested was only 5 years with no moving window (just one period in a sample: 2005-2010), which does not allow to generalise the results of this work with confidence.

In order to analyse investment strategies, the scholars chose various research methods: absolute values of the rate of return (Marshall, 2006; Ramsey, 2010), the rate of return adjusted for risk (Leggio & Lien, 2003; Arendáš & Kušnir, 2019), or another

methodology described by the author in work (Choe & Ban, 2020; Majumdar, Puthiya, & Bendarkar, 2021). Aiming to compare the study results of all the sources found, in this work, an absolute value of the rate of return without risk adjustment is be used.

In addition, some assumptions have been introduced in scientific papers for research purposes. Firstly, all transaction costs are ignored. Moreover, taxation issues as well as dividends are also not taken into account. Finally, additional interest income from the formation of a reserve fund when using the VA investment strategy is not included.

In their works, scholars also consider various investment scenarios. The main differences are reduced to the frequency of investment, as well as the investment horizon. As for the investment frequency, these are mainly monthly contributions, but some studies consider quarterly ones. More rare investment contributions are not assessed. If talk about the investment horizon, there are three different periods: short-term, medium-term and long-term. Short-term period – investment ceases within a year; medium-term period – from a year to five years; long-term period – more than five years.

The conclusions reached in some of the most substantial computational studies of the rate of return of Dollar-Cost Averaging and Value Averaging investment strategies conducted recently are summarised in Table 3.

One can notice certain patterns. Regardless of the data and the time period under study, in the conditions of a short-term time horizon, DCA demonstrates a greater rate of return compared to VA. Nonetheless, if the investment horizon is increased to a long-term, the return of VA begins to exceed DCA. It is likely that in the medium-term horizon, VA will also outperform DCA; however, there are some sources who claims that this is not the case (Arendáš & Kušnir, 2019; Choe & Ban, 2020). In addition, one can witness the peculiarity of the Indian stock market, where the VA investment strategy shows greater profitability even in a short-term time period (Majumdar, Puthiya, & Bendarkar, 2021). The investment frequency highly likely does not affect the observed patterns.

Thus, further research is required to determine in which investment period VA shows greater return compared to DCA, as well as to determine whether the pattern of VA superiority persists in long-term conditions in different international markets. Moreover, it is necessary to establish whether the investment frequency does not actually affect the overall result in the application of two investment strategies.

Table 3

Studies on computational analysis of the rate of return of Dollar-Cost Averaging and Value Averaging investment strategies

Authors (year of publication)	Data	Period tested	Investment horizon	Investment frequency	Greater rate of return (DCA or VA)
Marshall (2000)	S&P 500	1966-1989	Medium-term (5 years)	Quarterly	VA
Leggio and Lien (2003)	S&P 500	1926-1999	Short-term (1 year)	Monthly	DCA
Marshall (2006)	S&P 500 Dow Jones FTSE	1871-2002 1932-2002 1984-2002	Medium- and Long-term (5, 10 and 20 years)	Quarterly	VA
Chen and Estes (2010)	Data simulation	-	Long-term (30 years)	Monthly	VA
Ramsey (2010)	Vanguard Total Stock Market Index S&P 500 MSCI Emerging Markets	2005-2010 2005-2010 2005-2010	Medium-term (5 years)	Monthly	VA
Panyagometh (2013)	SET TRI	2000-2010	Long-term (25, 30 and 35 years)	Monthly	VA
Arendáš and Kušnir (2019)	S&P 500	2007-2018	Medium-term (5 years)	Monthly	DCA
Choe and Ban (2020)	Korean Fund Market	2001-2019	Short- and Medium-term (1 and 5 years)	Monthly	DCA
Majumdar, Puthiya and Bendarkar (2021)	Indian Equity Market	2010-2019	Medium- and Long-term (3, 5 and 10 yaers)	Monthly	VA

Source: Compiled by author

2. Empirical appraisal of Dollar-Cost Averaging and Value Averaging investment strategies rate of return in different markets.

2.1. Data and methodology

In order to conduct a comprehensive empirical analysis, a multi-stage action plan was developed. The reader can see its summary in Figure 2.

Figure 2

Stages of empirical analysis
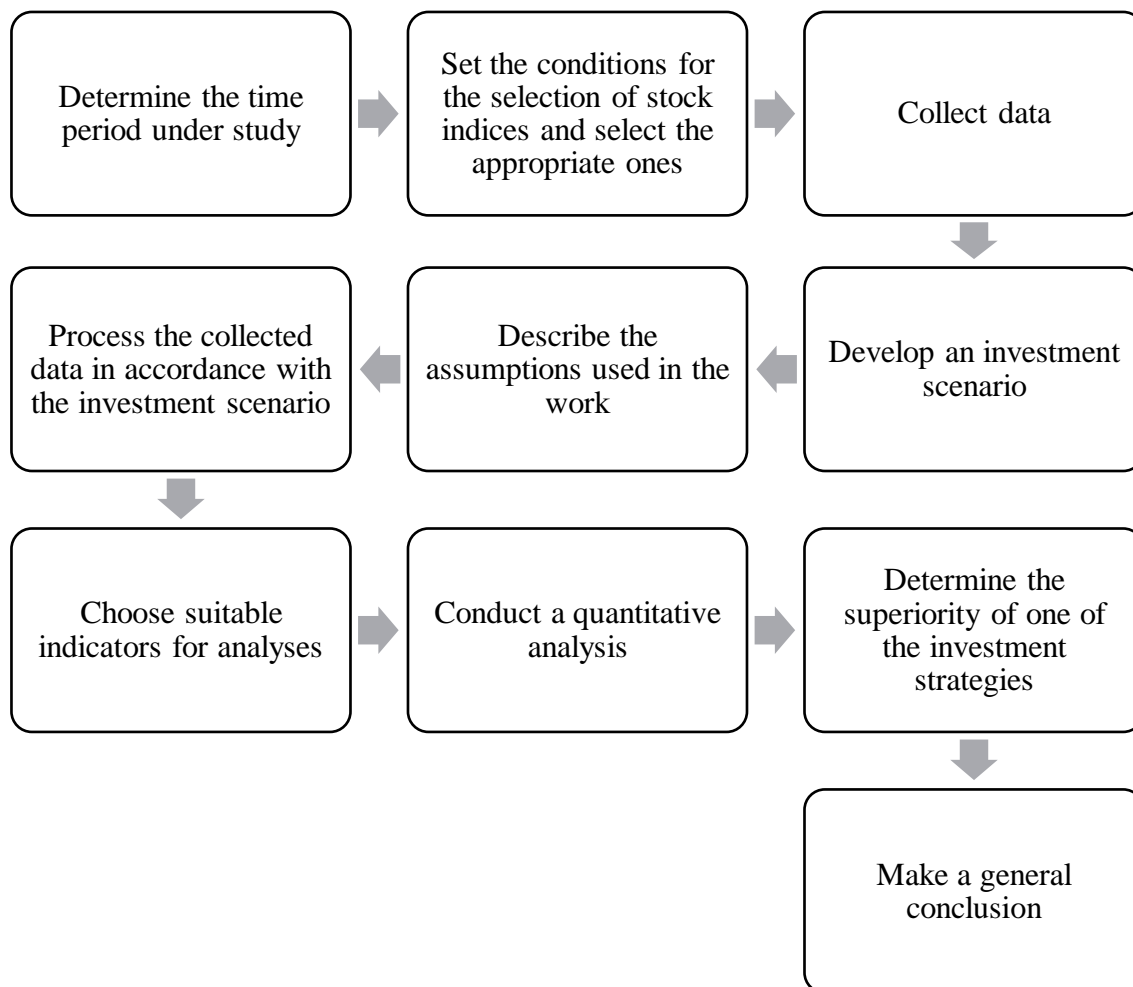
Source: Compiled by author

The choice of the stock indices of various countries was based on the capitalization of the leading stock exchanges (World Federation of Exchanges database, 2020) in order to obtain a representative view of the distribution of capital in the world. In addition, the exchange to be chosen should have had a sufficient history of existence since the selected period of calculation was from January 1, 1995 to March 10, 2022 (a total of 27 years and 3 months). This period was chosen due to the fact that it includes several crises, both local (Asian Financial Crisis, European sovereign debt crisis) and international (Dot-com bubble,

Global financial crisis, Covid crisis), as well as growth series. The approach with the inclusion of various market shocks is consistent with past studies (Marshall, 2000) since it allows to determine how much the revealed pattern in the superiority of the rate of return of one of the investment strategies will persist in different market conditions: rising, declining and fluctuating market. In total, fourteen international indices were selected that met the set conditions. The data was collected from three publicly available sources: Yahoo! Finance, Seeking Alpha and Market Watch. In Table 4 one can find all the stock indexes used in the study.

Table 4

Stock indexes used in the study

Index title	Country	Stock exchange	Currency
Bovespa Index	Brazil	B3 – Brazil Stock Exchange	Brazilian real
CAC 40	France	Euronext Paris	Euro
DAX (German Stock Index)	Germany	Frankfurt Stock Exchange	Euro
Financial Times Stock Exchange 100 Index	United Kingdom	London Stock Exchange	Pound sterling
Hang Seng Index	China	Hong Kong Stock Exchange	Hong Kong Dollar
IBEX 35	Spain	Bolsa de Madrid	Euro
Korea Composite Stock Price Index (KOSPI)	Korea	Korea Exchange	Korean Republic won
Nikkei 225	Japan	Tokyo Stock Exchange	Japanese yen
Standard and Poor's 500	USA	New York Stock Exchange NASDAQ Cboe BZX Exchange	US Dollar
S&P BSE SENSEX	India	Bombay Stock Exchange	Indian rupee
S&P/ASX 200	Australia	Australian Securities Exchange	Australian dollar
S&P/TSX Composite Index	Canada	Toronto Stock Exchange	Canadian dollar
SMI (Swiss Market Index)	Switzerland	SIX Swiss Exchange	Swiss franc
Taiwan Capitalization Weighted Stock Index (TAIEX)	Taiwan	Taiwan Stock Exchange	New Taiwan dollar

Source: Compiled by author

When reviewing the scientific literature (see Table 3, p. 14), some gaps in the topic under study were identified. Moreover, to the ambiguity in the superiority of the DCA and

VA investment strategies in various international markets, such inconsistencies in the results as investment horizon and investment frequency were identified. Based on these gaps, the following investment scenario was developed.

This study assumes that the investor regularly (once a month or once a quarter) replenishes his or her account with the same amount in the national currency. Deposits occur on the 10th day of each month (Majumdar, Puthiya, & Bendarkar, 2021) since the investor will be able to settle all mandatory payments by this time and have precise information about the funds available to him or her in the current period. They then purchase any of the represented ETFs on the stock index of the country of residence. If the exchange is closed on that day, the purchase takes place on the first possible day before the 10th. In order to select the necessary values, the data was pre-processed.

For the sake of research, 1 000 units of currency were used for the convenience of calculation. To calculate the purchase price, the closing price of the selected stock index on the day of purchase is considered. The amount in actual investment practice does not matter because it will not affect the return in any way. So, when applying the selected strategies, any investment amount can be used. The national currency was chosen for the purpose of levelling the currency revaluation of assets: to remove the effect of strengthening or weakening the exchange rate of the local currency in relation to other world currencies, which could affect the calculated rate of return when using investment strategies.

An investor may have different savings goals. It implies three different investment periods:

- short-term – 1 year: e.g. purchasing household appliances, vacations, expensive household or personal items;
- medium-term – 5 years: e.g. car purchase;
- long-term – 10 years: e.g. purchase of the real estate, children's education, retirement savings.

As in previous studies, this research ignores transaction costs, tax burden and tax benefits, as well as possible dividends, what should be considered the main limitations in the work. Also, the additional interest income from the formation of the reserve fund when using the VA investment strategy is not taken into account. Moreover, it is also allowed to purchase a fraction of an asset (ETF shares in the case). The purchase amount of underlying assets is rounded down to four decimals. In the case of monetary units, rounding occurs to two decimal places. Although, while scholars in previous works were forced to accept this as an

assumption, nowadays, there are services that allows one to purchase a fractional share of an asset (e.g. Revolut, Robinhood, Trading 212). It should also be noted that in the case of using VA investment strategy, the formed reserve fund does not bring additional profitability but is simply on the current account.

In order to increase the sample, the beginning of the investment period is always shifted one month ahead until the end of the investigated horizon (until March 10, 2022). So, there are 1-, 5- and 10-year moving windows. The total number of observations depending on the investment frequency and investment horizon is presented in Table 5.

Table 5

The number of observations depending on the investment frequency and investment horizon

Frequency Horizon	Short-term	Medium-term	Long-term
Month	315	269	207
Quarter	315	269	207

Source: Compiled by author

The absolute periodic rate of return (APRR, see equation 1) is used as one of the indicators to compare the superiority of a particular investment strategy under study. When using it, we measure a point to point return of a period (without studying the rates within the framework of the investment period), representing only the final result of an investment. Such a measure will allow a private investor to have a clearer perception of what the expected value of the portfolio could be at the end of the investment period: how much was invested and how much could be received at the end.

$$APRR = \frac{CSV_{t+1} - PV_t}{PV_t} \times 100\% \qquad (1)$$

APRR – Absolute Periodic Rate of Return

CSV$_{t+1}$ – Current Sale Value

PV$_t$ – Purchase Value

So that the reader does not get a false impression of the research's results from the fact that APRR does not consider the time value of money, an analysis will be as well carried out using the Extended Internal Rate of Return (XIRR). The XIRR formula provides one with an extended rate of return which considers cash flows and discount rates, as well as the corresponding dates (unlike a simple Internal Rate of Return, which does not take into account when cash flow exactly occurred), resulting in a more accurate return on investment percentage.

Two indicators will be used to compare the superiority of a particular investment strategy. The first is the percentage of greater APRR as well as XIRR of one investment

technique over another. In the case of a negative yield (loss), the superiority is given to the smallest negative value (the smallest loss). The second is the Mean Excess Return (MER, see equation 2) of Value Averaging investment technique over Dollar-Cost Averaging.

$$MER = \sum_{i=1}^{n} \frac{(VA\ XIRR_i - DCA\ XIRR_i)}{N} \tag{2}$$

MER – Mean Excess Return

VA XIRR$_i$ – Extended Internal Rate of Return resulting from applying Value Averaging

DCA XIRR$_i$ – Extended Internal Rate of Return resulting from applying Dollar-Cost Averaging

N – Number of Observations

Also, to check the statistical significance of the fact that there is a difference in the rates of return when using DCA and VA investment strategies, two tail one-sample t-test with a 0,05 level of significance is used. The data sets of excess return for each investment frequency and horizon with the following hypotheses are tested:

- H$_0$: μ_{MER} = 0; There is no statistically significant difference in the rate of return when using DCA and VA investment strategies
- H$_1$: μ_{MER} ≠ 0; There is a statistically significant difference in the rate of return when using DCA and VA investment strategies

Such a statistical test will allow one to determine if the internal rate of return provided by these strategies is the same or not. The assumption of normally distributed data is neglected due to the big enough sample in every case. The spreadsheet tool Microsoft Excel was used to conduct all quantitative studies.

2.2. Results and Discussion

Quantitative analysis in accordance with the established investment scenario was carried out for each of the selected stock indexes. Detailed results can be found in Appendix A-N. In this part, a general analysis of the results will be performed. The analysis will be built per three investment horizons: short-term, medium-term and long-term, where the topic of their impact on investment strategies will be touched. Within each period, the impact of the investment frequency on the return when using DCA and VA investment strategies will be determined. After a fragmentary analysis, a general conclusion will be made about the superiority in the rate of return of one of the investment strategies studied.

2.2.1. Short-term performance

If we consider the short-term period, the VA strategy showed the best result, though it can be judged that the it could not fully realise its potential. Despite the fact that the share of VA superiority over DCA was consistently higher in all cases (meaning all exchanges and the frequency of investment), the share of DCA superiority remained relatively high. This becomes especially noticeable if we take into account the time value of money: XIRR superiority of DCA in 22 cases (out of 28; both investment frequencies) was 10% or higher. That is, if an investor decides to stick to the VA strategy, the chance that he will miss out on additional return remains rather high.

However, if the investor was lucky enough when using the VA strategy, and in their investment period, it surpassed the DCA, it can be stated that they received additional income. In all the markets studied, MER was positive, which is an additional argument in favour of VA superiority. An investor could expect an average of 1.13% additional return on all markets under study, regardless of the investment frequency. Furthermore, in all cases, the statistical test confirmed that MER is not zero: zero hypothesis should be rejected and first hypothesis should be accepted. So, based on this parameter, it can be argued that VA on average gives greater return than DCA.

Also, using Mean Excess Return, it can be analysed how the investment frequency affected the return when using either DCA or VA. Figure 3 shows MER in the markets depending on the frequency of investment.

Figure 3

Mean Excess Return of different investment frequency in the short-term.
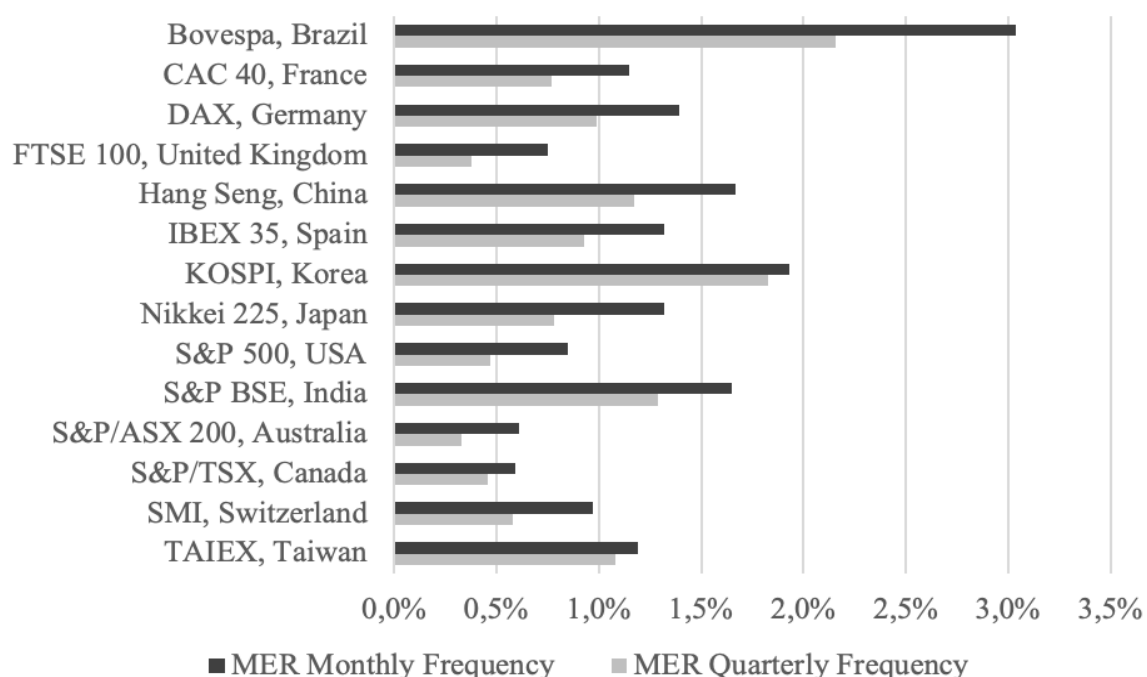
Note. MER$_{\text{VA-DCA}}$

Source: Compiled by author

On the graph, the reader can observe that the MER Monthly Frequency curve is always above the MER Quarterly Frequency curve. This means that with a more frequent investment in a short period, the VA strategy can provide investors with additional returns compared to DCA.

In general, in a short-term period, VA is superior but it cannot be unequivocally stated that the VA strategy will consistently outperform the DCA strategy. Of course, the probability of higher returns when using the first strategy in different markets is higher, but one should not rely only on luck. The differences in the results of past scientific work can just be explained by choice of different stock indices: the processing of different data has led to different results. When choosing an investment strategy for the short-term, the investor should instead be guided by their properties (see Table 1, p. 8) and choose the one that best meets their needs. However, it is safe to say that in the short term, it is worth increasing the frequency of investment to expect higher returns: prefer monthly to quarterly one.

2.2.2. Medium-term performance

Medium-term investors are more likely than short-term ones to expect higher returns when using a VA strategy instead of a DCA. When considering an absolute periodic rate of return, the share of VA superiority in all cases is greater than 98%. The number of cases where the XIRR superiority of DCA was more than 10% decreased to 15.

The pattern of MER being positive preserved and at the same time the gap between the effectiveness of the two strategies has also increased up to 1.6%. The statistical test confirms that H_0 should be rejected and H_1 should be accepted. Reviewing Figure 4, however, it can be seen that the gap in the difference MER, depending on the frequency of investment, has decreased. Moreover, there are two cases where the return of quarterly investment exceeded monthly.

Figure 4

Mean Excess Return of different investment frequency in the medium-term.
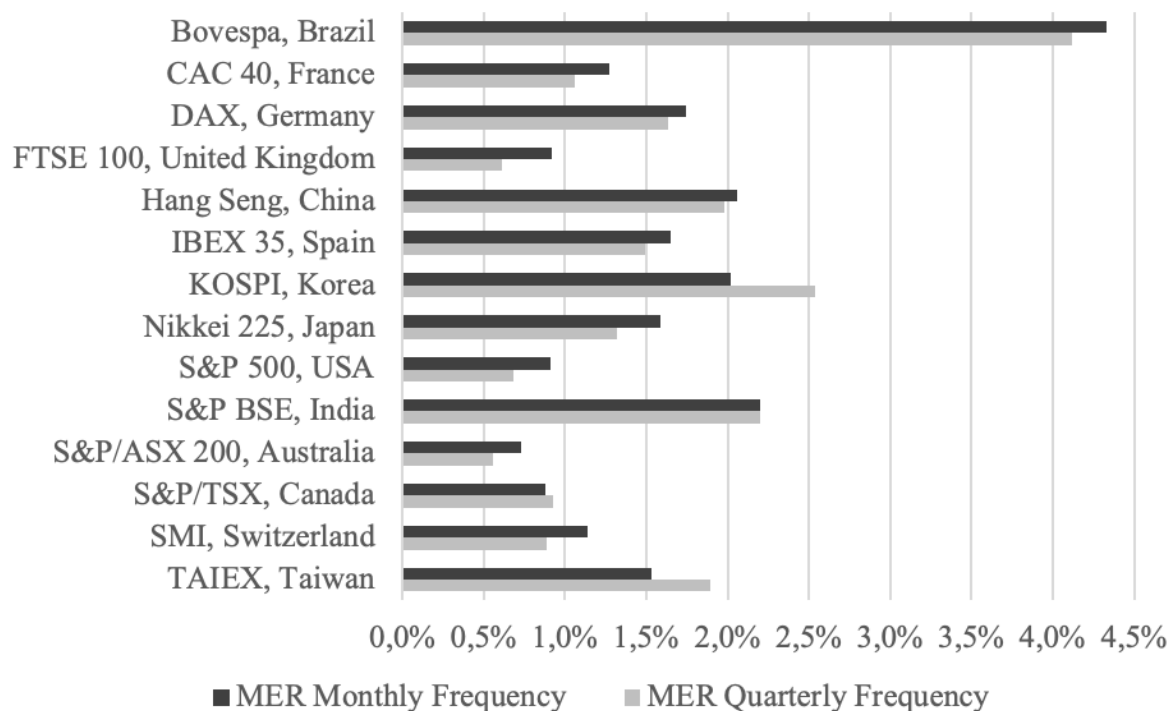
Note. MER$_{VA-DCA}$

Source: Compiled by author

Summarizing the information about the medium-term period, it can be argued that VA surpasses DCA in this period of time, which is consistent with most previous studies (Marshall, 2006; Majumdar, Puthiya, & Bendarkar, 2021). It is also interesting to note that the Korean market is the most out of the general pattern. Perhaps this was the reason for the false conclusions about the overall superiority of the DCA strategy over the VA in the Choe and Ban (2020) study. Perhaps these scholars should have tested their model in other markets before making categorical statements. As for the frequency of investments, one can observe how its role decreases. Nonetheless, investors should still stick to the monthly frequency of investing in order to receive even a small but additional income.

2.2.3. Long-term performance

According to previous scholars' articles, VA will outperform DCA under a long-term investment scenario (Panyagometh, 2013; Majumdar, Puthiya, & Bendarkar, 2021). This research can confirm this conclusion. The cases when the XIRR superiority of DCA exceeded more than 10% decreased to 4. The trend of the increasing gap in MER continued, which amounted to 1.73% and statistical tests have confirmed that MER is not zero. Now a clear trend of MER growth with an increase of investment horizon can be confirmed (see Figure 5).

Figure 5

Average Mean Excess Return trend with the growth of the time horizon
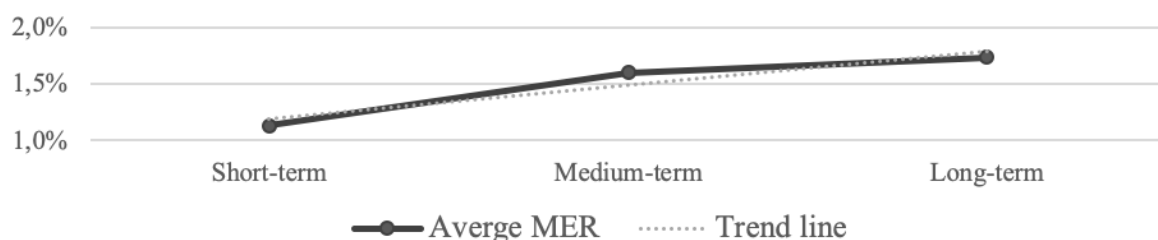
Note. MER_VA-DCA

Source: Compiled by author

As for the frequency of investing (see Figure 6), the difference in results is blurred: the superiority in MRE has been divided roughly in half. That is, in the long-term, the frequency of investment ceases to play a significant role.

Figure 6

Mean Excess Return of different investment frequency in the long-term.
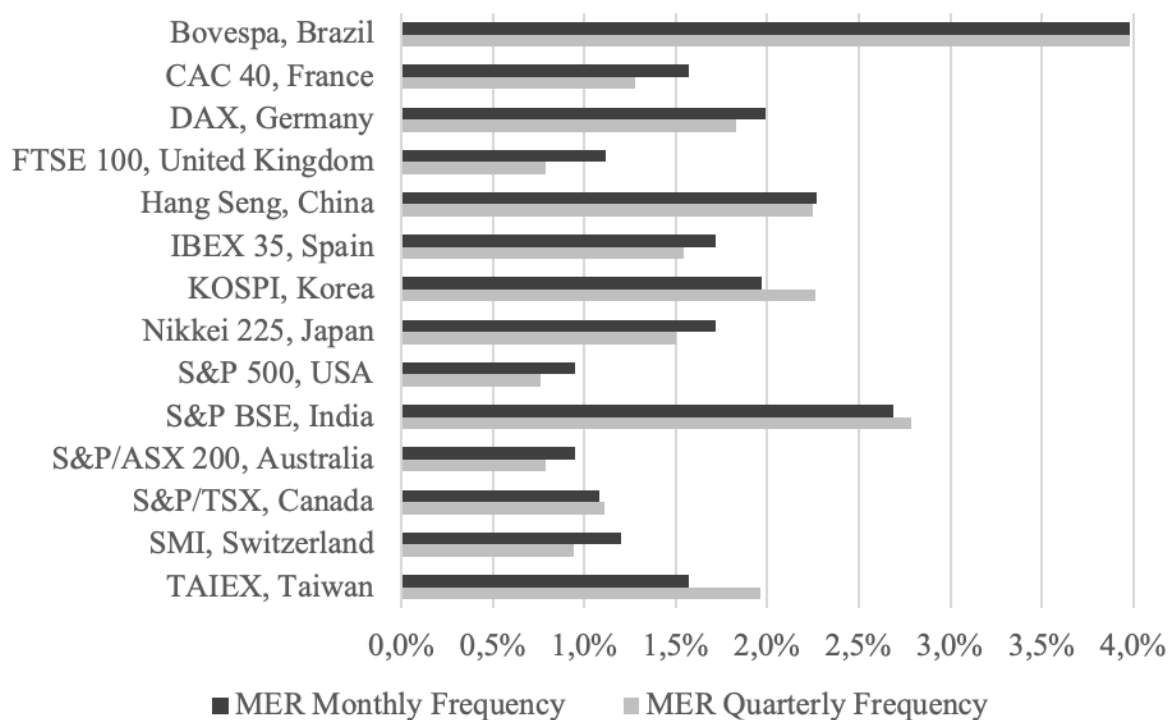
Note. MER_VA-DCA

Source: Compiled by author

Without a doubt, in the long-term, the investor should adhere to the VA investment strategy. By following fairly simple rules, one can most likely provide oneself with additional return. Furthermore, an investor can equally choose between monthly or quarterly investment frequencies, depending on their preferences, since the difference will be either insignificant or even zero.

Conclusion

This paper considered two popular investment strategies – Dollar-Cost Averaging and Value Averaging. First of all, they were characterised and positioned among other investment strategies. It was noted that combining a passive approach to investing which does not require much effort from the investor, as well as the regularity of investments, which prevented the need for a large amount of money at one particular moment, these strategies could become an ideal choice when applied by individuals and institutions.

Moreover, the author identified the main advantages and disadvantages of DCA and VA, both in comparison with other investment strategies and with each other. It was established that these strategies could be suitable for most investors. Of the main advantages, it is worth noting the university of the strategies studied, as well as their help in countering market sentiment. However, at the same time, DCA and VA obviously cannot take into account the individual characteristics of the investor, which requires preliminary preparation of an investment plan.

The analysis of the results of previous scientific papers was carried out, which allowed one to identify some patterns in the application of the investment strategies under study. For example, in the scientific literature there is an ambiguous impression of the use of DCA and VA in the short-term, and, on the contrary, there is unanimity in the superiority of VA in the long-term period of investment. Based on the literature review, areas of further research that were not touched upon in previous scientific works were identified. Namely, the issue of the superiority of the rate of return of DCA and VA during different investment horizons, as well as investment frequencies in various international markets, needed to be studied.

In the empirical part of the work, the author strived to determine whether there is a superiority in the rate of return of DCA and VA investment strategies even when using them in different markets. In addition, an attempt to clarify the functioning of these investment strategies in conditions of various investment frequencies and time horizons was made. Fourteen international stock indices representing the current distribution of capital globally were selected for analysis. The period from January 1, 1995 to March 10, 2022 was chosen to include various patterns of market movement: growth, stagnation and recession.

In general, it can be argued about the superiority of VA over DCA: in all cases, the primacy of VA in the superiority of absolute rate of return and extended internal rate of return, as well as positive mean excess return, could be observed. Additionally, another confirmation of the superiority of the VA investment strategy can be the rather surprising fact

that the MER in all analysed cases in every market has never been negative: VA XIRR, on average, is always greater than DCA XIRR.

Analysing the details, two patterns can be noted. Firstly, the longer the investment horizon, the better VA reveals itself, increasing the rate of return gap with DCA. Secondly, the greater the frequency of investment when using VA, the greater the return should be expected. This pattern was most pronounced in the short-term scenario. However, it is worth noting that the frequency loses its significance with the increase in the investment duration.

To sum up, both investment strategies offer investors a tool for sustained opposition to market sentiment, allowing them not to succumb to market panic. Combining passive and regular investment approaches, DCA and VA can be a good choice for both households and organizations. Nevertheless, following the relatively uncomplicated additional rules of VA investment strategy, the investor will enjoy an additional interest income in various markets compared to DCA.

Further research on this topic could be aimed at determining whether the trend of growth in the rate of return gap persists when applying DCA and VA investment strategies with an increase in the time horizon. In addition, it is possible to increase the data sample with the inclusion in the study of the stock indices of exchanges founded after 1995. Moreover, future works may try to overcome the main limitations, meaning to include in the analysis transaction costs, country-specific tax issues, possible dividend payments, as well as additional interest income from the formation of a reserve fund when using the VA strategy. Finally, the idea of testing DCA and VA strategies when acquiring other assets is interesting: bonds, commodities or cryptocurrencies.

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Appendices

Appendix A

Analysis of the application of investment strategies Dollar-Cost Averaging and Value Averaging on the example of the Standard and Poor's 500 index

Monthly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	5.13%	0%	10.1%	6.67%	6.01%	100%	10.95%	92.7%	0.85%	Reject H_0, Accept H_1
Medium-term	19.53%	0%	6.3%	5.58%	27.56%	100%	7.22%	94.05%	0.91%	Reject H_0, Accept H_1
Long-term	41.31%	0%	6.07%	1.93%	74.09%	100%	7.02%	98.07%	0.95%	Reject H_0, Accept H_1

Quarterly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	5.96%	0%	10.03%	14.92%	6.59%	99.37%	10.5%	83.17%	0.47%	Reject H_0, Accept H_1
Medium-term	20.25%	0.74%	6.31%	11.15%	27.57%	99.26%	6.99%	88.85%	0.68%	Reject H_0, Accept H_1
Long-term	42.02%	0%	6.07%	5.31%	71.3%	100%	6.83%	94.2%	0.76%	Reject H_0, Accept H_1

Appendix B

Analysis of the application of investment strategies Dollar-Cost Averaging and Value Averaging on the example of the Nikkei 225 index

Monthly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	2.37%	0.64%	5.28%	10.79%	3.85%	99.36%	6.59%	89.21%	1.32%	Reject H_0, Accept H_1
Medium-term	10.84%	0%	2.62%	8.18%	22.53%	100%	4.22%	91.45%	1.59%	Reject H_0, Accept H_1
Long-term	27.44%	0%	3.81%	3.86%	62.3%	100%	5.53%	96.14%	1.72%	Reject H_0, Accept H_1

Quarterly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	2.7%	5.4%	5.07%	22.54%	3.76%	93.65%	5.85%	76.83%	0.78%	Reject H_0, Accept H_1
Medium-term	11.15%	0%	2.6%	8.55%	22.13%	100%	3.91%	91.45%	1.32%	Reject H_0, Accept H_1
Long-term	27.71%	0%	3.78%	5.31%	60.08%	100%	5.28%	94.69%	1.5%	Reject H_0, Accept H_1

Appendix C

Analysis of the application of investment strategies Dollar-Cost Averaging and Value Averaging on the example of the Hang Seng index

Monthly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	3.6%	0.32%	7.8%	12.06%	5.46%	98.73%	9.48%	87.94%	1.67%	Reject H_0, Accept H_1
Medium-term	13.86%	0%	4.46%	1.86%	28.15%	100%	6.52%	98.14%	2.06%	Reject H_0, Accept H_1
Long-term	30.27%	0%	4.84%	1.93%	75.85%	100%	7.11%	98.07%	2.27%	Reject H_0, Accept H_1

Quarterly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	4.2%	4.13%	7.61%	23.81%	5.59%	94.92%	8.78%	75.24%	1.17%	Reject H_0, Accept H_1
Medium-term	14.33%	0%	4.46%	4.83%	27.73%	100%	6.44%	95.17%	1.98%	Reject H_0, Accept H_1
Long-term	30.76%	0%	4.84%	2.42%	74.44%	100%	7.08%	97.58%	2.25%	Reject H_0, Accept H_1

Appendix D

Analysis of the application of investment strategies Dollar-Cost Averaging and Value Averaging on the example of the Financial Times Stock Exchange 100 index

Monthly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	2.16%	0%	4.42%	7.94%	2.86%	99.05%	5.17%	91.75%	0.75%	Reject H_0, Accept H_1
Medium-term	5.46%	0%	1.65%	11.15%	9.57%	100%	2.56%	88.85%	0.92%	Reject H_0, Accept H_1
Long-term	12.26%	0%	2.15%	0.48%	21.56%	100%	3.27%	98.55%	1.12%	Reject H_0, Accept H_1

Quarterly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	2.51%	22.22%	4.36%	6.03%	2.96%	90.79%	4.74%	76.19%	0.38%	Reject H_0, Accept H_1
Medium-term	5.69%	0%	1.67%	12.64%	9.03%	100%	2.28%	87.36%	0.61%	Reject H_0, Accept H_1
Long-term	12.48%	0%	2.15%	1.93%	19.41%	100%	2.94%	98.07%	0.79%	Reject H_0, Accept H_1

Appendix E

Analysis of the application of investment strategies Dollar-Cost Averaging and Value Averaging on the example of the S&P/TSX Composite index

Monthly investment frequency

| Investment horizon | Dollar-Cost Averaging | | | | Value Averaging | | | | Mean Excess Return | T-test |
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	3.99%	2.86%	7.95%	20.32%	4.75%	96.83%	8.55%	78.73%	0.59%	Reject H0, Accept H1
Medium-term	14.85%	0%	5.01%	8.55%	21.4%	100%	5.89%	91.45%	0.88%	Reject H0, Accept H1
Long-term	29%	0%	4.78%	1.45%	46.38%	100%	5.86%	98.55%	1.08%	Reject H0, Accept H1

Quarterly investment frequency

| Investment horizon | Dollar-Cost Averaging | | | | Value Averaging | | | | Mean Excess Return | T-test |
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	4.62%	5.4%	7.85%	24.44%	5.22%	92.7%	8.3%	73.02%	0.46%	Reject H0, Accept H1
Medium-term	15.37%	0%	5.02%	7.81%	22.02%	100%	5.95%	91.82%	0.93%	Reject H0, Accept H1
Long-term	29.53%	0%	4.78%	0.48%	47.06%	100%	5.89%	99.52%	1.11%	Reject H0, Accept H1

Appendix F

Analysis of the application of investment strategies Dollar-Cost Averaging and Value Averaging on the example of the German Stock Index

Monthly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	5.56%	0.32%	11.4%	8.57%	7.18%	99.68%	12.79%	91.43%	1.39%	Reject H_0, Accept H_1
Medium-term	20.81%	0%	6.28%	11.52%	39.93%	100%	8.02%	88.48%	1.74%	Reject H_0, Accept H_1
Long-term	42.01%	0%	6.6%	2.9%	82.36%	100%	8.58%	97.1%	1.99%	Reject H_0, Accept H_1

Quarterly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	6.46%	0.95%	11.22%	24.13%	7.71%	98.73%	12.21%	75.56%	0.99%	Reject H_0, Accept H_1
Medium-term	21.59%	0%	6.3%	10.41%	39.99%	100%	7.93%	89.59%	1.63%	Reject H_0, Accept H_1
Long-term	42.76%	0%	6.59%	0.48%	79.57%	100%	8.43%	99.52%	1.83%	Reject H_0, Accept H_1

Appendix G

Analysis of the application of investment strategies Dollar-Cost Averaging and Value Averaging on the example of the Swiss Market Index

Monthly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	3.91%	0.32%	7.95%	12.7%	4.96%	99.68%	8.93%	87.3%	0.97%	Reject H_0, Accept H_1
Medium-term	10.64%	0%	3.25%	11.15%	17.24%	100%	4.39%	88.85%	1.14%	Reject H_0, Accept H_1
Long-term	18.97%	0%	3.18%	0.48%	32.12%	100%	4.38%	99.52%	1.2%	Reject H_0, Accept H_1

Quarterly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	4.56%	5.4%	7.87%	27.62%	5.33%	92.7%	8.44%	70.16%	0.58%	Reject H_0, Accept H_1
Medium-term	11.06%	1.49%	3.27%	13.75%	17.17%	98.51%	4.16%	85.87%	0.89%	Reject H_0, Accept H_1
Long-term	19.32%	0%	3.18%	1.45%	30.39%	100%	4.12%	98.55%	0.94%	Reject H_0, Accept H_1

Appendix H

Analysis of the application of investment strategies Dollar-Cost Averaging and Value Averaging on the example of the CAC 40 index

Monthly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	3.89%	0%	8.09%	7.3%	5.17%	100%	9.24%	92.06%	1.15%	Reject H_0, Accept H_1
Medium-term	12.2%	0%	3.31%	13.75%	28.76%	100%	4.58%	85.87%	1.27%	Reject H_0, Accept H_1
Long-term	14.3%	0%	2.22%	0.48%	29.44%	100%	3.79%	99.52%	1.57%	Reject H_0, Accept H_1

Quarterly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	4.5%	0.32%	7.92%	22.54%	5.48%	99.05%	8.69%	76.19%	0.77%	Reject H_0, Accept H_1
Medium-term	12.67%	0.37%	3.32%	15.61%	29.59%	99.63%	4.38%	84.39%	1.06%	Reject H_0, Accept H_1
Long-term	14.58%	0%	2.23%	0.97%	27.61%	100%	3.51%	99.03%	1.28%	Reject H_0, Accept H_1

Appendix I

Analysis of the application of investment strategies Dollar-Cost Averaging and Value Averaging on the example of the S&P/ASX 200 index

Monthly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	3.2%	0%	6.33%	8.57%	3.82%	100%	6.93%	91.43%	0.61%	Reject H_0, Accept H_1
Medium-term	13.46%	0%	4.57%	12.27%	18.61%	100%	5.3%	87.73%	0.73%	Reject H_0, Accept H_1
Long-term	26.22%	0%	4.24%	10.63%	42.98%	100%	5.19%	89.37%	0.95%	Reject H_0, Accept H_1

Quarterly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	3.72%	2.86%	6.27%	22.22%	4.14%	96.19%	6.6%	76.51%	0.33%	Reject H_0, Accept H_1
Medium-term	13.92%	0%	4.57%	14.87%	18.42%	100%	5.13%	84.76%	0.56%	Reject H_0, Accept H_1
Long-term	26.69%	0%	4.24%	11.11%	41.77%	100%	5.03%	88.89%	0.79%	Reject H_0, Accept H_1

Appendix J

Analysis of the application of investment strategies Dollar-Cost Averaging and Value Averaging on the example of the Korea Composite Stock

Price Index

Monthly investment frequency

Investment horizon	*Dollar-Cost Averaging*				*Value Averaging*				*Mean Excess Return*	*T-test*
	Avg. APRR	*Sup. APRR*	*Avg. XIRR*	*Sup. XIRR*	*Avg. APRR*	*Sup. APRR*	*Avg. XIRR*	*Sup. XIRR*		
Short-term	4.81%	0%	10.87%	10.48%	8.4%	100%	12.81%	89.21%	1.93%	Reject H_0, Accept H_1
Medium-term	21.98%	0%	7.17%	5.2%	40.20%	100%	9.19%	94.42%	2.02%	Reject H_0, Accept H_1
Long-term	53.2%	0%	7.65%	0.97%	431.83%	99.03%	9.62%	98.98%	1.97%	Reject H_0, Accept H_1

Quarterly investment frequency

Investment horizon	*Dollar-Cost Averaging*				*Value Averaging*				*Mean Excess Return*	*T-test*
	Avg. APRR	*Sup. APRR*	*Avg. XIRR*	*Sup. XIRR*	*Avg. APRR*	*Sup. APRR*	*Avg. XIRR*	*Sup. XIRR*		
Short-term	5.6%	0%	10.49%	16.19%	8.93%	98.73%	12.31%	82.86%	1.83%	Reject H_0, Accept H_1
Medium-term	22.67%	0%	7.14%	3.35%	44.75%	100%	9.69%	95.91%	2.54%	Reject H_0, Accept H_1
Long-term	54.11%	0%	7.64%	1.45%	670.2%	100%	9.9%	98.55%	2.26%	Reject H_0, Accept H_1

Appendix K

Analysis of the application of investment strategies Dollar-Cost Averaging and Value Averaging on the example of the Taiwan Capitalization Weighted Stock Index

Monthly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	4.01%	0.63%	8.5%	12.06%	5.56%	99.37%	9.69%	87.94%	1.19%	Reject H_0, Accept H_1
Medium-term	11.01%	0%	3.39%	7.81%	19.75%	100%	4.93%	92.19%	1.53%	Reject H_0, Accept H_1
Long-term	24.87%	0%	4.05%	7.25%	47.88%	100%	5.62%	92.75%	1.57%	Reject H_0, Accept H_1

Quarterly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	4.59%	3.49%	8.17%	20.95%	5.93%	95.24%	9.26%	77.78%	1.08%	Reject H_0, Accept H_1
Medium-term	11.33%	0%	3.37%	6.69%	20.77%	100%	5.27%	92.94%	1.89%	Reject H_0, Accept H_1
Long-term	25.18%	0%	4.03%	7.25%	51.53%	100%	5.99%	92.75%	1.96%	Reject H_0, Accept H_1

Appendix L

Analysis of the application of investment strategies Dollar-Cost Averaging and Value Averaging on the example of the Bovespa Index

Monthly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	9.77%	1.27%	20.29%	18.41%	13.73%	98.1%	23.33%	81.27%	3.04%	Reject H$_0$, Accept H$_1$
Medium-term	44.68%	0%	12.81%	0.74%	262.75%	100%	17.15%	99.26%	4.33%	Reject H$_0$, Accept H$_1$
Long-term	104.06%	9.18%	11.91%	3.38%	357.37%	90.82%	15.89%	96.62%	3.98%	Reject H$_0$, Accept H$_1$

Quarterly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	11.35%	12.38%	19.9%	28.57%	14.43%	86.98%	22.06%	71.43%	2.16%	Reject H$_0$, Accept H$_1$
Medium-term	46.46%	0%	12.82%	3.35%	192.05%	100%	16.94%	96.65%	4.12%	Reject H$_0$, Accept H$_1$
Long-term	106.24%	8.21%	11.92%	3.38%	393.05%	91.79%	15.9%	96.14%	3.98%	Reject H$_0$, Accept H$_1$

Appendix M

Analysis of the application of investment strategies Dollar-Cost Averaging and Value Averaging on the example of the IBEX 35 index

Monthly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	3.35%	0%	7.23%	9.52%	4.89%	100%	8.55%	90.16%	1.32%	Reject H_0, Accept H_1
Medium-term	6.94%	0%	1.42%	11.52%	18.25%	100%	3.08%	88.48%	1.65%	Reject H_0, Accept H_1
Long-term	9.06%	0%	1.18%	5.8%	26.46%	100%	2.9%	94.20%	1.72%	Reject H_0, Accept H_1

Quarterly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR	Avg. APRR	Sup. APRR	Avg. XIRR	Sup. XIRR		
Short-term	3.92%	1.59%	7.07%	19.05%	5.09%	97.14%	8%	80.63%	0.93%	Reject H_0, Accept H_1
Medium-term	7.31%	0%	1.46%	12.64%	18.7%	100%	2.95%	87.36%	1.49%	Reject H_0, Accept H_1
Long-term	9.33%	0%	1.2%	4.35%	25.63%	100%	2.74%	95.17%	1.54%	Reject H_0, Accept H_1

Appendix N

Analysis of the application of investment strategies Dollar-Cost Averaging and Value Averaging on the example of the BSE SENSEX index

Monthly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	*Sup. APRR*	*Avg. XIRR*	*Sup. XIRR*	*Avg. APRR*	*Sup. APRR*	*Avg. XIRR*	*Sup. XIRR*		
Short-term	7.88%	1.27%	16.12%	19.05%	10.3%	98.73%	17.77%	80.32%	1.65%	Reject H_0, Accept H_1
Medium-term	43.16%	0%	12.71%	14.13%	214.56%	100%	14.91%	85.5%	2.2%	Reject H_0, Accept H_1
Long-term	116.41%	2.42%	14.08%	14.98%	704.67%	97.58%	16.77%	84.54%	2.69%	Reject H_0, Accept H_1

Quarterly investment frequency

Investment horizon	Dollar-Cost Averaging				Value Averaging				Mean Excess Return	T-test
	Avg. APRR	*Sup. APRR*	*Avg. XIRR*	*Sup. XIRR*	*Avg. APRR*	*Sup. APRR*	*Avg. XIRR*	*Sup. XIRR*		
Short-term	9.09%	5.71%	15.75%	28.25%	11.15%	93.33%	17.04%	71.43%	1.29%	Reject H_0, Accept H_1
Medium-term	44.69%	0%	12.68%	12.27%	297.52%	100%	14.88%	87.36%	2.2%	Reject H_0, Accept H_1
Long-term	118.53%	3.38%	14.06%	14.49%	692.45%	96.62%	16.84%	85.51%	2,79%	Reject H_0, Accept H_1

Resümee

OSTUHINNA JA VÄÄRTUSE KESKMISTAMISE

INVESTEERIMISSTRATEEGIATE TULEMUSLIKKUS - RAHVUSVAHELINE

VÕRDLUS

Ilia Zheltov

Selles töös käsitleti kahte populaarset investeerimisstrateegiat – ostuhinna keskmistamine ja väärtuse keskmistamine. Kõige pealt iseloomustati ja positsioneeriti käsitletavad strateegiad teiste investeerimisstrateegiate sekka .Tuli välja, et passiivne lähenemine investeerimisele, mis ei nõua palju ajalist panust investorilt, kombineerituna regulaarsete investeeringutega, mis ennetab vajadust investeerida korraga suuri summasid, võib osutuda parimaks investeerimisstrateegiaks majapidamistele ning üksikisikutele. Lisaks tõi autor välja peamised tugevused ja nõrkused ostuhinna keskmistamise ja väärtuse keskmistamise strateegiate kohta, nii võrdluses teiste strateegiatega kui ka omavahel. Tehti kindlaks, et need strateegiad võivad sobida enamikule investoritele. Eelmiste teaduslike tööde tulemuste analüüs viidi läbi, mis võimaldas välja selgitada uuritavate investeerimisstrateegiate rakendamise mõned mustrid.

Töö empiirilises osas püüdis autor välja selgitada, kas ostuhinna keskmistamise ja väärtuse keskmistamise investeerimisstrateegiateson erinevusi tootluses. Lisaks püüti selgitada nende investeerimisstrateegiate toimimist erineva investeerimissageduse ja -ajaga tingimustes. Analüüsimiseks valiti välja neliteist rahvusvahelist aktsiaindeksit, mis esindavad kapitali praegust jaotumist kogu maailmas. Ajavahemikul 1. jaanuarist 1995 kuni 10. märtsini 2022 valiti välja erinevad turuliikumise mustrid: kasv, seisak ja majanduslangus.

Üldiselt võib väita väärtuse keskmistamise strateegia paremuse kohta ostuhinna keskmistamise strateegia suhtes: kõikidel juhtudel võib täheldada väärtuse keskmistamise strateegia ülimuslikkust tootluse ja XIRR-i paremuses, samuti positiivset keskmist ülemäärast tootlust. Detaile analüüsides võib märgata kahte mustrit. Esiteks, mida pikem on investeerimisperiood, seda paremini väärtuse kesmistamise strateegia toimib, suurendades tulumäära erinevust ostuhinna keskmistamise strateegiaga. MER-i kasvutendents on selgelt näha koos investeerimisaja pikenemisega. Teiseks, mida suurem on investeeringute sagedus väärtuse keskmistamise strateegia kasutamisel, seda suuremat tulu saab oodata. Lühiajalises stsenaariumis oli see muster kõige selgem. Siiski väärib märkimist, et sagedus kaotab oma tähtsuse koos investeeringute kestuse suurenemisega.

Kokkuvõtteks võib öelda, et mõlemad investeerimisstrateegiad pakuvad investoritele vahendit turu meelestatusele püsivaks vastuseisuks, mis võimaldab neil turupaanikale mitte

alla vanduda. Ühendades passiivsed ja regulaarsed investeerimisviisid, võib ostuhinna keskmistamise ja väärtuse keskmistamise investeerimisstrateegiad olla hea valik nii majapidamistele kui ka organisatsioonidele. Sellest hoolimata saab investor väärtuse kesmistamise investeerimisstrateegia suhteliselt lihtsaid põhimõtteid järgides täiendavat intressitulu teenida võrreldes ostuhinna keskmistamise strateegiaga.

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